Adaptive Robot Navigation Protocol for Estimating Variable Terrain Elevation Data

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***Abstract*—Efficiently measuring environmental phenomena ($e.g.$, elevation, chemical composition, and mineral density) is a task typically reserved for the geoscience community. Recent robotic systems with the potential for addressing the task of sampling currently exist, yet their navigation strategies (and subsequently sampling strategies) are seldom a function of the spatial change in the measured phenomena of interest. Solutions are especially void for intelligent systems to which resource constraints are applied ($i.e.$, battery power and experimentation time) while complete coverage of an area is expected. In this paper, we discuss the implementation of a custom navigation strategy based on immediately-sensed data that, when combined with spatial interpolation techniques, yields a re-creation of the surveyed space with root mean squared error that meets accepted mapping standards. Our methodology employs an adaptive coverage algorithm which succeeds in lowering the RMS error when compared to other navigation techniques. Our results are validated in simulation by considering: 1) randomly-generated terrains and 2) realistic digital elevation map (DEM) data transposed from publically available terrain contour maps.**

***Index Terms*—Earth observing system (EOS), area under test (AUT), robotic survey system (RSS).**

I. Introduction

Providing more accurate information about the features of the Earth's surfaces including physical phenomena like elevation and mineral density is a prime application for robotic surveying. Furthermore, future federal mandates may require that Earth observing systems integrate data from more than just the remote sensing and static on-ground monitoring technology that are currently employed. Excerpts from a recent 2010 investigation by committees of the National Research Council of the National Academies report there is a strong need for a national geodetic infrastructure [1]. With that, comes a need to identify which technologies will further the acquisition of more accurate geodetic information. More specifically, the National Academies study investigates "geodetic observing systems" in the larger context of a geodetic infrastructure and includes sea level change monitoring, precision agriculture, civil surveying, earth quake monitoring, forest structural mapping and biomass estimation as areas of application [1]. Each of these areas requires the capabilities that advancements in mobile robotics offers. Our focus is in the navigation algorithms implemented to successfully acquire relevant geodetic information in the form of terrain elevation.

One current limitation that exists in measuring phenomena at different points of interest on the Earth is spatial resolution. While satellites, combined with high-performance sensing, can provide a global assessment of the change taking place at the surface of this planet (and others), scientists still require *in situ* validation of these measurements. Researchers in the various geospatial communities often project the need for this added perspective [2–4] while those in the robotics community have proposed solutions with varying degrees of success [5, 6]. Based on these advancements in robotics, a theory for how to systematically acquire this high-resolution information is best addressed through the task of sampling. In this paper, we introduce considerations salient to the development of such a sampling system and algorithms that best enable an area's successful spatial characterization.

II. Literature Review

It is the concept of sampling, specifically, around which we frame our robotic surveyor design. By combining knowledge from the sampling community with the progress made in robotic surveying, we are better prepared to outline the requirements for our system.

A. Sampling

Sampling is a necessity in many fields, including the geophysical sciences, due to the breadth of coverage required to estimate a particular phenomenon over a large area and the limited resources to achieve that coverage. From elevation to chemical concentrations in soil to mineral content or hazardous material levels in aquatic environments, there is no shortage of applications where sampling is not preferred over the costly alternative of exhaustive coverage. Inherent to this challenge of robotic surveying (and surveying in general) is the sampling problem. We refer to work by Ayeni and Wang *et al.* for a relevant discussion on the topic of sufficient sampling and appropriate schemes or patterns to apply across a space [7, 8]. Also relevant in their work is the topic of heterogeneous sample spaces. Wang connects the terms "non-stationary" and "spatial heterogeneity" to ground the discussion in the geosciences (see [8], p. 524, 527), providing the scope of the terrain types we consider in this paper. Ayeni emphasizes the importance of appropriately applying the right sample scheme to the right type of spatially distributed data.

B. Robotics

Example applications of robotics and sampling tend to fall under the category of "environmental monitoring", where there is a common interest in estimating either statically or dynamically changing phenomena in both time and space. Work by [9, 10] discuss a coordinated effort between robotic agents to characterize the phenomena of an unknown environment using strategies common to the sampling community ($i.e.$, raster scanning, simple random sampling, and stratefied random sampling). Low *et al.* do not consider alternative types of search spaces beyond those characterized by discrete concentrations [9] while Rahimi *et al.* do not discuss validation techniques to confirm their sampling methodology [10].

To the authors' knowledge, the earliest work pioneering the use of robotics to perform surveying tasks is by [11]. In the literature, a reigning theme is the value of several coverage patterns, quantified by a quality of performance (QoP) metric. Unfortunately, this metric is not designed to provide any measurement information about the sample space, only a relative measure of distance traveled by the agent. Another aspect of our approach, not considered by Tunstel *et al.*, is using interpolation methods to estimate data where samples could not be collected during a survey. This is important when evaluating both the resources required to sample an area as well as how those resources are allocated to achieve maximum coverage.

III. Approach

Our aim is to achieve complete coverage by way of sufficient sampling using effective navigation strategies. Regardless of the genre, current solutions from the robotics community are often left unpursued in hardware and practical implementation [12, 13]. Since efficient navigation to different sampling sites is a key objective of our work, the concept of coverage must be addressed.

A. Data Coverage

Traditionally, the success of a navigation application is measured by the total sequential (or Euclidean) distance traveled by an autonomous agent while executing a specific task. Some of the authors in the aforementioned work (see Section II-B) approach their various metrics with this in mind. As a sampling tool for scientists, the spatial importance of each sample collected can not be taken for granted, especially for data that is heterogeneous in its spatial distribution, $i.e.$, the types of information of greatest interest for our work (see Section II-A). The spatial heterogeneity characteristic of a sample space declares that the mean of the data collected is location dependent [8]. The traditional statistics ($e.g.$, N^{th}-order mean and variance) that a scientist infers from this information, therefore, will rely heavily on an evenly distributed sample set. We provide a spatially-relevant alternative for assessing coverage.

Instead of defining coverage as a function of the total distance traversed by an agent from one sample location to another, we define a science-centric type of coverage that is defined relative to the cumulative sum of distances from all possible sample locations to a reference location within the

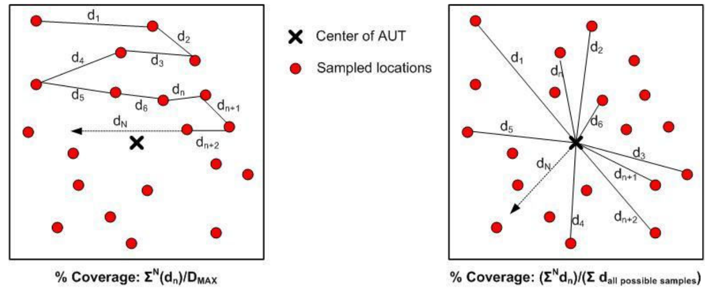


Fig. 1: Difference in percent coverage definition: Traditional (left), Non-Traditional (right).

area under test (AUT). Defining distance relative to a specific point emphasizes our goal of achieving distributed coverage through navigation and sampling (see Figure 1). This change in coverage definition improves upon the QoP metric, discussed in [11], as it places a theoretical upper bound on the number of possible samples reached by any adaptive navigation pattern considered and appends meaning to each sample acquired. As expressed in Equation 1, percent coverage is defined as a ratio of T_M, the sum of relative distances between actual samples, (x_m, y_m), and a reference location, (X_{ref}, Y_{ref}), to T_S, the sum of relative distances between all possible samples, (x_s, y_s), and that same reference, (X_{ref}, Y_{ref}).

$$\%Coverage = \frac{T_M}{T_S} = \frac{\sum_{m=1}^{M} \|(x_m, y_m) - (X_{ref}, Y_{ref})\|}{\sum_{s=1}^{S} \|(x_s, y_s) - (X_{ref}, Y_{ref})\|} \tag{1}$$

B. Navigation Methods

Given the expectations for our RSS (applicable environments and definitions of quality of coverage), we must consider specific approaches to navigating the AUT such that one approach's metrics yield acceptable performance over a comparable list of others. In the robotics community, exploration and coverage algorithms can be used to project success in obtaining complete coverage of an AUT in which a set of sampling points are carefully chosen based on design-specific conditions. Some of these algorithms perform well with respect to their sensor-specific solutions, yet de-emphasize the importance of strategic sensor placement within the environment, often relying on vision or close-range lidar to project best next-waypoints to follow [13]. Other approaches to coverage consider the goal of sensor placement, yet require some quantifiable data *a priori* that help guide the exploration process (and subsequently, the navigation) [10, 14]. Yet, the importance of local navigation based on immediately sensed information is favored in environmental monitoring applications because of the unique sampling operations that can only be performed at discrete locations, $e.g.$, temperature, humidity, and pressure. We therefore focus on a navigation strategy to accomplish sufficient quality coverage, while still obtaining locally-relevant detail in our measurements and requiring no prior information about the interior of the investigated area. This strategy is manifested as a specific navigation pattern that also allows for modifications (or adaptation) to its basic structure.

Demonstrated in theory, it was revealed, from work by Tunstel, that the pattern yielding the highest QoP was the parallel transect [11]. This lawnmower-like pattern is our baseline algorithm (Algorithm 1) that we improve upon. An example trajectory is shown in Figure 2a.

Algorithm 1 Lawnmower-traditional navigation scheme.

Require: Static navigation policy, $f(q)$ $(q = All\ northing\ positions\ across\ a\ swath)$
Require: Border dimensions, $dimx$, $dimy$; AUT, T
  $(P^{SL}) = T/Resources$ {Define vector of starting locations for each swath according to resources available}
  for $k \leq length(P^{SL})$ **do**
    $X^{Total}(P^{SL}(k), all) = T(P^{SL}(k), all)$ {Navigate across T, collecting all samples at designated Y-position, $P^{SL}(k)$}
  end for{Store total set of samples}
  return X^{Total}

The specific influence of our adapted methods came out of a hydrographic simulation survey system [15]. Bourgeois *et al.* simulate survey times for a Northeast coastal survey, adapting their own lawnmower-like navigation to the information gained from the quality and coverage of the coastal floor during each swath [15]. This, in effect, yields a set of parallel, non-linear swaths. Although the actual measurements, themselves, are not the cause of changes in the shape of these swaths, this furthered our understanding of how the lawnmower pattern could be adapted and inspired our modifications. Ideally, we want to preserve the benefit of collecting information iteratively, $i.e.$, continuing to execute parallel swaths across the area, but by altering the local trajectory of an agent's path across the terrain, we hope to do so more intelligently. Taking inspiration from the hydrographic surveying work [15], we found that we could mimic either a group of linear swaths with varying separation or a series of non-linear piecewise continuous trajectories ($i.e.$, Adaptive Parallel or Piecewise Continuous, respectively). In each case, how the characteristics of these patterns changed would be defined based on the information collected online by our agent.

1) Adaptive Parallel: Each successive swath executed according to the original Adaptive Parallel pattern is a function of the quality of bathymetry return information collected during the previous swath (see Section III-B). Since our system does not presume this type of scanning capability of our AUT, we needed to leverage our previously collected information in such a way that it could be translated into an estimate for how far along the northing direction the next parallel swath ought to be executed. We resolved that this estimate should be calculated based on the average change in slope information collected across all previous swaths. For the purpose of informing each successive swath, in Algorithm 2, the function $f(q)$ is defined based on the slope information collected by the previous swath.

Algorithm 2 Adaptive Parallel navigation scheme.

Require: Function for estimating successive swath width, $f(q)$
Require: Border dimensions, $dimx$, $dimy$; AUT, T
  $n = f_{sw}$ {Define the starting position along the Y-axis of the terrain based on initial slope information, sw}
  while $n \leq dimy$ **do**
    $n = n + f(q)$
    $X^{Total}_{(n,all)} = T_{(n,all)}$ {Navigate across T, collecting all samples at designated Y-position, n}
  end while
  return X^{Total}

To visualize the RSS' navigation within the AUT, Figure 2b shows an example trajectory of linear swaths unevenly spaced and asymmetrically distributed across the terrain.

2) Piecewise Continuous: We wanted to investigate if, by collecting the local information within the neighborhood of each swath of a symmetrically placed lawnmower pattern, a better estimate of the entire AUT would result. The next step, therefore, was to determine in what way that local information could be used to influence the navigation. Previous empirical tests we conducted showed that one feature common to navigation patterns yielding RMS error values lower than those sampled according to the lawnmower pattern was the presence of smaller sets of continuous paths scattered throughout the terrain. This is most likely the case because of the variety in sample placement in contrast to the stark linearity and uniformity of the lawnmower pattern. It was determined that the best way to generate these types of paths *in situ* was to consider steepest ascent/descent rules of navigation, allowing the RSS to navigate according to these control laws while operating within certain planimetric boundaries and under specific conditions. This approach to navigation produces the desired combination of terrain-specific detail and resource control lacking in other previously discussed work. The psuedo-code in Algorithm 3 details this process more formally.

Algorithm 3 Piecewise Continuous navigation scheme.

Require: Choice of navigation policy, $g(q)$ $(q = Steepest\ Ascent \,||\, Steepest\ Descent)$
Require: Border dimensions, $dimx$, $dimy$; AUT, T
  $(P^{SL}, bw) = T/Resources$ {Define vector of starting locations for each swath according to resources available} {Define bandwidth ($i.e.$, swath width) of allowable search range according to resources available for the AUT}
  for $k \leq length(P^{SL})$ **do**
    $Y_{Range} = [P^{SL}_k \pm bw]$
    $X_s = X_s + Apply\ g(q)\ at\ T^{P^{SL}_k}_{(x,y)}\ for\ (1 \leq x \leq dimx, y = Y_{Range})$
    {Apply primary navigation policy to terrain along a single swath and store recorded samples}
  end for
  $X^{Total} = X_s$ {Store total set of samples}
  return X^{Total}

By the end of navigating according to the Piecewise Continuous method, the RSS has collected a series of sample sets that are both representative of local change and achieved within the confines of allowable resources (see Figure 2c). This method is the most promising as it provides a simple data-driven navigation strategy. Furthermore, the segmented sets of samples produced by the Piecewise Continuous algorithm are spatially relevant to the heterogeneous data types under investigation. Made possible with this navigation strategy, the decision function, $g(q)$, allows the accumulation of multiple, highly correlated data sets to be attained. This is advantageous during post-analysis when different interpolation options are considered.

3) Lawnmower Random: We added one more sampling methodology to test the feasibility of previous approaches in randomness considered by both the sampling and robotics communities [7, 10]. The concept of random sampling for our application is implemented in much the same way as the Piecewise Continuous approach except the agent's heading (ψ) towards its next sample along a swath is defined as a uniformly random function, rather than $g(q)$. This method provides an even sampling distribution and would appear to be an ideal navigation strategy. The one caveat to this approach is that the RSS must actually navigate to these randomly selected waypoints and therefore will likely incur a hefty penalty in total Euclidean distance traveled. Although this measure of distance is not our RSS' primary reference metric, as was discussed at length in Section III-A, it will be seen later that, when compared to the other navigation methods, it is not reasonable to consider its use from a resources point of view.

C. RMS error

Our principle performance metric is root mean squared (RMS) error, $\hat{\theta}$. This is expressed in Equation 2, where Z_0 is our truth data while $\hat{Z}_0$ is the estimated data generated as a result of the samples collected according to our navigation policy.

$$\text{RMS error} = \hat{\theta} = \sqrt{E[(Z_0 - \hat{Z}_0)^2]} \tag{2}$$

Quantifying error in this way is a convenience, as it provides a single error estimate, which is relative to each location in the sample space, while also representative of the entire AUT. This succinct value allows us to test a larger number of DEMs and make sound observations about the effectiveness of our navigation algorithms on heterogeneous elevation data, irrespective of its local features.

IV. Results & Discussion

To effectively assess the performance of these navigation algorithms, it was important that a suitable range of sample DEMs were tested so that no one method outperformed the others due to a common feature (*e.g.*, periodicity of hills throughout the terrain). To accommodate this need for unique terrains, a DEM generator was created that produces terrain maps based on computer graphics techniques [16]. An example terrain is found in Figure 3.

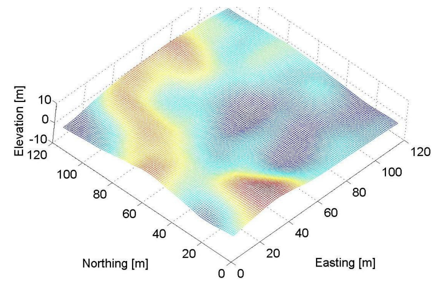


Fig. 3: Example terrain with heterogeneous elevation data.

Per each algorithm outlined in Section III-B, a set of reference swaths were defined within the sample space of each generated terrain. Each navigation method used these reference locations as both a starting point and northing boundary while navigating from east to west across the AUT. The original Lawnmower pattern was tested as our baseline sampling method. All of the methods are designated in the results as follows: traditional Lawnmower (LM), Adaptive Parallel (AP), Piecewise Continuous (PW-Cont), Lawnmower Random (LM-Rand).

When we compile the RMS error data over 50 different randomly-generated terrains, we find that the PW-Cont navigation strategy is most dominant over the range of zero to eight percent coverage per our definition in Section III-A (see Figures 4 and 5). Also dominant is the decrease in variation of the RMS error over all 50 terrains tested.

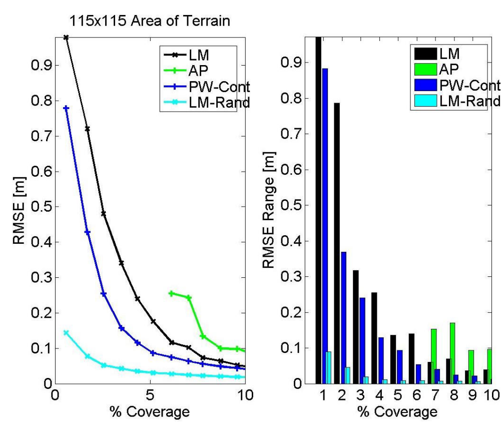
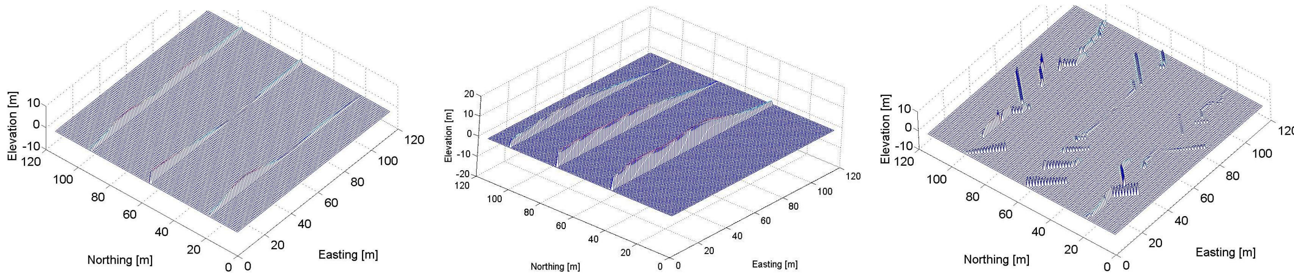


Fig. 4: RMS error comparing traditional Lawnmower, Adaptive Parallel, Piecewise Continuous, and Lawnmower-Random. Average RMS error (left), Range of Error (right).

Additionally, as seen in Figure 4, the overall range of error of PW-Cont shows a significant improvement (decrease) over the LM pattern, especially at two, four, and six percent coverage. At first glance, it appears as though the LM-Rand strategy is the preferred method, yet, when the trend data from the Euclidean distance plot of Figure 5 is considered, we find that the LM-Rand scheme (as well as AP) yield a much greater total Euclidean distance than PW-Cont as the number of samples increases and therefore cannot realistically be considered when attempting to minimize resource usage.



(a) Traditional Lawnmower navigation example. (b) Adaptive Parallel navigation example (c) Piecewise Continuous navigation example.

Fig. 2: Patterns of navigation strategies (Traditional Lawnmower, Piecewise Continuous, and Adaptive Parallel) as applied to simulated DEM data.

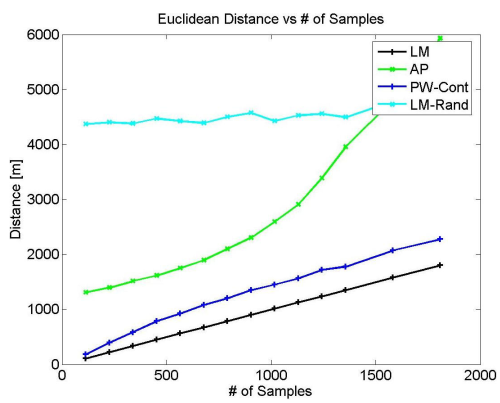


Fig. 5: Trend of Euclidean distance versus the number of samples.

The AP approach was insufficient for three reasons: 1) It assumes the slope information perpendicular to the path of navigation ($i.e.$, the *roll* angle) across the AUT will be uniform. This is not the case since our terrain is heterogeneous and can vary in any number of directions. 2) Even if the terrain satisfied the aforementioned characteristics, our function defining successive swath widths would have to be bounded to a pre-defined limit so that no information, such as a hill or valley, would be missed between swaths. After multiple trials, it was discovered that varying this pre-defined width only led to sampling of the AUT that demonstrated subtle improvements over that of the LM pattern. 3) There is no built-in regulation of resources, $i.e.$, the navigation would only terminate if the calculated start position of a successive swath exceeded the dimension of the AUT. A solution is needed that allows the user to specify limits on how far or for how long an agent may navigate during sample collection. With this in mind, we needed to consider more dynamic paths that exhibited successive irregular trajectories but with quantifiable control over available resources.

Aside from data presented in the context of our definition of percent coverage, a broader perspective reveals the importance of sample placement. Figure 6 confirms a trend similar to that of Figure 4 where PW-Cont is the most practical navigation option over others considered with the exception of LM-Rand, understanding that the cost for executing LM-Rand, in terms of Euclidean distance, is far beyond realistic consideration.

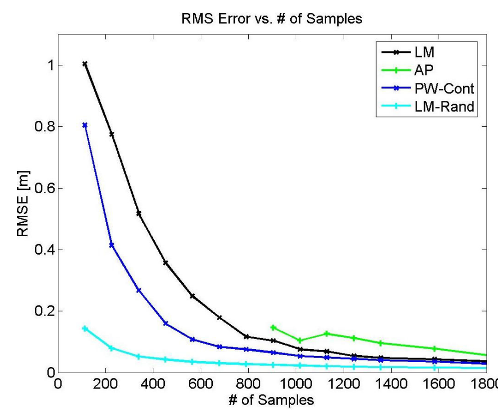


Fig. 6: RMS error versus total # of samples.

As a final analysis, we tested these same algorithms on a DEM generated from a contour map outlining a real terrain. As expected, the PW-Cont method is consistent in outperforming the LM pattern with a lower RMS error as coverage increases while maintaining a conservative increasing trend in total Euclidean distance traveled throughout the AUT (see Figures 7 and 8).

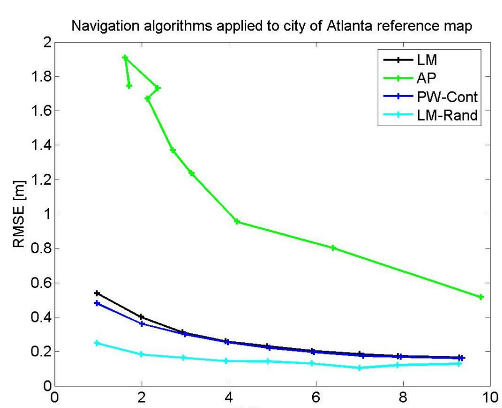


Fig. 7: RMS error comparing traditional Lawnmower, Adaptive Parallel, Piecewise Continuous, and Lawnmower-Random.

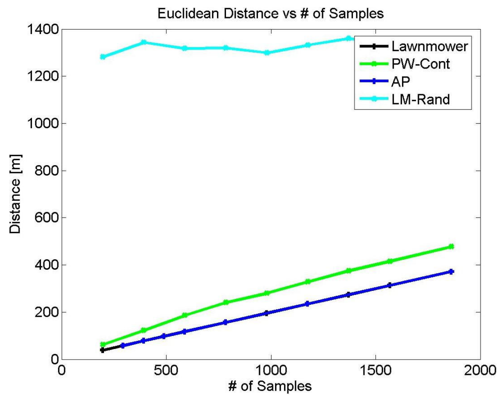


Fig. 8: Trend of Euclidean distance versus # of samples.

Since the performance of the PW-Cont method on real data begins to converge with that of LM method at coverages greater than three percent, we would like to see improvement ($i.e.$, lower absolute RMS error) at coverages less than three percent based on the definition provided in Equation 1, as well in order to establish a dominant lower range of successful coverages.

We are encouraged by the results presented in Figures 4 - 8, particularly in how, by sacrificing a slight increase in total distance for the sake of local exploration, a lower RMS error for coverages less than 10 percent is achieved. When considering mapping standards like that of the American Society of Photogrammetry and Remote Sensing (ASPRS), the performance of the PW-Cont navigation using simulated data at coverages greater than four percent satisfies requirements for the generation of Class 1 maps [17].

One proposed explanation for the smaller disparity between PW-Cont and LM patterns when tested on the real terrain DEM is the simplicity of the terrain itself. While the data of Figures 4 and 5 was a statistical average performance of 50 terrains, the results of Figures 7 and 8 were based on only one terrain, which is, at best, depicted as a sloping hill. By comparison, the database of terrains tested in simulation consist of much greater spatial complexity.

V. Conclusions and Future Work

The work presented here offers concrete insight into how an Earth science team can approach collecting necessary and sufficient *in situ* data for environmental monitoring applications using mobile robotic technology. While spatially static parallel transect trajectories provide the least cost in terms of point-to-point distance traveled, better estimates of areas between transects can be gained. This is accomplished by considering the sacrifice of minimal additional resources for the purpose of local exploration, centered around these purely linear swaths and driven by the measurements. Two points highlight the value of this work: 1) a scientist can approach an unchartered area with a robust robotic platform for sampling purposes confident that less than 25 percent coverage will be necessary to achieve a useful approximation of the terrain and 2) this can be achieved by considering non-traditional approaches. Future work includes considering how to navigate within additional types of phenomena not easily characterized as undulating terrain, but instead as more complex ecological elements requiring high-precision measurements. We will also supplement the results presented here with field trials comparing the execution of these algorithms in future work.

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