3.2 The Shape of the Earth *(Jonathan W. Chipman, December 2022)*

Geodesy is the science of measurement of the size and shape of the Earth. It provides an essential foundation for the fields of cartography and geovisualization. As measurement technologies have improved, our model of the Earth's shape has evolved. This shape can be described with increasing precision and increasing complexity, first as a sphere, then more precisely as a flattened sphere or ellipsoid, and finally as an irregular surface that diverges from the ellipsoid due to local anomalies in the Earth's gravitational field. Both the overall flattening and the finer-scale irregularities are small in comparison to the size of the Earth, but their effects are very important for surveying and mapping.

Humans have known, or hypothesized, that the Earth approximates a sphere for over 2000 years. Its exact size was more difficult to establish; one of the earliest measurements, by the Greek geographer Eratosthenes, was remarkably accurate (Box 3.2.1), but many other pre-modern estimates were too small or too large. With the development of modern surveying methods in the 1700s, newly empowered geodesists undertook to more precisely establish the size and shape of the Earth, by measuring the length of arcs of meridians of longitude in Lapland, Peru, and France and Spain (Hoare 2005; Alder 2002). These estimates were slowly refined from 1800 to the mid-20th century. After 1950, new technologies such as satellite radar altimeters, satellite gravimetry missions, and exponential increases in computing power led to the development of the new discipline of satellite geodesy and provided vastly more information about the exact shape of the Earth and how it changes over time (Box 3.2.2).

Box 3.2.1 Early measurement of the Earth by Eratosthenes

Born in the Greek city of Cyrene (in modern Libya) in the 3rd century BCE, Eratosthenes was a pioneering geographer, as well as a mathematician, astronomer, and head of the Library of Alexandria.

To estimate the circumference of the Earth, Eratosthenes began by comparing the solar zenith angle on the northern summer solstice at Alexandria and Syene (modern Aswan, Egypt), finding that the angles differed by about 7° or 1/50th of the circumference of a circle. The linear distance (measured by a surveyor walking between the cities) was reported to be 5000 stadia or 788 km[1]. Multiplying the latter by 50 gave a circumference for the Earth of approximately 39,400 km, within 2% of the true value (40,008 km for a circumference crossing the poles). Sources of error include measurement of the zenith angle and distance, and the incorrect assumption that Alexandria was due north of Syene.

While this early and surprisingly accurate measurement of the shape of the Earth is his most widely remembered accomplishment, Eratosthenes made many other important contributions, including first usage of the term geography (geographika), in a three-volume work covering physical, mathematical, and human geography. He also provided the first known angular system of meridians and parallels for the Earth, ancestors of modern longitude and latitude.

Portrait courtesy Österreichische Nationalbibliothek/ europeana.eu.

[1] The length of the classical distance unit "stadion" or "stade" was poorly defined; the conversion used here is approximate.

Box 3.2.2 Gladys Mae (Brown) West calculates the geoid

During her 42-year career as a mathematician and project leader for the US Navy, Dr Gladys West analyzed satellite radar altimeter data to derive the shape of the Earth's geoid. She developed software to accomplish the complex calculations needed for this process, then was named Project Manager for the Seasat radar altimetry mission. She wrote the data processing standards for the subsequent Geosat mission. The geoid models derived from her work on these two altimeters provide the foundation for all modern satellite geodesy, and for the global positioning system (GPS) and other global navigation satellite systems (GNSSs).

Born in 1930 into a rural Virginia farming family, Dr West overcame extraordinary obstacles to pursue her career in mathematics and geodesy. As valedictorian of her high school, she was awarded a scholarship to Virginia State College, one of the US's historically Black colleges and universities (HBCUs), where she obtained Bachelors and Masters degrees in Mathematics. She was hired as a mathematician and programmer at the Dahlgren Naval Proving Ground, only the second Black woman to work there. Despite the extraordinary significance of her work on measuring and modeling the geoid from space, Dr West's role in the field was largely unrecognized outside the Navy until long after her retirement. She subsequently completed a PhD in Public Administration at Virginia Tech. In recent years, with increased understanding of her contributions to our knowledge of the shape of the Earth and the foundations of GPS/GNSS technology, Dr West has been the recipient of a wide range of awards and honors.

Gladys West at work at the Naval Proving Ground, Dahlgren, Virginia, 1985. Photo courtesy US Navy.

When seen from space, the Earth appears perfectly spherical. It has a mean radius of 6371 km and diameter of 12742 km (Moritz 2000). If the Earth were scaled down and floating in front of us with an apparent diameter of one meter, even the largest mountains and valleys would barely alter its smooth, spherical surface. On this one-meter globe, Mt Everest, the highest mountain on Earth, would be only 0.7 mm tall.

Because of its rotation, the Earth is slightly flattened at the poles. This flattening is about 1 part in 298 or 0.335% (Moritz 2000), still too small to be perceptible to the eye but large enough to affect navigation, surveying, and other activities dependent on precise measurement. The cause of this flattening is often referred to as "centrifugal force", but it is not a true force, merely the effect of inertia acting on a rotating body. This **polar flattening** was predicted by Isaac Newton and confirmed by long-distance surveying expeditions in the 18th and 19th centuries.

Geodesists model the Earth as an **ellipsoid** (or oblate spheroid), whose **polar axis** (or **minor axis**) is slightly shorter than its **equatorial axis** (or **major axis**). Figure 3.2.1 shows a cross-section through this ellipsoid, though it is not drawn to scale; the flattening is greatly exaggerated. The **semimajor axis** extends from the Earth's center to its surface at the equator, and the **semiminor axis** from the center to the north or south pole. Standard values for the lengths of the semi-axes are 6378 km for the semimajor (equatorial) axis, and 6357 km for the semiminor (polar) axis (Moritz 2000). The prefix "semi" reflects the fact that these are half the length of the full axes, and are thus analogous to the radius rather than the diameter of a sphere.

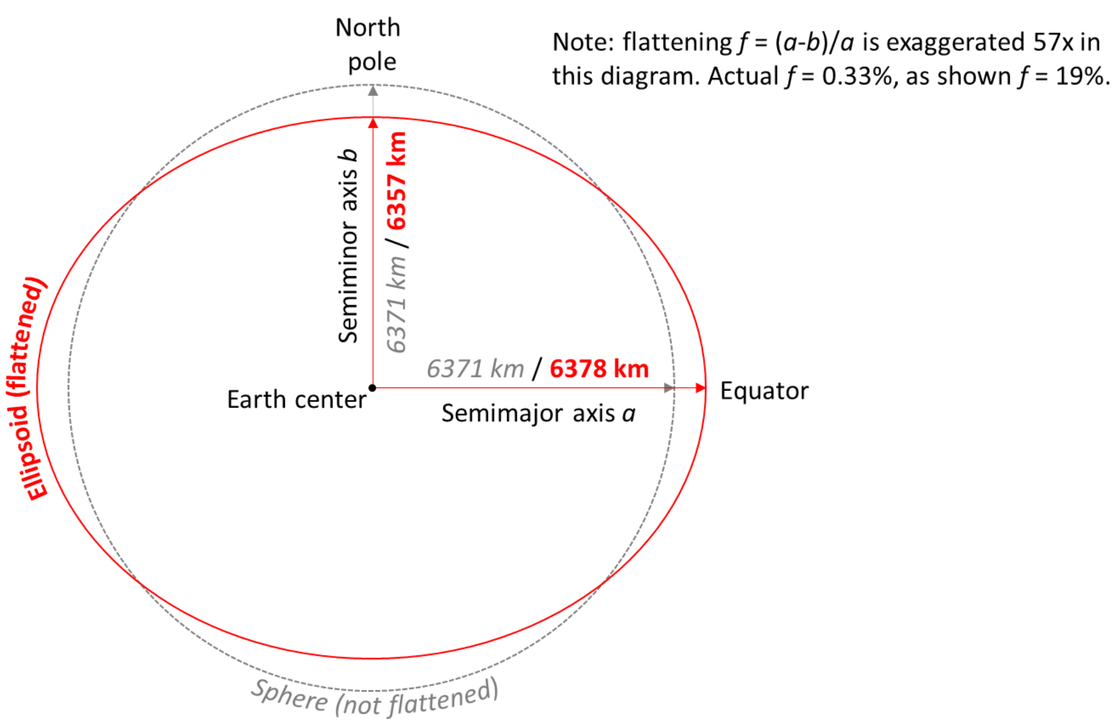

Figure 3.2.1 Cross-section through models of the Earth as a sphere (gray) and ellipsoid (red).

The ellipsoid flattening *f* can be calculated from the two semi-axes as shown in equation 3.2.1:

$$f = \frac{(a-b)}{a} \qquad (3.2.1)$$

where

f = flattening

a = semimajor axis (equatorial)

b = semiminor axis (polar)

Because of the irregular distribution of mass within the Earth, the actual shape of Earth is also irregular. This shape, called the **geoid**, is a surface of equal gravitational potential (an **equipotential surface**). It is the shape that the ocean surface would have if there were no land, winds, currents, tides, or differences in ocean temperature and salinity. One might imagine that this geoid – represented by sea level – would be a perfect sphere. If sea level is X meters from the center of the Earth at one location, surely it must also be X meters from the Earth's center everywhere else, or water would flow down from the higher point to the lower? However, the Earth's gravitational field is not uniform. For example, because continental crust is lighter than oceanic crust and mantle rock, the geoid tends to bulge outward over continents and inward over oceans. Within the continents (or ocean basins) there are also large regional variations, such as a large negative undulation near Sri Lanka where the geoid is pulled inward because the force of gravity is stronger, and a large positive undulation in the northeast Atlantic where slightly weaker local gravitational force causes the geoid to bulge outward.

Figure 3.2.2 shows one particular geoid model, the Earth Gravity Model 2008 (EGM2008). More precisely, it shows the difference between the geoid and the ellipsoid. Bulges and depressions in the geoid are greatly exaggerated in this illustration. Note that the color scale shows geoid/ellipsoid separations ranging from +90 to -110 **m**, a variation that would be visually imperceptible if not for vertical exaggeration, given the 6378 **km** semimajor axis of the ellipsoid.

Since it is impractical to measure the actual height of the geoid continuously over the entire Earth, geodesists develop models to represent the geoid. Just as the ellipsoid is a mathematical model of the flattening of the Earth due to its rotation, the geoid can be modeled mathematically in various ways, all of which are far more complex than the ellipsoid model. The EGM2008 geoid model shown in Figure 3.2.2 is based on **spherical harmonics**, a set of periodic functions on the surface of a sphere that interact to produce a more complex shape. This process is analogous to the use of Fourier analysis to model a complex signal using a set of waveforms with different frequencies.

At any location, the local force of gravity is perpendicular to the surface of the geoid. Using traditional surveying methods, measurements need to be corrected for deflection of the vertical so that they will be consistent with astronomical measurements (Robinson et al. 1984). Measurements made on the geoid are then transferred to the ellipsoid, which is the basis for the latitude/longitude coordinate system and all projected coordinate systems derived from it.

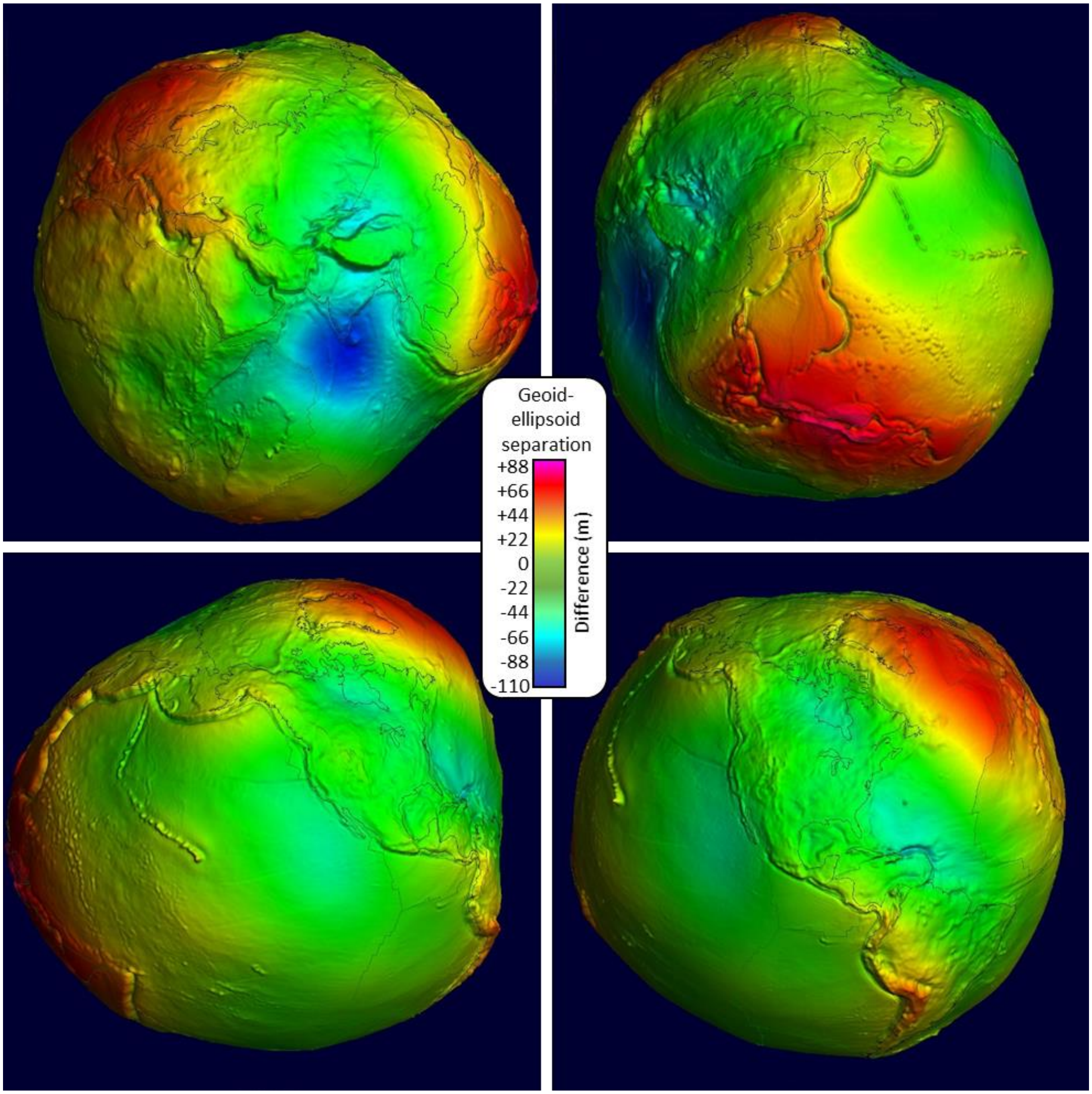

Figure 3.2.2 A model of the geoid (EGM2008) seen from four viewpoints around the globe. Color and distortion of shape both indicate geoid-ellipsoid separation. Land-water boundaries and tectonic plate boundaries faintly outlined. Models courtesy Ince et al. (2019).

References

Alder K. 2002. *The measure of all things: the seven-year odyssey and hidden error that transformed the world*. Free Press, New York, NY.

Hoare MR. 2005. *The quest for the true figure of the Earth: ideas and expeditions in four centuries of geodesy*. Ashgate Publishing Ltd, Aldershot, UK.

Ince ES, Barthelmes F, Reißland S, Elger K, Förste C, Flechtner F, Schuh H. 2019. ICGEM – 15 years of successful collection and distribution of global gravitational models, associated services and future plans. *Earth System Science Data*, 11, pp. 647-674, DOI: http://doi.org/10.5194/essd-11-647-2019.

Moritz H. 2000. Geodetic reference system 1980. *Journal of Geodesy*, 74(1), pp.128-133.

Robinson AH, Sale RD, Morrison JL, Muehrcke PC. 1984. *Elements of cartography*, 5th edition. John Wiley & Sons, New York, NY.