Bremsstrahlung: Light Meets Matter

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(Dated: October 12, 2023)

The study of radiation in the late nineteenth and early twentieth centuries brought us our modern understanding of high-energy electromagnetic waves and their interactions with matter. Without this crucial understanding, practices like x-ray imaging would not be possible. This paper outlines the original discovery of x-rays and the properties of x-ray spectra that make these practices possible today.

The modern era of medical technology would be very different without the discovery of x-rays and their special interactions with matter. Practices like x-ray spectroscopy and radiation imaging owe their discovery to the great physicists of the late nineteenth and early twentieth centuries. During this time radiation was all the rage, particularly the cathode ray. Cathode rays were defined at the time as "the electric current created in highly rarefied gases by . . . very high tension electricity" inside a tube containing a cathode and anode separated by a gap [2]. On November 8, 1895, Wilhelm Röntgen was studying one such discharge tube as they later became known. He noticed that if the tube was shielded from external light in a dark room, a plate covered on one side with a barium platinocyanide coating placed outside of the tube would sometimes become fluorescent. The cathode ray was enclosed in the tube, so this fluorescence on an external plate came as a great surprise. A completely different kind of ray was being emitted from the inside of the tube. As Röntgen moved the plate into the path of these new invisible rays he discovered that the rays were causing the fluorescence from a significant distance away. This could be up to two meters away from the discharge tube. Röntgen began experimenting with these new rays, and eventually, he discovered that objects in the path of the rays did not seem to stop them right away. The rays were passing through matter, but not completely. He found that objects of different thicknesses showed a corresponding transparency in the fluorescence on the plate behind them. That is only very thick or dense objects could block the rays from passing through them [2]. Famously, he developed a plate with an image of his wife's hand blocking the rays. (See FIG 1). In later experiments, Röntgen was able to show that these new rays were produced by the cathode rays impacting material objects. Because the nature of these invisible light rays was so unknown, he decided to call them by the name we still use today, x-rays. It was later discovered that x-rays are a form of high-energy electromagnetic radiation with higher frequency compared to visible light [2]. This frequency range is responsible for the type of matter interactions that Röntgen witnessed for the first time in his lab.

FIG. 1. Orginal x-ray image of Röntgen's wife's hand [3]. This type of image is created by placing an object in the path of an x-ray spectrum illuminating a barium platinocyanide plate. The plate behind the object becomes fluorescent when exposed to x-rays, but the object's density will hinder the ability of low-frequency x-rays to penetrate it. The obstructed path areas appear darker because only a smaller proportion of x-rays were able to pass through and illuminate the plate in that location. The image left behind is a shadow of sorts, whose darkness is characterized by the density of the object casting it. Close observers may notice that modern x-ray-created images show dense matter as white, and transparent regions as black, which is the opposite of Röntgen's image. Modern x-ray imaging uses a film impregnated with silver halide crystal, which reacts chemically with the film if exposed to x-rays [1]. This chemical reaction turns the film black. Though the image may appear a different color, modern x-ray imaging still depends on the specific energy and size of x-rays predicted by the Duane-Hunt Law as discussed later.

This lab investigates the form of x-ray generation that Röntgen was using, which is now known as bremsstrahlung. The name translates roughly to breaking radiation, which describes the process fairly well for just one word. The electric current of the cathode ray is actually a stream of electrons flowing in a current. Electrons are massive particles with a negative electric charge. When this stream of electrons comes near a material object, the electric charge and masses of the individual nuclei that make up the object interact with the electrons. This process is not completely understood, but

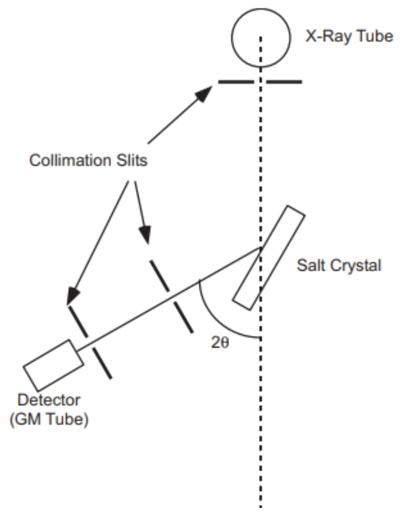

FIG. 2. Schematic drawing of lab setup. The x-ray tube contains a cathode and copper anode separated by a small gap of free space. A potential difference can be applied across this gap to force a current of electrons to slam into the anode. The interactions between these electrons and the anode create bremsstrahlung x-rays. The collimation slit channels the x-rays toward the diffraction crystal. The crystal is oriented at a fixed angle on a rotating platform, which is attached to the detector arm. The detector arm rotates at a two-to-one ratio with the diffraction crystal. This is designed so that the detector arm is always positioned at an angle that is exactly twice the angle of incidence that the x-rays make with the crystal [4].

its consequences can be predicted using an energy conservation argument. As an individual electron passes a nucleus, it is deflected by the charge and mass interactions between it and that nucleus. The nucleus absorbs the change in momentum of the electron from the charge and mass interactions, which upholds the principle of energy conservation. However, energy conservation also requires an accelerating charge to give off energy. The rapid acceleration change of the electron causes it to emit a high-energy photon. This high-energy photon is what we call a bremsstrahlung x-ray. The current consists of many electrons, and no two electrons will be traveling in exactly the same manner toward any of the given nuclei. Again, the *exact* behavior of an individual electron is not understood, but this is not necessary for large-scale observations. The different behavior of individual electrons suggests that some electrons will experience more change in acceleration than others, and different frequency photons will be emitted as a result. One can expect a nearly continuous spectrum of frequencies by measuring all of these photons at a macro level [5].

The first experiment of this lab seeks to empirically demonstrate that bremsstrahlung produces a nearly continuous spectrum of electromagnetic radiation. This requires a setup that can both produce x-rays as well as sort them by frequency. This particular experiment makes use of a Teletron Tel-X-ometer. This device contains a small x-ray tube itself containing a cathode and copper anode. The device also contains a diffraction crystal and a Geiger counter detector arm connected by a rotational assembly. This rotational assembly assures that the detector arm is always at an angle of twice the angle of incidence of the diffraction crystal. FIG 2 shows a schematic diagram of the Teletron Tel-X-ometer. The experiment begins by selecting a desired potential difference across the cathode-anode gap in the x-ray tube. Inside the tube, a current of electrons flows from the cathode on a collision course with the anode, which produces a spectrum of bremsstrahlung x-rays. These x-rays radiate outward in all directions, but some will pass through the collimation slit and be directed toward the diffraction crystal. As the spectrum of x-rays passes through the crystal, each frequency will diffract at a unique angle described by the Bragg diffraction constructive interference condition equation 1.

$$n\lambda = 2d \sin\theta \quad (1)$$

The inter-atomic spacing ”d”, is not changed during the experiment, and ”n” simply represents a whole number interval. The factor of two is a consequence of the specific geometry of the detector arm’s rotational assembly [4]. The detector arm can pivot through a wide range of angles, which allows one to measure the count rate of x-rays at any specific angle. Since each angle will produce a unique wavelength by equation 1, each count rate measurement will represent something proportional

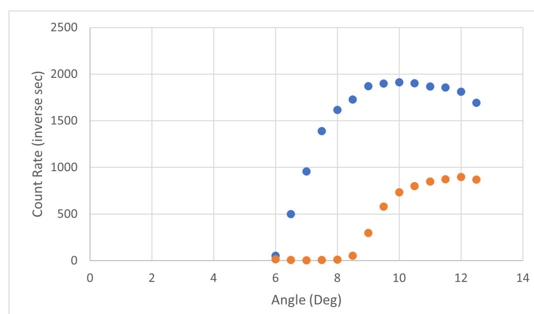

FIG. 3. 30kV (blue) and 20 kV (Orange) bremsstrahlung x-ray spectra. These spectra are represented in terms of diffraction scattering angle vs counts per time. Due to the Bragg diffraction constructive interference condition equation 1, the diffraction crystal will separate out x-ray wavelengths by scattering angle. By taking measurements at a specific angle one is measuring a value proportional to a specific wavelength. The counts per time are proportional to the intensity or number of photons with a specific wavelength per area per time at any given angle. Both lines have a sharp cut-off at the x-axis, which is known as the cut-off wavelength. The 20kV (orange) line plateaus on the x-axis because the detector picks up background radiation at wavelength values smaller than the bremsstrahlung x-rays.

to the relative intensity of that particular x-ray wavelength. This gives an idea of what proportion of photons of a specific wavelength are radiated per unit of time. Graphing each measurement for a range of angles then should produce a continuous spectrum with relative intensity. FIG 3 shows the results for a 20kV and 30kV potential difference in the x-ray tube.

Just as predicted, we see a nearly continuous spectrum for both lines. One can also observe a sharp cut-off in both lines at the left side limits. This cut-off is known as the cut-off wavelength, and it represents the minimum wavelength given off by a decelerating electron in the anode. Physically, that is the maximum acceleration possible, or an electron being slowed down so much that it loses all its velocity. Energy conservation suggests an electron cannot lose more energy than it had to begin with, so this is the maximum amount of energy any one electron can give off. The photon emitted from this event will be the most energetic (i.e. shortest wavelength) photon possible given the circumstances [5]. This wavelength size is directly linked to the strange behavior that Röntgen noticed in 1895. The interaction properties of electromagnetic radiation and matter are governed by the size of the wavelength [5]. The cut-off wavelength gives an idea of the limit on minimum size for bremsstrahlung x-rays, but what about this size is so special?

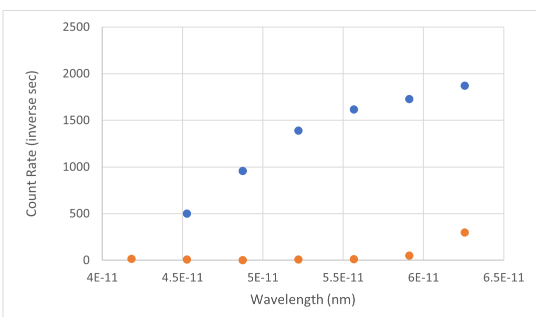

FIG. 4. The Bragg diffraction equation for constructive interference 1, suggests that the wavelength of the measured x-ray will be proportional to two times the inter-atomic spacing times the sine of the angle of incidence for the $n = 1$ condition. The inter-atomic spacing of the diffraction crystal used is $2.01 \times 10^{-10}m$ or $0.2nm$. Composing the domain of FIG 3 for theta gives the domain back in terms of wavelengths. The domain of this graph is the result of this transformation. The plot area is focused on the intercept values to showcase the cut-off wavelength.

Investigating this question will require some strategic analysis. The energy of a photon is equal to Planck's constant times the frequency of that photon [5]:

$$E = h\nu \tag{2}$$

As mentioned before, photons carrying the cut-off wavelength must have been emitted from the electrons of the maximum kinetic energy decelerating to zero kinetic energy. The energy carried by these photons must then be $20kV \times e$ and $30kV \times e$ respectively for the 20kV and 30kV potential difference trials. Using this knowledge and the fact that frequency $\nu = \frac{c}{\lambda}$, one can rewrite equation 2 for the 20kV and 30kV trials as:

$$eV_0 = \frac{hc}{\lambda_{min}}$$

$$20kV \times 1.602 \times 10^{-19}C = h\frac{c}{\lambda_{min}} \tag{3}$$

$$30kV \times 1.602 \times 10^{-19}C = h\frac{c}{\lambda_{min}}$$

This equation is known as the Duane-Hunt Law [5]. It suggests that the size of the cut-off wavelength and the energy of an emitted photon are related through Planck's constant and the speed of light. These simple universal constants control the photon's characteristics, and by extension electromagnetic interaction with matter. Bremsstrahlung x-rays are generated by following this incredibly simple law. To prove this statement one can use Planck's constant and the known values of the potential difference to estimate the minimum cut-off wavelengths or use the empirical cut-off values and the known potential difference to estimate Planck's constant. Performing both of these operations demonstrates the universality of electromagnetic-matter interaction, and confirms the validity of the data collected during this experiment.

The uncertainty of the potential difference for this lab is five percent [4], and the excepted value for Planck's constant is $6.626 \times 10^{-34}m^2kg/s$ [5]. Solving equation 3 for λ_{min} gives:

$$\lambda_{min} = \frac{hc}{eV_0}$$

$$\lambda_{min} = \frac{6.626 \times 10^{-34}m^2kg/s \times 3 \times 10^8 m/s}{20kV \times 1.602 \times 10^{-19}C} \tag{4}$$

$$\lambda_{min} = \frac{6.626 \times 10^{-34}m^2kg/s \times 3 \times 10^8 m/s}{30kV \times 1.602 \times 10^{-19}C}$$

Given a five percent uncertainty for the voltage and taking all other terms to be constant yields an uncertainty for the cut-off wavelength of:

$$(\frac{\sigma_{\lambda_{min}}}{\lambda_{min}})^2 = (\frac{\sigma_V}{V})^2$$

$$\sigma_{\lambda_{min}} = \sqrt{(\frac{20kV \times 0.05}{20kV})^2 \times \lambda_{min}^2} \quad (5)$$

$$\sigma_{\lambda_{min}} = \sqrt{(\frac{30kV \times 0.05}{30kV})^2 \times \lambda_{min}^2}$$

This yields $6.204 \times 10^{-11}m \pm 3.102 \times 10^{-12}m$ and $4.136 \times 10^{-11}m \pm 2.068 \times 10^{-12}m$ for the 20kV and 30kV trails respectively. FIG 4 performs an affine transformation on the x-axis of FIG 3 to produce units of wavelength via equation 1. This transformation preserves the shape of the curves in FIG 3. The plot area focuses on the intercept values, which are the cut-off wavelengths. Close examination shows that the 20kV line has measurements beyond the cut-off point. These are due to background radiation being picked up by the detector and are not relevant to this analysis. The Duane-Hunt predicted values seem to coincide nicely with the intercept values of FIG 4. The best way to test whether or not the empirical values are reasonable is to compare them to these predicted values with their respective uncertainty.

A safe estimate for the intercept values of FIG 4 is $5.95 \times 10^{-11}m$ and $4.18 \times 10^{-11}m$ for 20kV and 30kV respectively. These estimates are the result of tracing a linear estimate by hand near the intercept values. Both of these values fall within the Duane-Hunt predicted value uncertainties of equation 5. So Planck's constant successfully predicts the wavelengths of bremsstrahlung x-ray photons. Now the process can be reversed, using the uncertainties of the empirical values this time to estimate Plank's constant. This will be a more rigorous process because both the data values and the potential difference values have their own uncertainties. Solving equation 3 for Planck's constant gives:

$$h = \frac{eV_0 \times \lambda_{min}}{c}$$

$$h = \frac{20kV \times 1.602 \times 10^{-19}C \times 5.95 \times 10^{-11}m}{3 \times 10^8 m/s} \quad (6)$$

$$h = \frac{30kV \times 1.602 \times 10^{-19}C \times 5.95 \times 10^{-11}m}{3 \times 10^8 m/s}$$

The uncertainties for the empirical data are $5.95 \times 10^{-11}m \pm 1.5 \times 10^{-12}$ and $4.18 \times 10^{-11}m \pm 1.5 \times 10^{-12}m$ for 20kV and 30kV respectively. Again these are the result of tracing a linear estimate by hand and estimating the uncertainty for this process. The uncertainty for the potential difference is still five percent. This yields a total uncertainty for Planck's constant of:

$$(\frac{\sigma_h}{h})^2 = (\frac{\sigma_V}{V})^2 \times (\frac{\sigma_{\lambda min}}{\lambda_{min}})^2$$

$$\sigma_h = \sqrt{(\frac{20kV \times 0.05}{20kV})^2 + (\frac{1.5 \times 10^{-12}m}{5.95 \times 10^{-11}m})^2 \times (h^2)} \quad (7)$$

$$\sigma_h = \sqrt{(\frac{30kV \times 0.05}{30kV})^2 + (\frac{1.5 \times 10^{-12}m}{4.18 \times 10^{-11}m})^2 \times (h^2)}$$

Finally one arrives at $6.355 \times 10^{-34}m^2kg/s \pm 3.558 \times 10^{-35}m^2kg/s$ and $6.696 \times 10^{-34}m^2kg/s \pm 4.121 \times 10^{-35}m^2kg/s$ for the 20kV and 30kV trials respectively. With the excepted value of Planck's constant being, $6.626 \times 10^{-34}m^2kg/s$ both trials predict the constant within uncertainty. The accuracy of this result is incredible given its relative simplicity. The properties that Röntgen witnessed in 1895 are built into the structure of the universe. Planck's fundamental constant can be used to predict wavelengths, as well as be derived from empirical evidence. This special value determines photon characteristics and thereby controls the behavior of electromagnetic interactions with matter.

The special properties of high-energy electromagnetic radiation allowed Röntgen to create the x-ray image shown in FIG 1. The Duane-Hunt Law of equation 3 predicts the energy, wavelength, and limit of continuity that are responsible for Röntgen's image. The subtle gradient in transparencies that make up the outline of the hand is only possible because of the continuous spectrum. The density of a given portion of the hand stops x-rays up to a certain energy, while higher energies will still pass through. The x-rays with enough energy to pass through dense regions will have shorter wavelengths by the Duane-Hunt Law. These short wavelengths allow the x-rays to still produce fluorescence, but not like the bright white that the whole spectrum would produce. Finally, the regions of pure darkness lie behind matter so dense that the energy required to pass through is beyond that which the cut-off wavelength is capable of producing.

[1] https://www.glaciermedicaled.com/courses/301/10_Darkroom.htm
[2] https://www.nobelprize.org/prizes/physics/1901/rontgen/biographical/
[3] https://collection.sciencemuseumgroup.org.uk/objects/co134691/photograph-of-a-radiograph-of-hand-taken-by-wilhelm-conrad-rontgen-germany -1895-photograph
[4] J. McCoy, C. Conover, D. Tate, L. Lessard. Bremsstrahlung and X-Ray Diffraction. (Lab Manual) (2022)
[5] R. Eisberg and R. Resnick, Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles (John Wiley and Sons, New York, 1985)