Joint Operation Optimization of the Interdependent Water and Electricity Networks

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Abstract—With the rapid deployment of smart technologies and the growing complexity in our modern society, there is a huge demand for coordination in day-to-day operation of the critical infrastructure networks. The coordination between water and electricity networks particularly stands out and is urgently demanding as (i) water system is one of the most energy-intensive critical infrastructure, and (ii) water unavailability, if experienced, swiftly translates into a health, safety, and national security concern. This paper proposes a comprehensive day-ahead optimization framework for joint operation of the interdependent power and water systems. Different from the conventional paradigms where the power and water systems are independently and individually operated by their respective operators, the proposed optimization framework integrates the Optimal Power Flow (OPF) models in power grids with innovative models of the water distribution systems. The nonlinear hydraulic operating constraints in the proposed optimization models are linearized, resulting into a mixed-integer linear programming (MILP) model formulation. The proposed framework is applied to three 15-node water distribution systems, operated within the IEEE 9-bus test system. The simulation results demonstrate a significant cost saving that will be achieved when the proposed approach is applied for joint operation of power and water networks.

Index Terms—Operation optimization; optimal power flow (OPF); water-energy nexus; water distribution network; mixed-integer linear programming (MILP).

NOMENCLATURE

A. Sets

\( n \in \Omega_B \) Set of system buses.
\( g \in \Omega_G \) Set of system generating units.
\( k \in \Omega_L \) Set of system transmission lines.
\( r \in \mathbb{R} \) Set of water system reservoirs.
\( p \in \mathbb{P} \) Set of water system pumps.
\( s \in S \) Set of water system pipes.

B. Variables and Functions

\( Q_{b,t} \) Water flow rate for each bus \( b \) at time \( t \).
\( Q_{m,n}^{i,j} \) Vector of reservoirs’ water inflow rate for each bus \( b \) at time \( t \).
\( Q_{b,j}^{p} \) Water flow rate through pipe \( j \) for each bus \( b \) at time \( t \).
\( Q_{b,t}^{p} \) Water flow rate through pump \( p \) for each bus \( b \) at time \( t \).

\( h_{b,t} \) Pressure heads for each bus \( b \) at time \( t \).
\( h_{n,b,t}^{i,j} \) Pressure heads associated with pipe \( j \) and node \( n \) for each bus \( b \) at time \( t \).
\( h_{n,b,t}^{p} \) Pressure heads associated with pump \( p \) and node \( n \) for each bus \( b \) at time \( t \).
\( h_{b,t}^{r} \) Pressure heads associated with reservoir \( r \) for each bus \( b \) at time \( t \).

\( \text{Sign}(.()) \) Sign function.
\( \Delta E_{b,t} \) The difference of tanks’ inflow/outflow rate for each bus \( b \) at time \( t \).
\( T_{in,b,t} \) Vector of water inflow to tanks for each bus \( b \) at time \( t \).
\( T_{out,b,t} \) Vector of water outflow to tanks for each bus \( b \) at time \( t \).
\( V_{b,t} \) Volume of stored water in tanks for each bus \( b \) at time \( t \).
\( W_{b,t} \) Pumps’ speed for each bus \( b \) at time \( t \).
\( P_{b,t}^{p} \) Power consumption for pump \( p \) and bus \( b \) at time \( t \).
\( P_{b,t}^{p,new} \) Vector of water electricity consumption in bus \( b \) at time \( t \).
\( X_{b,i}^{i,j} \) Continuous decision variable for pressure head breakpoint \( i \) associated with pipe \( j \) for each bus \( b \) at time \( t \).
\( X_{b,u,m}^{u,m,p} \) Continuous decision variable for pressure head breakpoint \( u \) associated with pump \( p \) for each bus \( b \) at time \( t \).
\( \theta_{i,t} \) Voltage angle for bus \( i \) at time \( t \).

C. Binary Variables

\( Y_{b,t}^{i,j} \) Binary variable for pressure head breakpoint \( i \) associated with pipe \( j \) for bus \( b \) at time \( t \).
\( h_{s,m,p,b,t}^{\text{Upper}} \) Binary variable for the upper triangle in the rectangle at time \( t \).
\( h_{s,m,p,b,t}^{\text{Lower}} \) Binary variable for the lower triangle in the rectangle at time \( t \).

D. Parameters

\( P_{d,t} \) Total electricity demand at time \( t \).
\( P_{dn,t}, P_{dn} \) Maximum/Minimum electricity demand.
fuels and generating electricity. Hence, the inter-dependency
erly operate. On the other hand, water is essential for refining
electricity from power systems in order to normally and prop-
systems need to be operated jointly and interdependently in
designed as two independent and uncoupled systems, those
years to come due to the sharp increase in the population of
i.e., water treatment, water purification, cooling, wastewater,
infrastructures due to their pivotal role in back-boning
ATER and energy are considered as the most critical
for a high percentage of the total electricity demand [1]. To
put a figure on this, the electricity consumption by water in-
structure is approximated around 4% of the total electricity
consumption in the United States [2]. It, therefore, calls for
an improved reliability and efficiency of the water systems, i.e.,
water treatment, water purification, cooling, wastewater,
etc., since such systems will be further electrified in the next
two to come due to the sharp increase in the population of
the modern societies.

While water and power systems have traditionally been
designed as two independent and uncoupled systems, those
systems need to be operated jointly and interdependently in
real-world applications [3]. Water facilities need to receive electricity from power systems in order to normally and prop-
erly operate. On the other hand, water is essential for refining
fuels and generating electricity. Hence, the inter-dependency
of power and water systems is critical in the case of limited
availability of either water or electricity. For instance, in the
case where a shortage in cooling water for conventional steam
power plants is realized, water distribution system may not
be supplied with sufficient amount of electricity needed to
pump the water through transmission pipelines; this could, in
turn, lead to a failure in both systems. This closely-intertwined
ecosystem of water and power networks is commonly known
as water-energy nexus (WEN) [4]–[7].

WEN has been widely investigated in the literature. Inter-
actions between WEN and the climate change, water rights,
and food supplies have been investigated in [8]–[10]. Effects of
climate change on the operation of hydro power plants and wa-
t reservoir management were studied in [11]. Reference [12]
provides a literature review on the water system optimization
with special considerations to WEN. WEN linkage analysis is
employed in [13] to demonstrate the impacts of considering
coupled water-power networks on different economic sectors.
Impacts of the battery storage facilities on the optimal dispatch
of power and water in WENs are studied in [14]. Power and
water economic dispatch approach for the supply side has been
developed in [6]. Reference [7] presents a mathematical co-
dispatch model for the optimal network flows in both water
and power systems. Daily hydro-thermal operation scheduling
using robust optimization is investigated in [15] taking into
consideration only water network constraints, while ignoring
those of the power transmission networks. Different techniques
using a physics-based modeling of WEN are used in [16] to
model the interdependence structure of water, waste water,
and power systems. Demand response and frequency regulation
for Water distribution system is studied in [17]. Energy flexibility
through coordination of water and power systems is introduced
in [18]. Another scope of research in this area focuses on cost
minimization using strategies for load management in water
and wastewater loads and on the efficiency of passive energy
to help maintaining the reliability of the power grid [19]–[21].
Economic impact of water facilities load on power system op-
eration using different energy efficiency programs is explored
in [22]. Focusing only on the operation of water pumps and
excluding the power system constraints, has been a wide and
common area of research in the literature [23], [24]; studies
to model and integrate the interactions between the water and
power systems are found scarce. Modeling such interactions
and interdependencies is critical as separate management of
the networks individually will result in sub-optimal solutions
in one another. An example of such a sub-optimal solution
can be seen in many references in the literature where a first
optimization focuses solely on water networks that provides
the electricity consumption of the water facilities to power
system operators, and the second optimization is managed by
power system operators considering the submitted demand by
water network infrastructure.

To the best of the authors’ knowledge, there is a limited
number of efforts for modeling the integrated power-water sys-
tems. To bridge the gap, this paper proposes a joint operation
optimization model for interdependent water-power systems
operated by an independent system operator. The presented
model aims to enhance not only the flexibility of both electric

\[ x_k \] Reactance of transmission line \( k \).
\[ P_{\text{max}} \] Maximum power flow limit of line \( k \).
\[ P_{\text{max}}^{\text{g}} \] Maximum capacity limit of generating unit \( g \).
\[ P_{\text{min}}^{\text{g}} \] Minimum capacity limit of generating unit \( g \).
\[ D_{b,t} \] Vector of water demand \((m^3/h)\) for each bus \( b \) at time \( t \).
\[ \hat{h}_{b} \] Reservoirs’ geographical height at each bus \( b \).
\[ V_{\text{min},b} \] Tanks’ minimum volume at each bus \( b \).
\[ V_{\text{max},b} \] Tanks’ Maximum volume at each bus \( b \).
\[ \Delta E_{\text{min},b} \] Minimum charging/discharging difference for
tanks at each bus \( b \).
\[ \Delta E_{\text{max},b} \] Maximum charging/discharging difference for
tanks at each bus \( b \).
\[ r_p \] Pipe parameter.
\[ h_{\text{min},b} \] Minimum nodal pressure heads at each bus \( b \).
\[ h_{\text{max},b} \] Maximum nodal pressure heads at each bus \( b \).
\[ Q_{\text{max/min},b} \] Maximum/Minimum water flow rate to the
network at each bus \( b \).
\[ P_{\text{max/min},b}^p \] Maximum/Minimum power consumption for
pump \( p \) at each bus \( b \).
\[ q_{i,b}^p \] Water flow rate of breakpoint \( i \) for pump \( p \) at each
bus \( b \).
\[ q_{i,b}^j \] Water flow rate of breakpoint \( i \) for pipe \( j \) at each
bus \( b \).
\[ w_{m,b}^p \] Pump speed breakpoint \( m \) at each bus \( b \).
\[ c_{0,t} \] Fixed cost of generating unit \( g \) at time \( t \).
\[ c_{g,t} \] Linear cost of generating unit \( g \) at time \( t \).
\[ c_{p,t} \] Operation cost of pump \( p \) at time \( t \).
\[ c_{r,t} \] Operation cost of reservoir \( r \) at time \( t \).
\[ w_{i,b}^p \] Speed breakpoint \( i \) for pump \( p \) at each bus \( b \).
\[ a_{1,2,3}, b_{1,2} \] Performance parameters for pumps.
\[ B \] Incidence matrix of pumps’ location.
\[ L_p \] Pipe length.
\[ g \] Gravity.
\[ d \] Pipe diameter.

I. INTRODUCTION

WATER and energy are considered as the most critical
infrastructures due to their pivotal role in back-boning
the modern society and human life. Electricity usage for
pumping water through water distribution systems accounts
for a high percentage of the total electricity demand [1]. To
put a figure on this, the electricity consumption by water in-
structure is approximated around 4% of the total electricity
consumption in the United States [2]. It, therefore, calls for
an improved reliability and efficiency of the water systems, i.e.,
water treatment, water purification, cooling, wastewater,
etc., since such systems will be further electrified in the next
years to come due to the sharp increase in the population of
the modern societies.

While water and power systems have traditionally been
designed as two independent and uncoupled systems, those
systems need to be operated jointly and interdependently in
real-world applications [3]. Water facilities need to receive electricity from power systems in order to normally and prop-
erly operate. On the other hand, water is essential for refining
fuels and generating electricity. Hence, the inter-dependency
and water systems, but also elevates the resiliency of both systems in the face of harsh environments. The proposed framework dynamically receives the input information from both networks and minimizes the energy consumption of the intensive water network infrastructure, e.g., pumps. The forecasted day-ahead demanded water as well as electricity prices are employed to minimize the total operation cost of both systems.

The rest of the paper is organized as follows. Section II introduces the proposed integrated optimization framework for power-water systems and the corresponding mathematical formulations. Numerical case studies and simulation results on a 9-bus test system integrated with three 15-node water networks is presented in Section III. The paper is concluded in Section IV.

II. PROPOSED METHODOLOGY

In this section, the proposed formulation for the integrated modeling of power and water systems is presented. The components of the water network (e.g., reservoirs, pipes, pumps, and tanks) are mathematically modeled based on the a directed graph $G = (N, A)$, i.e., a set of nodes connected together by arcs directed from one node to another; where $N$ represents the set of nodes consisting of water sources (i.e., reservoirs or tanks) or consumers. Pipes and pumps are reflected by $A$ arcs. Positive and negative values for the water flow rate $Q$ define the direction of each arc. The schematic diagram of a water system and its hydraulic components are illustrated in Fig. 1.

A. Water Flow Balance Constraints

Water system demand is delivered to consumers from the reservoirs and tanks through a network of pipes and pumps. The water flow balance constraints are modeled as follow:

$$Q_{b,t}^n - D_{b,t} - Q_{b,t} \leq \Delta E_{b,t} = 0 \quad \forall b, \forall t$$  \hspace{1cm} (1a)

$$Q_{\text{max},b} \leq Q_{b,t} \leq Q_{\text{max},b} \quad \forall b, \forall t$$  \hspace{1cm} (1c)

$$Q_{b,t}^p \geq 0 \quad \forall b, \forall t$$  \hspace{1cm} (1d)

Water flow balance equation in water network is modeled in (1a). The difference between the input and output water flow of the tank $\Delta E_{b,t}$ is modeled in (1b), while the Water flow through pipes $Q_{b,t}$ is limited in (1c). Water flow through pumps is bounded in constraint (1d).

B. Nodal Pressure Head Models for Pipes

Water flow through pipes are resulted by the pressure level difference between the water nodes $N$. The nodal pressure head for pipes is modeled as follow:

$$h_{n,b,t}^j - h_{n+1,b,t}^j = r_p \cdot Q_{b,t}^j \cdot 1.852 \cdot \text{Sign}(Q_{b,t}^j) \quad \forall t, \forall b$$  \hspace{1cm} (2a)

$$r_p = \frac{8H_p}{\pi^2 gD_p^2}$$  \hspace{1cm} (2b)

$$h_{\text{min},b}^j - h_b = 0 \quad \forall t, \forall b$$  \hspace{1cm} (2c)

$$h_{\text{min},b}^j \leq h_b \leq h_{\text{max},b} \quad \forall t, \forall b$$  \hspace{1cm} (2d)

The flow of water through pipes is modeled in (2a) by the Hazen–Williams formula, where the coefficient $r_p$ depends only on the water flow as presented in (2b). Pressure head of the water reservoirs are fixed by its geographical heights in (2c), as reservoirs are considered as unlimited sources of water. Constraint (2d) bounds the nodal pressure at each node in the network.

A piece-wise linear formulation [17], [25] is applied to guarantee a feasible solution of the optimization problem. This is because constraint (2a) is, in nature, a polynomial nonlinear function. In order to apply the piece-wise linearization, $Q_{b,t}^j$ is divided into several breakpoints ($q_1^j, q_2^j, ..., q_k^j$). $q_1^j$ and $q_2^j$ are set at the extremes (i.e., at the maximum and minimum values for $Q_{b,t}^j$). Let $Y_{b,t}^i,j$ be a binary variable for each pipe at each bus which is associated with the $i$th interval (i.e., $q_1^j, q_k^j+1$). Note that $Q_{b,t}^j$ is forced to be associated with a pair of consecutive breakpoints where $X_{b,t}^i,j \in [0,1]$ is introduced as a continuous variable for each breakpoint. Dummy variables for the binary $Y_{b,t}^i,j$ are set at zero such that $Y_0 = Y_{N} = 0$. Finally, the pressure head difference $\Delta h_{b,t}^j(q_{k}^j)$ can be approximated through the following constraints:

$$\sum_{i=1}^{I-1} Y_{b,t}^{i+1,j} = 1 \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3a)

$$X_{b,t}^{i+1,j} = Y_{b,t}^{i,j} + Y_{b,t}^{i+1,j} \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3b)

$$\sum_{i=1}^{I} X_{b,t}^{i,j} = 1 \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3c)

$$X_{b,t}^{i,j} \leq Y_{b,t}^{i+1,j} \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3d)

$$X_{b,t}^{i,j} \leq Y_{b,t}^{i,j} \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3e)

$$Q_{b,t}^j = \sum_{i=1}^{I} X_{b,t}^{i,j} q_{i+1}^j \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3f)

$$h_{n,b,t}^j - h_{n+1,b,t}^j = \sum_{i=1}^{I} X_{b,t}^{i,j} \Delta h_{b,t}^j(q_{k}^j) \quad \forall b, \forall j, \forall t$$  \hspace{1cm} (3g)

Fig. 1. Schematic diagram of a typical water network.
Constraint (3a) enforces only one binary variable to assume the value 1, while (3b)-(3e) imply that only values other than zero are chosen for $X^{i,j}_{b,t}$ and $X^{i+1,j}_{b,t}$. Constraints (3f)-(3g) ensure that the pressure difference for each pipe is properly chosen for the accurate computation of the approximated values.

C. Tank and Pump Operation Constraints

Tanks and pumps are the most challenging components to be modeled in water networks. Tank is used to smoothen the pumpage demands during peak hours and to assist the water network during emergency conditions. Tank dynamic operation is modeled as follows:

$$V_{b,t+1} = V_{b,t} + \Delta E_{b,t} \quad \forall b, \forall t$$ (4a)

$$V_{b,t} = S_a h_{b,t} \quad \forall b, \forall t$$ (4b)

$$V_{\text{min},b} \leq V_{b,t} \leq V_{\text{max},b} \quad \forall b, \forall t$$ (4c)

$$\Delta E_{\text{min},b} \leq \Delta E_{b,t} \leq \Delta E_{\text{max},b} \quad \forall b, \forall t$$ (4d)

Flow balance equation for tanks is governed by constraint (4a). The pressure head at tank nodes is driven by the water stored in the related tanks as formulated in (4b). Constraint (4c) bounds the volume of each tank to its minimum and maximum capacity. The difference between the charging and discharging water flow in tanks is limited in constraint (4d).

Water pump increases the pressure $\Delta h^p_{b,t}$ by a controlled positive amount (i.e., $\Delta h^p_{b,t} = h^p_{b,t+1} - h^p_{b,t}$). The increase in water pump pressure is modeled in (5a). The electricity consumption for each pump is formulated in (5b) and the pump electricity consumption is limited in (5c).

$$\Delta h^p_{b,t} = W_{b,t} \left( a_1 - a_2 \left( \frac{Q_{b,t}^p}{W_{b,t}} \right)^{a_3} \right) \quad \forall b, \forall t$$ (5a)

$$P^p_{b,t} = W_{b,t}^3 \left( b_1 - b_2 \left( \frac{Q_{b,t}^p}{W_{b,t}} \right)^{b_3} \right) \quad \forall b, \forall t$$ (5b)

$$P^p_{\text{min},b} \leq P^p_{b,t} \leq P^p_{\text{max},b} \quad \forall b, \forall t$$ (5c)

Non-linearity is present in the water pressure equations for pump and pump power consumption constraints (5b) and (5c), respectively. The triangle technique is used to approximate the bi-variate functions $\Delta h^p_{b,t}$ and $P^p_{b,t}$. First, breakpoints are selected by dividing the $x$ and $y$ axes, respectively, into $U$ and $M$ points (i.e., $Q^{q_1,p}_{b,t}, Q^{q_2,p}_{b,t}, \ldots, Q^{q_{U},p}_{b,t}, w^{p}_{1,b}, w^{p}_{2,b}, \ldots, w^{p}_{M,b}$). The breakpoints $(q^{q_1,p}_{b,t}, q^{q_2,p}_{b,t})$ and $(w^{p}_{1,b}, w^{p}_{2,b})$ represent the minimum and maximum water flows through pump $Q^p_{b,t}$ and speed of pump $W_{b,t}$, respectively. Continuous variable $X^{u,m,p}_{b,t} \in [0,1]$ associated with each $(u,m)$ is introduced. Also, dummy variables for all upper and lower binary $h$ have to be set at zero such that $h^{Upper}_{0,s} = h^{Upper}_{s,0,s} = h^{Lower}_{s,M,s} = 0$ and $h^{Upper}_{0,s} = h^{Lower}_{s,0,s} = h^{Lower}_{s,M,s} = 0$.

The approximations of the two nonlinear constraints (5a) and (5b) are formulated as follows [25]:

$$\sum_{u=1}^{U} \sum_{m=1}^{M} X^{u,m,p}_{b,t} = 1 \quad \forall p, \forall b, \forall t$$ (6a)

$$Q^p_{b,t} = \sum_{u=1}^{U} \sum_{m=1}^{M} X^{u,m,p}_{b,t} Q^p_{u,b} \quad \forall p, \forall b, \forall t$$ (6b)

$$W_{b,t} = \sum_{u=1}^{U} \sum_{m=1}^{M} X^{u,m,p}_{b,t} W^p_{m,b} \quad \forall p, \forall b, \forall t$$ (6c)

$$\Delta h^p_{b,t} = \sum_{u=1}^{U} \sum_{m=1}^{M} \Delta h^p_{b,t}(q^p_{u,b},w^p_{m,b}) X^{u,m,p}_{b,t} \quad \forall p, \forall b, \forall t$$ (6d)

$$P^p_{b,t} = \sum_{u=1}^{U} \sum_{m=1}^{M} P^p_{b,t}(q^p_{u,b},w^p_{m,b}) X^{u,m,p}_{b,t} \quad \forall p, \forall b, \forall t$$ (6e)

$$\sum_{u=1}^{U} \sum_{m=1}^{M} \left( h^{Upper}_{u,m,p,b,t} + h^{Lower}_{u,m,p,b,t} \right) = 1 \quad \forall p, \forall b, \forall t$$ (6f)

$$X^{u,m,p}_{b,t} \leq h^{Upper}_{u,m,p,b,t} + h^{Upper}_{u+1,m,p,b,t} + h^{Upper}_{u,m,p+1,b,t} + h^{Lower}_{u+1,m,p,b,t} + h^{Lower}_{u,m,p+1,b,t} + h^{Lower}_{u,m,p,b,t} \quad \forall u, \forall m, \forall p, \forall b, \forall t$$ (6g)

The weights of the convex combination for the selected triangle is introduced in (6a). Constraints (6b) and (6c) represent the linear combinations of any values for $Q^p_{b,t}$ and $W_{b,t}$, respectively. The bi-variate nonlinear functions for the pressure difference in each water pump ($\Delta h^p_{b,t}$) and the power consumption of pumps ($P^p_{b,t}$) are approximated in (6d) and (6e), respectively. Constraint (6f) ensures that only one triangle is used for the convex combination. Constraint (6g) enforces that only non-zero values of $X^{u,m,p}_{b,t}$ can be associated with the three vertices of the triangle.

D. Water-Power Network Integration Constraints

Power system’s DC optimal power flow (DCOPF) mechanism is integrated with the water network through the following constraints.

$$P^p_{b,t}^{\text{new}} = \sum_{p=1}^{P} B_{p,b,t} P^p_{b,t} \quad \forall b, \forall t$$ (7a)

$$P_{d,t} = P_{d,t} + P^p_{b,t}^{\text{new}} \quad \forall n, \forall t$$ (7b)

$$P_{dn} \leq P_{d,t} + P^p_{b,t}^{\text{new}} \leq P_{dn} \quad \forall n, \forall t$$ (7c)

$$\sum_{g=1}^{G} P_g - \sum_{m \in \Omega^g} P_{km,t} = P_{d,t} \quad \forall n, \forall t$$ (7d)

$$P_{k,t}^{\text{max}} = \frac{\theta_{k,t} - \theta_{k,t-1}}{x_k} \quad \forall k, \forall t$$ (7e)

$$-P^p_{k,t}^{\text{max}} \leq P_{km,t} \leq P^p_{k,t}^{\text{max}} \quad \forall k, \forall t$$ (7f)

$$P_{g,t}^{\text{min}} \leq P_g \leq P_{g,t}^{\text{max}} \quad \forall t$$ (7g)
A new parameter for pump power consumption is introduced in \((7a)\) to adjust the dimension of \(P_{b,t}^p\). Constraint \((7b)\) sums the total electricity demand for power and water networks together. Electricity consumption is bounded in \((7c)\). Power balance constraint at each bus is enforced in \((7d)\). Constraint \((7e)\) sets the power flow in transmission lines. The power flow in each transmission line is bounded to the minimum and maximum limits in \((7f)\). Output power of system generating units is limited to the minimum and maximum capacities in \((7g)\).

The complete mixed-integer linear programming (MILP) optimization model of joint operation of power grids integrated with water networks is formulated as follows:

\[
\begin{align*}
\min \quad & \sum_{g=1}^{G} \sum_{t=1}^{NT} c_{0,t} + c_{g,t} P_{g,t} + c_{p,t} P_{b,t}^p + c_{r,t} Q_{b,t}^{in} \\
\text{s.t.} \quad & (1a - 1d), (2c - 4d), (5c - 6g) \\
& (7a - 7g) \\
& X_{ij}^{t,j} \in [0, 1], q_{ij}^{t,b} \in [-Q_{ij}^{max}, Q_{ij}^{max}], \forall j \in S \\
& X_{ij}^{in-p} \in [0, 1], q_{ij}^{p,b} \in [0, Q_{ij}^{max}], \forall p \in P \\
& \Delta h_{b,t}^P \in [0, \Delta h_{b,t}^{P \text{max}}], \forall p \in P
\end{align*}
\]

The objective function minimizes the total electricity consumption in the power-water network. The first two terms in the objective function are the fixed costs and the linear costs of the generating units. The third term is added to minimize the total consumption of electricity needed to pump the water in the network. The last term reflects the purchased water from the reservoirs. The objective function is subject to several power-water integrated constraints considering the linearized hydraulic operating constraints for water networks.

III. NUMERICAL CASE STUDIES

A. System Descriptions, Data, and Assumptions

The proposed formulation for joint operation optimization of the power and water systems is implemented on the IEEE 9-bus test system connected to three water distribution systems as illustrated in Fig. 2. The 9-bus test system consists of three generating units, nine transmission lines, and three load points. Each water distribution system consists of 15 nodes and is connected to a power grid load point. Note that each water system includes 11 pipelines, 3 pumps, and 2 tanks. Details on the location of each component in the water system (e.g. pumping stations, water tanks, nodes, etc.) are demonstrated in Fig. 4. Reservoir is treated as unlimited source of water and tanks are set to be empty at the initial time. In this model, each water system is characterized with different parameters (e.g., tank volume, demands etc.). All system data (i.e., the hourly generation and load profiles, transmission line parameters, water demand, pipeline parameters, etc.) are provided in [26]. All simulations are performed in A Mathematical Programming Language (AMPL) environment [27], using a Dell PowerEdge R815 with 4 AMD Opteron 6174 Processors (48 2.2 GHz cores) and 256 GB of Memory running CentOS 5.7. CPLEX solver is used to simulate and solve the model.

B. Results and Discussions

In order to illustrate the performance of the proposed optimization model, two different case studies are presented in this section. First test case (TC1) assumes that the use of electricity for water pumps and the OPF in power systems are co-optimized by two separate operators in respective domain. The optimization model of TC1 is simulated in two steps. Firstly, water system operators optimize the electricity usage...
of water pump as well as the purchased water and send the required electricity demand in the water sector to power system operators. Secondly, DCOPF is performed by power system operators considering the electricity demand to operate water pumps. The second test case (TC2) represents the proposed framework in which water and power systems are jointly operated where DCOPF and water hydraulic constraints are efficiently merged. The optimization engine in TC2 runs once by a single system operator.

The system operation cost as well as the computational time in both test cases are presented in Table I and Table II, respectively. Comparing the results in TC1 and TC2, one can conclude that TC2 is more efficient in terms of the total operation cost saving. TC1 results in a total operation cost of $38,419.55 for water and power system operation. In TC2, the optimal operation cost for the integrated power and water infrastructures is reported $20,174.71. The computation time for TC1 and TC2 are reported 66.1 seconds and 85.4 seconds, respectively. Even though the computation time for TC1 is lower, the time needed to coordinate the exchange of data between the two networks is not included. The proposed approach in TC2 is still computationally-attractive to be used in daily operation of the two networks.

Figure 4 shows the load profile for three different water systems supplied by the studied power system at load point 5, 7, and 9. According to Fig. 4, it can be seen that the total demand required to operate the water systems, when the two models operate jointly (TC2), is lower compared to a case when both systems are operated separately (TC1). Figure 5 shows the sum of the total electricity demand of all three water systems in TC1 and TC2. One can see in Fig. 5 that the total demand to operate the water networks is lower in TC2 compared to that in TC1.

IV. CONCLUSION

This paper has presented a comprehensive optimization framework for day-ahead optimal joint operation of water and power systems by an independent system operator. The integration of DC optimal power flow (DCOPF) and hydraulic water system operation has been taken into account to evaluate the applicability and effectiveness of the proposed framework in managing the interdependent critical infrastructure. The piece-wise linearization technique was used to approximate the nonlinear functions of the water pumping operations as well as the pressure head loss between different nodes. The non-convex optimization is, hence, transformed to a tractable mixed-integer linear programming (MILP) formulation which can be quickly solved by commercial off-the-shelf solvers. The simulation results revealed that under the joint operation of interdependent water and power infrastructures, the total operation cost can be significantly reduced. This is achieved through optimizing the electricity consumption by electric pumps used in the water networks and storing water in tanks during the peak electricity demand.
REFERENCES


