Chapter 4

Centricity, Referential Collections, and Triadic Post-Tonality

Tonality

Traditional common-practice tonality, the musical language of Western classical music from roughly the time of Bach to roughly the time of Brahms, is defined by six characteristics:

1. Key. A particular note is defined as the tonic (as in "the key of C♯" or "the key of A") with the remaining notes defined in relation to it.
2. Key relations. Pieces modulate through a succession of keys, with the keynotes often related by perfect fifth, or by major or minor thirds. Pieces end in the key in which they begin.
3. Diatonic scales. The principal scales are the major and minor scales.
4. Triads. The basic harmonic structure is a major or minor triad. Seventh chords play a secondary role.
5. Functional harmony. Harmonies generally have the function of a tonic (arrival point), dominant (leading to tonic), or predominant (leading to dominant).
6. Voice leading. The voice leading follows certain traditional norms, including the avoidance of parallel perfect consonances and the resolution of intervals defined as dissonant to those defined as consonant.

In common-practice tonality, these six attributes interact in a variety of mutually reinforcing ways. It is perfectly possible, however, for music to have only a few or even just one of these attributes without having all of them. Music like that has a clear connection to common-practice tonality without actually being tonal, strictly speaking.

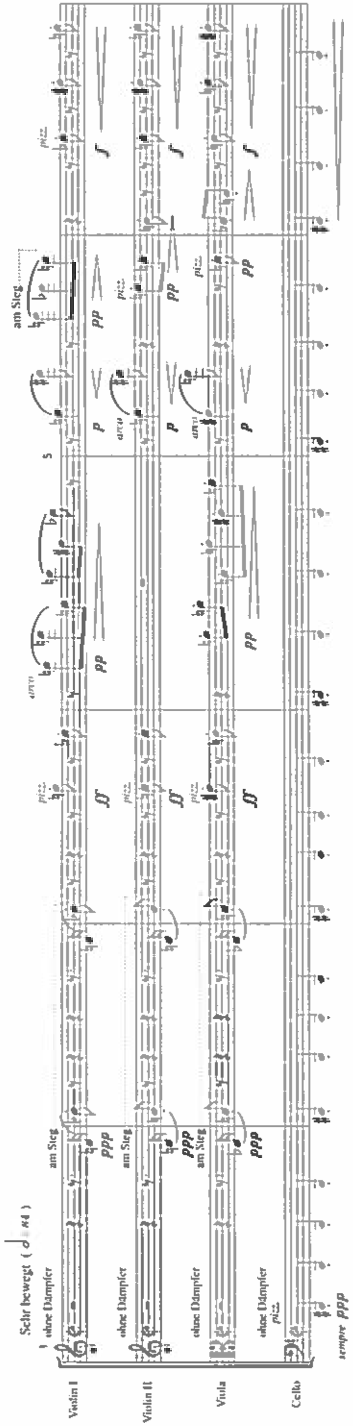
Of these six attributes, the first four characterize a significant body of post-tonal music, although often in nontraditional ways. Post-tonal music has various ways of creating a sense of focus on a particular note or harmony—among the most important of these is inversional symmetry. Post-tonal music has various ways of creating a sense of large-scale harmonic motion, often following the path of an interval cycle. Post-tonal music often makes use of diatonic scales, and a number of other common scales as well, including octatonic, hexatonic, and whole-tone scales. Post-tonal music frequently uses triads, although they are commonly combined in nontraditional ways. The last two attributes, however, play negligible roles. Functional harmony—harmonies with a tonic, dominant, or predominant function—remain important in a variety of popular musics, but not, generally speaking, in music of the Western art tradition. As a result, Roman numerals and function symbols will not be of much use in the analyzing the repertoire discussed in this book. Traditional voice leading, with its avoidance of parallel perfect consonances and its normative resolutions of dissonances to consonances, has largely been abandoned by both the art and popular traditions.

Centricity

All tonal music is centric, focused on specific pitch classes or triads, but not all centric music is tonal. Even without the resources of tonality, music can be organized around referential centers. A great deal of post-tonal music focuses on specific pitches, pitch classes, or pitch-class sets as a way of shaping and organizing the music. In the absence of functional harmony and traditional voice leading, composers use a variety of contextual means of reinforcement. In the most general sense, notes that are stated frequently, sustained at length, placed in a registral extreme, played loudly, and rhythmically or metrically stressed tend to have priority over notes that don't have those attributes.

For a simple example, consider the opening of the third of Webern's Movements for String Quartet, Op. 5 (see Example 4–1). A C♯ pedal runs through the passage. By brute repetition, the C♯ is established as an important pitch center in the passage. We inevitably hear the other events in the passage in relation to it. The C♯ receives special treatment throughout the piece and is the last note of the piece, played in octaves, triple-forte, by all four instruments. Though the piece is by no stretch of the imagination in C♯ major or C♯ minor, the C♯ certainly has a centric function. In principle, a pitch-class set, or even a set class, can also act as a referential center if it is sufficiently reinforced.

In post-tonal music, a sense of key or pitch-class center is often present only fleetingly. Consider, for one last time, the opening of Schoenberg's Piano Piece, Op. 11, No. 1 (look back at Example 2–15). It has a lyrical melody, complete with expressive appoggiaturas, and a chordal accompaniment. It has a traditional feel, and was written in 1908, in the earliest stages of Schoenberg's interest in writing music without a key signature. But a tonal hearing and a tonal analysis are virtually impossible to sustain. There exist published analyses of the passage, by three respected authorities, in three different keys. One says it's in E; one says it's in F♯, and one says it's in G. In a sense, they are all right, and there are other possible keys to be heard here as well. But no key shapes the structure in any deep or reliable way. Rather, tonality operates in this piece like a ghost, haunting the structure with its presence, but impossible to pin down in any satisfactory way. A complete account of the work, and others like it, will have to take into account the ghostly remnants of traditional tonality, but for a thorough analytical account and a richly satisfactory understanding, our traditional tonal analysis simply won't be of much use.



Example 4–1 C♯ as pitch center (Webern, Movements for String Quartet, Op. 5, No. 3).

Discussing pitch centricity in post-tonal music is more complicated than identifying the tonic of a tonal piece. In post-tonal music, we can talk about an entire spectrum of centric effects. At one extreme, represented by much twelve-tone music, there is little or no sense of centricity. Even so, of course, the pitch classes are not treated identically, and it is important to be sensitive to any kind of special treatment accorded to pitch classes or pitch-class sets. At the other extreme, many post-tonal pieces are deeply preoccupied by questions of centricity.

Inversional Axis

Centricity in post-tonal music can be established by various kinds of direct emphasis and reinforcement: centric pitches are usually stated longer, louder, more often, and higher (or lower) than noncentric pitches. In addition, centricity in post-tonal music can be based on inversional symmetry. An inversionally symmetrical set has an *axis of symmetry*, a midpoint around which all of the notes balance. An axis of symmetry may function as a pitch or pitch-class center.

In the beginning of the passage from Sofia Gubaidulina's String Trio shown in Example 4–2a, all three instruments establish the pitch B4 as a central tone.

a.

(continued)

Example 4–2 Inversional symmetry (Gubaidulina, String Trio, first movement).

b.

Example 4–2 Continued

As new notes are added, they are usually balanced around that central note (Example 4–2b): A♮, one or thirteen semitones below is balanced by C, one or thirteen semitones above; A3 is balanced by C♯6; G♯3 is balanced by D6; and G3 is balanced by D♯6. The only exceptions to the pattern are that F4 is balanced against F♯5 instead of F5 and that the last two low notes, F♯3 and F3, lack partners. Otherwise, B5 is the axis of symmetry and literally the central tone in the passage.

In some passages, notes radiate outward from a central tone in an *expanding wedge*. In the ninth of Ligeti's Ten Pieces for Wind Quintet, the three instruments (the horn and bassoon are tacet) are in canon (Example 4–3a). The shared canonic line begins on E♭ and then radiates outward (Examples 4–3b).

a.

*Throughout the piece the attacks are "level". That is, attack without special accentuation, sustain the tone ff, break off suddenly to breathe (without diminuendo), re-enter "level" just as suddenly, etc. Always take a good breath (breathing can be clearly audible).

Example 4–3 Expanding wedge (Ligeti, Ten Pieces for Wind Quintet, No. 9).

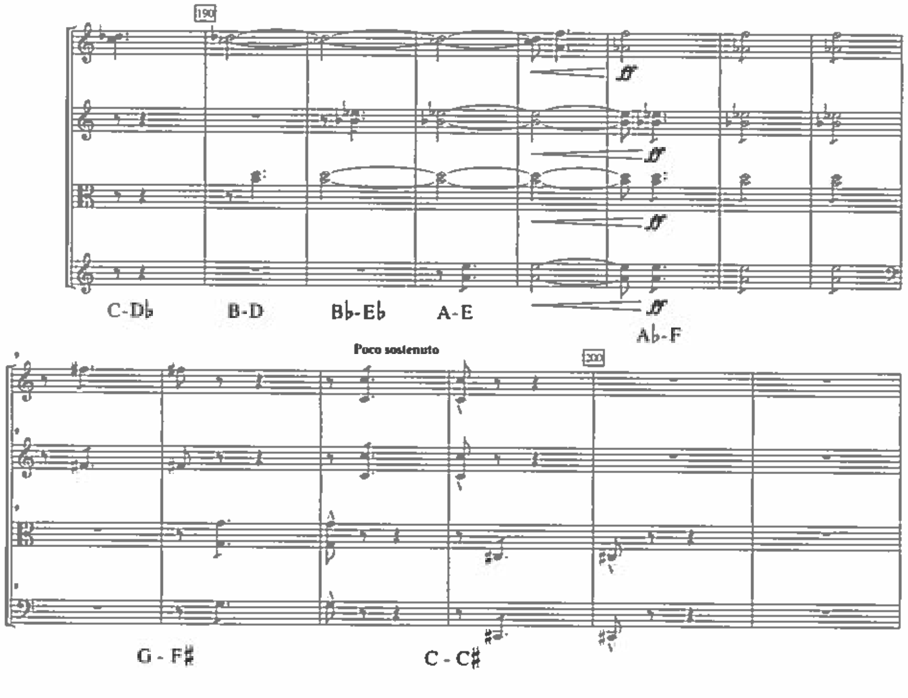
b.

Example 4–3 Continued

A *contracting wedge*, where everything converges on the axis note, can have strong cadential force, as at the end of the first movement of Bartók's String Quartet No. 5 (Example 4–4a). The lines begin on E in four different octaves, and then converge on the cadential B♭ (Example 4–4b).

Example 4–4 Contracting wedge (Bartók, String Quartet No. 5, first movement).

Something similar happens in the fifth movement of the same string quartet, but now the symmetry is realized in pitch-class space rather than in pitch space (Example 4–5a). The passage begins with the dyad C–D♭ and expands in a symmetrical wedge. When the wedge reaches F♯–G, both notes are doubled at the octave. Finally, the original dyad C–D♭ returns, but in a different octave from its first statement. Because of octave displacements, it is easier to grasp the inversional symmetry with a pitch-class clockface (Example 4–5b). On such a clockface, the axis of a.



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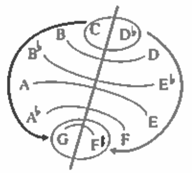
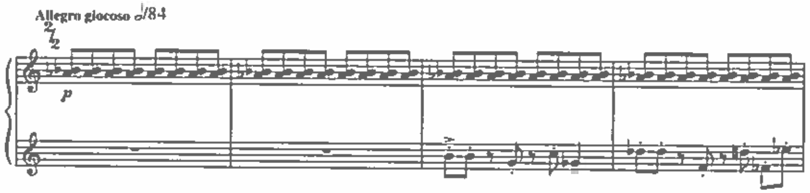


Example 4–5 Inversional axis (Bartók, String Quartet No. 5, fifth movement).

symmetry consists of two notes (or two pairs of notes) a tritone apart—these are the *poles* of the axis.

Bartók makes similar use of an inversional axis in pitch-class space in his Bagatelle, Op. 6, No. 2 (see Example 4–6a). The piece begins with repeated A♭s and B♭s in the right hand to which an expanding melodic wedge is added in the left hand: B and G (a semitone above and below the repeated figure); C and G♭ (two semitones above and below); D♭ and F (three semitones above and below); D and F♭ (four semitones above and below); and finally E♭, a pitch-class that can be understood to be five semitones both above and below. The only one of the twelve pitch classes that has not been heard is A, which lies right in the middle of the repeated figure, a kind of silent

a.



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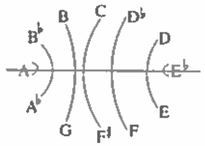


c.

Example 4–6 Inversional Axis (Bartók, Bagatelle, Op. 6, No. 2).

center around which everything balances (see Example 4–6b). A is the pitch axis, but A–E♭ is the pitch-class axis. The E♭ does not play much of a centric role in this opening phrase, but later in the piece the opening music returns transposed at T_6 (Example 4–6c). At that point, the pitch-class axis is still A–E♭, but now it is the E♭ that is particularly emphasized.

There are twelve axes of inversion, and these can be identified by either the sum of the balancing pairs of notes or by the axis notes themselves (see Figure 4–1). For each sum n, the inversional axis will pass through n/2 and n/2 + 6. For example, if the sum of the balanced notes is 8, the inversional axis will be 4–10. If the sum is odd, the axis will pass between two pairs of notes. For example, if the sum of the balanced notes is 7, the inversional axis passes between 3 and 4 and between 9 and 10. We will write that axis as 1/2–7/8.

The twelve inversional axes have the potential to function like the twelve major/minor keys of traditional tonality, including the possibility that music might "modulate" from one axis to another. The beginning of Bartók's *Sonata for Two Pianos and Percussion,* shown in Example 4–7a, features a seven-note chromatic motive that is symmetrical around its first note. The first statement (in measure 2, repeated an octave higher in measure 3) is symmetrical around F♯ while the second

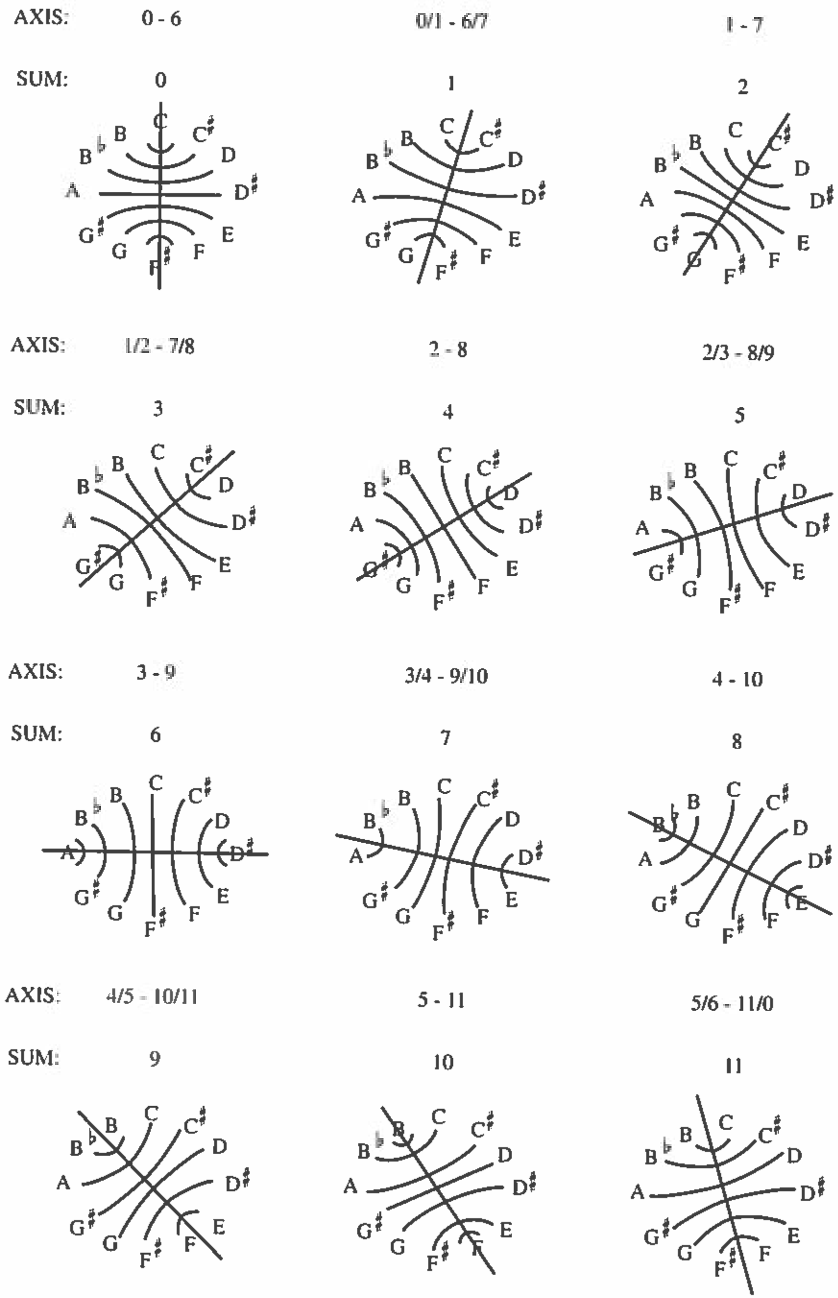


Figure 4–1

Example 4–7 Shifting axes of inversion (Bartók, Sonata for Two Pianos and Percussion, first movement, percussion parts omitted).

statement (measure 5) is symmetrical around C. These share the same axis of symmetry: 0–6 (sum 0). The two overlapped entries of the motive in measure 8–9 are symmetrical on G and then D♭. These share a different axis of symmetry: 1–7 (sum 2). The passage has shifted its centricity from one axis to another (Example 4–7b).

The Diatonic Collection

Composers of post-tonal music often use certain large sets as sources of pitch material. By drawing all or most of the smaller sets from a single large referential set, composers can unify entire sections of music. By changing the large referential set, the composer can create a sense of large-scale movement from one harmonic area to another. Many large collections are available, but four in particular have attracted extensive compositional and theoretical attention: the diatonic, octatonic, hexatonic, and whole-tone collections. Each of these collections has remarkable structural properties and harmonic resources and each is associated with a distinctive sound world.

The *diatonic collection* is any transposition of the seven "white notes" of the piano. It is set class 7–35 (013568T). This collection is, of course, the basic referential source for all of Western tonal music. A typical tonal piece begins within one diatonic collection, moves through other transposed diatonic collections, then ends where it began. All the major scales, (natural) minor scales, and church modes are diatonic collections. Diatonic collections also are common in twentieth-century music. Large stretches of music by Stravinsky and others can be referred to one or more diatonic collections. In post-tonal music, however, the diatonic collection is used without the functional harmony and traditional voice leading of tonal music.

Example 4–8 illustrates nonfunctional, static diatonicism in Stravinsky's *Petrushka.* Although the centricity of the passage is clear (on G for the first eight measures, then shifting to A), it is not traditionally tonal—just try analyzing it with Roman numerals! It does, however, use only diatonic collections. These collections define distinct harmonic areas. In the first eight measures, only the "white notes" are used. In measure 9, F♮ is replaced by F♯, resulting in a different diatonic collection, a transposition of the first. With the change in collection, we have a sense of large-scale shift from one area to another. The change coincides with a change in centricity, creating a clear musical articulation.

The opening of *Petrushka* moves from G-Mixolydian (G–A–B–C–D–E–F–G) to A-Dorian (A–B–C–D–E–F♯–G–A). There are seven possible orderings of the diatonic collection: Ionian (equivalent to the white notes from C to C), Dorian (from D to D), Phrygian (E–E), Lydian (F–F), Mixolydian (G–G), Aeolian (A–A), and Locrian (B–B). Each of these orderings can begin on any of the twelve pitch classes. In analyzing post-tonal diatonic music, we will usually want to know both the centric tone and the scalar ordering.

In some cases, a centric tone can be difficult to determine or musically irrelevant. Then, it may be necessary to refer to the diatonic collections in some more neutral way, without reference either to centric tone or ordering, by simply stating the number of accidentals needed to write the collection. In this terminology, the twelve diatonic collections are: 0-sharp, 1-sharp, 2-sharp, 3-sharp, 4-sharp, 5-sharp, 6-sharp or 6-flat, 5-flat, 4-flat, 3-flat, 2-flat, and 1-flat. For example, C-Lydian, G-Ionian, E-Aeolian, and F♯-Locrian, among seven different scalar orderings, all represent the 1-sharp collection. Conversely, the 2-flat collection, for example, might be represented by B♭-Ionian, C-Dorian, D-Phrygian, etc. There are twelve different diatonic

Example 4–8 Two diatonic collections (Stravinsky, *Petrushka,* Russian Dance).

collections, each of which can be ordered in seven different ways (Figure 4–2 summarizes the possibilities).

The diatonic collection provides a strong link to earlier music, but it acts in a new way, as primarily a referential source collection from which surface motives are drawn. In tonal music, the diatonic collection is usually divided up (partitioned) vertically into triads. In post-tonal diatonic music, triads are used but other harmonies also occur. For example, 4–23 (0257) and 3–9 (027) are diatonic subsets that occur in tonal music only infrequently and as dissonant by-products of voice leading. In Stravinsky's diatonic music, however, they are particularly common. Example 4–9 shows a diatonic passage from the beginning of Stravinsky's opera *The Rake's Progress.*

As we observed back in Chapter 1 (Example 1–12), on virtually every beat in this passage one finds either A–B–E or A–D–E, two forms of set class 3–9 (027).

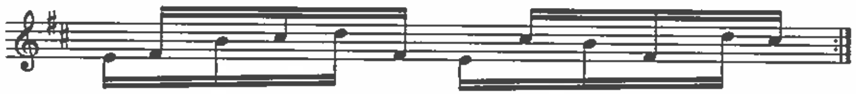
Collection name	*Possible orderings (scales)*
0-sharp (or 0-flat)	C-Ionian, D-Dorian, E-Phrygian, F-Lydian, G-Mixolydian, A-Aeolian, B-Locrian
1-sharp	C-Lydian, D-Mixolydian, E-Aeolian, F♯-Locrian, G-Ionian, A-Dorian, B-Phrygian
2-sharp	C♯-Locrian, D-Ionian, E-Dorian, F♯-Phrygian, G-Lydian, A-Mixolydian, B-Aeolian
3-sharp	C♯-Phrygian, D-Lydian, E-Mixolydian, F♯-Aeolian, G♯-Locrian, A-Ionian, B-Dorian
4-sharp	C♯-Aeolian, D♯-Locrian, E-Ionian, F♯-Dorian, G♯-Phrygian, A-Lydian, B-Mixolydian
5-sharp	C♯-Dorian, D♯-Phrygian, E-Lydian, F♯-Mixolydian, G♯-Aeolian, A♯-Locrian, B-Ionian
6-sharp (or 6-flat)	C♯-Mixolydian, D♯-Aeolian, E♯-Locrian, F♯-Ionian, G♯-Dorian, A♯-Phrygian, B-Lydian
5-flat	C-Locrian, D♭-Ionian, E♭-Dorian, F-Phrygian, G♭-Lydian, A♭-Mixolydian, B♭-Aeolian
4-flat	C-Phrygian, D♭-Lydian, E♭-Mixolydian, F-Aeolian, G-Locrian, A♭-Ionian, B♭-Dorian
3-flat	C-Aeolian, D-Locrian, E♭-Ionian, F-Dorian, G-Phrygian, A♭-Lydian, B♭-Mixolydian
2-flat	C-Dorian, D-Phrygian, E♭-Lydian, F-Mixolydian, G-Aeolian, A-Locrian, B♭-Ionian
1-flat	C-Mixolydian, D-Aeolian, E-Locrian, F-Ionian, G-Dorian, A-Phrygian, B♭-Lydian

Figure 4–2

Example 4–9 Static, nontriadic diatonic music (Stravinsky, *The Rake's Progress*).

Together, they form set class 4–23 (0257), a favorite of Stravinsky's. The passage is clearly centered on A and on the perfect fifth A–E. But Stravinsky fills in that fifth not with the traditional third, but with seconds and fourths, creating the sonorities most characteristic of his own music. The music is diatonic, but it is neither triadic nor tonal. Rather, the collection acts as a kind of harmonic field from which smaller musical shapes are drawn.

A significant amount of music that is often described as "minimalist" makes use of the diatonic collection as Stravinsky does, in nontraditional and harmonically nonfunctional ways. Example 4–10a shows the basic pattern from Steve Reich's *Piano Phase*, composed in 1967. It consists of five notes of the two-sharp collection, [B, C♯, D, E, F♯], although scalar order is hard to specify (E–Dorian? B–Aeolian?). The collection is presented as an interlocking of two figures, a three-note pattern (E–B–D) and a two-note pattern (F♯–C♯). A rhythmic sense of three-against-two is thus built in.

a. 

b. 

c. 

Example 4–10 A diatonic collection (Reich, *Piano Phase*).

Piano Phase is written for two pianists. The first player just repeats the pattern of Example 4–10a throughout the entire piece. The second player enters in unison and then, after a while, gradually increases the tempo until the second part has moved one-sixteenth note ahead. At this point, the second part locks back into the tempo of the first and together they play the music shown in Example 4–10b—a canon at the distance of one sixteenth-note. After a while, the second part moves ahead again until it locks back in two sixteenth-notes ahead, as in Example 4–10c—a canon at the distance of two sixteenth-notes. The entire piece involves moving in and out of phase in this manner until, after twelve stages, the original unison is restored.

From a collectional point of view, the piece is incredibly static—only five diatonic pitch classes are used throughout. But there is also remarkable variety produced by the gradual rhythmic shifts. For example, it is interesting to compare the harmonic dyads in Examples 4–10b and 4–10c. In Example 4–10b, a note from one of the two original figures (E–B–D and F♯–C♯) always sounds with a note from the other, and the intervals that result are often dissonant. In Example 4–10c, a note from one of the two original figures always sounds with a note from the same figure, and the intervals that result are generally consonant. This alternation of relatively consonant and relatively dissonant stages persists throughout the piece. The underlying five-note diatonic collection never changes, but its internal relationships are always shifting, always caught up in a dynamic process of change.

The Octatonic Collection

The *octatonic collection* has been another post-tonal favorite, particularly in the music of Bartók and Stravinsky. This collection, 8–28 (0134679T), has many distinctive features. First, it is highly symmetrical, both transpositionally and inversionally. It maps onto itself at four levels of transposition and four levels of inversion. As a result, it has only three distinct forms (just like its complement, the diminished-seventh chord). Figure 4–3 shows the three octatonic collections. The three octatonic

$OCT_{0,1}$	[0,1,3,4,6,7,9,10]
$OCT_{1,2}$	[1,2,4,5,7,8,10,11]
$OCT_{2,3}$	[2,3,5,6,8,9,11,0]

Figure 4–3

collections are identified by the numerically lowest pitch-class semitone that uniquely defines them. So $OCT_{0,1}$ is the octatonic collection that contains C and C♯, $OCT_{1,2}$ contains C♯ and D; and $OCT_{2,3}$ contains D and E♭.

Each octatonic collection contains two of the diminished-seventh chords and excludes the third (see Figure 4–4). When written out as a scale, the octatonic collection consists of alternating 1s and 2s (unlike the diatonic scale, where 2s predominate and the 1s are asymmetrically placed). It can be written in only two different ways, either beginning with a 1 and alternating 1–2–1–2–1–2–1 or beginning with a 2 and alternating 2–1–2–1–2–1–2.

Its subset structure is comparably restricted and redundant. Like the octatonic collection itself, many of its subsets are inversionally and/or transpositionally symmetrical. Each subset can be transposed at T_0, T_3, T_6, and T_9 without introducing any notes foreign to the collection. Conversely, it is possible to generate the octatonic collection by successively transposing any of its subsets at T_0, T_3, T_6, and T_9. If you take a major triad, for example, and combine it with its transpositions at T_3, T_6, and T_9, you create an octatonic collection. The partial list of subsets of $OCT_{0,1}$ in Figure 4–5

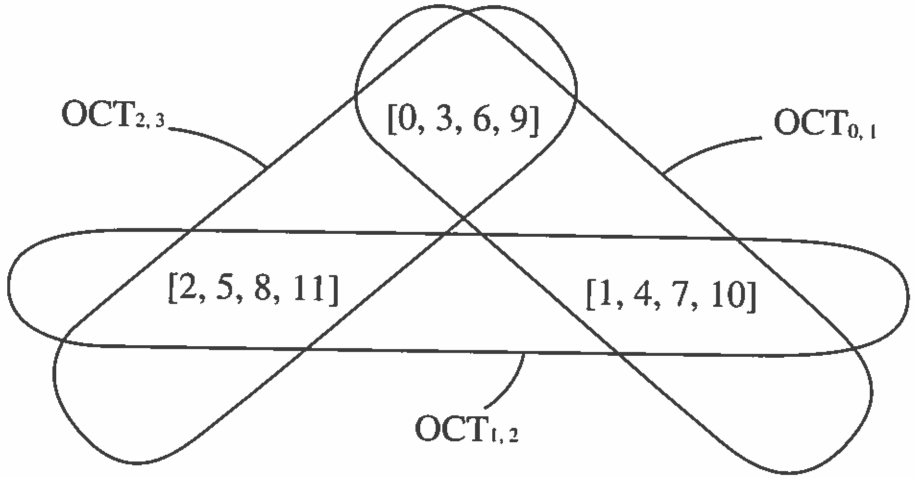


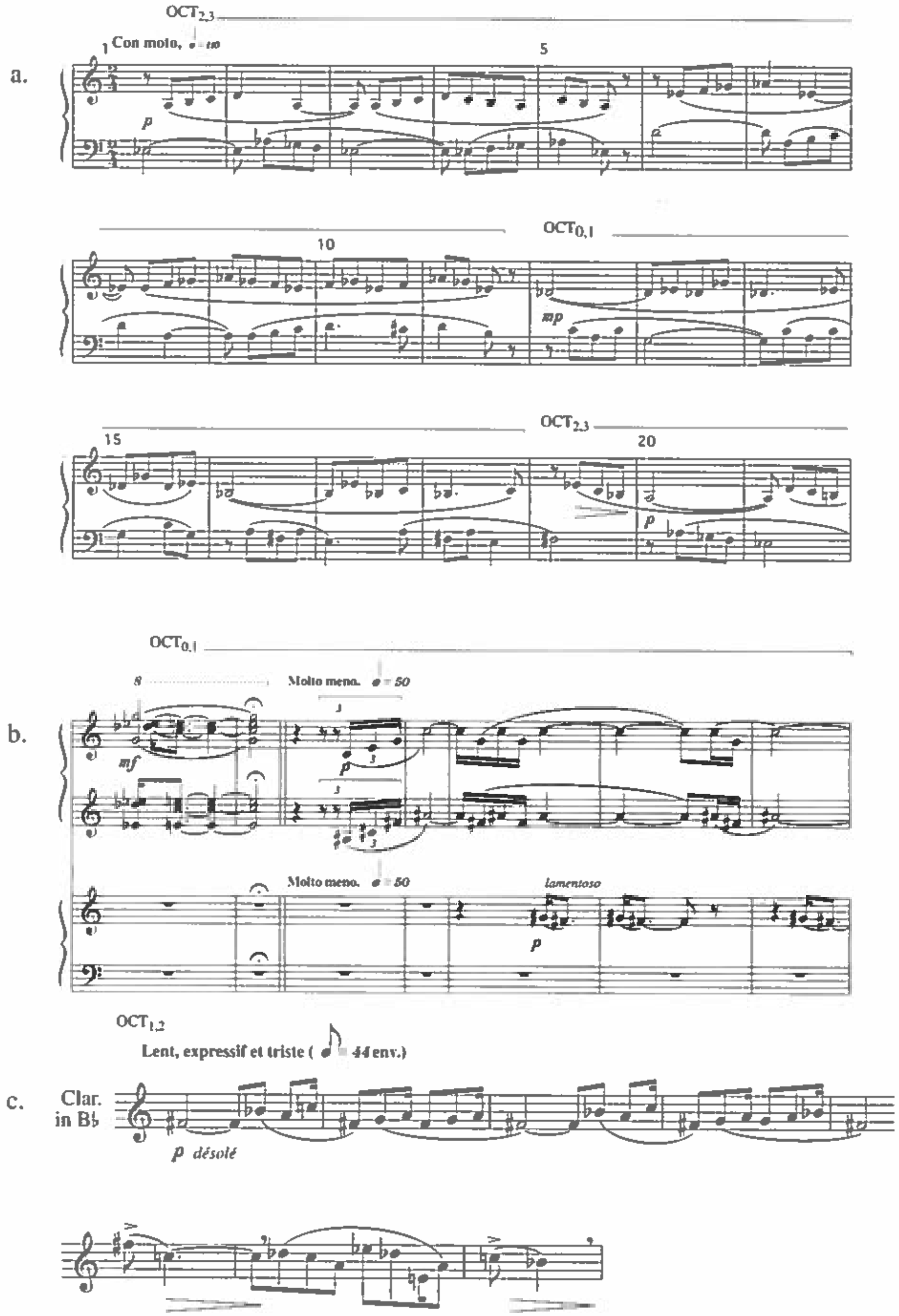
Figure 4–4

Set class	*Members*
3–2 (013)	[C, C♯, D♯], [D♯, E, F♯], [F♯, G, A], [A, A♯, C] Also, in inversion, [C♯, D♯, E], [E, F♯, G], [G, A, A♯], [A♯, C, C♯]
3–11 (037) (major or minor triad)	[C, D♯, G], [D♯, F♯, A♯], [F♯, A, C♯], [A, C, E] Also, in inversion, [C, E, G], [D♯, G, A♯], [F♯, A♯, C♯], [A, C♯, E]
4–3 (0134)	[C, C♯, D♯, E], [D♯, E, F♯, G], [F♯, G, A, A♯], [A, A♯, C, C♯]
4–10 (0235) (minor or dorian tetrachord)	[C♯, D♯, E, F♯], [E, F♯, G, A], [G, A, A♯, C], [A♯, C, C♯, D♯]
4–26 (0358) (minor seventh chord)	[C, D♯, G, A♯], [D♯, F♯, A♯, C♯], [F♯, A, C♯, E], [A, C, E, G]
4–27 (0258) (dominant or half-diminished seventh chord)	[C, E, G, A♯], [D♯, G, A♯, C♯], [F♯, A♯, C♯, E], [A, C♯, E, G] Also, in inversion, [C, D♯, F♯, A♯], [D♯, F♯, A, C♯], [F♯, A, C, E], [A, C, D♯, G]
4–28 (0369) (diminished seventh chord)	[C, D♯, F♯, A], [C♯, E, G, A♯]

Figure 4–5

shows that the octatonic collection contains many familiar formations, and that these always occur multiple times.

Example 4–11 contains brief octatonic passages by Bartók, Stravinsky, and Messiaen. The Bartók begins by combining one form of 4–10 (0235) in the left



Example 4–11 Three octatonic passages (Bartók, *Mikrokosmos*, No. 101, "Diminished Fifth"; Stravinsky, *Petrushka*, Second Tableau; Messiaen, *Quartet for the End of Time*, third movement, "Abime des Oiseaux").

hand with its tritone transposition in the right hand. The result is the complete $OCT_{2,3}$. In measure 12, the larger collection shifts to an incomplete $OCT_{0,1}$, now created by tritone-related forms of 3–7 (025). In measure 19, the music shifts back to $OCT_{2,3}$. The large-scale harmonic organization of the passage, and of the rest of the work, is determined by the motion among the octatonic collections. The passage by Stravinsky combines two major triads a tritone apart to create an incomplete $OCT_{0,1}$. The passage by Messiaen does not parse quite so easily, although it too is concerned with transposing its constituent subsets, such as 3–2 (013) by T_3 and T_6. In post-tonal music, and even in earlier music, octatonic collections frequently emerge as by-products of transpositional schemes involving minor thirds and tritones.

The Whole-Tone Collection

In addition to the diatonic and octatonic collections, the whole-tone collection, set class 6–35 (02468T), also occurs frequently in post-tonal music. The whole-tone collection has the highest possible degree of symmetry, both transpositional and inversional, and its set class contains only two distinct members. We can refer to them as WT_0 (the whole-tone collection that contains pitch-class C) and WT_1 (the whole-tone collection that contains pitch-class C♯). They also are sometimes called the "even" or "odd" collections, because all of the pitch-class integers in WT_0 are even (0, 2, 4, 6, 8, 10), while those in WT_1 are odd (1, 3, 5, 7, 9, 11).

The intervallic and subset structures of the whole-tone collection are predictably restricted and redundant. It contains only even intervals: interval-classes 2, 4, and 6. It contains only three different trichord-classes, 3–6 (024), 3–8 (026), and 3–12 (048), three different tetrachord-classes, 4–21 (0246), 4–24 (0248), and 4–25 (0268), and a single pentachord-class, 5–33 (02468). Figure 4–6 presents this information in the form of an *inclusion lattice*—set classes are connected to those they either contain or are contained within, with the whole-tone collection itself written at the top.

Example 4–12 shows two passages that are based on a single whole-tone collection: WT_0. The Debussy features descending whole-tone scales in parallel 4s. The bass B♭ in measures 5–7 asserts itself as a centric tone. The Cowell is from a wild piece called "The Banshee" that is played on the strings of the piano. According to the composer's performance instructions, A "indicates a sweep with the flesh of the finger from the lowest string up to the note given" and B tells the performer to "sweep lengthwise along the string of the note given with the flesh of the finger." At the top of each glissando, the goal note belongs to WT_0, and these descend through the entire whole-tone scale.

Both passages in Example 4–13 involve contrast between the two whole-tone collections. In the Bartók, a melody from one of the whole-tone collections is paired with an accompaniment from the other. In the Crumb, a vocalise from one of the whole-tone collections gives way to a final distant echo from the other.

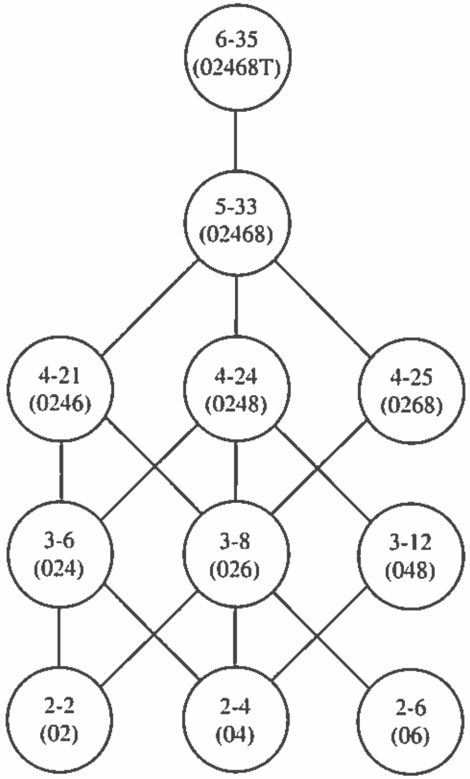
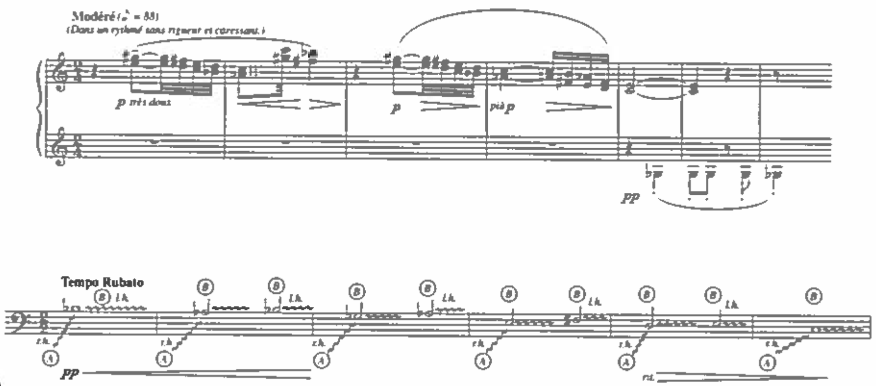


Figure 4–6

a.

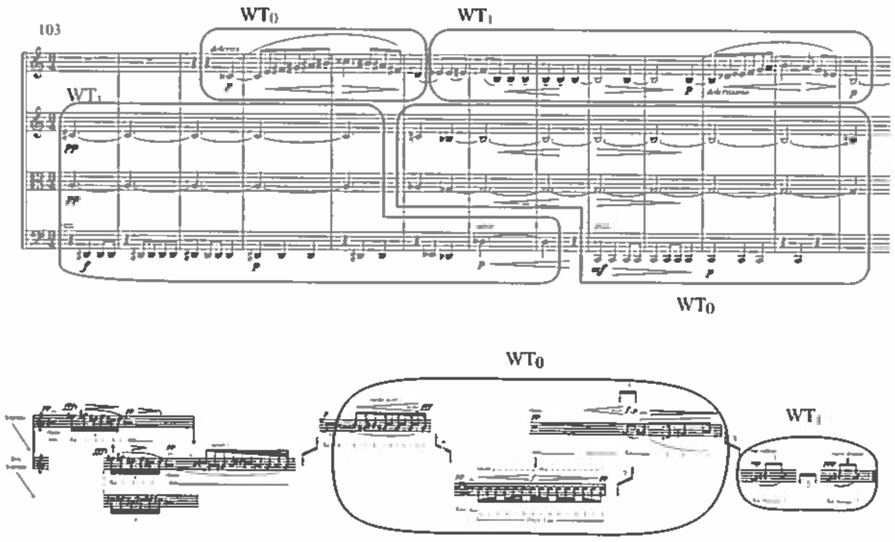
b.



Example 4–12 A whole-tone collection (Debussy, "Voiles"; Cowell, "The Banshee").

a.

b.



Example 4–13 Two whole-tone collections (Bartók, String Quartet No. 1, first movement; Crumb, *Ancient Voices of Children*).

The Hexatonic Collection

Like the octatonic and whole-tone collections, the hexatonic collection, set-class 6–20 (014589), occurs frequently in post-tonal music and has many striking properties. It is both transpositionally and inversionally symmetrical at three different levels. As a result, there are only four distinct members of this set class, identified by the numerically lowest pitch-class semitone that uniquely defines them (see Figure 4–7).

$HEX_{0,1}$ [0, 1, 4, 5, 8, 9]
$HEX_{1,2}$ [1, 2, 5, 6, 9, 10]
$HEX_{2,3}$ [2, 3, 6, 7, 10, 11]
$HEX_{3,4}$ [3, 4, 7, 8, 11, 0]

Figure 4–7

When written out as a scale, the hexatonic collection alternates 1s and 3s (compare the octatonic scales, which alternate 1s and 2s). The hexatonic collection can be thought of as a combination of two augmented triads related by semitone—each hexatonic collection contains two of the four augmented triads and excludes the other two (see Figure 4–8).

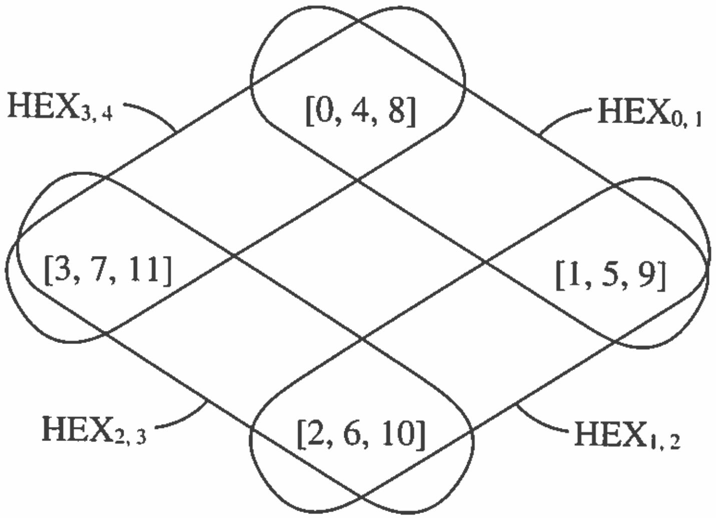


Figure 4–8

Because of its internal symmetries and redundancies, the hexatonic collection has a limited subset structure—see the inclusion lattice provided in the previous chapter as Figure 3–11. Among its subsets are some familiar formations, like the major seventh chord, the major or minor triad, and the augmented triad. As a result, it is possible to write music that is hexatonic, but nonetheless has a somewhat traditional feel.

Example 4–14 shows three hexatonic passages. Schoenberg's six-note chord represents HEX$_{2,3}$—its highest three and lowest three notes are both augmented triads. In the Bartók, the upper three parts arpeggiate an augmented triad, F–A–C♯, up into the stratosphere. Far below, the cello plays a melody that centers on an A-minor triad, A–C–E. Together, melody and accompaniment project HEX$_{0,1}$. In the Babbitt, first violin and viola play only HEX$_{0,1}$ while second violin and cello play only the complementary HEX$_{2,3}$. Both of these hexatonic collections are arranged to feature combinations of i3 and i4.

Collectional Interaction

The four collections discussed thus far (diatonic, octatonic, whole-tone, and hexatonic) often occur in productive interaction with each other. Music may shift from one to another and musical passages can be understood in terms of the interpenetration of one by another. The diatonic and octatonic collections make a particularly effective pair because, despite their obvious structural differences, they share many subsets. The octatonic collection is rich in triads—it contains four major triads and four minor triads. It also contains other diatonic harmonies, including the scale-

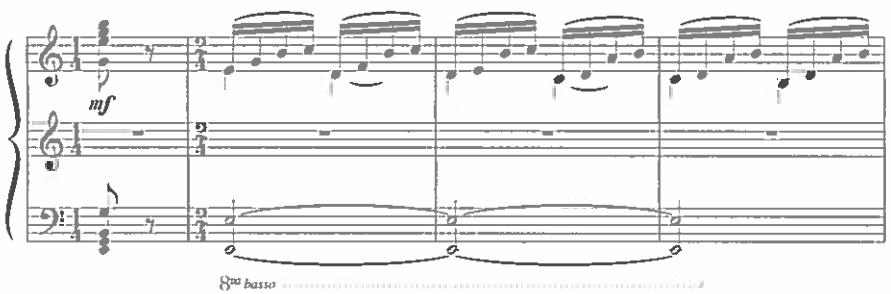
Example 4–14 Hexatonic collections (Schoenberg, *Little Piano Piece*, Op. 19, No. 2; Bartók, String Quartet No. 2, first movement; Babbitt, String Quartet No. 2).

segment 4-10 (0235) and minor, half-diminished, and dominant seventh chords. All of these harmonies can create points of intersection in music that uses both diatonic and octatonic collections.

The beginning of Stravinsky's *Symphony of Psalms* makes extensive use of $OCT_{1,2}$ beginning on E: E–F–G–A♭–B♭–B–C♯–D. Set class 4–3 (0134), featured in this ordering, was described by Stravinsky as the basic idea for the entire work. He referred to that set class as "two minor thirds joined by a major third." The famous opening chord, known as the "Psalms chord," is immediately followed by music drawn from the octatonic collection that contains it (see Example 4–15).

Example 4–15 The "Psalms chord" heard as a subset of an octatonic collection on E.

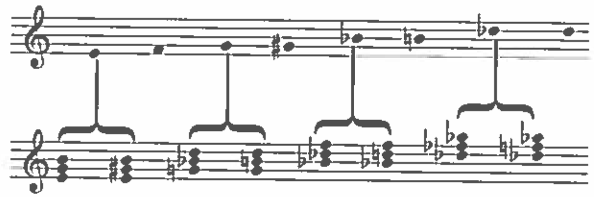
At rehearsal no. 2, the chord is stated again (for the fourth time). Now, however, the chord is followed by music drawn from a *diatonic* collection (E-Phrygian) that contains it (see Example 4–16).



Example 4–16 The "Psalms chord" heard as a subset of E-Phrygian.

The chord is an element common to $OCT_{1,2}$ on E and to E-Phrygian. It links the contrasting types of music in this movement. The large-scale harmonic organization of the movement involves contrasting octatonic and diatonic sections, with important interactions and links between the two.

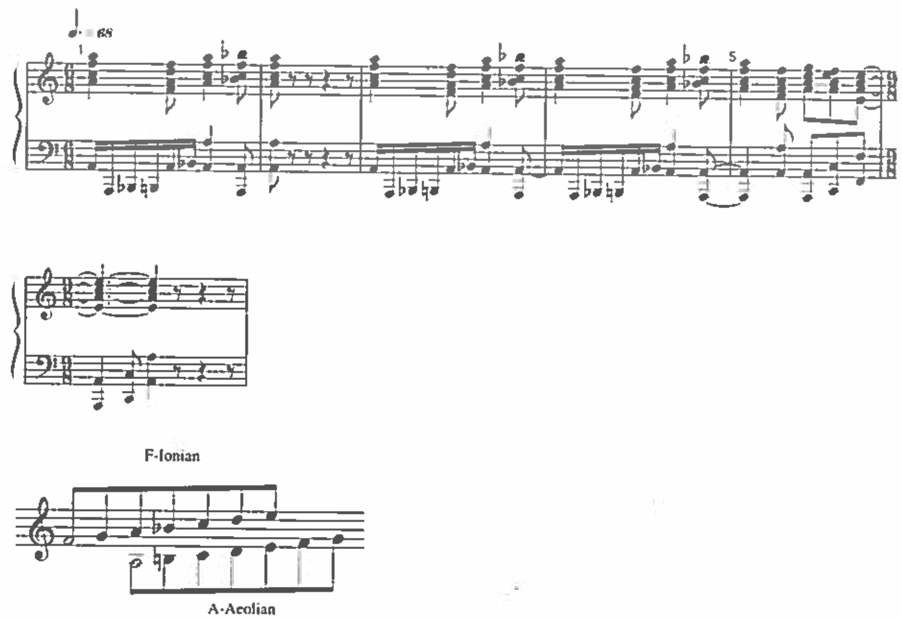
Because of the extreme symmetry of the octatonic collection, it often produces a centric conflict. Consider, for example, the position of triads within the collection. If it is ordered to begin with a semitone, major and minor triads can be constructed on the first, third, fifth, and seventh degrees of the scale (see Example 4–17).



Example 4–17 The triadic resources of the octatonic collection.

Because the triad can be used to reinforce pitch classes, this symmetrical disposition frequently results in a static polarity of competing centers. Sometimes tritone-related pitch classes are poised against one another, competing for priority. Sometimes, as in the first movement of the *Symphony of Psalms*, the competing centers are pitch-class interval 3 apart. In that movement, E and G compete for centric priority. Their competition can be heard even in the first chord. That chord has E in the bass, but G is the note that is most heavily doubled. A tension between E and G continues throughout the movement, with G winning out in the end. This kind of centric polarity is typical of octatonic music, and the polarity is reinforced here by the nature of the octatonic-diatonic interaction.

It also is possible for one octatonic collection to interact with another, or for one diatonic collection to interact with another. When diatonic collections interact, composers often use collections that share six common tones. In the passage in Example 4–18, Stravinsky combines A-Aeolian with F-Ionian. The F and A compete



Example 4–18 A combination of A-Aeolian and F-Ionian (Stravinsky, Serenade in A).

as centers, as do the F-major and A-minor triads. Look at the sonority formed on the downbeat of each of the first five measures. It has A in the outer voices, but F is always present in an inner voice. The triad is F major, but the A is the most prominent tone. On the last beat of measure 5, the right hand arrives on an A-minor triad, but beneath it the bass insists on F. That sonority combines triads on A and F just as the passage as a whole combines diatonic collections on A and F.

Large collections may interact and interpenetrate over the course of a passage or a piece. In analyzing post-tonal music, one must be sensitive not only to the motivic interplay of the surface, but to the larger referential collections that lurk beneath the surface.

Interval Cycles

We can gain a useful perspective on the diatonic, octatonic, whole-tone, and hexatonic collections, and on other important collections, by concentrating on the intervals that can generate them. Figure 4–9 shows what happens if we start on any pitch class and move repeatedly by any interval, thus creating an *interval cycle.*

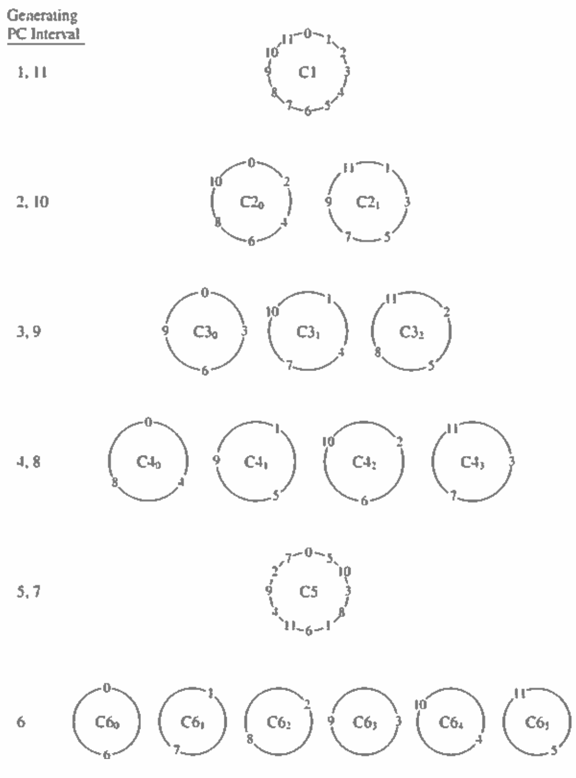


Figure 4–9

If we move by i1 or i11, we get the cycle of semitones, or C1. Moving around the cycle clockwise involves motion by i1; moving counterclockwise involves motion by i11. Either way, the cycle takes us through all twelve pitch classes before returning to its starting point. As a result, there is only one C1-cycle. There are two C2-cycles, however, one moving through the even pitch classes and one through the odd pitch classes. There are three C3-cycles, corresponding to the three diminished-seventh chords, and four C4-cycles, corresponding to the four augmented triads. The C5-cycle is the familiar "circle of fourths or fifths." As with C1, there is only one C5-cycle, because C5 takes us through all twelve tones before returning to its starting point. There are six C6-cycles, each corresponding to one of the six tritones.

Many important collections have their origins in the interval cycles. The chromatic scale corresponds to C1. The two whole-tone collections correspond to the two C2-cycles. The octatonic collection results from a combination of any two of the three C3-cycles. The hexatonic collection results from combining either of the even C4-cycles ($C4_0$ or $C4_2$) with either of the odd C4-cycles ($C4_1$ or $C4_3$). (Combining two even or two odd C4-cycles produces a whole-tone collection.) The diatonic collection corresponds to any contiguous seven-note segment of C5 (and the pentatonic collection corresponds to any five-note segment of C5). The C6-cycles can be combined in various ways to produce whole-tone collections or octatonic collections. From this cyclic point of view, we can imagine the four referential collections studied in this chapter as creating a distinctive sound world from generation by a single interval: the whole-tone from i2, the octatonic from i3, the hexatonic from i4, and the diatonic from i5.

A *cyclic set* is one that consists of an entire cycle or a segment of a cycle. A diatonic collection is a cyclic set because it consists of a seven-note segment of C5. Other, smaller cyclic sets also play a prominent role in post-tonal music. Bartók's String Quartet No. 3, for example, begins with a four-note segment of C1 (C♯–D–D♯–E) and ends with a four-note segment of C5 (C♯–G♯–D♯–A♯) (see Example 4–19). That both sets are in a sense generated from C♯ is no coincidence—Bartók himself thought of this quartet as "in C♯."

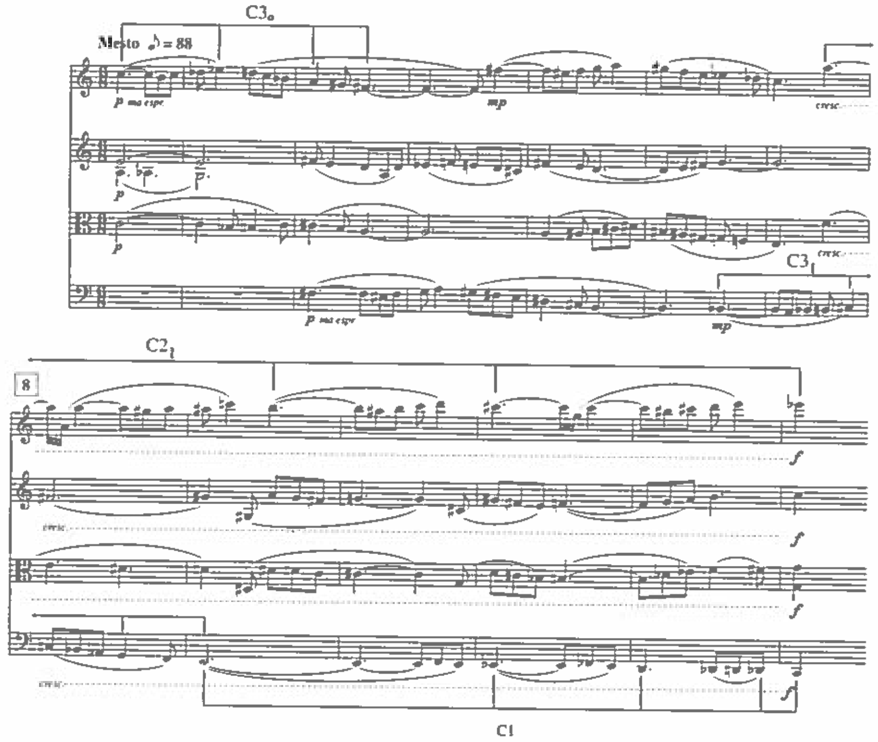
In the phrase from an Ives song shown in Example 4–20, the melody consists of five-note segments of the two C2-cycles (accidentals affect only the notes they immediately precede). The accompanying chords, in contrast, are generally five-note segments of C5, and they are arranged registrally to reveal their cyclic origins.

The interval cycles can also be used to guide large-scale motions in post-tonal music. In the fourth movement of Bartók's String Quartet No. 6, the music is headed for a cadence on a C-minor triad in measure 13 (see Example 4–21). That cadential goal is approached in the cello by a descending 1-cycle (E–E♭–D–D♭–C) and in the first violin by an ascending 2-cycle (A–B–C♯–E♭). The 3-cycle, although not part of this cadential progression, shapes the main theme of the movement (and of the quartet as a whole).

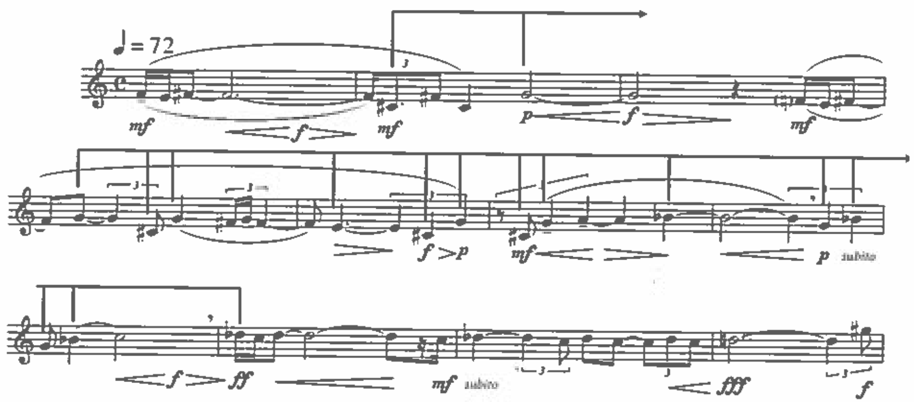
The 3-cycle also shapes the opening section of Varèse's *Density 21.5* for solo flute (Example 4–22). For the first ten measures, the principal melodic boundary tones are those of $C3_1$, C♯–E–G–B♭, and the melody gradually rises an octave from C♯4 to D♭5. The arrival on D5 in measure 11, marked *fff,* signals a shift of harmonic orientation.

Example 4–19 Cyclic sets (Bartók, String Quartet No. 3, first six and last four measures).

Example 4–20 Cyclic sets (Ives, "The Cage," from *114 Songs*).



Example 4–21 Interval cycles (Bartók, String Quartet No. 6, fourth movement).

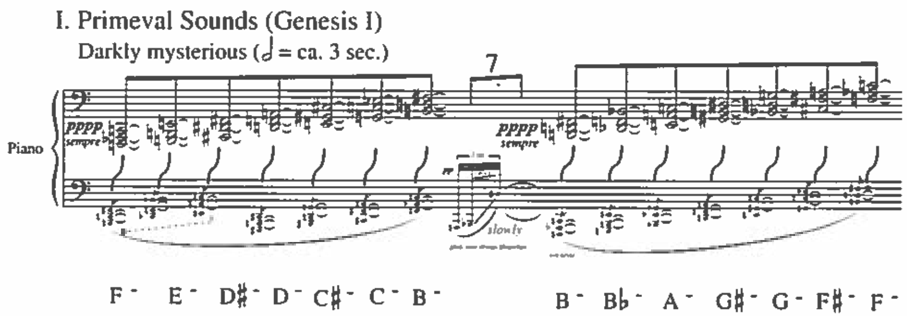


Example 4–22 An interval cycle (Varèse, *Density 21.5*).

Triadic Post-Tonality

Familiar major and minor triads are basic harmonies in several different post-tonal styles of composition, including neoclassicism, neotonality, and minimalism. In many cases, we find extended progressions of triads that are not constrained by the norms of traditional tonality. In particular, the triads do not relate to each other functionally, as predominants, dominants, or tonics. Such music is triadic, but still distinctively post-tonal.

There are two principal types of triadic progression in post-tonal music. The first is *motivic:* the triads are projected along certain well-defined pathways. One such motivic pathway involves the interval cycles. In Example 4–23, the opening of Crumb's *Makrokosmos,* all twelve minor triads are arranged to follow the 1-cycle.



Example 4–23 Minor triads projected through complete 1-cycle (Crumb, *Makrokosmos*).

The roots of the triads in the right-hand part descend by semitone through an entire octave (the roots are given in capital letters with a + or – symbol to indicate triad quality). At the same time, the actual notes gradually ascend from the depths, with each note moving upward to the nearest available position in the next chord. Each right-hand triad is preceded by a grace-note triad a tritone lower in the left hand. These left-hand triads also trace a 1-cycle, in parallel tritones with the right hand.

A different kind of motivic patterning is evident in Example 4–24, the end of an orchestral song by Britten. Here, the triads are all major, and they are arranged symmetrically around D-major. $C\sharp^+$ and $E\flat^+$ surround D^+ symmetrically and always wedge toward it. The C^+ at the cadence balances the E^+ in measure 30. The key signature and the final chord both suggest that D^+ is the tonic, but its tonicity is created by inversional symmetry, not by traditional harmonic functions.

A third kind of motivic pathway is illustrated in Example 4–25, from a string quartet by Shostakovich. At the end of the passage, we hear a progression of four minor triads: G^-, $A\flat^-$, $B\flat^-$, and D^-. The triads are preceded by a twelve-note melody in the cello. The final notes of that melody (D, E♭, and F) come back as the highest notes of the triads. In that sense, the triads simply harmonize, in a parallel and nonfunctional way, a previous melody. There is an additional, more subtle motivic link

Example 4–24 Symmetrical arrangement of major triads around D (Britten, Serenade for Tenor, Horn, and Strings, Op. 31, "Sonnet").

between the melody and the triads. A member of sc(0137) is embedded in the melody, and the succession of triadic roots describes another member of the same set class. So both the soprano melody and the root progression follow a motivic path.

The second principal kind of harmonic succession in post-tonal music involves *triadic transformations* that connect triads of different quality (major goes to minor and vice versa). Triadic transformations are defined by two qualities: *voice-leading parsimony* and *contextual inversion.* Voice-leading parsimony means that the triads are connected in the smoothest possible way, with the voices moving as little as possible. The most parsimonious voice leading involves two voices motionless (there are two common tones) and the voice that does move does so by only one semitone. Slightly less parsimonious voice leading might involve two voices motionless and one moving by two semitones, or one voice motionless and two voices moving by one semitone each. Contextual inversion means that to get from one triad to the next, you invert around one or two of the notes in the first triad—remember that a major triad and a minor triad can always be understood as related by pitch-class inversion.

Figure 4–10 illustrates four triadic transformations. The first is called P, and it relates a major and minor triad that contain the same perfect fifth and share the same root, like C^+ and C^-. When C-major moves to C-minor (or vice versa), E moves to E♭ (or vice versa)—a single voice moving by semitone. These two triads are related by the inversion that maps C and G onto each other. The second transformation is called L,

Example 4–25 Motivic progression of triads (Shostakovich, String Quartet No. 12, second movement).

and it relates a major and minor triad that contain the same minor third—the third of the major triad becomes the root of the minor triad (and vice versa). In moving between C⁺ and E⁻, for example, you invert around their shared minor third, and that produces parsimonious voice leading, with C moving a semitone to B. The third transformation is called R, and it relates a major and minor triad that contain the

Triadic Transformations

Name	Description	Contextual Inversion	Parsimonious Voice Leading	Example
P (Parallel)	Major and minor triad share the same root	Invert around the shared perfect fifth	One voice moves by 1 semitone	P: C⁺ ↔ C⁻, I^{C}_{G}
L (Leading-tone)	The third of a major triad becomes the root of a minor triad	Invert around the shared minor third	One voice moves by 1 semitone	L: C⁺ → E⁻, I^{E}_{G}
R (Relative)	The root of a major triad becomes the third of a minor triad.	Invert around the shared major third	One voice moves by 2 semitones	R: C⁺ → A⁻, I^{C}_{E}
SLIDE	Major and minor triad share the same root	Invert around the note that is the third of both triads	Two voices move by 1 semitone	SLIDE: C⁺ → C♯⁻, I^{E}_{E}

Figure 4–10

same major third. Inverting around that shared major third will cause one voice to move by two semitones. Moving from C⁺ to A⁻, for example, involves the inversion that exchanges C and E and moves G to A.

These three transformations are widely used in post-tonal music (and in late-nineteenth-century chromatic music as well) Their theoretical description originates with the theorist Hugo Riemann and has been elaborated in contemporary neo-Riemannian theory. To these, we add one additional transformation that Lewin calls SLIDE. This transformation relates a major and minor triad that share the same third, like C⁺ and C♯⁻. This transformation involves contextual inversion around a single note (rather than a pair of notes) and it causes two voices to move, although they each move only by semitone.

These transformations can occur independently, or they can be combined into larger progressions. Figure 4–11 illustrates one important abstract possibility. If we start with C⁺ and move by P and L in alternation, we create a cycle of six triads: major and minor triads built on C, E, and A♭. Each move around the cycle involves changing a single note by one semitone, so the voice leading as we move around the cycle is as parsimonious as possible. Each triad differs from the adjacent triads by only a single semitone. The cycle as a whole involves only six different pitch classes, and these correspond to one of the hexatonic collections, $HEX_{3,4}$. The triads that lie opposite each other on the cycle (like C⁺ and G♯⁻) are known as *hexatonic poles*—they have no notes in common and together they exhaust the hexatonic collection. The four *hexatonic systems*, or *LP-cycles*, are illustrated in Figure 4–11.

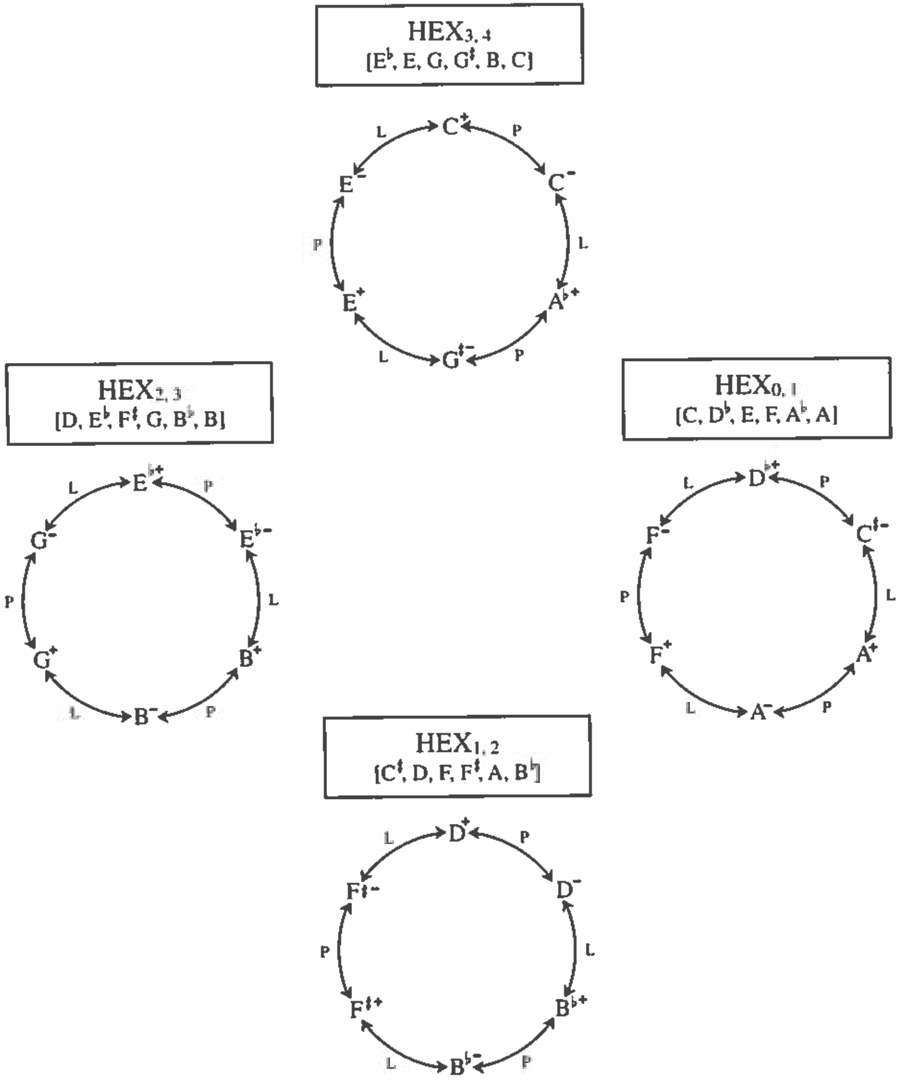


Figure 4–11

A different cycle, a *PLR-cycle,* is illustrated in Figure 4–12. This cycle consists of the six triads (three major and three minor) that all share a single pitch class, in this case pitch-class C. Each triad in the cycle differs from its neighbors by only one or two semitones. Two of the triads lying opposite each other are related by SLIDE.

John Adams, whose music is often triadic, uses SLIDE to connect two triads in *Harmonium* (see Example 4–26). E♭-major and E-minor share G as their third, and

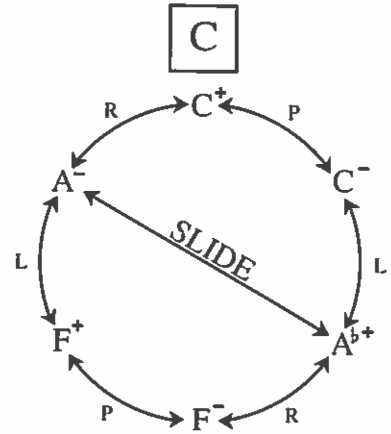


Figure 4–12

that shared note, along with the smooth motions in the other voices, binds together these tonally distant chords.

In his opera *Nixon in China,* Adams uses L to connect triads (see Example 4–27). All four chords in the two L-progressions are drawn from the same hexatonic system, $HEX_{2,3}$.

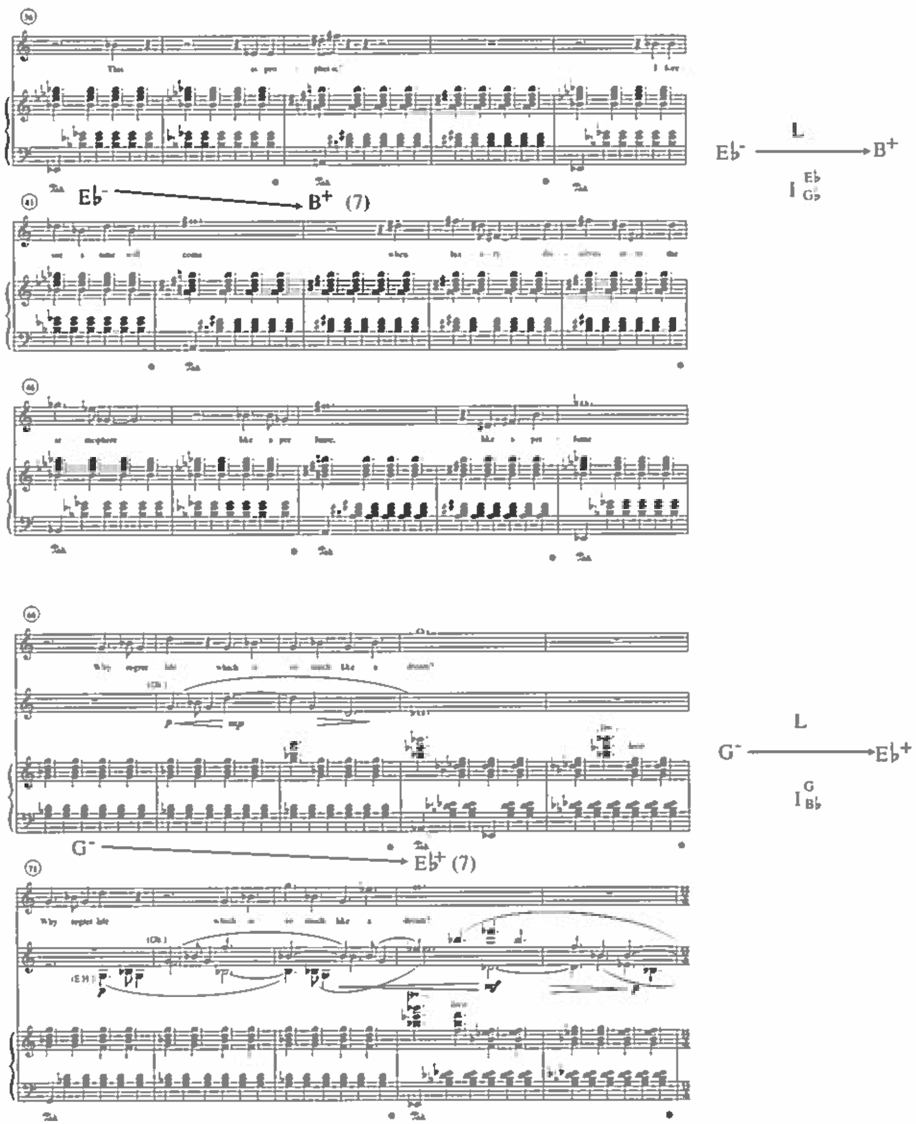
In another minimalist opera, Glass's *Einstein on the Beach,* triads are often related by contextual inversion and parsimonious voice leading. In the progression shown in Example 4–28, one that occurs with some frequency in the opera, the first three chords belong to the same hexatonic system—the first two are related by L and the second and third by a combination of P-followed-by-L. The first and third chords are hexatonic poles. From the A-major chord, the progression concludes in tonally normal fashion: IV–V–I. But even here there is a sense of inversional symmetry—just as the A^+ is approached by triads whose roots are four semitones above and below it, the cadential E^+ is approached by triads whose roots are five semitones above and below it.

Shostakovich's music could hardly be described as minimalist in any sense, but it also uses progressions of triads shaped by contextual inversion and parsimonious voice leading. In the passage in Example 4–29, A-major is juxtaposed with F-minor. These harmonies are hexatonic poles: they lie opposite each other in a hexatonic system and thus together exhaust the full six-note collection. The movement from which the passage is taken ends just a few measures later on F-major, another triad from the same hexatonic system.

Post-tonal progressions of triads can be lengthy, as in the opening measures of Zwilich's *Chamber Concerto* (see Example 4–30). There are eight overlapping triads here, alternating major and minor. The initial alternation of P and SLIDE results in a chain of triad roots descending by semitone: B♭–A–A♭. When $A♭^+$ has been reached, the progression moves around one of the hexatonic systems until it reaches the hexatonic pole, E^-.

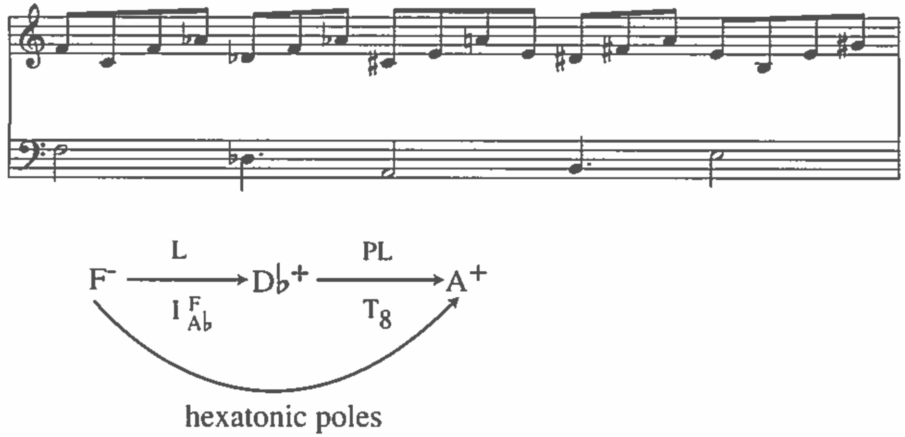
Example 4–26 Progression of triads via SLIDE (Adams, *Harmonium*, Part I).

There is a significant group of works, by Stravinsky and others, that create some sort of centric ambiguity between triads that are related by P, L, or R. In the passage from Stravinsky's *Symphony in C* shown in Example 4–31, two L-related triads (C⁺ and E⁻) compete for priority. The accompaniment consists only of repeated Es and Gs, notes that are common to both triads. That accompaniment could support either the melodic B (producing E⁻) or C (producing C⁺), but the contour and rhythm

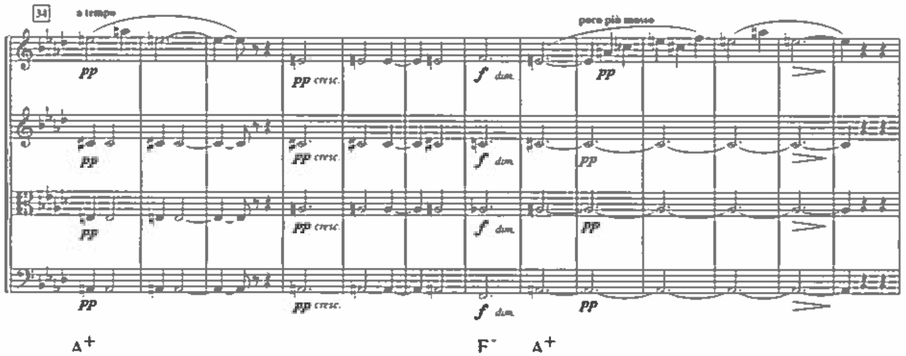


Example 4–27 Two progressions with a hexatonic system (Adams, *Nixon in China*, Act II).

of the melody make it hard to tell which is structural and which embellishing. A tension between C-major and E-minor triads is crucial to this passage and to the entire movement. There is no progression between the triads; rather, they are poised against each other in a state of unresolved tension.



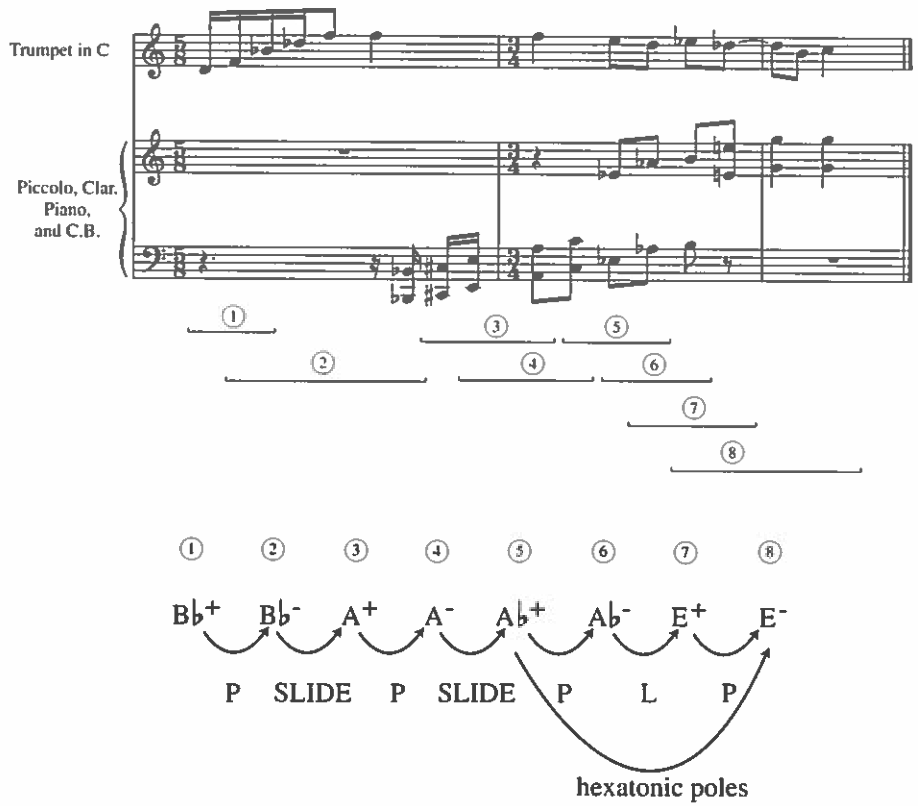
Example 4–28 Progression with a hexatonic system (Glass, *Einstein on the Beach*).



Example 4–29 Hexatonic poles (Shostakovich, String Quartet No. 4, second movement).

BIBLIOGRAPHY

Attempts to analyze post-tonal music in terms of tonal theoretical categories like key and functional harmony have produced predictably uneven results. Symptomatic of the problems inherent in such an enterprise are three published analyses of Schoenberg's Piano Piece, Op. 11, No. 1, in three different keys: Will Ogdon, "How Tonality Functions in Schoenberg's Opus 11, No. 1," *Journal of the Arnold Schoenberg Institute* 5 (1982), pp. 169–81, analyzes it in G; Reinhold Brinkmann (*Arnold Schönberg: Drei Klavierstücke Op. 11: Studien zur frühen Atonalität bei Schönberg* (Wiesbaden: Franz Steiner Verlag, 1969) analyzes it in E; William Benjamin (*Harmony in Radical European Music, 1905–20*, paper presented to the Society of Music Theory, 1984) analyzes it as a prolongation of F♯ as the dominant of B. Similar fundamental disagreements have bedeviled tonally oriented analyses of Schoenberg's Little Piano Piece,



Example 4–30 Progression of triads (Zwilich, *Chamber Concerto for Trumpet and Five Players*).

Op. 19, No. 2: Hugo Leichtentritt (*Musical Form* (Cambridge, Mass.: Harvard University Press, 1951)) analyzes it in B; Hermann Erpf (*Studien zur Harmonie- und Klangtechnik der neueren Musik* (Leipzig, 1927; reprinted 1969) analyzes it in C.

Analyses rooted in the tonal theories of Heinrich Schenker have met with varying degrees of success. See Felix Salzer, *Structural Hearing: Tonal Coherence in Music* (New York: Dover, 1962); Roy Travis, "Toward a New Concept of Tonality?" *Journal of Music Theory* 3 (1959), pp. 257–84; Roy Travis, "Directed Motion in Schoenberg and Webern," *Perspectives of New Music* 4 (1966), pp. 84–88; Robert Morgan, "Dissonant Prolongations: Theoretical and Compositional Precedents," *Journal of Music Theory* 20 (1976), pp. 49–91; Paul Wilson, "Concepts of Prolongation and Bartók's Opus 20," *Music Theory Spectrum* 6 (1984), pp. 79–89; Allen Forte, "Tonality, Symbol, and Structural Levels in Berg's *Wozzeck*," *Musical Quarterly* 71 (1985), pp. 474–99; James Baker, "Voice-Leading in Post-Tonal Music: Suggestions for Extending Schenker's Theory," *Music Analysis* 9/2 (1990), pp. 177–200; James Baker, "Post-Tonal

Example 4–31 Combination of L-related triads (Stravinsky, *Symphony in C*, first movement).

Voice-Leading," in *Models of Musical Analysis: Early Twentieth Century Music*, ed. Jonathan Dunsby (Oxford: Basil Blackwell, 1993), pp. 20–41; Steve Larson, "A Tonal Model of an 'Atonal' Piece: Schoenberg's Opus 15, Number 2," *Perspectives of New Music* 25/1–2 (1987), pp. 418–33; Charles Morrison, "Prolongation in the Final Movement of Bartók's String Quartet No. 4," *Music Theory Spectrum* 13/2 (1991), pp. 179–96; David Neumeyer and Susan Tepping, *A Guide to Schenkerian Analysis* (Englewood Cliffs, NJ: Prentice Hall, 1992), pp. 117–24; Edward Pearsall, "Harmonic Progressions and Prolongation in Post-Tonal Music," *Music Analysis* 10/3 (1991), pp. 345–56; and Olli Väisälä, "Concepts of Harmony and Prolongation in Schoenberg's Op. 19/2," *Music Theory Spectrum* 21/2 (1999), pp. 230–59. The prolongational approach is critiqued in James Baker, "Schenkerian Analysis and Post-Tonal Music," in *Aspects of Schenkerian Theory*, ed. David Beach (New Haven: Yale University Press, 1983) and Joseph N. Straus, "The Problem of Prolongation in Post-Tonal Music," *Journal of Music Theory* 31/1 (1987), pp. 1–22. From a non-Schenkerian point of view, Fred Lerdahl has developed and widely applied a theory of prolongation for atonal music: *Tonal Pitch Space* (Oxford: Oxford University Press, 2001).

The centricity induced by inversional balance is a central theme of George Perle: see *Twelve-Tone Tonality* (Berkeley: University of California Press, 1977) and *The Listening Composer* (Berkeley: University of California Press, 1990). See also Elliott Antokoletz, *The Music of Bela Bartók: A Study of Tonality and Progression in Twentieth-Century Music* (Berkeley: University of California Press, 1984) and Philip Lambert, "On Contextual Transformations," *Perspectives of New Music* 38/1 (2000), pp. 45–76.

There are several prominent referential collections that are not discussed in this chapter, including the nondiatonic minor scales (melodic and harmonic minor)—see Dmitri Tymoczko, "Stravinsky and the Octatonic: A Reconsideration," *Music Theory Spectrum* 24/1 (2002), pp. 68–102.

Most discussions of the octatonic collection in Stravinsky's music, and its interaction with the diatonic collection, have taken as their point of departure Arthur Berger's seminal article "Problems of Pitch Organization in Stravinsky," *Perspectives of New Music* 2/1 (1963), pp. 11–42. The definitive treatment of this subject is Pieter van den Toorn, *The Music of Stravinsky* (New Haven: Yale University Press, 1983). See also Richard Taruskin, *Stravinsky and the Russian Traditions: A Biography of the Works Through Mavra* (Berkeley: University of California Press, 1996). For a discussion of Bartók's octatonicism, see Elliott Antokoletz, *The Music of Béla Bartók* and Richard Cohn, "Bartók's Octatonic Strategies: A Motivic Approach," *Journal of the American Musicological Society* 44/2 (1991), pp. 262–300. The octatonic collection is one of Olivier Messiaen's "modes of limited transposition." See *The Technique of My Musical Language*, trans. J. Satterfield (Paris: Alphonse Leduc, 1956), pp. 58–63.

George Perle has written extensively about interval cycles. See his "Berg's Master Array of the Interval Cycles," *Musical Quarterly* 63 (1977), pp. 1–30 and *The Operas of Alban Berg, Volume Two: Lulu* (Berkeley: University of California Press, 1985). See also Elliott Antokoletz, "Interval Cycles in Stravinsky's Early Ballets," *Journal of the American Musicological Society* 34 (1986), pp. 578–614; Dave Headlam, *The Music of*

Alban Berg (New Haven: Yale University Press, 1997); J. Philip Lambert, "Interval Cycles as Compositional Resources in the Music of Charles Ives," *Music Theory Spectrum* 12/1 (1990), pp. 43–82; and Richard Cohn, "Properties and Generability of Transpositionally Invariant Sets," *Journal of Music Theory* 35/1–2 (1991), pp. 1–32.

A concern with parsimonious voice leading and contextual inversion underpins the recent outpouring of neo-Riemannian theory. Among the foundational publications in this area are four articles by Richard Cohn: "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," *Music Analysis* 15/1 (1996), pp. 9–40; "Neo-Riemannian Operations, Parsimonious Trichords, and Their Tonnetz Representations," *Journal of Music Theory* 41/1 (1997), pp. 1–66; "Square Dances with Cubes," *Journal of Music Theory* 42/2 (1998), pp. 283–96; and "Introduction to Neo-Riemannian Theory: A Survey and Historical Perspective," *Journal of Music Theory* 42/2 (1998), pp. 167–80. The last of these introduces a special issue of the *Journal of Music Theory* dedicated to neo-Riemannian theory.

Exercises

THEORY

I. Inversional axis: Inversionally symmetrical sets map onto themselves under T_nI. The axis of symmetry for such a set is n/2 – n/2 + 6.

1. For each of the following sets, determine if they are inversionally symmetrical. If they are, find the axis (or axes) of symmetry:
   a. [1,4,5,8]
   b. [10,0,1,2,4]
   c. [1,2,3,4,8,9]
   d. [9,10,11,3,5]
   e. [4,6,11]
   f. [1,2,5,6,9,10]

2. Construct at least two pitch-class sets that are symmetrical around the following axis (or axes). Give your answer in normal form.
   a. 4–10
   b. 2/3–8/9
   c. 1–7
   d. 1–7 and 4–10

II. Referential collections:

1. For each of the large collections discussed in this chapter (the diatonic, octatonic, whole-tone, and hexatonic collections), do the following:
   a. Compare their interval vectors.

   b. Compare them with regard to transpositional and inversional symmetry.
   c. Compare them with regard to the trichords they contain. To do this, you will have to extract all of the trichordal subsets and identify the set classes to which they belong.

2. Write out the following scales.
   a. D-Mixolydian
   b. E♭-Phrygian
   c. G♯-Locrian
   d. $OCT_{0,1}$ beginning on G
   e. $OCT_{1,2}$ beginning on G
   f. WT_1 beginning on B
   g. $HEX_{1,2}$ beginning on A

3. Identify each of the following collections, using nomenclature presented in this chapter:

III. Triadic transformations: Major and minor triads can be connected with a variety of transformations that conjoin contextual inversion with voice-leading parsimony.

1. Apply triadic transformations as indicated:
   a. P (A⁺)
   b. R (C♯⁻)
   c. L (F♯⁻)