Recall: $a^m \cdot a^n = a^{m+n}$

Product Property:

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

↑ product ↑ sum

Ex 1) Express $\log_3(9 \cdot 27)$ as a sum of logarithms:

$$\log_3(9 \cdot 27) = \log_3(9) + \log_3(27)$$

↑ product

$3^? = 9$ $3^? = 27$

$2 + 3$

$= \boxed{5}$

Ex 2) Express $\log_2 p^3 + \log_2 q$ as a single logarithm:

$$\log_2 p^3 + \log_2 q = \log_2(p^3 q)$$

↑ sum of logs base 2 ↑ product

single log

Recall $(a^m)^n = a^{m \cdot n}$

Power Property:

$$\log_a(x^y) = y\log_a(x)$$

power → x^y ; product → $y\log_a(x)$

NOT: $(\log_a x)^y$

Ex 3) Express each as a product:

(a) $\log_a 11^{-3}$

$-3\log_a 11$

(b) $\log_a \sqrt[4]{7}$

$\log_a 7^{1/4}$

$\frac{1}{4}\log_a 7$

(c) $\ln x^6$

$6\ln x$

Recall: $\frac{a^m}{a^n} = a^{m-n}$

Quotient Property:

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

Quotient → $\frac{x}{y}$; Difference → $\log_a(x) - \log_a(y)$

Ex 4) Express $\log_t \frac{8}{w}$ as a difference of logs:

$$\log_t\left(\frac{8}{w}\right) = \log_t(8) - \log_t(w)$$

↑ quotient

Ex 5) Express as a single logarithm: $\log_b 64 - \log_b 16$

$$\log_b 64 - \log_b 16 = \log_b\left(\frac{64}{16}\right)$$

$$= \log_b(4)$$

Ex 6) Express as a sum of logs: $\log_b x^3 y^2 z$

$$\log_b(x^3 y^2 z) = \log_b(x^3) + \log_b(y^2) + \log_b(z)$$

↑ product property

$$= \boxed{3\log_b x + 2\log_b y + \log_b z}$$

↑ power property

Ex 7) Express as a product: $\ln \sqrt[3]{4}$

$$\ln 4^{1/3} = \frac{1}{3}\ln 4$$

↑ power property

Ex 8) Express as a single log & simplify:

$\log 10{,}000 - \log 100$

$\log 10{,}000 - \log 100 = \log\left(\frac{10{,}000}{100}\right)$

$= \log_{10}(100)$

$= 10^{?} = 100 \quad = \boxed{2}$

Ex 9) Express each of the following in terms of sums & differences of logs:

(a) $\log_a \frac{x^2 y^5}{z^4}$

Quotient Prop:

$\log_a (x^2 y^5) - \log_a (z^4)$

Product Prop:

$\log_a (x^2) + \log_a (y^5) - \log_a (z^4)$

Power Prop:

$2\log_a x + 5\log_a y - 4\log_a z$

(b) $\log_a \sqrt[3]{\frac{a^2 b}{c^5}}$

$\log_a \left(\frac{a^2 b}{c^5}\right)^{1/3}$

Power Prop: $\frac{1}{3}\log_a \left(\frac{a^2 b}{c^5}\right)$

Quotient Prop:

$\frac{1}{3}\left(\log_a (a^2 b) - \log_a c^5\right)$

Product Prop:

$\frac{1}{3}\left(\log_a (a^2) + \log_a b - \log_a c^5\right)$

Power Prop:

$\frac{1}{3}\left(2\log_a a + \log_a b - 5\log_a c\right)$

(c) $\log_b \frac{a y^5}{m^3 n^4}$

Quotient Prop:

$\log_b (a y^5) - \log_b (m^3 n^4)$

Product Prop:

$\log_b a + \log_b y^5 - (\log_b m^3 + \log_b n^4)$

Power Prop:

$\log_b a + 5\log_b y - (3\log_b m + 4\log_b n)$

$\log_b a + 5\log_b y - 3\log_b m - 4\log_b n$

$\log_a a$

$a^? = a$

1

$\frac{1}{3}(2 \cdot 1 + \log_a b - 5\log_a c)$

$\frac{2}{3} + \frac{1}{3}\log_a b - \frac{5}{3}\log_a c$

Ex 10) Express as a single logarithm:

$5\log_b x - \log_b y + \frac{1}{4}\log_b z$

product — difference — sum — product

Power Property: $\log_b x^5 - \log_b y + \log_b z^{1/4}$

Quotient Property: $\log_b \left(\frac{x^5}{y}\right) + \log_b z^{1/4}$

Product Property: $\log_b \left(\frac{x^5}{y} \cdot z^{1/4}\right) = \log_b \left(\frac{x^5 z^{1/4}}{y}\right)$

Ex 11) Express as a single log: $\ln(3x+1) - \ln(3x^2 - 5x - 2)$

Quotient Prop: $\ln\left(\frac{3x+1}{3x^2-5x-2}\right)$

$3x^2 - 5x - 2 = 3x^2 - 6x + x - 2$

$3x(x-2) + 1(x-2)$

$(x-2)(3x+1)$

ac: -6, $-6 \cdot 1$, -5; b: $+$

$\ln\left(\frac{3x+1}{(x-2)(3x+1)}\right)$

$= \ln\left(\frac{1}{x-2}\right)$

$\frac{1}{x-2} = (x-2)^{-1}$

$\ln(x-2)^{-1}$

$= -\ln(x-2)$

Ex 12) Simplify each of the following:

(a) $\log_a a^8$

$a^? = a^8$

$\boxed{8}$

(b) $\ln e^{-t}$

$\ln X = \log_e X$

$e^? = e^{-t}$

$\boxed{-t}$

(c) $\log 10^{3k}$

$\log_{10} 10^{3k}$

$10^? = 10^{3k}$

$= \boxed{3k}$

Ex 13) Given that $\log_a 2 \approx 0.301$ & $\log_a 3 \approx 0.477$, Find the following, if possible:

(a) $\log_a 6$

$= \log_a (2 \cdot 3)$ Product Prop

$= \log_a 2 + \log_a 3$

$\approx 0.301 + 0.477$

$= \boxed{0.778}$

(b) $\log_a \frac{2}{3}$

$= \log_a 2 - \log_a 3$

$\approx 0.301 - 0.477$

Quotient Prop

$\approx \boxed{-0.176}$

(c) $\log_a 81$ Product Prop ↓

$= \log_a (3 \cdot 3 \cdot 3 \cdot 3)$

$= \log_a (3^4)$

Power Prop

$= 4 \log_a 3$

$\approx 4(0.477)$

$= \boxed{1.908}$

(d) $\log_a \frac{1}{4}$

Quotient Prop

$\log_a 1 - \log_a 4$

↓

(e) $\log_a 5$

Product ?
Quotient ?
Power ?
} NO!

(F) $\frac{\log_a 3}{\log_a 2}$

quotient

$\log_a 4$

$-\log_a(2^2)$ power prop

$-2\log_a 2 \approx -2(0.301)$

$\approx \boxed{-0.602}$

$\log_a(3+2)$ ~~$\log_a 3 + \log_a 2$~~ can't simplify

$\log_a 5$

$\approx \frac{0.477}{0.301}$

$= \boxed{1.585}$

of logs NOT log of a quotient

Ex 14) Given $\log_7 2 \approx 0.356$. Find & simplify:

$\log_7 7 = 1$

(a) $\log_7 8$

$= \log_7(2^3)$ power prop

$= 3\log_7 2$

$\approx 3(0.356) \approx \boxed{1.068}$

(b) $\log_7 14$

$= \log_7(2 \cdot 7)$ product prop

$= \log_7 2 + \log_7 7$

$\approx 0.356 + 1$

$\approx \boxed{1.356}$