4.1 A: Polynomial Functions

Polynomial Functions : $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Coefficients : $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers

Exponents : $n, n-1, \ldots, 1, 0$ are whole numbers

$$P(x) = \underbrace{a_n x^n}_{\text{Leading Term}} + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Leading Coefficient $a_n \neq 0$

degree

$x^0 = 1$

Polynomial Function	Example	Degree	Leading Term	Leading Coefficient
Constant	$f(x) = 2$ $2x^0 = 2(1) = 2$	0	2	2
Linear	$f(x) = \frac{1}{3}x - 5$ $\frac{1}{3}x^1 - 5$	1	$\frac{1}{3}x$	$\frac{1}{3}$
Quadratic	$f(x) = 4x^2 - x^1 + 1$	2	$4x^2$	4
Cubic	$f(x) = x^3 + 2x^2 - x + 3$	3	x^3	1
Quartic	$f(x) = -x^4 + 2.1x^3 - 3x^2 + 0.1x + 10$	4	$-x^4$	-1

* ex) $f(x) = 0$ * $= 0x^{15}$ or $0x^0$ or $0x^3 \Rightarrow$ No degree

ex) $f(x) = \frac{2}{x} + 1$ $\left(\frac{2}{x} = 2x^{-1}\right)$ $f(x) = 2x^{-1} + 1$

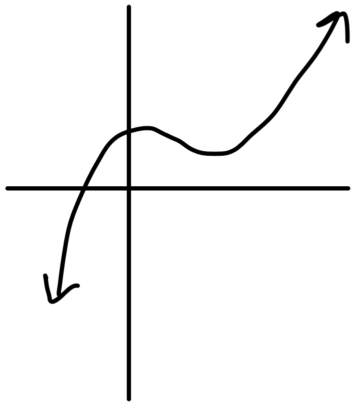
$\Rightarrow$ Not a polynomial (-1 is not a whole #)

ex) $f(x) = \sqrt{x} + 3$

$= x^{1/2} + 3 \Rightarrow$ Not a polynomial ($\frac{1}{2}$ is not a whole #)

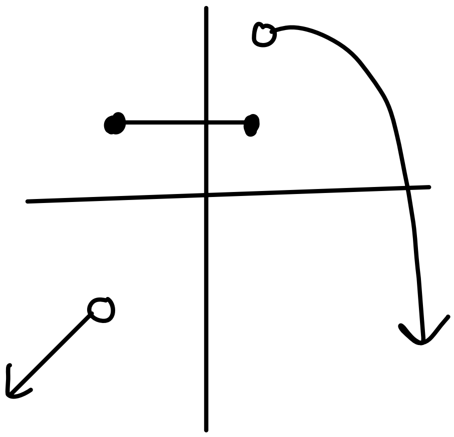
Polynomial Graphs are continuous

↳ No holes or breaks — smooth



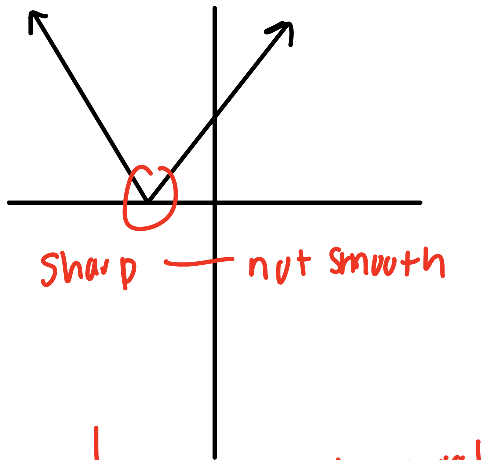
Continuous

↳ polynomial



Discontinuous

↳ not polynomial



↳ not polynomial

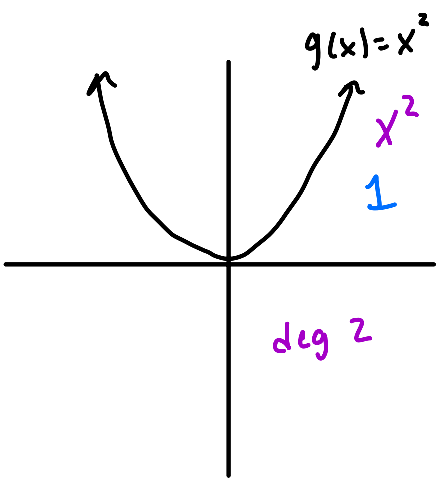
⟹ Domain of polynomial Functions: $(-\infty, \infty)$

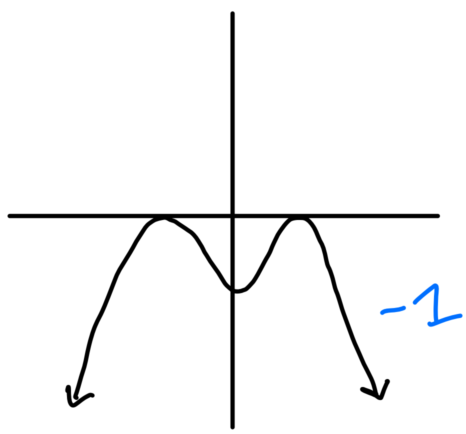
Leading Term Test:

Leading Term → end behavior (as $x \to -\infty$, $x \to +\infty$)

Leading Terms

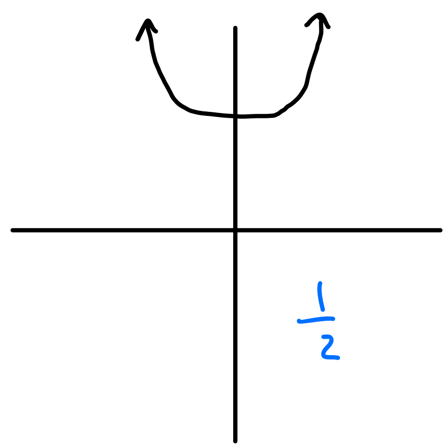
Even





$g(x) = -x^4 - 2x^3 + x - 1$

$-x^4$ deg 4

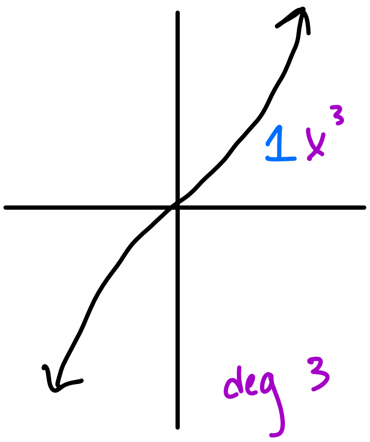


$g(x) = 3 + \frac{1}{2}x^6$

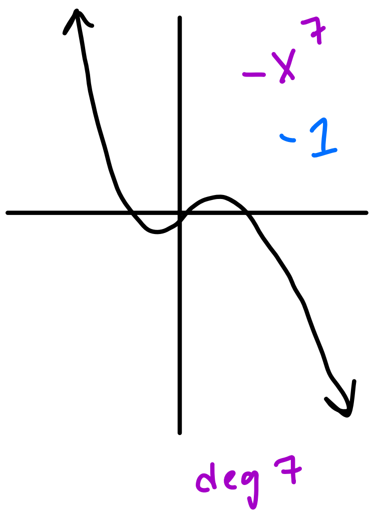
$\frac{1}{2}x^6$ deg 6

odd

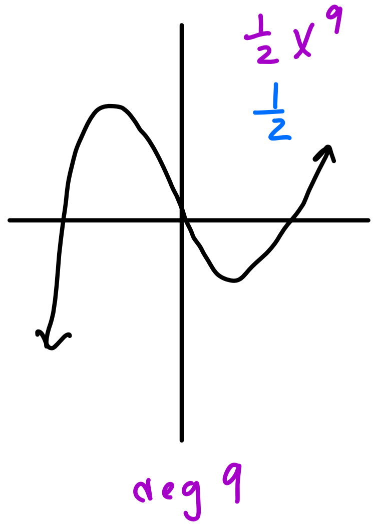
$f(x) = x^3$

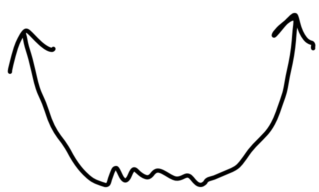
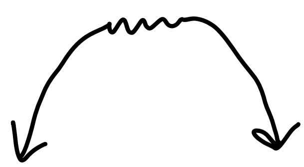
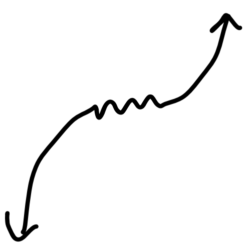
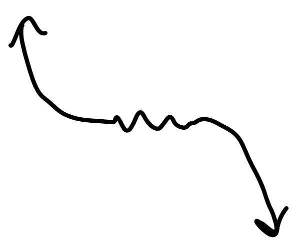


$f(x) = -x^7 - 2x^2$



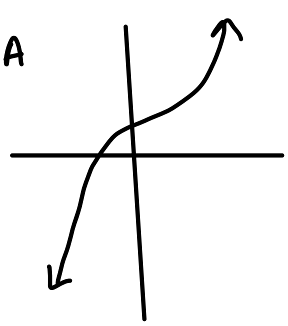
$f(x) = \frac{1}{2}x^9 - 20x + 1$



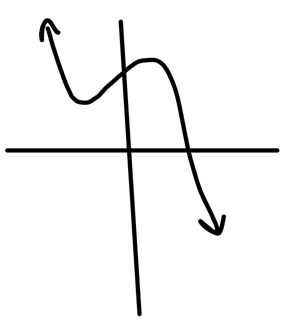
Degree	coefficient > 0	coefficient < 0
Even		
Odd		

Ex 1) Match each function with one of these graphs

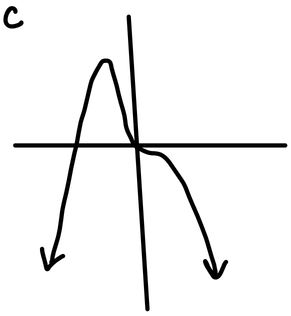
A



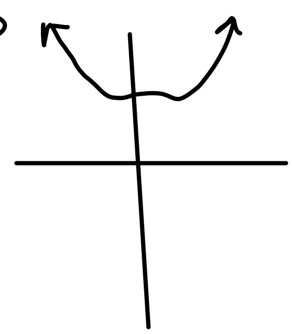
B



C



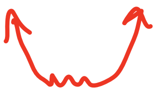
D



(a) $f(x) = 3x^4 - 2x^3 + 3$

$3x^4$ deg 4 → even

$3 > 0$

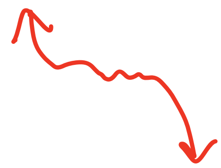


D

(b) $f(x) = -5x^3 - x^2 + 4x + 2$

$-5x^3$ deg 3 → odd

$-5 < 0$

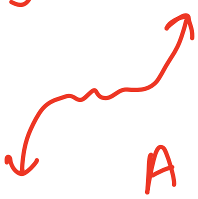


B

(c) $f(x) = x^5 + \frac{1}{4}x + 1$

x^5 deg 5 → odd

$1 > 0$



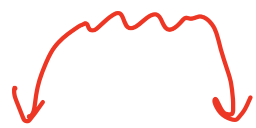
A

(d) $f(x) = -x^6 + x^5 - 4x^3$

C

$-x^6$ deg 6 → even

$-1 < 0$



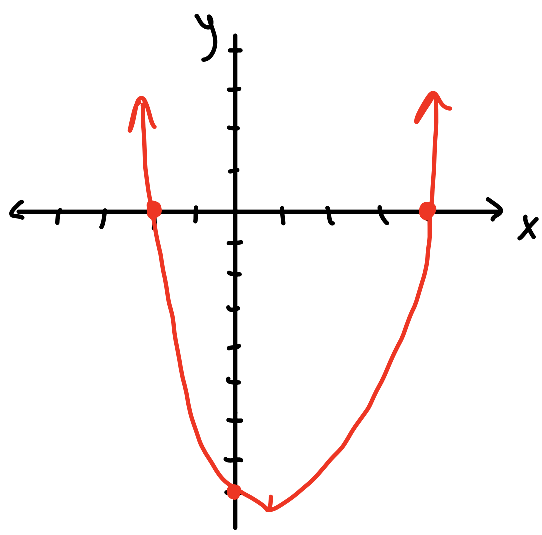
Zeros of Polynomials

ex) $g(x) = x^2 - 2x - 8$

$(x-4)(x+2) = 0$

$x = 4, \quad x = -2$

x-intercepts: $(4,0)$
$(-2,0)$



LT: x^2 → deg 2 = even

coeff: $1 > 0$ → opens up

y-int: $(0,-8)$

Ex 2) $P(x) = x^3 + x^2 - 17x + 15$

Determine whether $2, -5$ are zeros of $P(x)$

Plug in for x & see if we get 0

$x = 2$: $P(2) = (2)^3 + (2)^2 - 17(2) + 15$

$8 + 4 - 34 + 15 = -7 \neq 0$

so 2 is NOT a zero of $P(x)$

$x = -5$: $P(-5) = (-5)^3 + (-5)^2 - 17(-5) + 15$

$-125 + 25 + 85 + 15$

$-100 + 100 = 0$

so -5 is a zero of $P(x)$

Ex 3) Find the zeros of $f(x) = 5(x-2)^3(x+1)$ ← Factored Form

St Form: $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$

Set $f(x) = 0$: $5(x-2)^3(x+1) = 0$

$(x-2)^3(x+1) = 0$

$(x-2)^3 = 0$ $\quad$ $x + 1 = 0$

$\boxed{x = 2}$ $\quad$ $\boxed{x = -1}$

how often solution repeats

multiplicity of 3

$5(x-2)^3(x+1)^1$

Ex 4) Find zeros of $g(x) = -(x-1)^2(x+2)^2$ Factored Form

Set $g(x)=0$: $-(x-1)^2(x+2)^2 = 0$

$(x-1)^2(x+2)^2 = 0$

$x = 1$, $x = -2$

mult. 2 mult. 2

Ex 5) Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$

↳ standard Form

Set $f(x)=0$: $\underbrace{x^3-2x^2}\underbrace{-9x+18} = 0$

Factor by Grouping: $x^2(\underline{x-2}) - 9(\underline{x-2}) = 0$

$(x-2)(x^2-9) = 0$

$(x-2)(x+3)(x-3) = 0$ ← Factored Form

$$\boxed{\begin{array}{ccc} x=2 & x=-3 & x=3 \\ \text{mult } 1 & \text{mult } 1 & \text{mult } 1 \end{array}}$$

Ex 6) Find the zeros of $f(x) = x^4 + 4x^2 - 45$

↳ standard Form

$x^4 + 4x^2 - 45$

$(x^2)^2 + 4x^2 - 45$ $u = x^2$

$u^2 + 4u - 45 = 0$

-45

9 -5

4

$(u+9)(u-5) = 0$

$u = -9$, $u = 5$

$u = x^2 \Rightarrow \sqrt{u} = x$

$x = \pm\sqrt{-9}$ $x = \pm\sqrt{5}$ each w/ mult 1

$x = \pm 3i$

$(x^2+9)(x^2-5) = 0$

$\sqrt{x^2} = \sqrt{-9}$ $x^2 = 5$

$x = \pm 3i$ $x = \pm\sqrt{5}$

4.1 B: Polynomial Models

Ex 7) A gymnast dismounts the uneven parallel bars. Her height, h, in feet, depends on the time, t, in seconds, that she is in the air as follows: $h(t) = -16t^2 + 8t + 8$

(a) How high is she in the air after 0.25 seconds?

$t = 0.25 = \frac{1}{4}$ → plug in for t in $h(t)$

$$h\left(\tfrac{1}{4}\right) = -16\left(\tfrac{1}{4}\right)^2 + 8\left(\tfrac{1}{4}\right) + 8$$
$$= -16\left(\tfrac{1}{16}\right) + 2 + 8$$
$$= -1 + 10 = \boxed{9 \text{ ft}}$$

(b) $\underbrace{\text{How long}}_{\text{solve for } t}$ until the gymnast $\underbrace{\text{reaches the ground}}_{h(t)=0}$?

set $h(t) = 0$ & solve for t:

$$-16t^2 + 8t + 8 = 0$$
$$-8(2t^2 - t - 1) = 0$$
$$2t^2 - t - 1 = 0$$
$$\underbrace{2t^2 - 2t} + \underbrace{t - 1} = 0$$

ac
-2
-2
1
-1
b
+

$2t(t-1)+1(t-1)=0$

t = 1 second

$(t-1)(2t+1)=0$

$t-1=0 \qquad 2t+1=0$

$\boxed{t=1} \qquad 2t=-1$

$t=-\frac{1}{2}$ ← can't have negative time

1c) When will the gymnast be 8 ft above the ground?

set h(t) = 8 & solve for t:

$-16t^2+8t+8=8$

$\quad -8 \quad -8$

get into st. form

$-16t^2+8t=0$

$-8t(2t-1)=0$

$-8t=0 \qquad 2t-1=0$

$\boxed{t=0} \qquad \boxed{t=\frac{1}{2}}$