Simulating Alternative Course Election Systems with Auction Theory

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Abstract

This project addresses the inefficiencies and anxieties associated with the current course election system at Dartmouth College. To provide students with more control and transparency, we propose an alternative course election system using auction theory. Students are allocated course election points, and the course election process is simulated as an auction. We study the effectiveness of different auction mechanisms and compare them with the allocation model currently in use. We associate different bidding strategies with student preferences and conduct auctions based on these strategies. The simulation involves multiple rounds, utility score calculations, and strategy transitions for low-performing students. Identified limitations include the lack of strategy transitions between rounds, which could be addressed through incorporating survey data or adopting a Moran process approach. Future research directions include exploring other auction mechanisms, refining assumptions, and tweaking simulation parameters. This simulation has the potential to revolutionize course allocation systems, providing students with greater control and fairness in the process.

Introduction

Problem Statement: College course allocation is an essential process for students, but the current system of course election at Dartmouth College can be inefficient and lead to a lot of anxiety. The process is often randomized, with priority given to factors such as academic major and class year, leaving students with little control over the process. The current course election system is not transparent in its algorithm, making it difficult for students to develop effective election strategies. Students often have knowledge on the popularity of courses and the importance of taking certain courses due to their major plans and D plans, but this lack of transparency, coupled with the randomized nature of the current system, often results in students not being able to secure the courses they need or want. To address these issues, we propose a simulation of alternative course election systems using auction theory. By allocating an equal number of course election points to students at the beginning of each course election cycle and simulating course election as an auction, we aim to provide students with more agency in the process. We will study the effectiveness of different auction mechanisms and compare them with a strictly randomized allocation model that Dartmouth currently implements. Our proposed simulation using auction theory has the potential to revolutionize the course election process, providing students with more control and transparency in course allocation. The following sections will detail the methodology used in this study, as well as the results and implications of our findings.

Design

The course election system designed for Dartmouth College adopts an auction-based mechanism that seeks to optimize student welfare in the course selection process. This auction model allocates a total of one bid point to each student, which they can allocate among at least one and at most three courses in each round. The highest k bids are selected for each course, with k being the availability of the course, and the winning bidders pay the bid of the highest lost bid, like in a second price auction. The system conducts three rounds of auction, and students can proceed to the next round if they have bids left and do not have three courses. One significant aspect of this course election system is that it only considers welfare and not revenue, because the College does not gain from the bidding points. The auction system focuses on maximizing the satisfaction of students in terms of course selection. By only allocating one bid point to each student, the system ensures fairness and equal opportunity for every student. Moreover, the three-round auction system enables students to refine their choices and increase their chances of securing the courses they desire. This welfare-centric approach aligns with the combinatorial auction type, which seeks to allocate goods among bidders in a manner that maximizes their overall welfare. Another important feature of the course election system is that all bidders have a limited budget. This budget constraint compels students to make rational and strategic decisions in allocating their bids across different courses. In this way, students prioritize their course preferences and avoid overspending their limited budget. The budget constraint aligns with the limited budget auction type, which is common in scenarios where bidders have a finite amount of money to allocate to different items. Moreover, the auction mechanism allows bidders to bid on combinations of courses, but they may get only a few of them. This combinatorial auction type encourages students to bid on course packages that best align with their academic and career goals. Students can allocate their bids strategically to increase their chances of getting at least one of the courses they desire. This design also aligns with the combinatorial auction type, where bidders bid on combinations of goods. Another unique feature of this auction system is that students can enter the next round if they have bids left and do not have three courses. This design enables students to refine their choices and increase their chances of securing the courses they desire. In each round, students can allocate their bids strategically to increase their chances of winning the desired course.

This system aligns with the open bid auction type, where bidders have the opportunity to participate in subsequent rounds of bidding. In conclusion, our course election system is a unique auction design that takes into account the limited budget of the bidders, the demand and properties of the goods, and the visibility of bidding. It is inspired by the second price auction, combinatorial auction, and sealed bid auction, but with modifications to better suit the specific needs of our system. Our design promotes the welfare of the bidders by ensuring that all students get a course that they desire, as long as they are willing to pay the price. Our sole focus on welfare enables the system to maximize utility of the students. With its unique features, our course election system has the potential to revolutionize the way course elections are conducted in colleges and universities.

Methods & Implementation

Course selection is simulated via a series of auctions in which randomly generated students represent varied strategies for determining allocation of course "preference point". 2021 Dartmouth College spring course election data, including enrollment, section size, and median GPA points, is used as a basis for approximating class "desirability". This desirability is calculated by dividing the total enrollment of the course by the number of students electing courses that term and is henceforth referred to as popularity probability. Popularity probability data from a randomly selected m classes is then used to determine the likelihood of each of n individual simulated students being interested in the course. This interest as either a 0 (not interested in the course) or 1 (interested in the course) is noted in a binary $m \times n$ matrix. For each student interested in less than 3 courses, we select "not interested" cells at random, changing them to "interested" until there are exactly 3 cells of interest. These cells are then altered in a similar fashion to express student-wise unique integer interest in each class such that cells of previous values $[1, 1, 1, \cdots]$ are now $[1, 2, 3, \cdots]$. We then normalize each student interest column. Once we have successfully generated a normalized preference matrix for all students and classes, we associate auction strategies with each individual's preference. Our study associates three possible bidding strategies with class preference - *aggressive* (A), *truthful* (T), and *non-aggressive* (NA). These three strategies are outlined below.

Let us assume that there are 2-students with 3 different potential strategies competing for 3 courses. Let us denote the student's true value preferences as follows,

$$pref_A = pref_B = [x, y, z]$$

Where, $x + y + z = 1$ and we assume $x > y > z$ where $x, y, z \in [0, 1]$. Now, each student has 3 strategies which we define as follows,

- $\mathbf{T}$ - *bidding as per true value* : In this case, the students would bid exactly equal to their valuation
- $\mathbf{A}$ - *aggressive bidding* : In this case, students would bid as if they are risk seeking. In this case, their bids would be represented as follows : $[\frac{x^r}{x^r+y^r+z^r}, \frac{y^r}{x^r+y^r+z^r}, \frac{z^r}{x^r+y^r+z^r}]$ where r is the aggressiveness factor.
- $\mathbf{NA}$ - *non-aggressive bidding* : In this case, students would bid as if they are risk averse. In this case, their bids would be represented as follows : $[\frac{x^s}{x^s+y^s+z^s}, \frac{y^s}{x^s+y^s+z^s}, \frac{z^s}{x^s+y^s+z^s}]$ where s is the non-aggressiveness factor.

At the start of the simulated course election, student strategies are randomly assigned to individual student preferences via a list of fixed hyperparameters specifying the frequency of each strategy's occurrence. Each student is assigned a fixed amount of "currency" with which they can bid upon courses. For each student, the currency is initially allocated in accordance with the assigned strategy for their preferred three classes (i.e. for truthful strategy students, the currency allocated to each individual class would be exactly equal to the normalized preference for that class times the total amount of currency allocated to that student). Courses are assigned sealed-bid auction-style. For each course of enrollment size j, the j highest bids for the course are immediately assigned to the class and the associated bid currency subtracted from that student's overall currency. If a student receives the class (via bidding one of the j highest bids or because the class does not fill), it is elected. Once a student has either elected all three of their preferred classes or exhausted their currency, they are removed from the allocation bidding process. After each round, the student receives a utility score associated with their elected courses based on their preference matrix. The average utility score for each of the three strategies across all students is calculated. Students with a utility score in the lowest 20% of the population are re-assigned a strategy at random. This does leave the possibility that the students may be re-assigned to their previously unsuccessful strategy - this is intended to simulate students that fail to change their allocation strategy despite limited satisfaction with their course election results. Based on data surveyed from current Dartmouth students regarding their intended strategies for this proposed course election, we ran the majority of our simulations with an initial $1:5:4$ (A: NA: T) ratio of strategy prevalence. For selected simulations, primarily those with greater student populations, we used an initial $3:3:4$ ratio of strategy prevalence to approximate an equal distribution of starting strategies.

Results

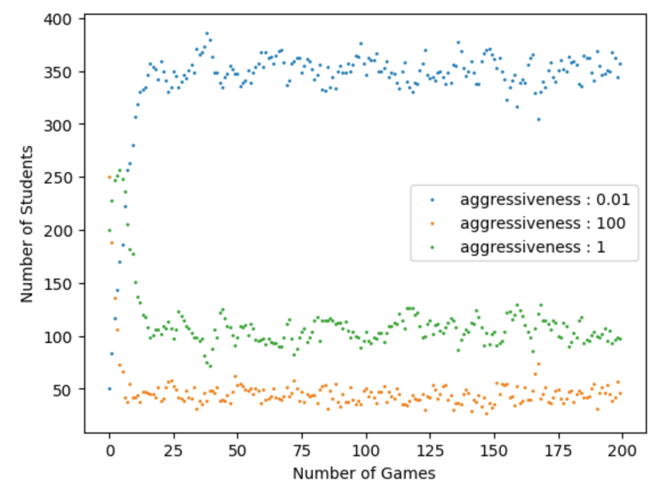

Figure 1: Low population, Low competition, Broad interest (n = 500, m = 40, 50-year scale, Starting ratio: 1:5:4, NA >> T > A)

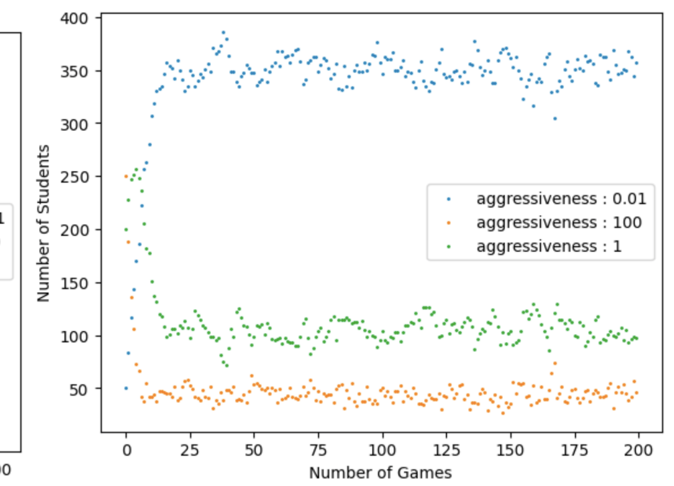

Figure 2: Low population, Medium competition, Broad interest (n = 500, m = 30, 50-year scale, Starting ratio: 1:5:4, NA >> T > A)

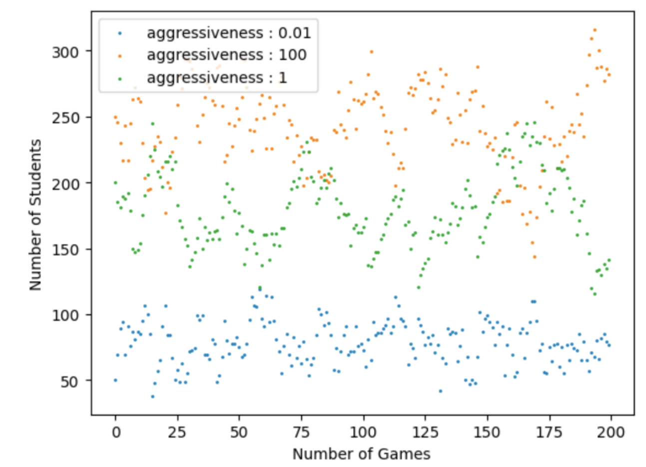

Figure 3: Low population, High competition, Medium interest (n = 500, m = 20, 50-year scale, Starting ratio: 1:5:4, NA >> T > A)

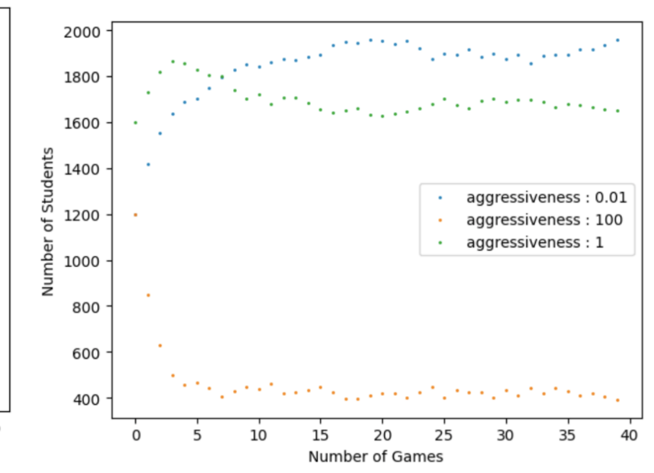

Figure 4: High population, Real-life competition (n = 4000, m = 500, 10-year scale, Starting ratio: 3:3:4)

Figure 1 demonstrates that the NA strategy ($r = 0.01$) is most popular strategy over time (70%), T strategy ($r = 1$) is second most prevalent (~20%) and A ($r = 100$) is least prevalent (~10%).
Figure 2 demonstrates that NA and A strategies prevalence ($r = .01, r = 100$ respectively) are strongly inversely correlated and oscillate between ~30% and ~60% prevalence. T strategy ($r = 1$) is stable and least prevalent (~10%).
Figure 3 demonstrates that T and A strategies prevalence ($r = 1, r = 100$ respectively) are loosely inversely correlated and oscillate between ~30% and ~60% prevalence. NA strategy ($r = 0.01$) is stable and least prevalent (~15%).
Figure 4 demonstrates that NA and A strategies' prevalence ($r = .01, r = 100$ respectively) are loosely inversely correlated and oscillate between ~40% and ~50% prevalence. Truthful ($r = 1$) is stable and least prevalent (~10%).

Discussion & Conclusion

Moving forward, one focus area will be experimenting with various winner determination auction strategies. For instance, an auction mechanism where winners pay the second highest bid could be implemented. This method encourages honest bidding, improving the efficiency and equity of course allocation. Investigating the influence of these auction styles on student tactics and overall results can yield crucial insights.We can also optimize our surveying and modeling assumptions. Gathering broader data on student choices, limitations, and decision influences through a more extensive survey can enhance our simulation's realism and precision. Fine-tuning the modeling techniques and principles within our auction theory framework will offer a truer depiction of the course selection process.Addressing existing shortcomings and examining the effects of various factors will involve adjusting parameters in our simulation. Changing the round numbers or initial ratios of course election points given to students could be beneficial. Such systematic changes can provide a deeper understanding of how these variables affect student behavior, course allocation results, and overall system efficiency.One compelling future research area is examining intra-game strategy shifts. This allows students to adapt their tactics considering others' choices and the changing availability of courses. Incorporating these shifts can portray the competitiveness of course selection, offering a complete view of student decision-making strategies.