Mathematics of the Electoral College

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Overview

Is the US President elected directly?

- No.

- The president is elected by "electors" who are elected in each state and meet in January to elect the President.

- These electors are called the **Electoral College**.

- Washington DC was not included until 1961 (23rd Amendment).

- Residents of Puerto Rico and other territories still have no vote for President

Why did the framers of the Constitution devise an Electoral College?

- It was 1787.

- The US was vast and sparsely populated (13 states).

- Communication and transportation were primitive.

- Campaigning was seen as inappropriate.

- Direct democracy was regarded, by many, as a dangerous idea.

Are senators elected directly?

- Now, *yes*, but not in original Constitution!
  - Senators *were* appointed by state legislatures.
- This was changed by the 17th Amendment (1913).
- This "mistake" started a "decline in so-called states rights" (Late Justice Antonin Scalia, November 13, 2010, Texas Tech. Law School).

What is there to like about the EC?

- It is traditional and a unique American institution (like inches and feet).

- It is believed to favor "small" states & rural areas (so big states and cities don't dominate). One needs ¾ of states to ratify amendment. Most states are small.

- Supports the idea of "federalism".

- Localizes logistical issues with elections (Russians need to work all 50 states!)

- Encourages 2-party system.

What is there not to like?

- Not every voter has same "power" (degree of influence over the outcome). The system is not **anonymous**.

- It violates the majority principle ("one person-one vote"). A candidate with a **majority** of votes need not win the election.

- A non-viable candidate can change the outcome. It fails the test of **independence**.

- Entrenches the 2-party system.

- Focus on swing states. Discourages voter turnout elsewhere.

One person one vote

Majority candidate need not win

When has EC differed from "Popular Vote"?

- **1876:** Rutherford B Hayes wins EC 185 to 184. Samuel J. Tilden wins 51.5% of popular vote (***majority***).

- **1888:** Benjamin Harrison wins EC 201 to 200. Grover Cleveland wins *plurality* in popular vote.

- **2000:** George W Bush wins EC 271 to 267. Al Gore wins *plurality* of popular vote (Supreme Court awards Florida's 25 votes to Bush).

- **2016:** Donald Trump wins EC 304 to 227. Hillary Clinton wins plurality 48% to Tump's 46%, with Libertarian Gary Johnson winning 3.3%.

Lack of "Independence"

Candidate with no chance of winning can "turn" the election

2000 Presidential Election

- Illustrates lack of independence: the "spoiler" phenomenon.
- Florida: 25 electoral votes to Bush and decide race.

	George Bush	Al Gore	Ralph Nader
Popular vote	47.9%	48.4%	2.74%
Electoral vote	271	266	0

2000 popular vote Florida

	George Bush	Al Gore	Ralph Nader	Others
Votes	2,912,790	2,912,253	97,488	40,575
Percent	48.847%	48.838%	1.635%	0.68%

- Bush beat Gore in Florida by only 537 votes.

- Nader won 97,488 Florida votes. Arguably otherwise Gore—not Bush—votes.

- Recount ended 5-4 by Supreme Court intervention, Bush confirmed as winner.

Independence
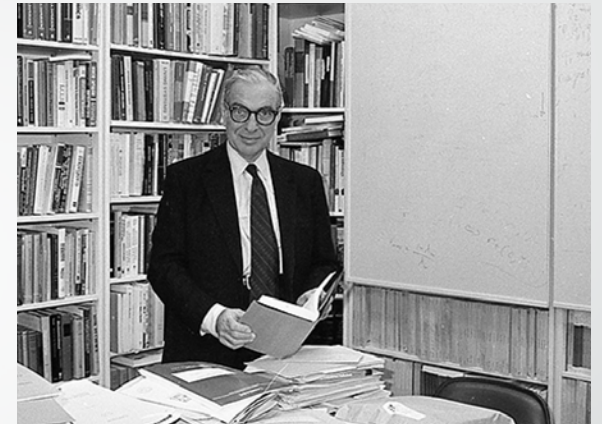
- Lack of independence also an issue with "popular vote" (plurality method) election.

- ***Arrow's Impossibility Theorem***: Any "reasonable" voting system with more then two candidates cannot satisfy independence, aka "Dictator Theorem".

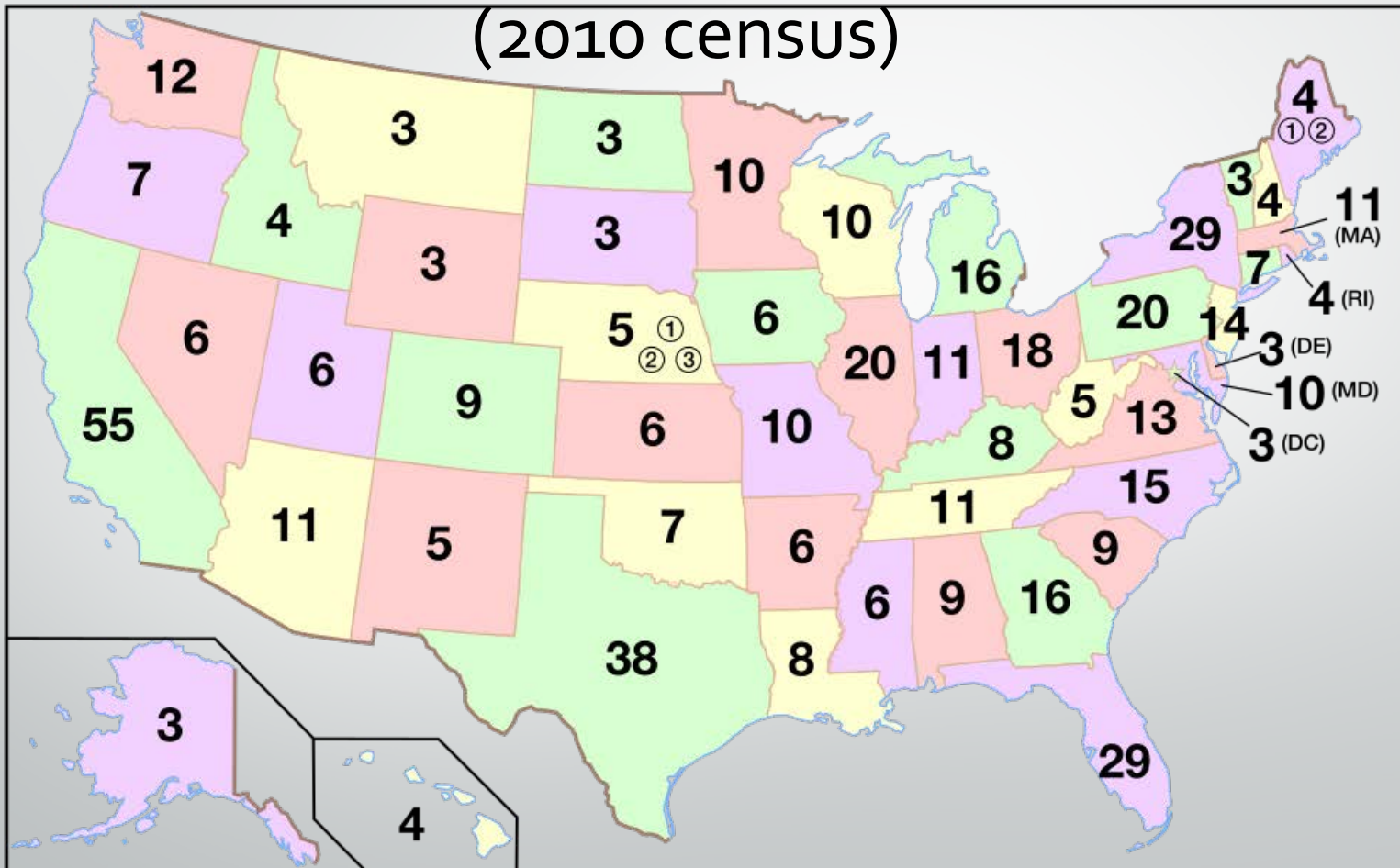- Kenneth Arrow (1921-2017). Economics Nobel Prize 2004.

Lack of Anonymity

Different voters have different power

Weighted voting

- Different voters have different numbers of votes (different voting "power").
  - Partners in a law firm.
  - Stockholders in a corporation.
  - Juniors and Seniors in a Fraternity or Sorority

# Electoral votes: weighted voting by states (2010 census)

How is number of electoral votes determined?

- Each state gets its number of House seats, plus 2 (corresponding to its Senate seats).

- House seats determined by *Census* (next Census 2020) and *apportionment* (Huntington-Hill method).

- DC gets exactly 3 votes (23rd Amendment, 1961).

- House: 435 + Senate: 100 +DC: 3 = 538 total.

- 270 is a *majority* (269 to 269 split is possible).

The "+2" phenomenon
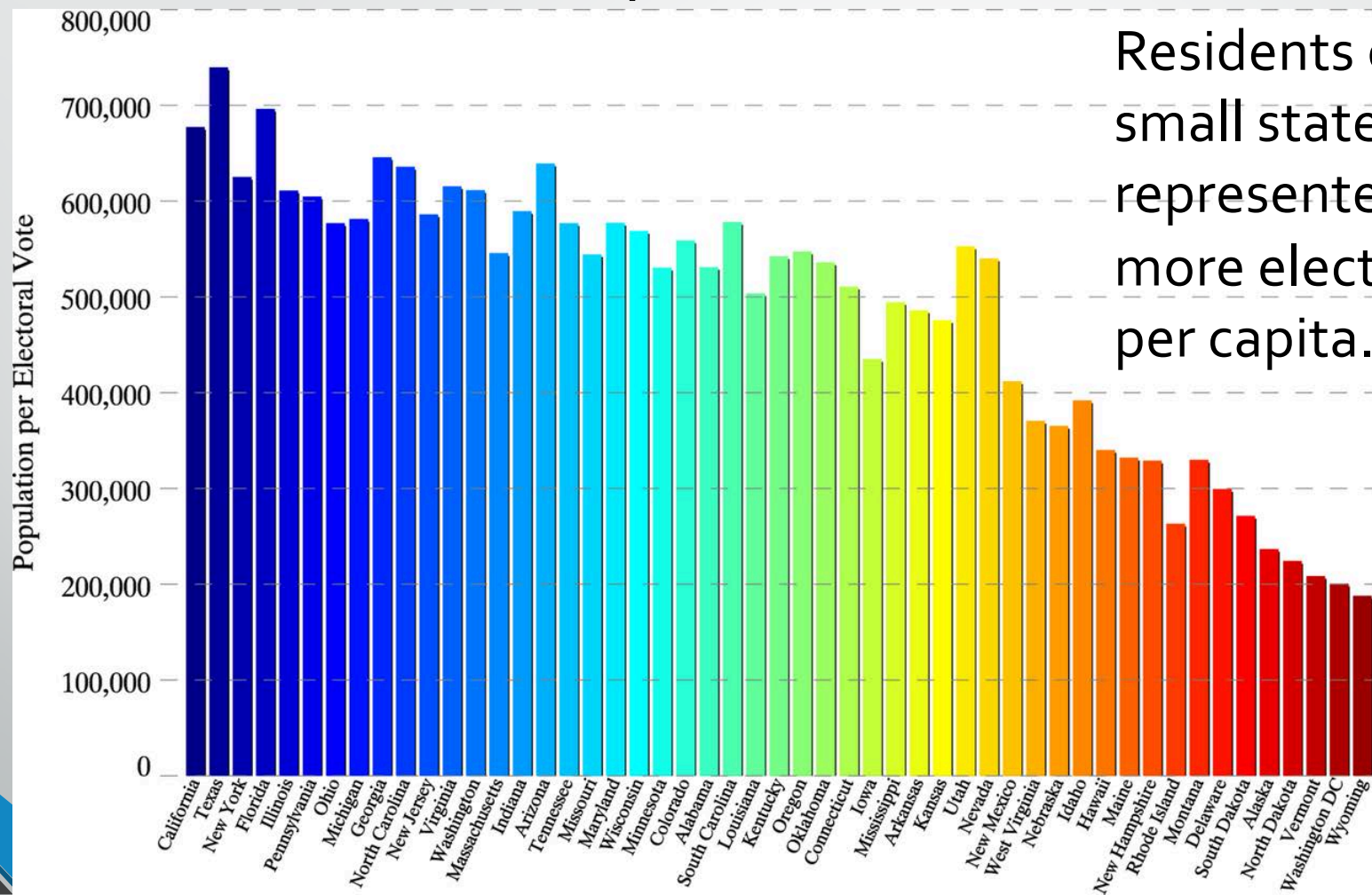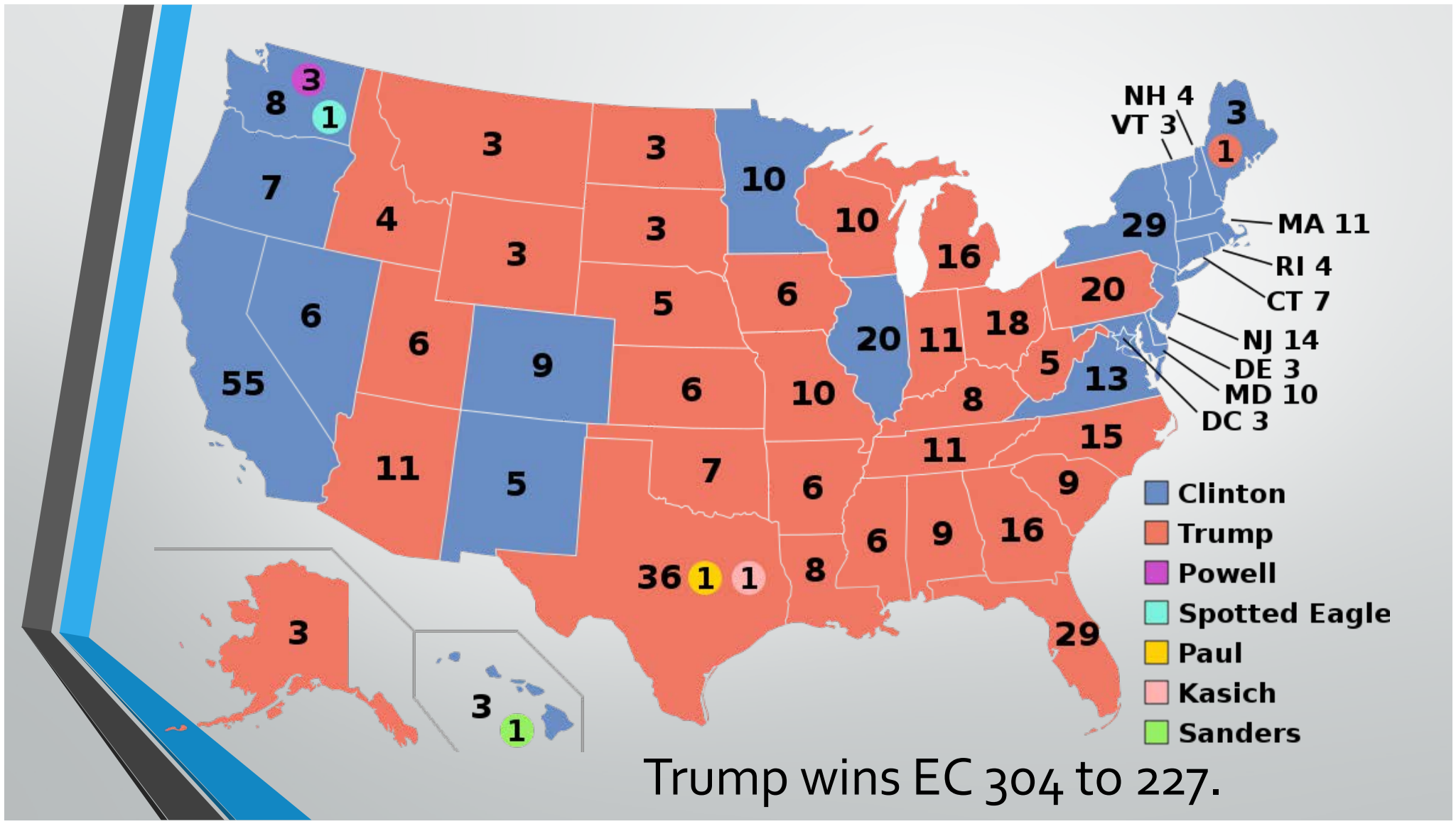

Residents of small states are represented by more electors, per capita.

Conventional wisdom vs John Banzhaf

- Conventional Wisdom:
  - The +2 phenomenon means that the **EC favors small states**.
- John Banzhaf III, GW Law professor:
  - "Should think of people, and not states, as voters".
  - His analysis suggests **EC favors voters in large states**.

Some Maps from 2016 Election

2016 Congressional Districts by Winning Presidential Party

EC without +2 effect

Hillary Clinton (205)
Donald Trump (230)
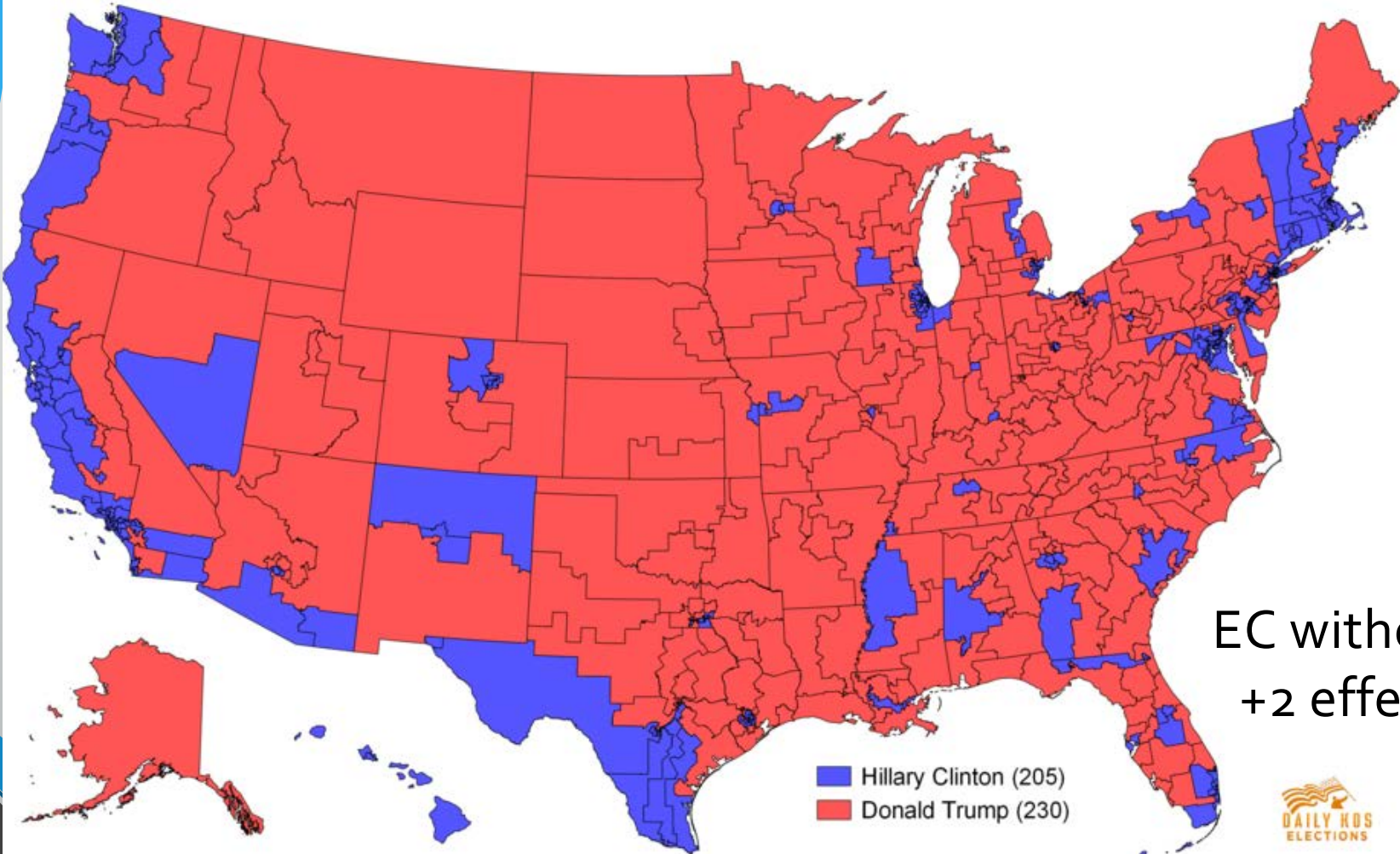
States scaled by electoral votes
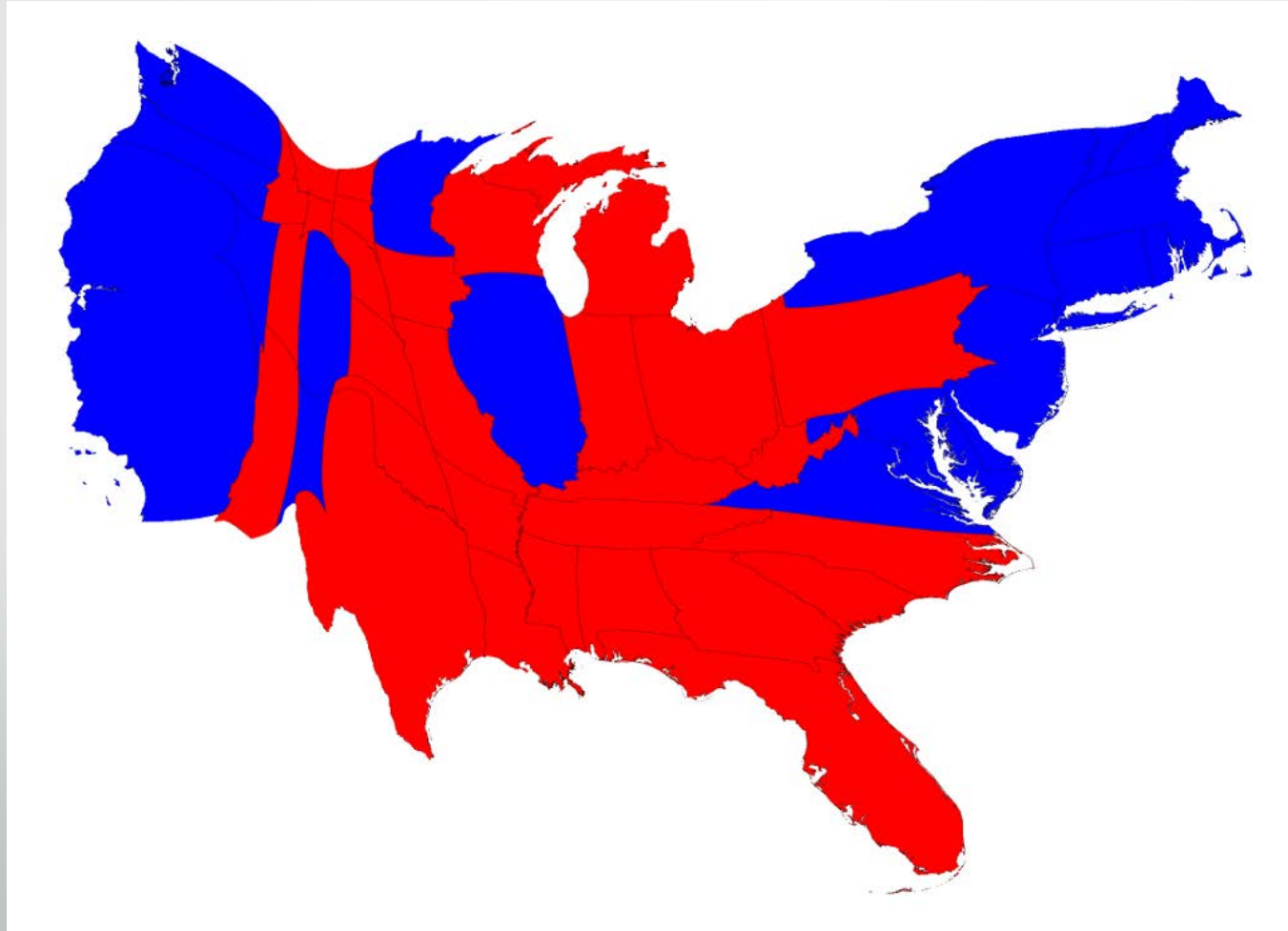
National Popular Vote Interstate Compact (began 2007).

Status as of May 2018:

One way to get rid of EC without amending the Constitution.
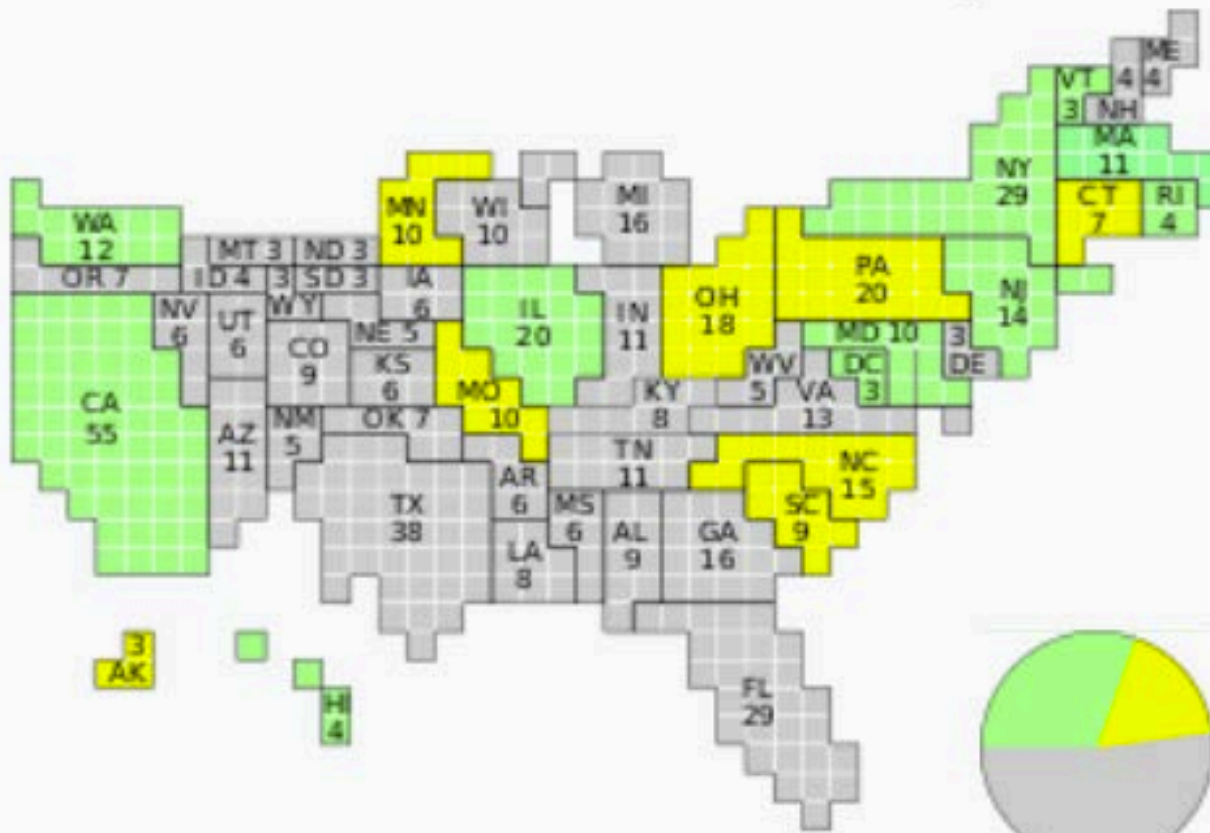
Enacted into law (165 electoral votes; 30.7% of EC)

Pending in current legislative session (92 EVs; 17.1%)

Not enacted and no bill pending (281 EVs; 52.2%)[1]

Banzhaf's Analysis

Banzhaf power index

- Invented by John Banzhaf III (GW Professor of Law) in the 1960's.

- Used to analyze Nassau County, NY Board of Supervisors.

- Lawsuit on behalf of some citizens who believed they were under-represented.

- Later applied analysis to Electoral College in several law review articles.

Idea

- Assume not all voters have same "power" (i.e., system not *anonymous*).

- Assume *two* candidates.

- Voters who favor one candidate form a ***coalition***.

- Coalition is ***winning*** if it has the votes to elect its candidate.

- Member of winning coalition is ***critical*** if his/her vote is necessary to win.

Example 1. European Economic Community of 1958 (the future EU).

Country	Votes	
France	4	
Germany	4	
Italy	4	
Belgium	2	
Netherlands	2	
Luxembourg	1	

12 votes needed to win.

Winning Coalitions

COALITION	WEIGHT	CRITICAL MEMBERS
FGIBNL	17	none
FGIBN	16	none
FGIBL	15	F, G, I
FGINL	15	F, G, I
FGIB	14	F, G, I
FGIN	14	F, G, I
FGIL	13	F, G, I
FGBNL	13	F, G, B, N
FIBNL	13	F, I, B, N
GIBNL	13	G, I, B, N
FGI	12	F, G, I
FGBN	12	F, G, B, N
FIBN	12	F, I, B, N
GIBN	12	G, I, B, N

European Economic Community of 1958

Country	Votes	Banzhaf Power
France	4	10
Germany	4	10
Italy	4	10
Belgium	2	6
Netherlands	2	6
Luxembourg	1	0

Example 2. Senate 2001

- Senate starts with 50 Democrats and 50 Republicans + 1 VP (Dick Cheney--Republican)

- Jim Jeffords (R-Vt) becomes Independent (I-Vt).

- New count: 50 Democrats, 50 Republicans, 1 Independent.

- 51 votes needed to pass bill.

Jim Jeffords' Senate

COALITION	WEIGHT	CRITICAL MEMBERS
DRI	101	none
DR	100	D, R
DI	51	D, I
RI	51	R, I

Senate 2001

Party	Votes	Banzhaf Power
Republicans	49+1	2
Democrats	50	2
Jim Jefford (I-Vt)	1	2

Banzhaf's analysis of electoral college

Two parts

Part 1. Theoretical

- If a state has population of N, your *probability* of being the critical voter on winning coalition in your state's popular election is *proportional to* $1/\sqrt{N}$.

- Your state's *probability* of being the critical vote in the EC is *proportional* to N (or really $N + 2$).

- Thus your *probability* of being the critical voter in the 2-step process is *proportional* to $\sqrt{N} = N \cdot \left(1/\sqrt{N}\right)$.

- Voters in bigger states have an advantage!

Census
2010

State	Population
California	37,253,956
Texas	25,145,561
New York	19,378,102
Florida	18,801,310
Illinois	12,830,632
Pennsylvania	12,702,379
Ohio	11,536,504
Michigan	9,883,640
Georgia	9,687,653
North Carolina	9,535,483
New Jersey	8,791,894
Virginia	8,001,024
Washington	6,724,540
Massachusetts	6,547,629
Indiana	6,483,802
Arizona	6,392,017
Tennessee	6,346,105
Missouri	5,988,927
Maryland	5,773,552
Wisconsin	5,686,986
Minnesota	5,303,925
Colorado	5,029,196
Alabama	4,779,736
South Carolina	4,625,364
Louisiana	4,533,372
Kentucky	4,339,367

State	Population
Oregon	3,831,074
Oklahoma	3,751,351
Connecticut	3,574,097
Iowa	3,046,355
Mississippi	2,967,297
Arkansas	2,915,918
Kansas	2,853,118
Utah	2,763,885
Nevada	2,700,551
New Mexico	2,059,179
West Virginia	1,852,994
Nebraska	1,826,341
Idaho	1,567,582
Hawaii	1,360,301
Maine	1,328,361
New Hampshire	1,316,470
Rhode Island	1,052,567
Montana	989,415
Delaware	897,934
South Dakota	814,180
Alaska	710,231
North Dakota	672,591
Vermont	625,741
Wyoming	563,626
DC	601,723

For example

- South Carolina: with pop. 4,625,364 vs DC with 601,723.

  - A South Carolina voter should be $2.77 = \sqrt{7.7}$ times as influential as a DC voter.

- California: with pop 37,253,956 vs Wyoming with 563,626.

  - A California voter should be $8.13 = \sqrt{66.1}$ times as influential as an Alaska voter.

- But these numbers fail to take into account the "+2" effect.

Estimating the "+2" effect

- We compare electoral votes instead of population.

- South Carolina: with 9 electoral votes vs DC with 3.

  - A South Carolina voter should be $1.73 = \sqrt{3}$ times as influential as a DC voter.

- California: with 55 electoral votes vs Wyoming with 3.

  - A California voter should be $4.28 = \sqrt{18.3}$ times as influential as an Alaska voter.

- But all these calculations are only approximate….

Part II. Computational

- Banzhaf: "People and not states" are the voters in presidential election.
- EC is a 2-step process (voting method) from voters to the election of president.
    - Voters vote in their state, then—based on these votes—states vote in EC.
- Not exactly weighted voting, but still not "anonymous" (unlike the popular vote: plurality method).
- In theory one can still count critical voters in winning coalitions: but on national scale.
- However, the problem is *computationally challenging*!

Computer models

- In 1968, Banzhaf obtained access to early computer: IBM 360 with Fortran. He did "Monte Carlo" simulation of Banzhaf power for the 1960 Census.
  - Very approximate, but seemed to confirm his assertion that voters in big states benefit. Published in law review article.

- 1990's, Mark Livingston, Computer Scientist at US Naval Research Lab, Washington, DC used Sun Workstation with C for 1990 census (published online).

- 2010 Bobby Ullman, High School Student & son of my co-author Dan Ullman, used Dell desktop with Java for 2000 Census (published in 1st edition of our book).

- 2016 Dan Ullman, used Dell laptop with *Matlab*, for 2010 census, valid through 2020 election, published in 2nd edition of our book.

STATE	EVs	Index
California	55	3.38
Texas	38	2.57
New York	29	2.25
Florida	29	2.25
Pennsylvania	20	1.91
Illinois	20	1.90
Ohio	18	1.80
Michigan	16	1.73
Georgia	16	1.71
North Carolina	15	1.61
New Jersey	14	1.59
Virginia	13	1.53
Washington	12	1.53
Tennessee	11	1.45
Indiana	11	1.45
Minnesota	10	1.45
Arizona	11	1.43
Massachusetts	11	1.43
Wisconsin	10	1.41
South Carolina	9	1.38
Maryland	10	1.38
Alabama	9	1.38
Missouri	10	1.37
Wyoming	3	1.32
Colorado	9	1.31
Rhode Island	4	1.31

STATE	EVs	Index
Kentucky	8	1.29
Vermont	3	1.28
Louisiana	8	1.25
Connecticut	7	1.25
Dist. of Columbia	3	1.25
West Virginia	5	1.24
Nebraska	5	1.23
Oklahoma	7	1.20
Nevada	6	1.20
Kansas	6	1.19
Oregon	7	1.19
Utah	6	1.18
Arkansas	6	1.18
Alaska	3	1.18
North Dakota	3	1.18
New Hampshire	4	1.17
Mississippi	6	1.17
Maine	4	1.17
New Mexico	5	1.17
Iowa	6	1.15
Hawaii	4	1.13
South Dakota	3	1.10
Idaho	4	1.06
Delaware	3	1.05
Montana	3	1.00