The Implications of Fractal Fluency for Biophilic Architecture

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Abstract

Fractals are prevalent throughout natural scenery. Examples include trees, clouds and coastlines. Their repetition of patterns at different size scales generates a rich visual complexity. Fractals with mid-range complexity are particularly prevalent. Consequently, the 'fractal fluency' model of the human visual system states that it has adapted to these mid-range fractals through exposure and can process their visual characteristics with relative ease. We first review examples of fractal art and architecture. Then we review fractal fluency and its optimization of observers' capabilities, focusing on our recent experiments which have important practical consequences for architectural design. We describe how people can navigate easily through environments featuring mid-range fractals. Viewing these patterns also generates an aesthetic experience accompanied by a reduction in the observer's physiological stress-levels. These two favorable responses to fractals can be exploited by incorporating these natural patterns into buildings, representing a highly practical example of biophilic design

Keywords: Fractals, biophilia, architecture, stress-reduction, navigation.

Introduction

Due to their prevalence in natural scenery, fractal patterns are a central component of our daily visual experiences. Examples include lightning, clouds, trees, rivers and mountains. Fractals can also be generated artificially. These can be divided into two categories – 'exact' and 'statistical' fractals. Exact fractals, which are built by repeating a pattern precisely at different magnifications, have been explored by mathematicians since the 1860s. It wasn't until the 1980s that Benoit Mandelbrot published *The Fractal Geometry of Nature* in which he catelogued nature's statistical fractals and discussed using mathematical methods to replicate them [1]. 'Statistical' fractals introduce randomness into their construction, such that only the pattern's statistical qualities (e.g. density, roughness, complexity) repeat. Consequently, statistical fractals simply look similar at different size scales. Whereas exact fractals display the cleanliness of artificial shapes, statistical fractals capture the 'organic' signature of natural objects (Figure 1).
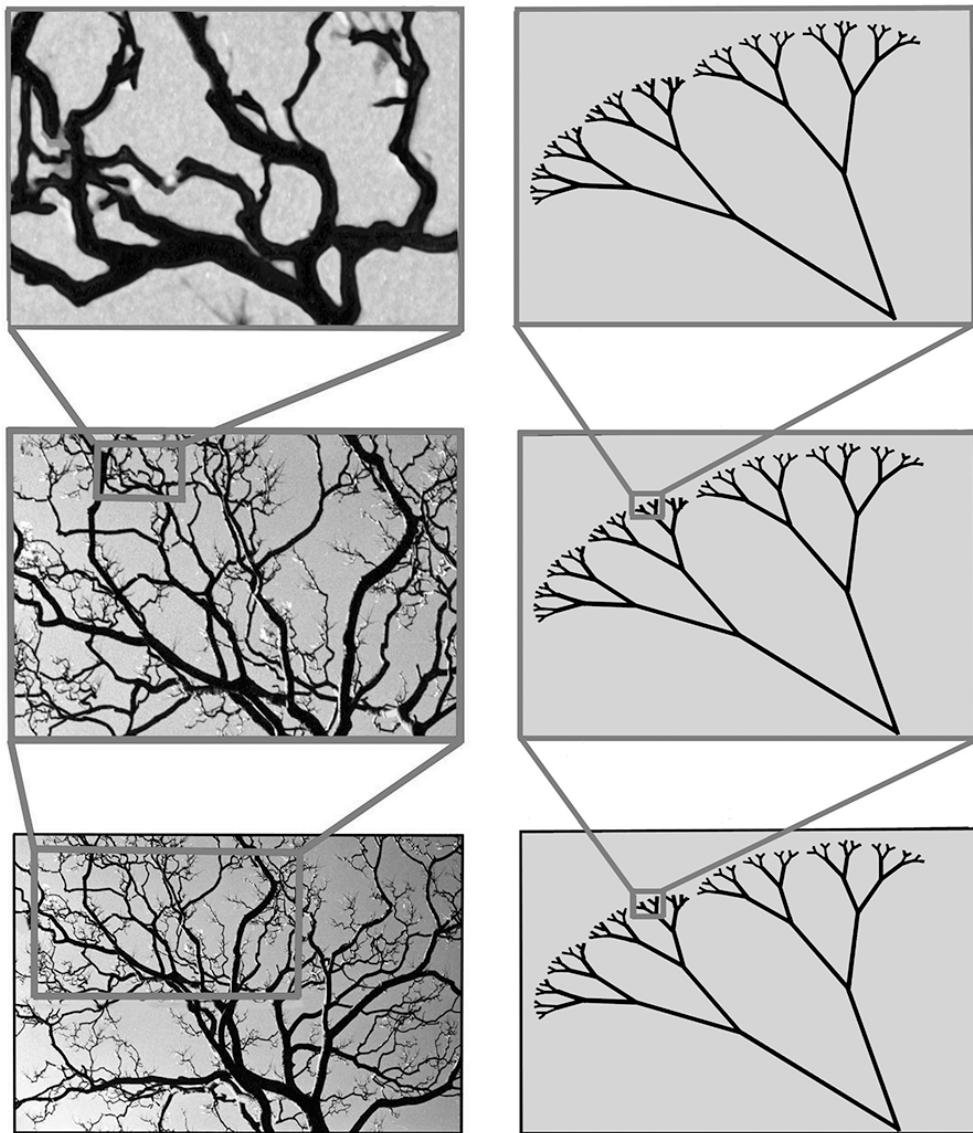

Figure 1. The branch patterns of an artificial tree repeat exactly at different magnifications (right column). In contrast, only the statistical qualities repeat for a real tree (left column). Images generated by RPT.

Statistical fractals are highly topical in the field of 'bio-inspiration,' in which researchers investigate the favorable functionality of natural systems and apply their findings to artificial systems. The growing role of fractals in the arts suggests that the repeating patterns might serve a vital bio-inspired function by capturing the aesthetic quality of nature. Previous studies have shown that exposure to natural scenery can have dramatic, positive consequences for the observer [2-4]. In particular, Roger Ulrich and colleagues showed that patients recover more rapidly from surgery in hospital rooms with windows overlooking nature. Although groundbreaking, these demonstrations of biophilic responses employed vague descriptions for nature's visual properties. Our research has built on these studies by testing a highly specific hypothesis – that the statistical fractals inherent in natural objects are inducing these remarkable effects [5-6].

As reflected in Figure 1, the repeating patterns of fractals generate highly complex images. However, some fractals are more complex than others depending on the amounts of coarse and fine patterns contributing to the fractal mixture. Because many of nature's fractals exhibit mid-range complexity, we proposed a 'fractal fluency' model for the human visual system in which it has adapted to efficiently process these mid-complexity patterns [5-6]. The model predicts that this 'effortless looking' will result in the enhanced performance of visual tasks and, accordingly, the patterns will assume an aesthetic quality. The question of fractal aesthetics holds special significance for the field of experimental aesthetics. When one of its early pioneers, George Birkhoff, introduced 'Aesthetic Measure' as a concept in the 1930s (the idea that aesthetics could be linked to measureable mathematical properties of the observed images) visual complexity was a central parameter in his proposals [7].

Here, we first provide an historical review of the manifestation of fractals in art and architecture and then discuss two studies that higlight the positive consequences of incorporating them into the built environment. The first focuses on people's enhanced ability to navigate within mid-complexity fractal environments and the second highlights their asethetic quality. Given that these positive impacts of fractals originate from their prevalence in nature, fractal architecture can be seen as a specific and highly practical example of 'biophilia' – a term made popular by the conservationist E.O. Wilson to emphasize "the urge to affiliate with other forms of life" [8].

Background: Fractals in Art and Architecture

Symbolic representations of fractals can be found in cultures across the continents spanning several centuries, including Roman, Egyptian, Aztec, Incan and Mayan civilizations. They frequently predate patterns named after the mathematicians who subsequently defined their visual characteristics. For example, although Helge von Koch is famous for developing *The Koch Curve* in 1904, a similar shape featuring repeating triangles was first used to depict waves in friezes by Hellenic artists (300 B.C.E.). In the 13th century, repetition of triangles in Cosmati Mosaics generated a shape later known in mathematics as *The Sierpinski Triangle* (named after Waclaw Sierpinski's 1915 pattern). Triangular repetitions are also found in the 12th century pulpit of *The Ravello Cathedral* in Italy. The lavish artwork within *The Book of Kells* (circa 800 C.E.) and the sculpted arabesques in *The Jain Dilwara Temple* in Mount Abu, India, (1031 C.E.) also both reveal stunning examples of exact fractals.

The artistic works of Leonardo da Vinci [9] and Katsushika Hokusai [10] serve as more recent examples from Europe and Asia, each reproducing the recurring patterns that they saw in nature. Da Vinci's sketch of turbulence in water, *The Deluge* (1571–1518), was composed of small swirls within

larger swirls of water. In *The Great Wave off Kanagawa* (1830–1833), Hokusai portrayed a wave crashing on a shore with small waves on top of a large wave. His other woodcuts from the same period also feature repeating patterns at several size scales: *The Ghost of Kohada Koheiji* shows fissures in a skull and *The Falls At Mt. Kurokami* features branching channels in a waterfall. From the 20th Century, Jackson Pollock's abstract paintings have been shown to be just as fractal as nature's scenery [11-14]. Although the fractal character of his poured patterns originated from the dynamics of his body motions (specifically an automatic process related to balance known to be fractal), he spent ten years consciously manipulating this process. His own reflections on the meaning of his patterns – "I am nature" and "my concerns are with the rhythms of nature" - suggest that he understood their link to nature. Other modern painters have also been shown to display fractal characteristics into their work [15-17].

An alternative strategy to relying on careful observation of nature's fractals is to employ mathematics to replicate them. The nautilus shell is one of the finest examples of a spiral found in nature, mathematics and art (Figure 2) [18]. Figure 2 shows an example by the artist Daniel Della-Bosca. Jacob Bernoulli was one of the first mathematicians to become fascinated by these spirals' properties: the size of the spiral decreases but its shape is unaltered with each successive curve, creating exact fractals. The spiral's rate of shrinking is set by the 'Golden Ratio' (1.618), which is also called the 'Divine Proportion' because of its proposed aesthetic qualities [19].
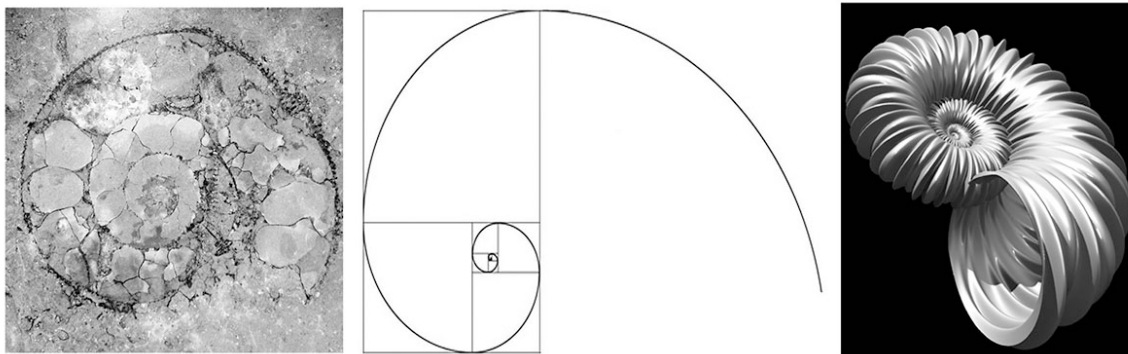

Figure 2. Left: A Nautilus fossil. Middle: A mathematical mapping of a Nautilus spiral. Right: Daniel Della-Bosca's Nautilus Sculpture.

The artist who integrated mathematics into art most effectively is M.C. Escher. Inspired by the Islamic tiles that he saw during a trip to Spain's Alhambra, Escher took the bold step of incorporating patterns that repeat at many size scales into his art. "Since a long time I am interested in patterns with 'motives' getting smaller and smaller till they reach the limit of infinite smallness," he said [20]. Escher's most famous prints, the *Circle Limit* series (1958–1960), reflect both the mathematical challenge and the troubled artistic road that he took to meet it (Figure 3) [21]. Making his patterns fit together required considerable thought and a helping hand from mathematics. He finally found the solution in the mathematical work of Harold Coxeter who declared: "Escher got it absolutely right to the millimeter" [22].

Escher's patterns have captured the imaginations of both artists and mathematicians for more than half a century. Along the way, the patterns' connection with nature has fallen by the way side. His work is often presented as an elegant solution to a purely academic exercise of mathematics — a clever visual game. In reality, Escher's interest lay in the fundamental properties of natural patterns [23]. He frequently sketched natural scenery, including the repeating patterns of tree branches and leaf veins. Escher declared: "We are not playing a game of imaginings — we are conscious of living

in a material, three dimensional reality." He replicated nature in what he referred to as the "deep, deep infinity" of his repeating patterns. Intriguingly, Escher's *Circle Limit* series predated Mandelbrot's *Fractal Geometry of Nature* by 20 years.
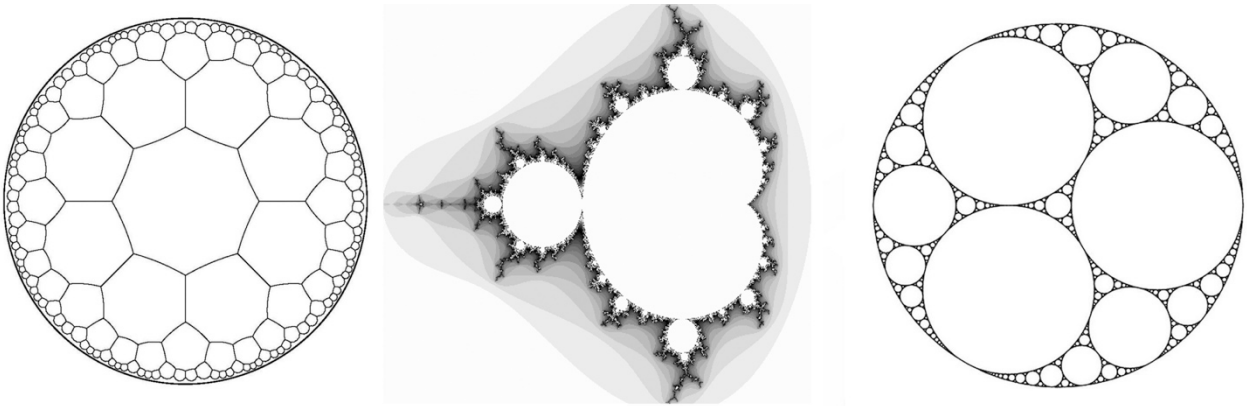
Figure 3. Examples of fractals: The repeating structure used in Escher's Circle Limit series (left), the Mandelbrot Set (middle), and an Apollonian Foam (right). Images generated by RPT.

Soon after Mandelbrot declared fractals to be nature's geometry, he and other mathematicians developed the most published image generated by any mathematician — *The Mandelbrot Set* (Figure 3). Part of its intrigue is that it contains exact fractals in the regions called the Misiurewicz Points and statistical fractals elsewhere. The underlying rules used to construct the image were astonishingly simple but nevertheless required computers to generate the complexity of layer upon layer of fractal patterns. Although similar equations had appeared earlier in the 20th century (such as those of the equally famous Julia Set, named after the mathematician Gaston Julia), it wasn't until the 1980s that Mandelbrot had the necessary computing power to generate the pictures from the equations. Just as microscopes and telescopes transformed biology and astronomy, the modern microprocessor radically expanded people's ability to explore and create fractal patterns. Today, there are many examples of computer art that use fractals - either exact or statistical - as the building blocks of their patterns. Computer technology continues to push fractal arts' creative boundaries. The Mandelbulb is a 3-dimensional analog of the Mandelbrot Set, first constructed in virtual space by Daniel White and Paul Nylander in 2009. Author and mathematician Rudy Rucker had proposed Mandelbulbs 20 years earlier, but he lacked the contemporary computing power to display them.

Although computers can fill virtual worlds with the rich patterning of fractals, in the physical world they are almost exclusively the trademark of nature. However, 3-dimensional printers now allow intricate patterns designed by computers to be printed ('contour-crafted') as physical objects. Della-Bosca used '3-*D*' printers to construct the fractal sculpture shown in Figure 2. Much like a walk through nature's forests, his sculptures surround you, invite you on a journey that is both visual and tactile. This physicality serves as the driving force behind the creation of his sculptures. He sees his 3-dimensional sculptures as an obvious approach to capturing nature's fundamental appeal. Mandelbrot has previously noted: "In order to understand geometric shapes, I believe that you have to see them" [24]. Della-Bosca has taken this thought one step further by asking "What happens if you touch them, too?" The leap from fractal sculpture to fractal architecture seems equally logical to Della-Bosca: "We require our environment to keep us physically, mentally, and emotionally fulfilled, so it is logical to assume that the built environment should not be filled with empty geometry but should be as rich and detailed as we can make it" [24].

Advocates of fractal architecture [25-30] are often inspired by the work of the architect Christopher Alexander who was a critic of conventional architecture and its lack of reflection of human aspirations and needs [31]. Yet, although fractals appear in the patterns generated by the skylines [32] and boundaries [33] of cities, fractal buildings remain conspicuously absent. So why aren't today's urban landscapes dominated by fractal buildings? The answer is as simple as it is daunting - with each layer of repeating pattern, the escalating costs send builders running back to the rectangular box. However, although fractal repitition has been considered too extravagant for buildings, there have been many cases of architecture attempting to symbolize it by incorporating a few repeating layers. The Borobodur temple built in Java during the 8th century (Figure 4) is an early example [34]. The Castel del Monte, designed and built by the Holy Roman Emperor Frederick II, has a basic shape of a regular octagon fortified by eight smaller octagonal towers at each corner. Gothic cathedrals of Europe (12th century) also exploit fractal repetition in order to deliver maximum strength with minimum mass. The fractal character also dominates the visual aesthetics of these buildings. A Gothic cathedral's repetition of different shapes (arches, windows, and spires) on different scales yields an appealing combination of complexity and order [35].

The Ryoanji Rock Garden in Japan represents an example from the 15th century [36]. Gustav Eiffel's tower in Paris (1889) is a more recent demonstration (Figure 4), highlighting the practical implications of fractal architecture. If the tower had been designed as a solid pyramid, it would have required a large amount of iron without significant added strength. Instead, Eiffel exploited the structural rigidity of a triangle at many different size scales. Frank Lloyd Wright's repetition of a triangle adds to the visual appeal of his Palmer House in Ann Arbour (USA, 1950–1951) [37]. The organic quality of Frank Gehry's contemporary architecture has also been discussed in terms of fractals [38]. Going beyond the design of individual buildings, complexes within African villages have been shown to follow a fractal plan [39].
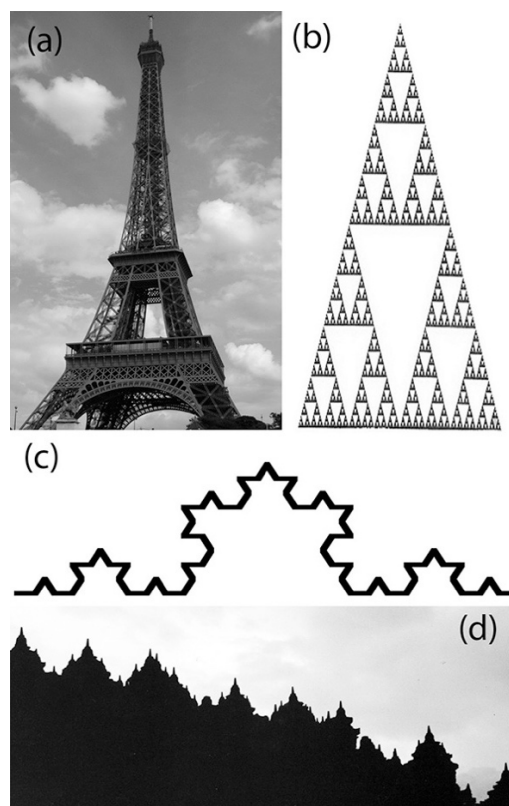

Figure 4. The Eiffel Tower (a) compared to a Sierpinski Triangle (b) and a Kock Curve (c) compared to the Borobodur Temple. Images generated/photographed by RPT.

More recently, explorations of bubble patterns led to a famous example of modern architecture [40]. Foams form intricate patterns that efficiently pack a range of bubble sizes into a small area. The Apollonian pattern (Figure 3) is an example of a fractal foam in which increasingly small bubbles are packed into the gaps that inevitably form between the larger bubbles. In 1993, Denis Weaire used computer simulations to determine the optimal packing pattern for foam [41]. The resulting structure served as the inspiration for the aquatic center at the Olympics in Beijing in 2008. The foam pattern of the so-called *Water Cube* is shaped by more than 22,000 steel beams. Shapes of varying size cram together into a pattern that appears to be disordered – but only superficially. The underlying structure follows the geometric order required by nature's rules of foam formation.

The above review highlights the prevalance of fractals in art and architecture throughout history. In the future, 3-dimensional printers will be able to move beyond these symbolic demonstrations in which a limited number of repetitions were employed. This revolutionary technology will print whole rooms, allowing assembly into buildings, making fractal architecture a practical proposition. With this in mind, in the next section we will review some of the advantages of adopting these fractal designs.

Fractal Fluency

To quantify the visual complexity of the fractal images used in our studies, we adopt a traditional mathematical parameter called the fractal dimension D, which describes how the patterns occurring at different magnifications combine to build the resulting fractal shape [1, 42]. For a smooth line (containing no fractal structure) D has a value of 1, while for a completely filled area (again containing no fractal structure) its value is 2. However, the repeating patterns of the fractal line cause the line to begin to occupy space. As a consequence, its D value lies between 1 and 2. By increasing the amount of fine structure in the fractal mix of repeating patterns, the line spreads even further across the two-dimensional plane and its D value therefore moves closer to 2.
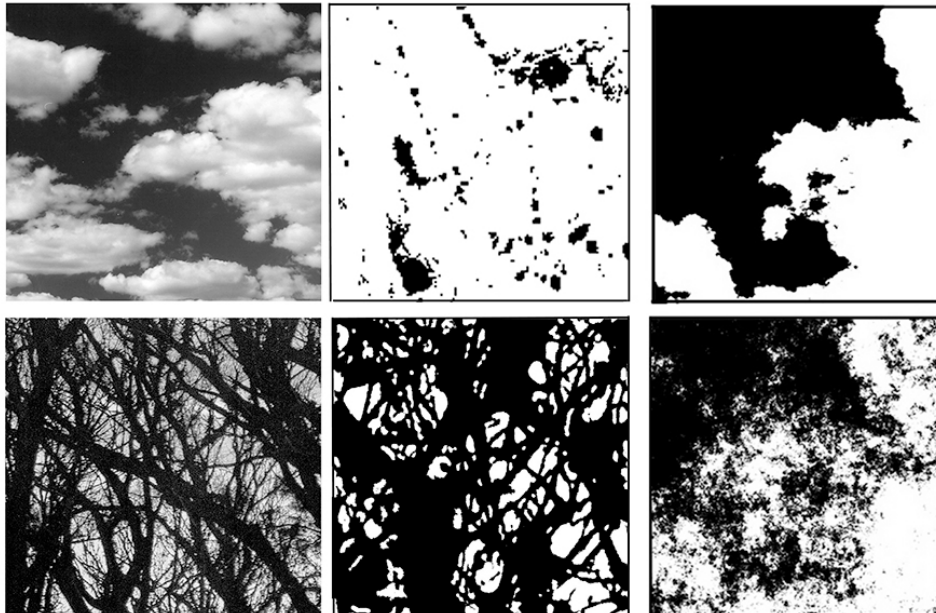
Figure 5. Fractal complexity in nature, art and mathematics. The left column shows clouds with D = 1.3 (top) and a forest with D = 1.9 (bottom). The middle column shows Jackson Pollock's Untitled 1945 with D = 1.1 (top) and Untitled 1950 with D = 1.89 (bottom). The right column shows computer-generated fractals with D = 1.2 (top) and D = 1.8 (bottom).

Figure 5 demonstrates how a statistical fractal's *D* value has a crucial effect on the visual characteristics of fractal patterns found in nature (photographs of clouds and trees), art (paintings generated by Jackson Pollock) and mathematics ((computer-generated images) [43]. For fractals described by low *D* values (ie closer to 1), the relatively small content of fine structure builds a very smooth sparse image. However, for fractals with *D* values closer to 2, the larger amount of fine structure builds an image full of intricate structure. More specifically, because the *D* value charts the ratio of coarse to fine structure, it is expected that *D* will serve as a useful measure of the visual complexity generated by the repeating patterns. Behavioral research by our group [44] and others [45] confirms that the complexity perceived by observers does indeed increase with the image's *D* value (Figure 6).
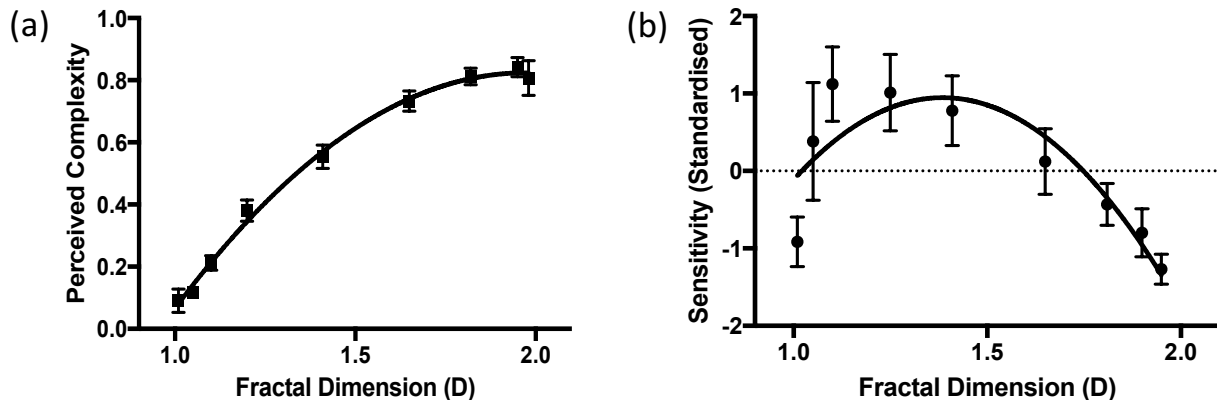

Figure 6. (a) Using fractal images similar to the computer-generated images shown in Figure 5, an investigation of 310 participants revealed that perceived complexity increases with the fractal's D value [44]. (b) An investigation involving 68 participants revealed a peak discrimination sensitivity for fractals with mid-D values [46].

The physical processes that build nature's fractals determine their *D* values. Although objects appearing in natural scenes are described by *D* values across the range $1.1 < D < 1.9$, the most prevalent fractals lie between 1.3 - 1.5. We therefore proposed a fluency model in which the human visual system has adapted to efficiently process the mid-complexity patterns of these prevalent *D* = 1.3 - 1.5 fractals [5, 6]. This model predicts that the increased processing capabilities should result in enhanced performances of visual tasks when viewing mid-*D* fractals. This model was confirmed using computer-generated fractal images that repeat over a magnification range of 100 (similar to typical natural fractals). Using these images, our behavioral studies demonstrated participants' heightened sensitivity to mid-*D* fractals [46]. As shown in Figure 6, participants displayed a superior ability to distinguish between fractals with different *D* values in the mid-*D* range [46]. Similarly, participants exhibited a peak in detection sensitivity for mid-*D* fractals. To demonstrate this, fractal patterns were displayed on a monitor and the contrast between the dark pattern and its light background was gradually reduced until the monitor displayed uniform mean luminance. We found that the participants were able to detect the mid-*D* fractals for much lower contrast conditions than the low and high *D* fractals [47].

A further demonstration of increased processing capabilities was identified by measuring participants' EEG responses to viewing fractals, which highlighted the ability to maintain attention when observing mid-*D* fractals [47, 48]. There is also evidence to suggest that pattern recognition capabilities increase for mid-*D* fractals. We are all familiar with percepts induced by clouds (Figure 7). A possible explanation is that our pattern recognition processes are so enhanced by these fractal clouds that the visual system becomes 'trigger happy' and consequently we see patterns that aren't actually there. Research reveals that mid *D* fractal images activate the object perception and

recognition areas of the visual cortex [49] and allow for a larger number of percepts to be formed [50]. This is consistent with behavioral studies in which the capacity to perceive shapes in fractal images was shown to peak in the low *D* range [51, 52].
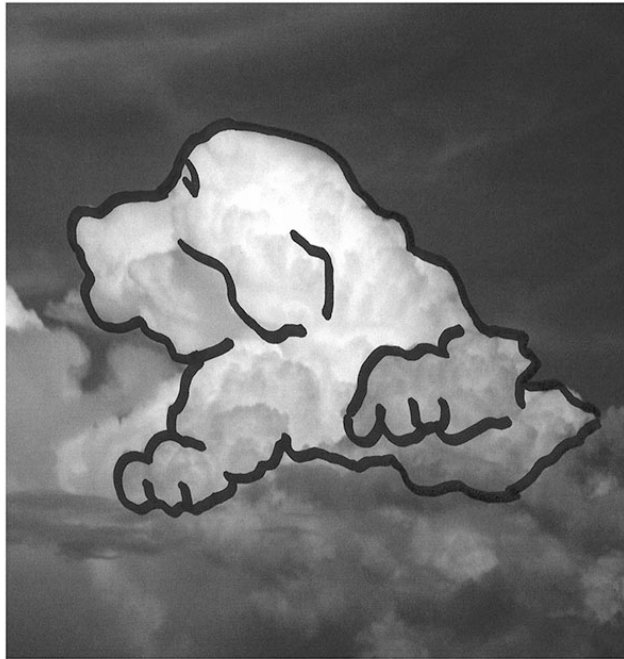
Figure 7. A photograph of clouds with the perceived image of a dog drawn on it. Photograph by RPT.

Does fractal fluency also lead to an enhanced processing of visual spatial information and therefore to a superior ability to navigate through environments characterized by mid-*D* fractals? To answer this important question for fractal architecture, we generated virtual fractal enviroments characterized by varying *D* values [53]. Virtual environments have been used in navigational research for several decades [54-55] and have been shown to be good approximations of physical environments for transferring navigational skills to their real-world equivalents [56-57]. Human performance in complex virtual environments has often been studied using regular geometric structures such as mazes [58-60]. However, such studies do not capture the fractal complexity inherent in natural environments. Other research has replicated the features of specific natural environments [61-63] which required time-consuming physical collection of environmental information and was expensive to carry out. In contrast, our approach generates controlled environments in which the generic fractal qualities can be tuned with precision and ease.

Generation of the virtual landscapes is described in detail elsewhere [53]. They each spanned 200m in virtual space and consisted of flat ground with protruding fractal hills of maximum height 50m. Figure 8 demonstrates the impact of varying *D*. Seventy-four participants navigated an avatar through the landscapes from a first-person perspective using a PlayStation controller and they could move their avatar not only around the flat surface but also over terrains with inclines of less than 45^0. They were instructed to search as quickly as possible for the goal (eg a coconut) randomly placed within the landscape. Various experimental conditions were investigated, including the effect of including a topographic map featuring the goal's location, the presence of a distractor goal (eg a second coconut), and making the goal invisible (eg by burying it). Including these conditions allowed for confirmation that the experimental conditions indeed measured navigational performance above and beyond the difficulties of simply moving around the features of the environments [53]. In each case, completion speeds and accuracy (the ratio of finding the goal before or after arriving at the distractor) were measured.
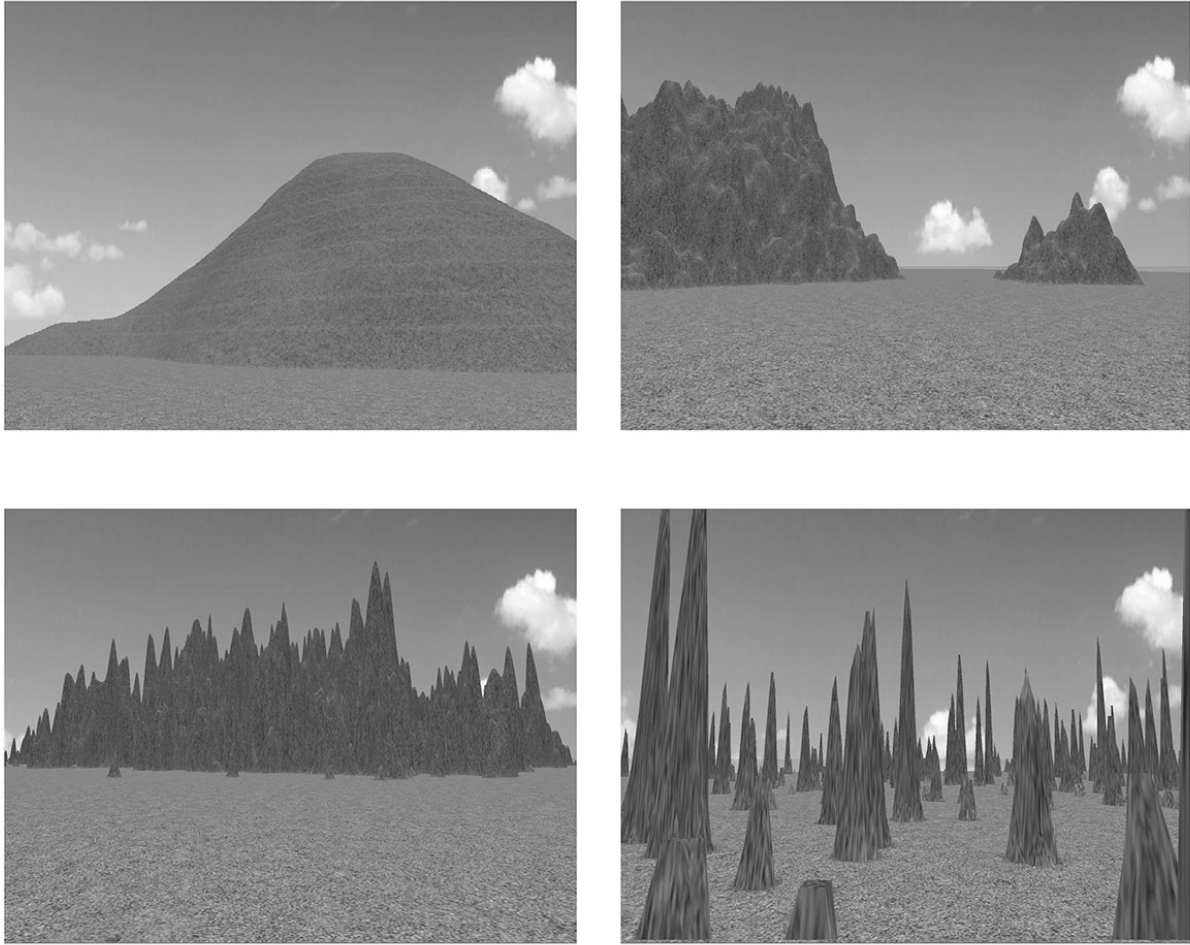
Figure 8. Examples of first-person perspective views during the navigation experiment. The D of the landscapes are 1.1 (top-left), 1.3 (top-right), 1.7 (bottom-left) and 1.9 (bottom-right).

Figure 9 shows an example result for the condition in which the goal was buried and the partipants read a map to guide them to the goal. The measure of accuracy was designed to convey the ability of participants to make precise localization judgments on a scale that ranged from 0 (designating chance performance) to 100 (designating perfect performance) [53]. In order to account for both accuracy as well as time-to-goal within a single construct, we also calculated a measure of overall performance and found that this too peaked at mid-*D* complexity. This navigation performance closely matches that expected from the fluency model.

In addition to effective navigation through a fractal environment, fractal fluency creates a unique aesthetic quality due to the relative ease with which fractals can be processed. In 1993, we conducted the first aesthetics experiments on fractals, showing that 95% of observers preferred complex fractal images over simple Euclidean ones [64]. Soon after, others employed computer-generated fractals to show that mid-*D* fractals were preferred over low and high *D* fractals [65]. Over the past two decades, fractal aesthetics experiments performed by ourselves and others have shown that preference for mid-*D* fractals is universal rather than dependent on specific details of how the fractals are generated. We showed that preference for mid-*D* patterns occurred for fractals generated by mathematics, art and nature using images similar to those shown in Figure 5 [43]. Whereas this experiment featured relatively simple natural images such as a tree or a cloud, this was soon

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broadened to include more complex natural scenes featuring many fractals [66] and also larger varieties of computer-generated fractals [44, 67].
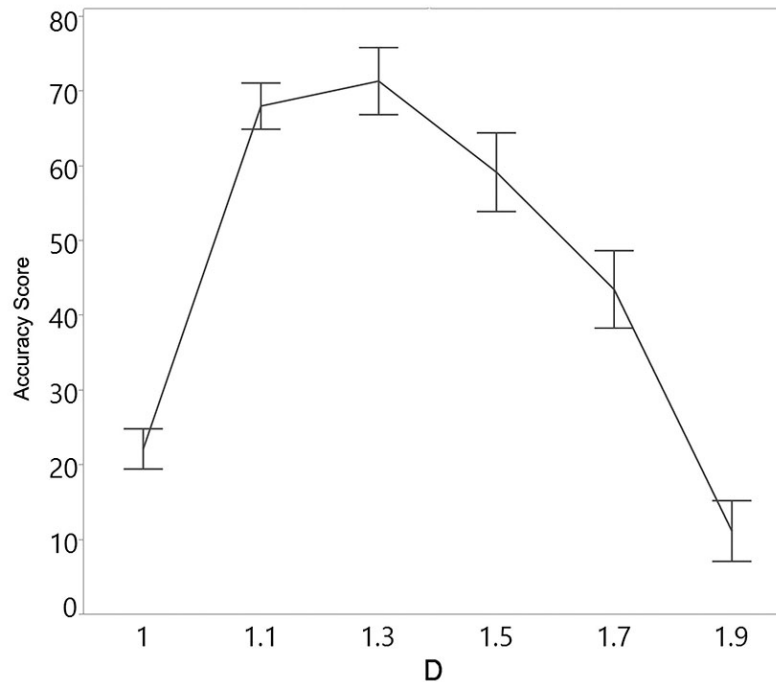

Figure 9. The relationship between D and the mean accuracy (see main text), revealing a peak in navigation performance at D = 1.3 [53].

Figure 10 shows preference results for 20 participants who veiwed computer-generated stimuli similar to those shown in Figure 5 [5]. The panels are for 4 different 'configurations' in which the computer used 4 different seed patterns to build the fractal images. The peak preference showed a remarkable consistency despite superficial variations in the 4 fractal seeds. Furthermore, this peak behavior for aesthetics follows that revealed in Figure 6 for the observer's processing abilities (as quantified by their abilities to detect and discriminate fractals). In addition to these laboratory-based behavioral experiments, others have used a computer server to send screen-savers to a large audience of 5000 people. New fractals were generated by an interactive process between the server and the audience, in which users voted electronically for the images they preferred [68]. In this way, the parameters generating the fractal screen-savers evolved with time, much like a genome, to create the most aesthetically preferred fractals. The results re-enforced the preference for mid-D fractals found in the laboratory-based experiments. This 'aesthetic resonance' for $D = 1.3 - 1.5$ fractals also induces the state of relaxation indicated by the peak in alpha response in the qEEG studies and by skin conductance measurements [47, 34]. Whereas the above experiments focused on the universal responses to fractals, more recent experiments have started to examine the subtle differences in responses between individuals [44, 69, 70] and also different forms of fractals (for example, exact versus statistical fractals) [70].
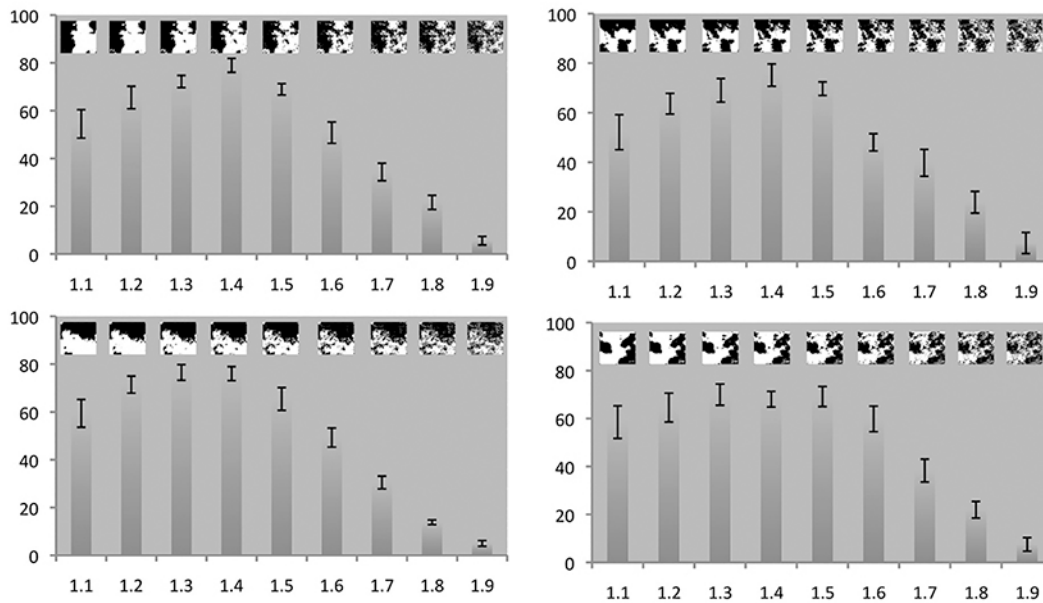
Figure 10. Visual preference for computer-generated fractal patterns. For each of the 4 panels, D is plotted along the x-axis and the preference on a scale 0-100 is plotted along the y-axis. Each of the 4 panels uses a different fractal configuration to investigate preference. The fractal images are shown as insets in each panel [5].

Conclusion

Our historical review of fractals highlights a natural inclination on the part of artists and architects to design buildings and environments which captue the visual essence of fractal geometry. With the advent of 3-*D* printing techniques, we expect this inclination will be more frequently transformed from imagination into practicality. Accordingly, we have reviewed our recent psychology experiments on fractal fluency to demonstrate that people will display enhanced visual capabilities in fractal environments characterized by mid-complexity. In particular, people will be able to navigate effectively through these spaces and will benefit from their aesthetic and stress-reducing effects. Given that job stress alone is estimated to cost American businesses many millions of dollars annually [71] the latter effect holds a huge potential benefit to society.

Finally, we note that there are other 'bio-inspiration' motivations for creating a building based on fractals. Fractals have large surface area to volume ratios. For example, trees are built from statistical fractals in order to maximize exposure to the sunlight. Similarly, bronchial trees in our lungs maximise oxygen absorption into the blood vessels. Possible advantages of this large surface area for fractal buildings include solar panels on the rooftops and windows that deliver a large amount of light to the building's interior. The repeating structures of fractals also dissipate the energy of impinging waves. For example, the energy of ocean waves crashing on the shoreline is dissipated by the fractal coastlines and this reduces erosion. For this reason, modern storm barriers feature fractal surfaces. Similarly, trees serve as effective windbreakers compared to flat surfaces because their fractal branches dissipate the wind's energy. Mathematicians have even shown that if the circular shape of a drum is replaced by a fractal, it will dissipate vibrations so effectively that it won't make a noise when struck by the drum stick [72]! As a consequence, fractal building designs will minimize noise and vibrations from traffic and earthquakes. When all of these physical advantages are coupled

with the visual impacts covered in this article, it becomes clear that artificial fractal environments have a bright future.

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