1. Find a real number \( x \) for which \( x[x] = 1234 \).

Note: \( [x] \) is the largest integer less than or equal to \( x \).

2. Let \( C_1 \) be a circle of radius 1, and \( C_2 \) be a circle that lies completely inside or on the boundary of \( C_1 \). Suppose \( P \) is a point that lies inside or on \( C_2 \). Suppose \( O_1 \) and \( O_2 \) are the centers of \( C_1 \) and \( C_2 \), respectively. What is the maximum possible area of \( \Delta O_1O_2P \)? Prove your answer.

3. The numbers 1, 2, \ldots, 99 are written on a blackboard. We are allowed to erase any two distinct (but perhaps equal) numbers and replace them by their nonnegative difference. This operation is performed until a single number \( k \) remains on the blackboard. What are all the possible values of \( k \)? Prove your answer.

Note: As an example if we start from 1, 2, 3, 4 on the board, we can proceed by erasing 1 and 2 and replacing them by 1. At that point we are left with 1, 3, 4. We may then erase 3 and 4 and replace them by 1. The last step would be to erase 1, 1 and end up with a single 0 on the board.

4. Let \( a, b \) be two real numbers so that \( a^3 - 6a^2 + 13a = 1 \) and \( b^3 - 6b^2 + 13b = 19 \). Find \( a + b \). Prove your answer.

5. Let \( m, n, k \) be three positive integers with \( n \geq k \). Suppose \( A = \prod_{1 \leq i \leq j \leq m} \gcd(n + i, k + j) \) is the product of \( \gcd(n + i, k + j) \), where \( i, j \) range over all integers satisfying \( 1 \leq i \leq j \leq m \). Prove that the following fraction is an integer

\[
\frac{A}{(k + 1) \cdots (k + m)} \binom{n}{k}.
\]

Note: \( \gcd(a, b) \) is the greatest common divisor of \( a \) and \( b \), and \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \).