Artistic Forms and Complexity

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We discuss the inter-relationship between various concepts of complexity by introducing a complexity triangle featuring objective complexity, subjective complexity and social complexity. Their connections are explored using visual and musical compositions of art. As examples, we quantify the complexity embedded within the paintings of the Jackson Pollock and the musical works of Johann Sebastian Bach. We discuss the challenges inherent in comparisons of the spatial patterns created by Pollock and the sonic patterns created by Bach, including the differing roles that time plays in these investigations. Our results draw attention to some common intriguing characteristics suggesting universality and conjecturing that the fractional nature of art might have an intrinsic value of more general significance.

I. HOW COMPLEXITY ENTERS INTO THE STUDY OF ART FORMS

Every artistic form is a system in the sense that it is an ensemble of inter-related parts forming a whole that is greater than the sum of those parts, a property referred to as emergence. The paint marks on a canvas, the notes of a symphony, and the words in a novel make the point clear. Artistic productions are characterized by system-theoretic concepts like entropy, self-organization, adaptation, emergent properties and, most importantly, complexity. When regarding an artistic form, the challenge to the complexity scientist is to address two basic questions: How do we define and characterize the system-theoretic fingerprints of an artistic form and how can we actually measure them? Can we use these characterizations to identify homological equivalences between artistic structures that on the surface appear different?

A central focus discussed here is the concept of the complexity triangle, composed of three vertices:
- Objective complexity: the intrinsic complexity of an artistic form; this is the complexity of a symphony, sculpture, novel or painting independent of an external observer but not independent of the structures creator.
- Subjective complexity: the complexity of the form as perceived by an external observer.
- Social complexity: the complexity of an artistic form as agreed upon by social consensus, such as the art community or society as a whole.

In order to obtain quantitative insight it is important (a) to devise meaningful ways to characterize and measure each of these forms of complexity, and (b) to investigate relationships these different forms of complexity have to each other, both within a given type of artistic structure (paintings, music, literature, for instance) and across different types. Here we explore the above concepts by comparing investigations in visual art and music.

We consider two artists whose work has been the focus of previous complexity analysis - the American painter Jackson Pollock (1912-1952) [Taylor et al, 1999] and the German composer Johann Sebastian Bach (1685-1750) [Boon & Decroly, 1995]. We emphasize that this choice of artists is not based on any specific expectation of shared artistic content in their works the two artists emerged from different artistic eras and environments, and certainly had different artistic goals. Rather, by placing these two studies of complexity back to back in the same perspective, our aim is to highlight generic similarities and differences that emerge when applying complexity analysis to visual and sonic compositions. The objective is that the questions raised by this essay will provide ample fuel for future research directions involving a range of artistic disciplines and artists.

The concept of time deserves special consideration in our attempts at paralleling the complexity of visual art and music. First consider the role of time in the artistic creation of the complexity of the work. The formulation of the concept of complexity, and its dependence on time, can be traced back to the late 19th century when the Austrian

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physicist Ludwig Boltzmann studied the evolution in time of physical systems. Boltzmann's theory of entropy provided a framework for the irreversible evolution of time towards equilibrium, where systems exhibit maximum complexity in the sense that at equilibrium a system has maximal occupation of the configuration space.

In this picture, time serves as an evolutionary factor the arrow of time by which natural phenomena can organize systems, where the organization materializes as the emergence of complexity. The concept of emergence was at the core of new insights highlighted in the 1972 article *More is different* [Anderson, 1972] which emphasized that an ensemble of simple elements is more than merely their sum, and that the whole can exhibit remarkable properties that cannot be predicted from the knowledge of the systems component elements [Anderson, 1972]. Complexity appears then as a characteristic of the ensemble of these elements.

By analogy, complexity in artistic pieces could be viewed as the materialization of the ensemble of elements entering artistic creation. Complexity might then be expected to materialize in unique ways for specific artists. For both Pollock and Bach creations we will therefore consider whether the degree of complexity of the finished artworks can be quantified and whether the quantification may inform us as how to characterize them and to distinguish their art from that of others working in the same genres.

How does this tell-tail complexity emerge during the time period of the creative process? Pollock was filmed during the creation of some of his best-known works and in this article we present an analysis of the evolution of the complexity of his painted patterns as the painting grew from its initial sparse paint marks to the completed piece. How would we attempt an analogous analysis of the emergence of complexity during Bachs creative process? This question highlights the difference that time plays in their finished works and in an audiences reaction to those works.

An audience experiences the complexity of Pollocks work through the patterns mapped out across the spatial dimensions of his canvases. For the completed artwork, these patterns are static in the sense that they dont evolve with time. In contrast, the audiences experience of a musical composition is not static. When listening to one of Bachs pieces, the audience will experience the works complexity through the evolution of the acoustic signal as a function of time.

Whereas this difference might, at first, appear to require different approaches, the universal nature of the technique that we employ to measure the objective complexity of the visual art and music allows for a quantifiable common evaluation. Building on previous successful applications of scaling analysis to spatial and temporal patterns in Nature [Mandelbrot, 1977], we adopt an analogous scaling analysis for assessing the complexity of Pollock [Taylor, 2010] and Bachs works [Boon & Decroly, 1995]. Scaling analysis quantifies the relationship between patterns at different scales and how they interact to build the resulting complexity of the whole pattern [Mandelbrot, 1977]. The details of this analysis will be covered in this article in two sections the first on Pollock, the second on Bach.

Whereas Pollocks patterns are assessed across a range of size scales, Bachs are assessed across time scales. Nevertheless, if the scaling parameters for the two forms of work turn out to be matched, this would indicate an identical level of objective complexity, even though one work traces out its complexity as a function of time and the other as a function of space. We will show that the spatial patterns created by Pollock at his creative peak exhibit a medium degree of objective complexity - a halfway state between order and disorder, between predictability and unpredictability. Intriguingly, we will show that this mid-complexity also dominates the temporal patterns within Bachs music.

This mathematical equivalence of objective complexity across the two forms of work doesnt take into account the impact of the complexity on the audience. This is the role of the remaining two vertices of the complexity triangle the subjective and social complexities. Visual and sonic complexities are processed by different physiological systems within the observer. Therefore, sonic and visual works with matched objective complexities might be expected to induce strikingly different emotive and perceptual responses, generating different subjective and social complexities for musical and visual art forms. This expectation is reduced if we take into account the psychological condition Synesthesia in which observers can translate experiences across senses, including vision and hearing [Campen, 1999]. These issues will be discussed in the Conclusion section.

The Conclusion section also highlights an intriguing direction for future research aimed at potentially narrowing this crucial difference that time plays for Pollocks paintings and Bachs music. Although Pollocks patterns are static, the way that the observer is exposed to them might not be. Pollocks canvases are sufficiently large that the observers eye has to wander across the vast canvas, gradually absorbing the complexity with time. This temporal experience of Pollocks complexity might therefore have more in common with hearing Bachs music than might initially be expected for visual and sonic art forms.

II. JACKSON POLLOCKS VISUAL COMPLEXITY

Consider the three examples of abstract computer-generated artwork (A, B and C), generated using the LISP programming language, shown in Figure 1, and their objective complexity: the intrinsic complexity of the three artworks. A common measure of objective complexity of an image is based on the length of the shortest computer

FIG. 1: Three computer-generated abstract art works (A, B, C from top to bottom).

program required to generate the image. It is impossible to strictly prove that a particular program is the shortest necessary to generate a particular image. Nevertheless, its not unreasonable to assume that the programs used are the shortest, or at least is very close to the shortest, since its difficult to see how to compress them and still have them generate the same pieces. By counting the number of characters in the programs (including spaces), we find what we might term the objective complexity of the artworks to be 186 (A), 264 (B) and 434 (C).

In order to compare the objective complexity measured in this way with the subjective and social complexities discussed above, we performed a perception experiment in which 24 participants were asked to rank the three images in terms of (1) their perceived complexity (subjective complexity), and (2) aesthetic preference (social complexity). The results of are shown in Table 1 where Complexity and Aesthetics show the average rating on a scale of 1 (low) to 3 (high).

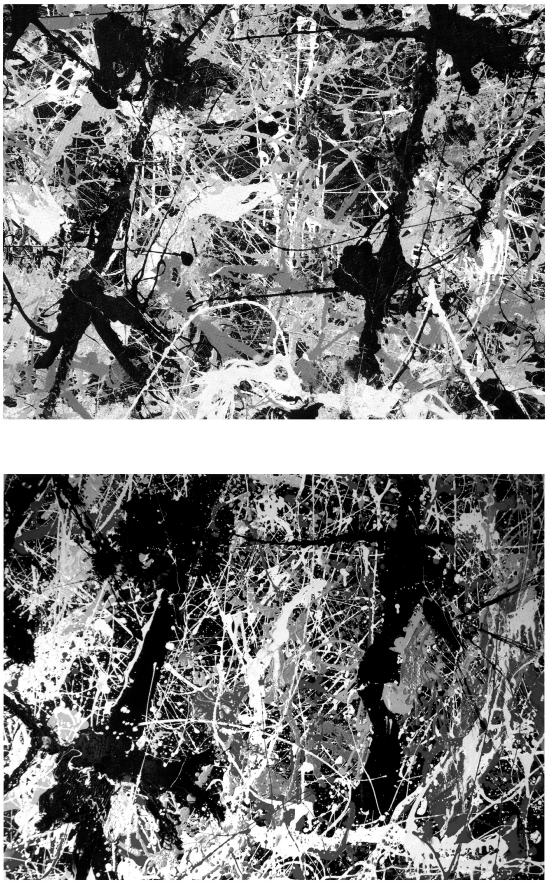

FIG. 2: Close-up sections of Blue Poles: Number 11, 1952 (The National Gallery of Australia) (top) and a replica (bottom).

LISP length : A 186 ; B 264 ; C 434
Complexity : A 1.75 ; B 2.08 ; C 2.17
Aesthetics : A 2.00 ; B 1.54 ; C 2.21

Table 1: The results of the experiment used to compare the complexity triangle.

While a sample size of 24 participants is highly limited, the above results provide an initial indication of a potential correlation between the objective complexity (LISP length) and subjective complexity. However, the correlation is weak and furthermore fails to reveal any additional correlation with social complexity (aesthetics). A likely reason for this lack of correlation is that visual variations unrelated to complexity dominate the aesthetic judgment of the art works. The results of Table 1 therefore emphasize the need to focus on art works for which the visual impact is dominated by the complexity of the artistic patterns. Figure 2 shows a section of Jackson Pollocks Blue Poles: Number 11, 1952 and a known replica of his work. In contrast to the broken lines painted by conventional brush contact with the canvas surface, Pollock poured a constant stream of paint onto his vast, horizontal canvases to produce patterns possessing immense visual complexity [OConnor & Thaw, 1978].

Can we develop quantifiable measures of objective complexity to distinguish between the two images shown in Figure 2? Given that the works werent generated by computer programs, we need an alternative method for assessing their objective complexity. Rather than quantifying the algorithmic generation process, we need to now consider the physical process that generated the complexity. Painted at his creative peak, the distinctive swirls of paint found in Blue Poles: Number 11, 1952 (210 by 486.8cm) evolved through a six-month period of repetitive paint deposition. This cumulative painting process is strikingly similar to the way patterns in Nature arise - for example, the way leaves fall day after day to build a pattern on the ground, or the way waves crash repeatedly on the shore to create the

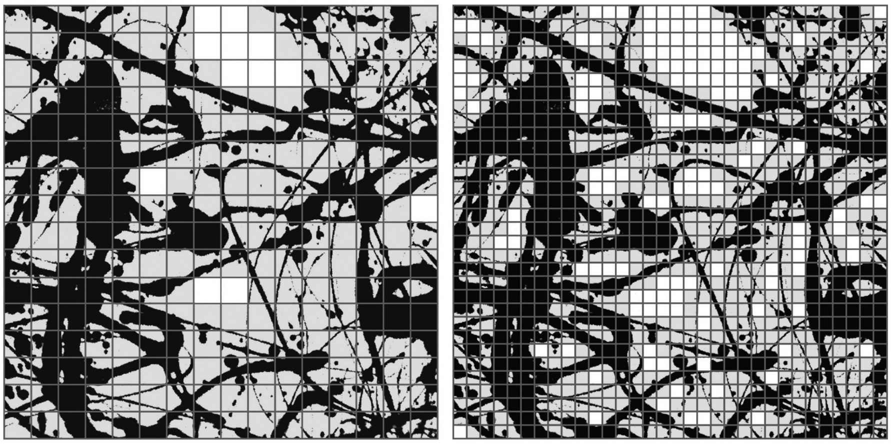

FIG. 3: A schematic representation of the technique used to detect the statistical characteristics of Pollock's patterns.

erosion patterns in the cliff face. In fact, during his peak years of 1947-52, Pollocks poured paintings frequently were described as 'organic', suggesting the imagery in his paintings alluded to Nature.

Since the 1970s, many of Nature's patterns have been shown to be fractal [Mandelbrot, 1977]. Examples include coastlines, clouds, flames, lightning, trees and mountain profiles. In contrast to the smoothness of artificial lines, fractals consist of patterns that recur on finer and finer magnifications, building up shapes of immense visual complexity [Mandelbrot, 1977]. Nature's fractals obey statistical self-similarity - the patterns observed at different magnifications, although not identical, are described by the same spatial statistics. Given the 'organic' appearance of Pollocks paintings, an important step towards quantifying the objective complexity of his work is to take the pattern analysis techniques used to identify fractals in Nature's scenery and apply the same procedure to his canvases.

Adopting this approach, 14 of Pollocks paintings were chosen to span the full variety of his poured painting catalog: from his first attempts of 1943 up to his final, mature works of 1952, from one of his smallest paintings (48.9 by 35.5 cm of Free Form) to one of his largest canvases (266.7 by 525.8cm of Autumn Rhythm: Number 30, 1950), and paintings created using different paint media (enamel, aluminum, oil, ink and gouache) and supports (canvas, cardboard, paper and glass) [Taylor et al, 2007, OConnor & Thaw, 1978]. Many of Pollocks paintings feature a number of different colored layers of paint. These paintings were electronically deconstructed into their constituent colored layers and a fractal analysis was performed on each layer. For example, for Blue Poles: Number 11, 1952, 5 layers were extracted for analysis (blue-black, aluminum, light yellow, dark yellow and orange). In total, 35 colored layers were extracted from the 14 paintings, and the pattern analysis confirmed that each of these constituent layers were consistent with natures fractal patterns [Taylor et al, 1999, Taylor et al, 2007].

The traditional method for detecting the statistical self-similarity of fractal patterning is shown in Figure 3 for a schematic representation of a Pollock painting. The scanned photograph of the painting is covered with a computer-generated mesh of identical squares (or boxes). By analyzing which squares are occupied by the painted pattern (shaded grey in Figure 3) and which are empty, the statistical qualities of the pattern can be calculated. Reducing the square size is equivalent to looking at the pattern at a finer magnification.

A crucial parameter for characterizing the objective complexity of a fractal pattern is the fractal dimension, D. This parameter quantifies the scaling relationship between the patterns observed at different magnifications [Mandelbrot, 1977]. For Euclidean shapes, dimension is a simple concept and is described by the familiar integer values. For a smooth line (containing no fractal structure) D has a value of 1, while for a completely filled two-dimensional area its value is 2. However, the repeating structure of a fractal line embedded in a two-dimensional space generates a non-integer D value that lies in the range between 1 and 2.

The D value of Pollocks fractal patterns can be determined by applying the so-called box-counting technique to the computer-generated mesh of squares discussed above. Specifically, if the number of occupied squares, N, is counted as a function of the square size, L, then for fractal behavior N(L) scales according to the power law relationship $N(L) \sim L^{-D}$ [Mandelbrot, 1977, Taylor et al, 1999]. This power law generates the scale invariant properties that are central to fractal geometry. The D value, which charts this scale invariance, can be extracted from the gradient of the scaling plot of log(N) plotted against log(L). Typical scaling plots demonstrating the specific fractal properties of Pollocks paintings are described in detail elsewhere [Taylor et al, 2007].

How does D set the objective complexity of Pollocks work? The power relationship $N(L) \sim L^{-D}$ quantifies the crucial role played by D in determining the patterns visual complexity. A high D value is a signature of a large N(L)

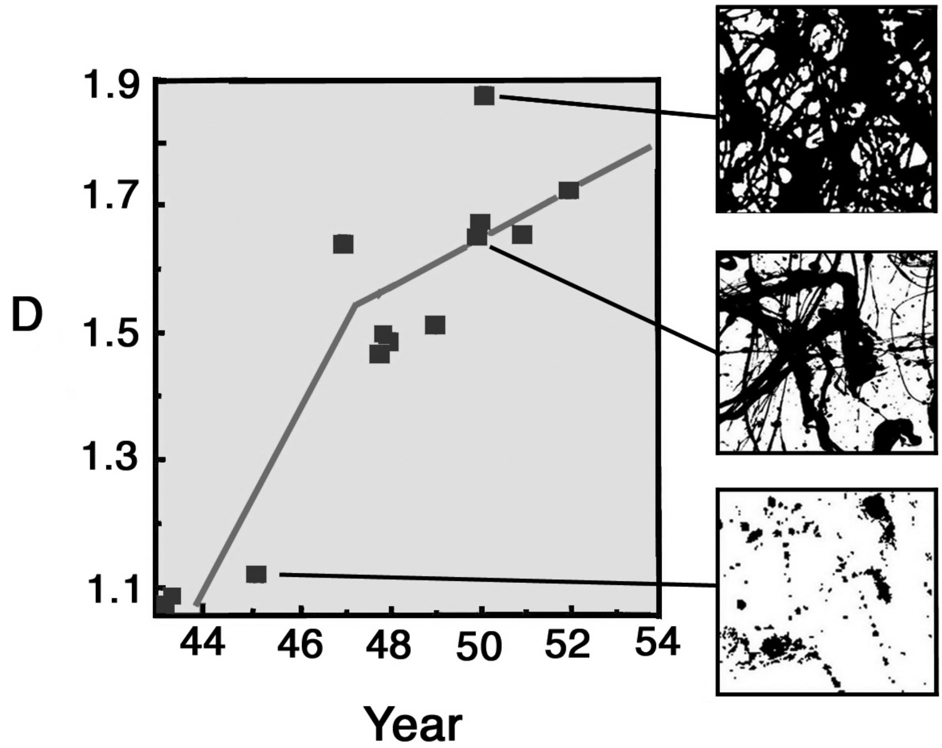

FIG. 4: The D value of Pollocks paintings plotted against the year in which he painted them.

value at small L and reflects the fact that many small boxes are being filled by fine paint structure. This can be seen, for example, for the three Pollock patterns shown in Figure 4. The fine features play a more dominant role for the high D paintings than for the low D paintings. Clearly, D is a highly appropriate mathematical tool for quantifying the objective complexity of fractal patterns. Traditional measures of visual patterns quantify objective complexity in terms of the ratio of fine structure to course structure. D goes further by quantifying the relative contributions of the fractal structure at all the intermediate magnifications between the course and fine scales. A low D value of 1.1 indicates a minimum objective complexity, while a high D value of 1.9 indicates a maximum.

Having declared our method for quantifying objective complexity, we return to a question raised in the introduction. Objective complexity might be expected to materialize in unique ways for specific artists. Can objective complexity therefore be used to distinguish Pollocks art works from others working in the poured genre? Crucially, fractal patterns are not an inevitable consequence of pouring paint it is possible to generate poured patterns that have a non-fractal composition [Taylor et al, 2007]. The distinctive fractal characteristics identified in Pollocks paintings are the product of the refined technique Pollock used to pour paint and all of the analyzed Pollock poured paintings have this fractal composition.

To examine the extent to which these fractal characteristics are unique to Pollock, 37 undergraduate students were invited to generate abstract paintings using the pouring technique. The painting parameters were deliberately chosen to match Pollocks black and white poured paintings such as Number 23, 1948 (57.5cm by 78.4cm). The students were asked to paint a single black layer on an area measuring 61.6cm by 91.5cm. The box-counting analysis revealed that none of the 37 paintings matched the fractal characteristics of Pollocks work [Taylor et al, 2007]. This 100% failure rate for non-Pollock paintings stands in contrast to the 100% success rate for Pollock paintings.

How does Pollocks unique objective complexity emerge during the creative process? During his creative peak in 1950, he was filmed while he painted and Figures 5(a-d) are processed images taken from this film that show the poured painting at different times during its evolution. The images allow us to examine the emergence of the fractal patterns as he painted. The box-counting technique was applied to each of the images (the details of the scaling plots can be found elsewhere [Taylor et al, 2002]), allowing us to chart D as a function of the creation process. Table 2 charts the D value of the painting as more paint is deposited. Significantly, Pollock destroyed this D = 1.9 painting soon after it was completed, suggesting that perhaps D = 1.9 was too high an objective complexity for his aesthetic tastes. This hypothesis is supported by the results shown in Figure 4, which charts the D values of his completed

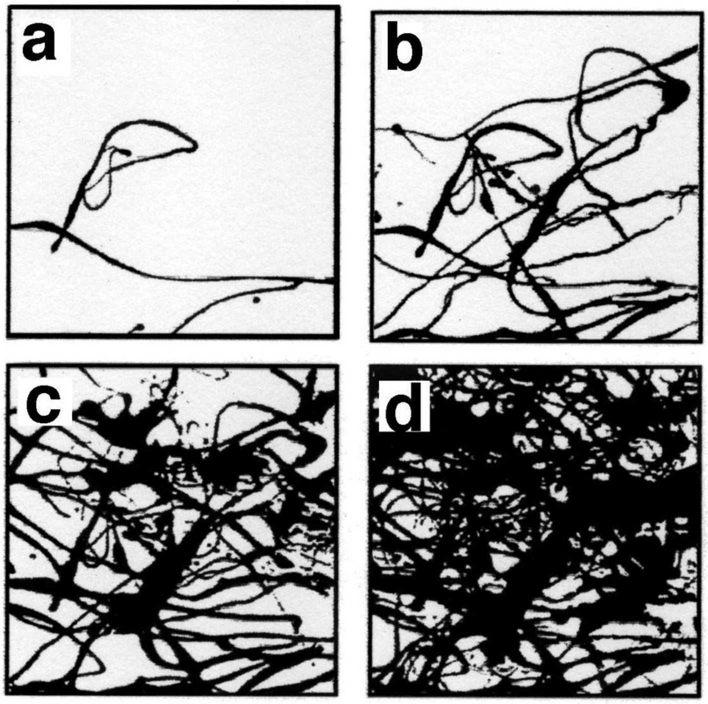

FIG. 5: (a-d) A drip painting shown at four different times t during the paintings creation: (a) 5, (b) 20, (c) 27 and (d) 47 seconds. The painting process was filmed by P. Falkenberg and H. Namuth in 1950.

paintings as he refined his artistic technique across the years. His paintings evolve from the low objective complexity (i.e. low D values) of his initial attempts of pouring paint in the early 1940s, and appear to gravitate towards a refined D value of 1.7. In particular, the D value of his masterpiece, Blue Poles: Number 11, 1952, completed at the peak of his career in the summer of 1952, is quantified by D = 1.72.

t: 5 sec ; D: non-fractal ; A: 3.3 %
t: 20 sec ; D: 1.52 ; A: 16.5 %
t: 27 sec ; D: 1.72 ; A: 42.5 %
t: 47 sec ; D: 1.89 ; A: 70.2 %

Table 2. A summary of the paintings D values as the amount of paint deposited on the canvas increased with time t. A is the percentage of the canvas surface covered with paint.

Pollocks preference for higher objective complexity brings us to the other two corners of the complexity triangle subjective and social complexity. Previous perception experiments performed on computer-generated fractal patterns confirm that raising the D value increases their perceived complexity [Cutting & Garvin, 1987, Gilden, 1993, Pentland, 1984]. In other words, there is a direct correlation between objective and subjective complexity. This can also be seen for the Pollock paintings shown in Figure 4: the low D painting appears less complex than the high D painting, which in comparison appears to be rich in intricate structure. This correlation is expected because perceived complexity is based on the ratio of fine to course structure in the fractal mixture [Cutting & Garvin, 1987, Gilden, 1993, Pentland, 1984] and, as discussed above, this is precisely what D quantifies.

We defined the third vertex of the triangle, social complexity, as the complexity of an artistic form as agreed upon by social consensus. To begin our investigation of the relationship between objective complexity and social complexity for Pollocks work, lets consider the aesthetic qualities associated with the visual complexity of his fractal patterns. Perception experiments were performed on 40 cropped images from Jackson Pollocks paintings, 10 each with D values of 1.12, 1.50, 1.66 and 1.89 [Spehar et al, 2002, Taylor et al, 2005]. The visual appeal of the patterns was investigated as a function of D, using a 'forced choice' visual preference technique. Participants were shown a pair of images with different D values on a monitor and asked to choose the most "visually appealing". Using this technique, all images were paired with each other and preference was quantified in terms of the proportion of times each image was chosen.

The results indicate that we can establish three categories with respect to aesthetic preference for D: 1.1 - 1.2 low preference, 1.3 - 1.5, high preference and 1.6-1.9 low preference [Spehar et al, 2002, Taylor et al, 2005]. These results were confirmed to hold for computer-generated fractal images and images of natural fractal scenes. These findings, based on 220 participants, therefore indicate that people display a common subjective response to the objective complexity of fractals as a whole, they prefer to view fractals with mid-complexity quantified by D = 1.3 - 1.5. It is interesting to note that fractal patterns with these D values are very common in natural scenery (note: although fractal objects such as clouds have a D value in the range $2 < D < 3$, their observed profiles have a D value in the range $1 < D < 2$). This raises the possibility that we prefer this level of objective complexity through an association with Nature. This social consensus about aesthetic dependence can be interpreted as a display of social complexity.

Could this also be the reason why Pollocks paintings have grown in popularity through the years [Taylor, 2010]? Perhaps the consensus on aesthetics revealed by perception experiments drives a social complexity that Pollocks fractal complexity possesses high aesthetic worth. If this appealing hypothesis is the case, then there is one puzzle to be solved. There is a distinct difference between the social complexity (quantified by D = 1.3 - 1.5) and the objective complexity of Pollocks masterworks (quantified by D = 1.7). The driving force behind Pollocks quest for this higher level of complexity forms the subject of future research but emphasizes the subtle interplay between objective, subjective and social complexity.

III. JOHANN SEBASTIAN BACHS MUSICAL COMPLEXITY

The arrow of time is intrinsic to musical expression: music emerges from silence and returns to silence. So in contrast to Pollocks art, its not possible to take a snapshot of a piece of music; if time is stopped, the music simply vanishes. In physical terms, a musical sequence can be considered as the time evolution of an acoustic signal. In both representations a piece of music can be cast in the form of a time series. Technically, this means the music can be expressed either in the form of a sequence of acoustic pulses or in the written form of the symbols of a music score. Thus, a piece of music can be cast as a set of data points distributed in a linear sequence having the dimension of time. Such a sequence, however complicated it may be, can always be coded as a string of binary digits 0s and 1s.

This idea leads us back to the concept introduced in the discussion of Figure 1 - that the objective complexity of this string can be quantified by the length of the shortest computer program required to generate it. This formal concept was formulated independently in 1965 by G.J. Chaitin [Chaitin, 1969] and A.N. Kolmogorov [Kolmogorov, 1965], who proposed an algorithmic (objective) definition of complexity. As an example, a finite sequence of 2N bits, where 1 and 0 alternate (e.g., 10101010101010101 ...), can be generated by a program of very short length (for example, Write the pair 01 N times). On the other hand, a sequence with the same number of 1's and 0's distributed randomly (1101000001110101000001011111 ...) can probably not be generated by a program shorter than simply writing out the complete string.

The length of the shortest program then measures the complexity of the bit string in question. An interesting aspect of the procedure is that one must scan the bits of the string sequentially in order to perform the computation. This is an operation carried out in time. So algorithmic complexity implies a measurement in time. Considering the string of data obtained by coding a piece of music, its algorithmic complexity is then a signature of the dynamics of that piece of music.

How can we apply these concepts to the analysis of music, and what insights would this offer to our perception of music? We take the printed score of a chosen piece of music and play it on a synthesizer interfaced to a computer [Boon & Decroly, 1995]. The pitch values are thus converted into digital data, which can then be stored in the computers memory. So the score is now converted into a time series, say X(t), for a single part of the score. Pieces with several parts are treated part-by-part to produce a collection of time series X(t), Y(t), Z(t), . . . Alternatively, one can access data sources where digitally coded musical scores are readily available. Once the music is in the form of time series, data is processed using the tools developed in dynamical systems theory.

One possibility is to apply the box-counting technique to the X(t) data. In practice one lays a mesh of boxes over the pattern created by plotting X against t. As with the Pollock patterns, one can then extract D from the resulting scaling plot of log N(L) versus log (L). Note the important fact that L is now measured in units of time! Thus, a high D value (which corresponds to a large number of filled small-time-scale boxes) indicates that the musical trace contains a large contribution of high frequency components. The scaling plot can therefore be seen as being equivalent to a spectral analysis in the sense that it measures the relative amounts of small and high frequency components in the data trace [Fairbanks & Taylor, 2010]. Indeed, it is possible to show that a spectral analysis of a fractal X(t) data series produces a power law relationship between the spectral power, S(f), and frequency, f, as follows: $S(f) \sim f^{-\alpha}$ [Fairbanks & Taylor, 2010]. The mathematical relationship between the power law exponent α and D is given by

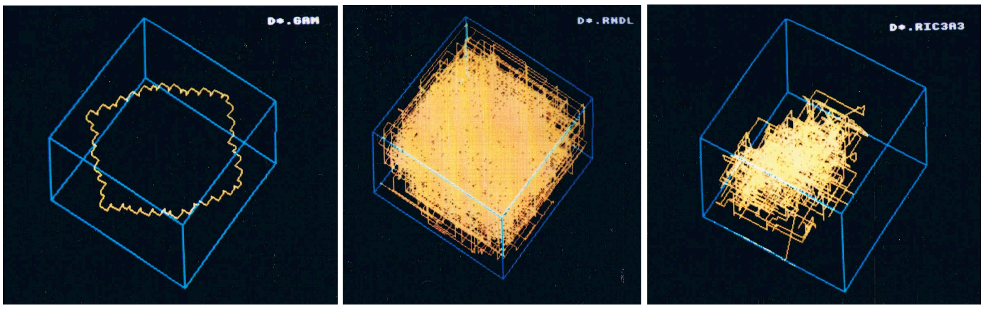

FIG. 6: Phase portraits: (a) Ascending and descending chromatic scale, quantified by D = 1; (b) Computer generated random music, quantified by D = 3; (c) Ricercar from Bach's Musical Offering, quantified by D = 1.72 (a, b, c from left to right).

D = (5 - α)/2 [Fairbanks & Taylor, 2010].

To investigate musical complexity further, we consider the concept of phase portraits. Suppose we want to analyze a string trio piece, and we have processed the violin part, X(t), the viola part, Y(t), and the cello part, Z(t). We can then construct a three-dimensional graph, called the phase space, where the three axes chart X(t), Y(t), Z(t). The X-axis represents the range over which the notes are played on the violin, and similarly for the viola along the Y-axis, and for the cello along the Z-axis. Suppose the piece starts with the violin playing the note A, the viola playing F, and the cello playing C simultaneously on the first beat of the first bar. Plotting the corresponding numerical values along each axis gives one point in the X-Y-Z phase-space. The next note will give the next point in phase-space, and so on until the completion of the piece. Joining the points yields a trajectory as illustrated in Figure 6(c), which shows the phase portrait of the three part-Ricercar of Bach's Musical Offering. The result is called the phase portrait, which gives a spatial representation (in the abstract phase space) of the temporal dynamics of the musical piece reconstructed from the time series obtained from the pitch variations as a function of time: the phase portrait maps the time evolution of a dynamical process onto a spatial representation.

Measuring characteristics of the spatial object provides a measure of the dynamics. In Figure 6, along with the phase portrait of the Bach piece, are shown two typical extreme examples: an elementary score constructed as a canon of three repeatedly ascending and descending chromatic scales and a piece of random music constructed with computer-generated white noise. Obviously these three pieces fill very different spatial regions. Earlier, we saw how the dimensionality measure in the two-dimensional space of Pollock's paintings provides a characterization of Pollock's style. The same type of dimensionality analysis is now performed for the phase portraits in music using the same box counting method described above.

To illustrate the procedure, consider the pictures shown in Figure 6. The repeated ascending and descending chromatic scales are periodic in time so their corresponding trajectory in space forms a closed loop which has dimensionality D = 1. In contrast, a piece of random music explores all possible combinations in the course of its time evolution and therefore the resulting trajectory fills the available space homogeneously so the resulting object has dimensionality D = 3. On the one hand, we have an almost totally predictable musical piece (the chromatic scales with minimal complexity) while on the other hand we have a sequence of unpredictable sounds (having maximal complexity).

Figure 7 shows the values of D obtained from the analysis of the phase portraits for various musical pieces composed over the broad time period from 1700 to 1950 (details can be found elsewhere [Boon & Decroly, 1995]). Their fractional values indicate that the musical pieces are fractal. However, there appears to be no systematic evolution in the D value in the course of time, with values ranging from $\sim$1.4 to $\sim$1.8. It is interesting to note that the average value (D $\sim$ 1.6) is just a little below Pollocks peak value of 1.7, suggesting that artists might have a universal preference for objective complexity that spans across plastic and musical compositions. In particular, the Bach piece shown in Figure 6 has a fractional dimensionality D =1.72, which precisely matches that achieved by Pollock for the painting Blue Poles: Number 11, 1952.

The dimension D computed by the box-counting method is referred to as the covering dimension, since it quantifies the amount of space covered by the pattern at different size scales. The mathematical equivalent of this measured dimension is called the Hausdorff dimension. These two equivalent dimensions characterize the structure of the complete phase trajectory - its value gives a quantitative measure of the global dynamics of the musical piece. Whereas the D values, and the associated global dynamics, of Pollocks paintings appear to be sufficiently unique to distinguish his poured paintings from imitations [Taylor et al 2005], this does not appear to be the case for Bach. To possibly distinguish Bachs complexity from that of other composers, it is also necessary to look at the local dynamics of the musical composition. A measure of the local dynamics can be obtained from the application of information

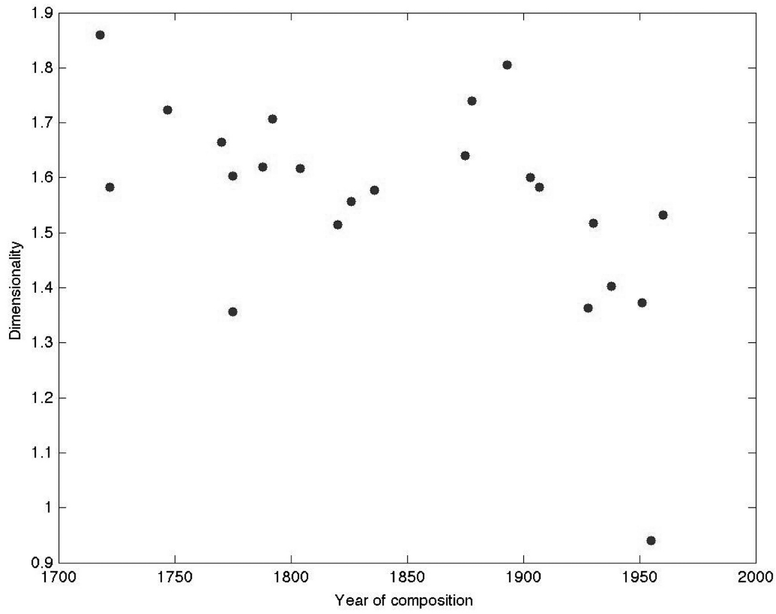

FIG. 7: The D values of 30 musical pieces sampled in the music literature from the 17th to the 20th century, plotted as a function of year in which the pieces were composed.

theory [Boon & Decroly, 1995].

The analysis proceeds along the lines of the discussion of the introductory part of this section. A sequence of notes is viewed as a string of characters and is analyzed from the point of view of its information content. The string is defined by straightforward coding of the pitch through the assignment of a symbol to each note. The entropy H, which measures the information content of a string of characters on the basis of their occurrence probability, is defined such that its value has an upper bound (= 1) for a fully random sequence. H_0 is the zeroth order entropy, which is a measure of the direct occurrence of each musical note, while the nth order entropies, $H_n(n \neq 0)$, follow from the successive conditional probabilities of increasing orders. In fact, the most relevant quantity is the probability of finding the note s(i+1) given that the previous note was s(i) (i.e. for n=1 in H_n).

In western music an important feature is the tonality T. One introduces a quantity defined as the parametric entropy H_1, which measures the information content of a musical sequence in order to quantify the transition probabilities from one note to the next, given that the transition can occur within the reference tonality (s(i) and s(i+1) in T), outside the tonality (s(i) and s(i+1) not in T), or from T to off T (s(i) in T, s(i+1) not in T), and vice-versa. The operational result is that a large value of the parametric entropy indicates frequent excursions away from the tonality, with transitions over intervals distributed over a large number of notes. But if the parametric entropy has a low value, then a note determines almost unambiguously the next one, especially when the next note remains in the range of tonality.

Dimensionality (D) and entropy (H_1) analyses were performed on eighty sequences chosen in the music literature from the 17th century (J.S. Bach) to the 20th century (E. Carter). An interesting result occurs when we plot the dimensionality D versus parametric entropy H_1. As shown in Figure 8, a trend appears indicating a correlation between D and H_1, that is, between local dynamics and global dynamics. While no analytical relation could be conjectured for this relation, Figure 8 suggests that a statistical analysis performed on a larger number of music pieces might provide a better quantification. We note that analogous local dynamics techniques can be applied to the Pollocks paintings. These investigations are underway in order to see if a similar correlation between global and local dynamics appears in visual art.

IV. CONCLUDING COMMENTS

We considered the relationship between objective, subjective and social complexity by focusing on the spatial complexity of Jackson Pollocks abstract paintings and the temporal complexity of Johann Sebastian Bachs music as well as about 30 music pieces by various composers. We measured the objective complexity using established tools of

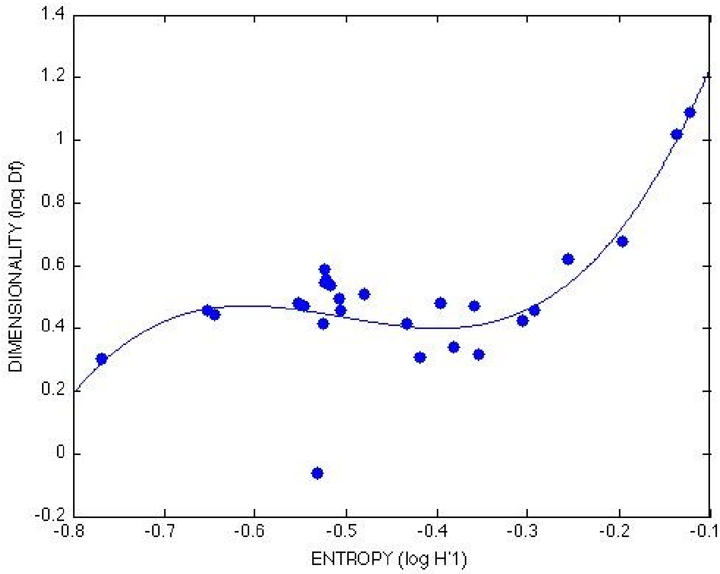

FIG. 8: Dimensionality (log D) versus entropy (log H_1) for 30 musical pieces (same as in figure 6). The curve is a third degree polynomial fit to the data shown as a guide to the eye.

complexity theory fractal dimension D to quantify the global dynamics and entropy H to measure the local dynamics. Notably, we found that over a period of ten years (1943-1952) Pollock refined his fractal construction process and appeared to be able to tune the D value and hence the objective complexity of his paintings. As a consequence, his masterworks gravitated to an objective complexity of 1.7. We found that the musical compositions of Bach have a similar level of objective complexity. Intriguingly, the D values of Pollocks work appear to be trademark signatures of his unique objective complexity, and might potentially help in distinguishing authentic Pollocks from imitations. This doesnt seem to be the case for musical compositions.

For spatial fractal patterns, previous research has shown that there is a clear correlation between objective complexity (D) and subjective (perceived) complexity of the pattern [Cutting & Garvin, 1987, Gilden, 1993, Pentland, 1984]. However, a different story emerges for social complexity, as judged by aesthetics. Whereas Pollock aimed for an objective complexity of 1.7, our perception experiments indicate that the social complexity has a lower value of 1.3-1.5. It would be interesting to expand our studies to investigate other artists who generated complex abstract art (most notably Pollocks colleague Willem De Kooning) to see if their work is characterized by similar values of objective complexity.

An equally important avenue for future experiments concerns the subjective and social complexity of musical compositions. Would an audience prefer musical compositions quantified by D = 1.3 - 1.5? Although musical and visual works with say D = 1.5 have mathematically equivalent objective complexities, this does not imply that two pieces with the same D value would necessarily have analogous subjective and social complexities. Given the different roles played by time in the two art forms, and the physiological differences of the eye and the ear, one might expect that these more human complexities will diverge for the two art forms.

However, any debate along these lines needs to incorporate the concept of Synesthesia. This is a neurologically-based condition in which stimulation of one sensory pathway leads to involuntary experiences in a second sensory pathway. This translation across the senses results in people being able to see noises or hear colors. The condition is most common among artists. Synesthetic art refers to either art created by synesthetes (the name for those who experience the effect) or art that attempts to convey the synesthetic experience. Only recently has science studied synesthesia in artists [Campen, 1999]. In particular, for deceased artists, research is limited to interpreting biographical information. However, influential artists such as Georgia O'Keeffe and Wassily Kandinsky have been associated with this effect, and there has been debate over whether the improvisational nature of jazz music that Pollock listened to when painting influenced the patterns that he produced [Taylor, 2010]. Within this context, it is worthwhile to speculate there might be shared preferences for particular levels of complexity across sonic and visual stimuli.

An appealing experiment is underway which attempts to compare social complexity for sonic and visual art. We plan to translate the visual fractal patterns found in Pollocks Blue Poles: Number 11, 1952 into musical fractals. The

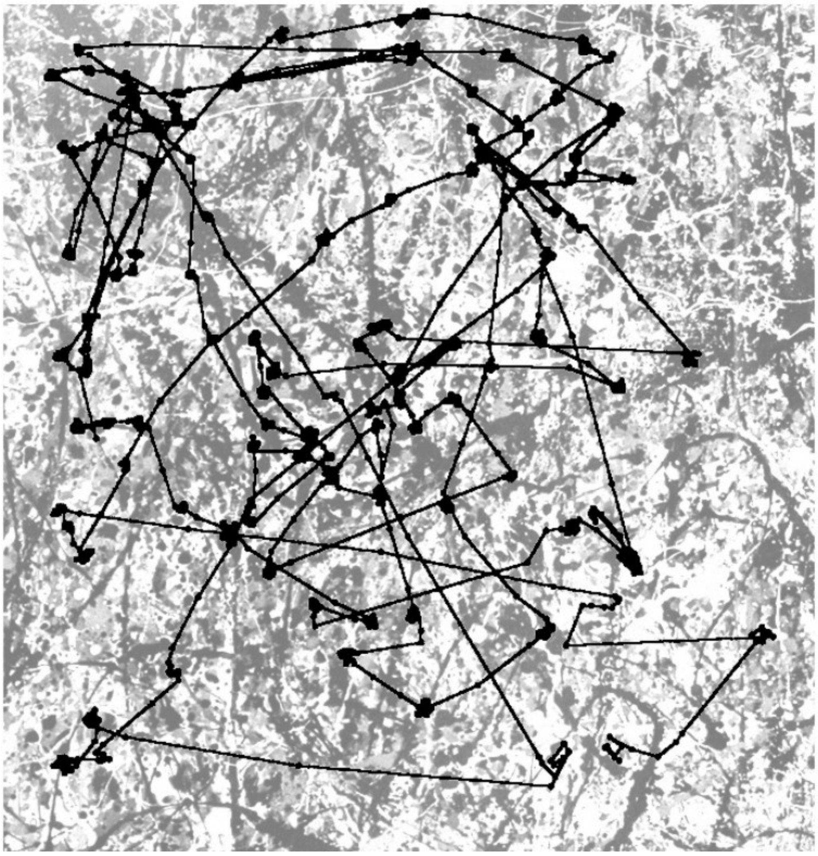

FIG. 9: An example of an eye-trajectory during an observation of one of Pollocks poured paintings (Autumn Rhythm: Number 30, 1950).

digital data sets of Pollocks work consist of a two dimensional array of pixels (where each pixel has green, red and blue components measured on a scale of 0 to 255) [Taylor et al, 2005]. One way of generating the Pollock music would be to treat the three colors as three musical instruments and their musical notes could then be set by the color ranges between 0 to 255. The resulting music could then be pictured using similar the phase portraits to those shown in Figure 6 for Bach.

To play the musical notes of Blue Poles: Number 11, 1952, we would have to consider the arrow of time. One option would be to conduct a raster scan across the canvas, sampling the pixels row by row. Although this seems logical, this is not the way the eye samples the visual fractals of Pollocks work. Figure 10 shows the results of an eye-tracking experiment in which a Pollock painting is displayed on a monitor and an infra-red camera follows the observers gaze [Fairbanks & Taylor, 2010]. The black trajectory in Figure 9 indicates how the eyes gaze moves across the Pollock image as the observer absorbs the complexity of Pollocks work. Intriguingly, the trajectory itself turns out to be fractal [Fairbanks & Taylor, 2010]. In terms of our musical performance of Blue Poles: Number 11, 1952, the pixels in Pollocks canvas could be sampled in the order determined by the typical eye trajectory of an observer. The musical piece would then evolve in an analogous way as the observer witnessing the painting itself. In this experiment, the musical and visual versions of Pollocks works will share an identical objective complexity, as quantified by their D value. Thus, asking participants to rate the aesthetic experience of hearing and seeing a Pollock work will provide an interesting comparison of how social complexity varies across the visual and sonic art forms.

The musical performance of Blue Poles: Number 11, 1952 is clearly just one of many possible future experiments. A central challenge to all future attempts at comparative studies across visual and sonic arts will be that of working out how to account for the differing roles played by time: there is no escaping the fundamental difference of complexity unfolding across the differing dimensions of space and time. However, the above eye-tracking experiments open up an interesting avenue for exploration. Like many of Pollocks masterworks, Pollocks Blue Poles: Number 11, 1952 is vast (measuring 210 by 486.8cm), forcing the eye to search through Pollocks spatial patterns. Consequently, Pollocks complexity unfolds as a function of space and time. This incorporation of time into the observational experience establishes a common link with the complexity unfolding during a piece such as Bachs Ricercare.

To close, we return to the two questions that we asked at the start of the article: how do we define and characterize the fingerprints of an artistic form and can we use these characterizations to identify equivalences between artistic structures that on the surface seem different? The three forms of complexity objective, subjective and social introduced here have been shown to provide useful fingerprints for quantifying the various impacts of art on an audience. We hope that these concepts serve as a springboard for future experiments designed to investigate the roles of complexity across the different forms of art.

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