

ADAMS-NOVIKOV CHARTS

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ABSTRACT. This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 90-stem.

This document contains large-format Adams-Novikov charts that compute the classical 2-complete stable homotopy groups. The charts are essentially complete through the 90-stem.

The charts are intended to be viewed electronically. The authors can supply versions that are suitable for printing.

Justifications for these computations appear in [1] and [3]. In essence, they are entirely determined by an analysis of the $\mathbb{C}$ -motivic Adams spectral sequences for the sphere spectrum and for the cofiber of τ . See also [4] and [5] for computations in a smaller range.

This document supersedes [2].

1. v_1 -PERIODIC ELEMENTS IN THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the v_1 -periodic part of the Adams-Novikov E_2 -page through the 110-stem. Traditionally, these elements have names involving the symbol α , but our notation for elements is incompatible with the Greek letter system. Rather, our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ .

We have separated the v_1 -periodic elements from the main Adams-Novikov E_2 -page for legibility, since they are entirely understood.

- (1) Solid dots indicate copies of $\mathbb{Z}/2$.
- (2) Open circles indicate copies of $\mathbb{Z}/2^k$, for some $k \geq 2$. The value of k is shown next to each circle.
- (3) Lines of slope 1 indicate h_1 multiplications.
- (4) Arrows of slope 1 indicate infinitely many h_1 multiplications.
- (5) Lines of slope $1/3$ indicate h_2 multiplications.
- (6) Blue lines of slope -3 indicate Adams-Novikov d_3 differentials.

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2. v_1 -PERIODIC ELEMENTS IN THE ADAMS-NOVIKOV E_∞ -PAGE

This chart shows the v_1 -periodic part of the Adams-Novikov E_2 -page through the 110-stem. Traditionally, these elements have names involving the symbol α , but our notation for elements is incompatible with the Greek letter system. Rather, our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ .

We have separated the v_1 -periodic elements from the main Adams-Novikov E_∞ -page for legibility, since they are entirely understood.

See Section 1 for instructions on interpreting the chart. In addition:

- (1) Green lines indicate hidden 2 extensions.

3. E_2 -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov E_2 -page through the 110-stem, excluding the v_1 -periodic elements. Our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ . Our notation is incompatible with the Greek letter system.

- (1) Black dots indicate copies of $\mathbb{Z}/2$.
- (2) Red dots indicate copies of $\mathbb{Z}/4$.
- (3) Blue dots indicate copies of $\mathbb{Z}/8$.
- (4) Green dots indicate copies of $\mathbb{Z}/16$.
- (5) Purple dots indicate copies of $\mathbb{Z}/32$.
- (6) Lines of slope 1 indicate h_1 multiplications.
- (7) Lines of slope $1/3$ indicate h_2 multiplications.
- (8) Magenta lines indicate that an extension equals 2 times a generator. For example, $h_2 \cdot h_2 d_0$ equals $2 \cdot h_0 g$ in the 20-stem.
- (9) With one exception, orange lines indicate that an extension equals 4 times a generator. For example, $h_1^3 \cdot \tau g$ equals $4 \cdot h_2 g$ in the 23-stem. The exception is that $h_2 \cdot \overline{P^2 h_1^2 h_6 c_0}$ equals $8 \cdot h_0^3 \cdot \Delta h_2^2 h_6$ in the 93-stem.

4. d_3 DIFFERENTIALS IN THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov d_3 differentials through the 90-stem, excluding the v_1 -periodic differentials. Our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ . Our notation is incompatible with the Greek letter system.

See Section 3 for instructions on interpreting the chart. In addition,

- (1) Blue lines of slope -3 indicate Adams-Novikov d_3 differentials.
- (2) Dashed blue lines indicate possible differentials.

5. E_4 -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov E_4 -page through the 90-stem, excluding the v_1 -periodic elements. The Adams-Novikov d_5 differentials are also indicated. Our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ . Our notation is incompatible with the Greek letter system.

- (1) Black dots indicate copies of $\mathbb{Z}/2$.
- (2) Red dots indicate copies of $\mathbb{Z}/4$.

- (3) Blue dots indicate copies of $\mathbb{Z}/8$.
- (4) Green dots indicate copies of $\mathbb{Z}/16$.
- (5) Lines of slope 1 indicate h_1 multiplications.
- (6) Lines of slope $1/3$ indicate h_2 multiplications.
- (7) Magenta lines indicate that an extension equals 2 times a generator. For example, $h_2 \cdot h_2 d_0$ equals $2 \cdot h_0 g$ in the 20-stem.
- (8) With one exception, orange lines indicate that an extension equals 4 times a generator. For example, $h_1^3 \cdot \tau g$ equals $4 \cdot h_2 g$ in the 23-stem. The exception is that $h_2 \cdot Ph_2 h_5$ equals $8 \cdot h_3^2 h_5$ in the 45-stem.
- (9) Blue lines of slope -5 indicate Adams-Novikov d_5 differentials.
- (10) Dashed blue lines indicate possible differentials.

For clarity, the chart also shows the possible d_3 differentials.

6. E_6 -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov E_6 -page through the 90-stem, excluding the v_1 -periodic elements. The Adams-Novikov d_r differentials for $r \geq 7$ are also indicated. Our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ . Our notation is incompatible with the Greek letter system.

See Section 5 for instructions on interpreting the chart. In addition,

- (1) Blue lines of indicate Adams-Novikov d_r differentials for $r \geq 7$.
- (2) Dashed blue lines indicate possible differentials.

For clarity, the chart also shows the possible d_3 and d_5 differentials.

7. THE E_∞ -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

This chart shows the Adams-Novikov E_∞ -page through the 90-stem, excluding the v_1 -periodic elements. Our names for elements are consistent with the names of elements in the $\mathbb{C}$ -motivic Adams spectral sequence for the cofiber of τ . Our notation is incompatible with the Greek letter system.

See Section 5 for instructions on interpreting the chart. In addition,

- (1) Red lines indicate hidden extensions by 2. The dashed red lines in the 54-stem indicate that there is a hidden 2 extension, but its target is not known precisely.
- (2) Blue lines indicate hidden extensions by η .
- (3) Green lines indicate hidden extensions by ν .
- (4) Dashed light blue lines indicate possible differentials.

Beyond the 64-stem, not all hidden extensions have been resolved; see [3] for more details.

For clarity, the chart also shows possible differentials as dashed lines.

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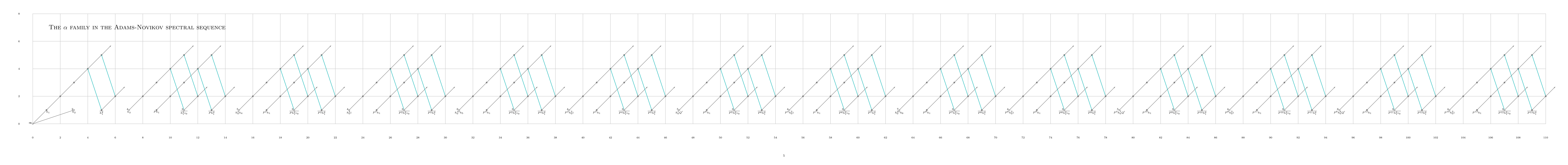
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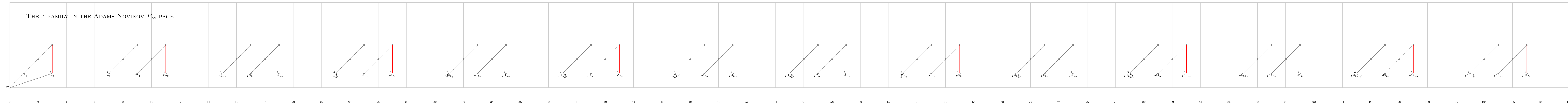
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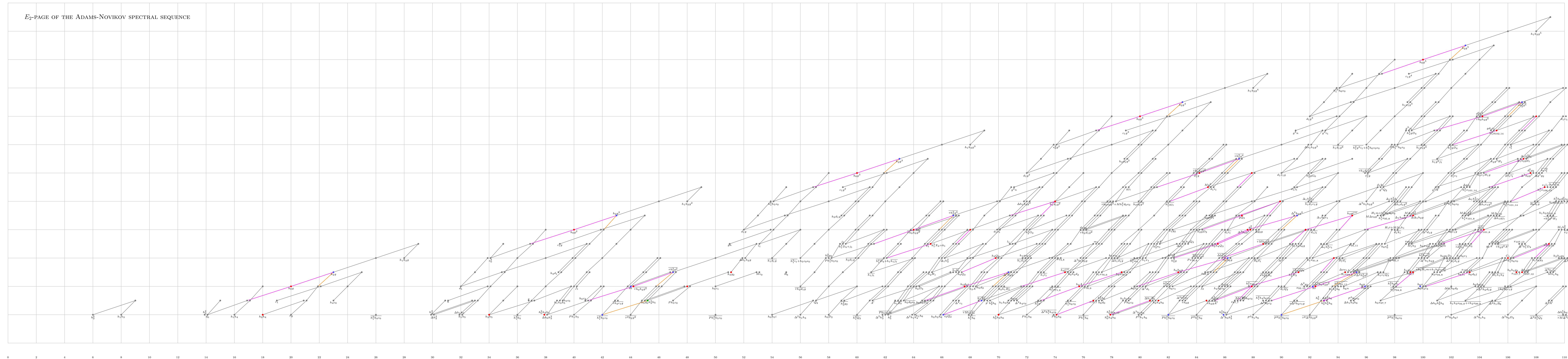
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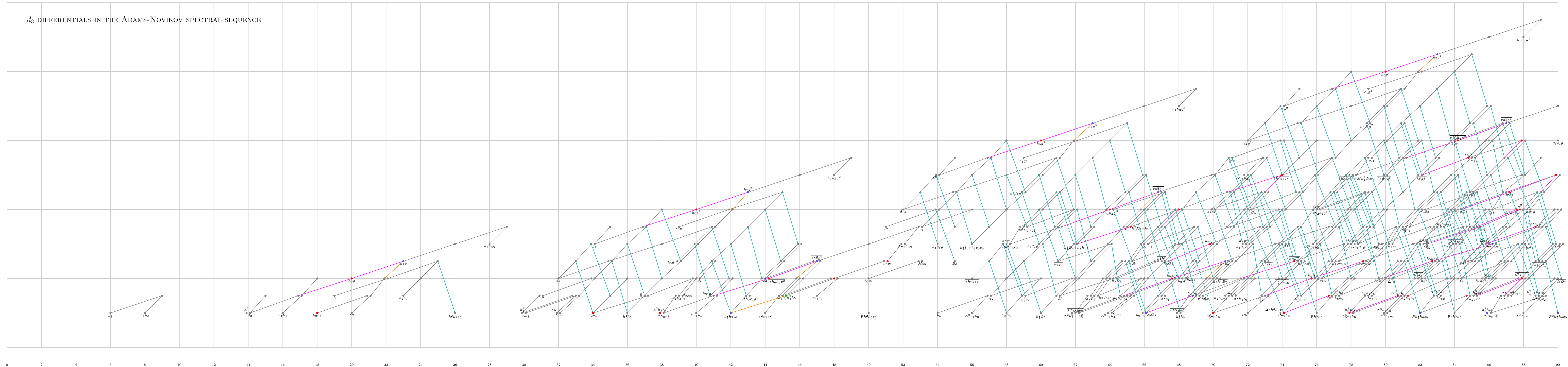
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THE α FAMILY IN THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

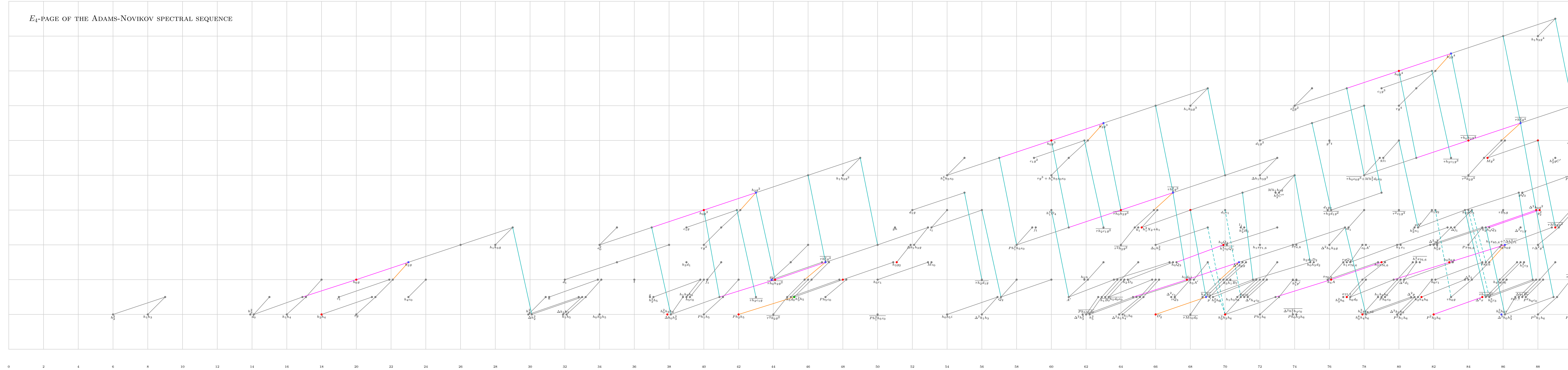








E_4 -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE



E_6 -PAGE OF THE ADAMS-NOVIKOV SPECTRAL SEQUENCE

