PRINT

- Roster
- Quiz 1 + Key
- Outline

Outline

- Quiz 1 (20 min) 4:30-4:50
- Activity 4
  - Individual : # 1 (warm-up) (2 min) 4:50-4:54
    Go over (2 min)
  - Indeterminants $\frac{0}{0}$, $\frac{\infty}{\infty}$ (6 min) 4:54 - 5
    + Steps
  - Together : # 2 (5 min) 5 - 5:05
  - In Groups : # 3 (3 min) 5:05 - 5:10
    Go over (2 min)
  - Together : # 8 (10 min) 5:10 - 5:20
    5 min break (5 min) 5:20 - 5:25
  - Together : # 13 (10 min) 5:25 - 5:35
  - In Groups : # 11, 12 (15 min) 5:35 - 6:05
    Go over # 11, 12 (15 min)

  IF time : # 6, 9

  Assigned : # 4, 5, 7, 10

Warm-Up

Work on warm-up individually (2.5 min)
Go over as class

S1.5 | Limits | common Sense & the Indeterminate

Instructions: Approximate the limit by using proper notation & Intuition to justify your answer.

1. Suppose f is defines as:

$$let\ f(x) = \frac{2^x - 1}{x+1}$$

$$Find\ \lim_{x\to 1} f(x)$$

As $x \to 1$, $\frac{2^x - 1}{x+1} \to \frac{2^1 - 1}{1+1} = \frac{1}{2}$

$$\boxed{\lim_{x\to 1} f(x) = \frac{1}{2}}$$

we get # answer.
Not always &
Not always right away...

What Do we mean?

$\frac{\#}{\#} = \#$:)

$\frac{0}{\#} = 0$:)

$\frac{\#}{0} = ??$

We need to be careful: remember not actually 0 but approaching 0

↳ so denominator is just really small which means our answer is very large

$\frac{\#}{0} = \pm \infty$

Ex) Let's consider $f(x) = \frac{1}{x}$. Find $\lim_{x \to 0} \frac{1}{x}$.

As $x \to 0$, $\frac{1}{x} \to \frac{1}{0}$

So denominator getting close to 0

↳ we are dividing by a really small #

but tricky: is it small (+) # or small (−) # ??

Need to take limit from both sides:

As $x \to 0^+$, $\frac{1}{x} \to \frac{1}{0^+} \to +\infty$

(think: $\frac{1}{0.00001}$ = big #)

So, $\lim_{x \to 0^+} f(x) = +\infty$

As $x \to 0^-$, $\frac{1}{x} \to \frac{1}{0^-} \to -\infty$

(think: $\frac{1}{-0.00001}$ = big (−) #)

So, $\lim_{x \to 0^-} f(x) = -\infty$

$\neq$

$\Rightarrow \lim_{x \to 0} f(x) = \text{DNE}$

$\frac{0}{\#} = \#$, $\frac{\#}{0} = \pm\infty$, but what about $\frac{0}{0}$??

Indeterminates

$\frac{0}{0}$, $\frac{\infty}{\infty}$

Steps

1. use common sense approach.
   - If you get #, 0, or $\pm\infty$: done :)
   - If you get indeterminate → step 2.
2. Do some Algebra : - Factor
   - conjugate
   - Absolute value (Piecewise function)
3. Try common sense approach again
   - If you get #, 0, or $\pm\infty$: done :)
   - If you get indeterminate → repeat.

2. Suppose f is defines as: Together as a class

$$let\ f(x) = \frac{x^2-4}{x-2}$$

$$Find\ \lim_{x\to 2} f(x)$$

(1) As $x \to 2$, $\frac{x^2-4}{x-2} \begin{matrix}\to \\ \to\end{matrix} \frac{0}{0}$ ∴ indeterminate

(2) Alg: Factor $\frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = x+2$

(3) As $x \to 2$, $x+2 \to 4$

$$\boxed{\lim_{x\to 2} f(x) = 4}$$

3. Suppose f is defines as:

work in groups
Go over as a class

$$let\ f(x) = \frac{x^2+x-2}{x-1}$$

$$Find\ \lim_{x \to 1} f(x)$$

(1) As $x \to 1$, $\frac{x^2+x-2}{x-1} \begin{matrix} \to \\ \to \end{matrix} \frac{0}{0}$ indeterminate

(2) Algebra: $\frac{x^2+x-2}{x-1} = \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = x+2$

(3) As $x \to 1$, $x+2 \to 3$

$$\boxed{\lim_{x \to 1} f(x) = 3}$$

Together as a class

8. $\lim\limits_{x \to -5} \frac{x+5}{\sqrt{11-x}-4}$ Sq. root

(1) As $x \to -5$, $\frac{x+5}{\sqrt{11-x}-4} \begin{matrix} \to \\ \to \end{matrix} \frac{0}{0}$ Indeterminate

(2) Algebra: Conjugate $\frac{x+5}{\sqrt{11-x}-4} \cdot \frac{(\sqrt{11-x}+4)}{(\sqrt{11-x}+4)}$

$$= \frac{(x+5)(\sqrt{11-x}+4)}{11-x-16} = \frac{(x+5)(\sqrt{11-x}+4)}{-x-5}$$

$$= \frac{\cancel{(x+5)}(\sqrt{11-x}+4)}{-\cancel{(x+5)}} = -(\sqrt{11-x}+4)$$

$$= -\sqrt{11-x}-4$$

(3) As $x \to -5$, $-\sqrt{11-x} - 4 \to -\sqrt{11-(-5)} - 4 = -\sqrt{16} - 4$
$= -4-4 = -8$

$$\boxed{\lim_{x\to -5} \frac{x+5}{\sqrt{11-x}-4} = -8}$$

Together as a class

13. $\lim\limits_{x\to 7^-} \dfrac{|7-x|}{x^2-x-42}$ Absolute value

(1) As $x \to 7^-$, $\dfrac{|7-x|}{x^2-x-42} \begin{matrix}\to\\ \to\end{matrix} \dfrac{0}{0}$ Indeterminate

(2) Algebra: Abs value $\to$ Piecewise

$$|7-x| = \begin{cases} 7-x & \text{if } 7-x \geq 0 \\ & x \leq 7 \\ -(7-x) & \text{if } 7-x < 0 \\ & x > 7 \end{cases}$$

$$\frac{|7-x|}{x^2-x-42} = \frac{7-x}{x^2-x-42}$$

(3) As $x \to 7^-$, $\dfrac{7-x}{x^2-x-42} \begin{matrix}\to\\ \to\end{matrix} \dfrac{0}{0}$ Indeterminate

(4) Algebra: Factor $\dfrac{7-x}{x^2-x-42} = \dfrac{7-x}{(x-7)(x+6)} = \dfrac{-(\cancel{x-7})}{(\cancel{x-7})(x+6)} = \dfrac{-1}{x+6}$

(5) As $x \to 7^-$, $\dfrac{-1}{x+6} \begin{matrix}\to\\ \to\end{matrix} \dfrac{-1}{13}$

$$\boxed{\lim_{x\to 7^-} \frac{|7-x|}{x^2-x-42} = \frac{-1}{13}}$$

11. $\lim\limits_{x \to 2^+} \dfrac{|x|-2}{x^2-4}$

12. $\lim\limits_{x \to 2^-} \dfrac{|x|-2}{x^2-4}$

Work in groups

Go over as a class

11) As $x \to 2^+$, $\dfrac{|x|-2}{x^2-4} \begin{matrix} \to \\ \to \end{matrix} \dfrac{0}{0}$ Indeterminate

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ $\qquad \dfrac{|x|-2}{x^2-4} = \dfrac{x-2}{x^2-4}$

As $x \to 2^+$, $\dfrac{x-2}{x^2-4} \to \dfrac{0}{0}$ Indeterminate

Factor $\dfrac{x-2}{x^2-4} = \dfrac{\cancel{x-2}^{\,1}}{\cancel{(x-2)}(x+2)} = \dfrac{1}{x+2}$

As $x \to 2^+$, $\dfrac{1}{x+2} \begin{matrix} \to \\ \to \end{matrix} \dfrac{1}{4}$ $\qquad \boxed{\lim\limits_{x \to 2^+} \dfrac{|x|-2}{x^2-4} = \dfrac{1}{4}}$

12) As $x \to 2^-$, $\dfrac{|x|-2}{x^2-4} \begin{matrix} \to \\ \to \end{matrix} \dfrac{0}{0}$ Indeterminate

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ $\qquad \dfrac{|x|-2}{x^2-4} = \dfrac{x-2}{x^2-4} = \dfrac{\cancel{x-2}^{\,1}}{\cancel{(x-2)}(x+2)} = \dfrac{1}{x+2}$

As $x \to 2^+$, $\dfrac{1}{x+2} \begin{matrix} \to \\ \to \end{matrix} \dfrac{1}{4}$ $\qquad \boxed{\lim\limits_{x \to 2^-} \dfrac{|x|-2}{x^2-4} = \dfrac{1}{4}}$

(11) & (12) : $\lim\limits_{x \to 2} \dfrac{|x|-2}{x^2-4} = \dfrac{1}{4}$

6. $\lim_{x\to 3} \frac{x^2-9x+18}{x^2-2x-3}$

In groups (if time)

(1) As $x \to 3$, $\frac{x^2-9x+18}{x^2-2x-3} \begin{matrix} \to \\ \to \end{matrix} \frac{0}{0}$ indeterminate

(2) $\frac{x^2-9x+18}{x^2-2x-3} = \frac{(x-6)\cancel{(x-3)}}{(x+1)\cancel{(x-3)}} = \frac{x-6}{x+1}$

(3) As $x \to 3$, $\frac{x-6}{x+1} \begin{matrix} \to \\ \to \end{matrix} \frac{-3}{4}$

$$\boxed{\lim_{x\to 3} \frac{x^2-9x+18}{x^2-2x-3} = \frac{-3}{4}}$$

9. $\lim_{x\to 2^+} \frac{|x-2|-3}{x-5}$

As $x \to 2^+$, $\frac{|x-2|-3}{x-5} \begin{matrix} \to \\ \to \end{matrix} \frac{-3}{-3} = 1$ # done :)

$$\boxed{\lim_{x\to 2^+} \frac{|x-2|-3}{x-5} = 1}$$