Quantum space-time crystals:

Interplay of symmetry breaking and symmetry restoration for both spatial and time dimensions in the wave functions of mesoscopic systems other than nuclei

Constantine Yannouleas and Uzi Landman

School of Physics, Georgia Institute of Technology

Rep. Prog. Phys.
70, 2067 (2007)

arXiv:1706.09006

Prospects on the microscopic description of odd mass nuclei and other multi-quasiparticle excitations with beyond-mean-field and related methods, ECT*, Trento, 25-29 Sep. 2017

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From Wikipedia

"A time crystal or space-time crystal is a structure that repeats periodically in time, as well in space. Normal three-dimensional crystals have a repeating pattern in space, but remain unchanged with respect to time; time crystals repeat themselves in time as well, leading the crystal to change from moment to moment.
The idea of a time crystal was first described by Nobel laureate and MIT professor Frank Wilczek in 2012"

Time reflection and time translational symmetries are broken Quantum Space Time Crystal (symmetry breaking in all four dimensions: space and time)

Time evolution phenomena in quantum mechanical finite systems

Unprecedented experimental control of few-body systems of trapped ultracold ions and neutral atoms

Physics

VIEWPOINT

How to Create a Time Crystal

A detailed theoretical recipe for making time crystals has been unveiled and swiftly implemented by two groups using vastly different experimental systems.

by Phil Richerme*

The story of time crystals—whose lowest-energy configurations are periodic in time rather than space—epitomizes the creative ideas, controversy, and vigorous discussion that lie at the core of the scientific process. Originally theorized by Frank Wilczek in 2012 [1] (see 15 October 2012 Viewpoint), time crystals were met with widespread attention, but also a healthy dose of skepticism [2]. This ignited a debate in the literature, culminating in a proof that time crystals cannot exist in thermal equilibrium, as originally imagined by Wilczek [3]. But the

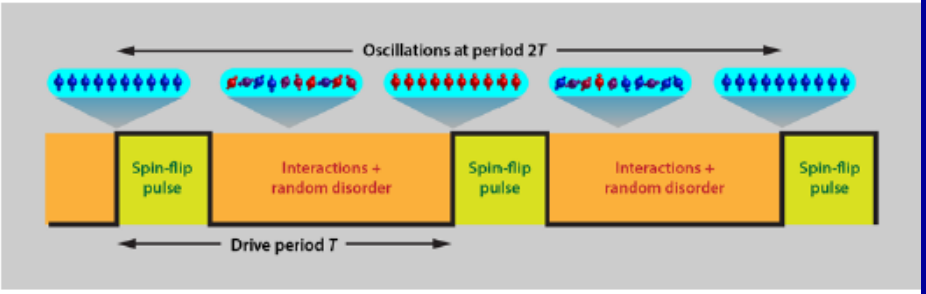
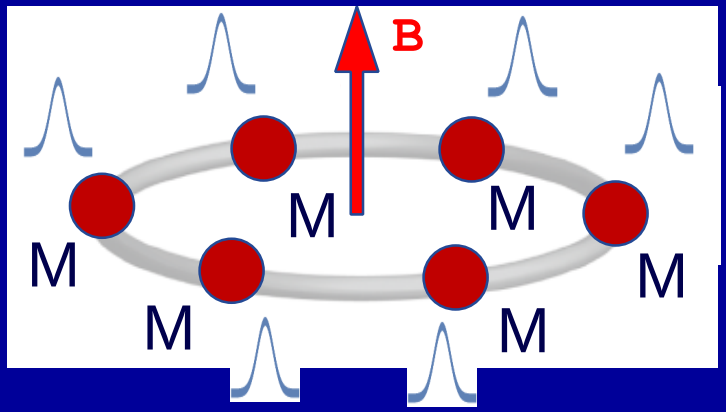
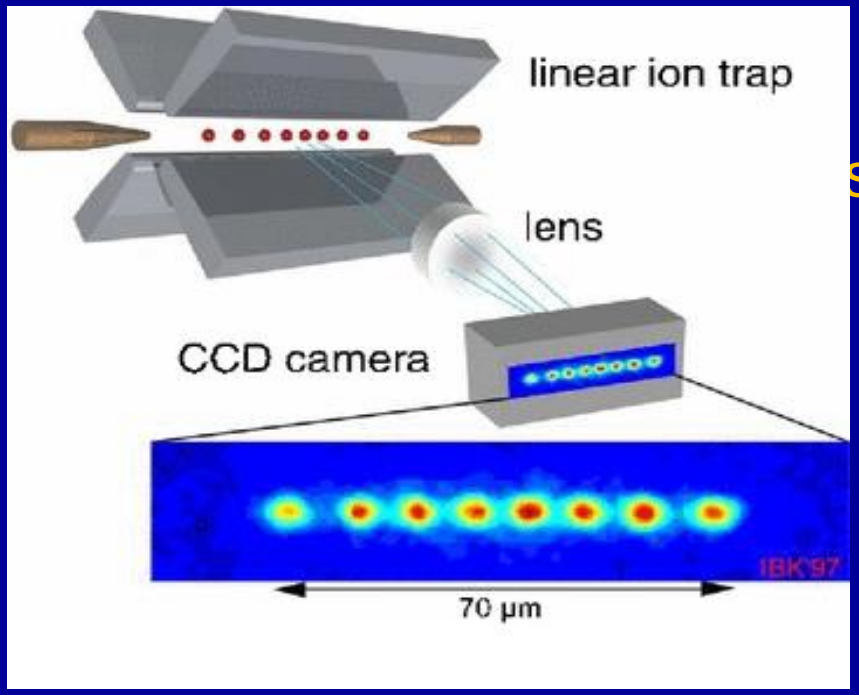


Figure 1: Yao *et al.* [7] have developed a blueprint for creating a time crystal and a method for detecting it, which has been followed by two experimental groups [8, 9]. Quantum spins are subjected to imperfect spin-flip driving pulses and then allowed to interact with each other in the presence of strong random disorder in the local

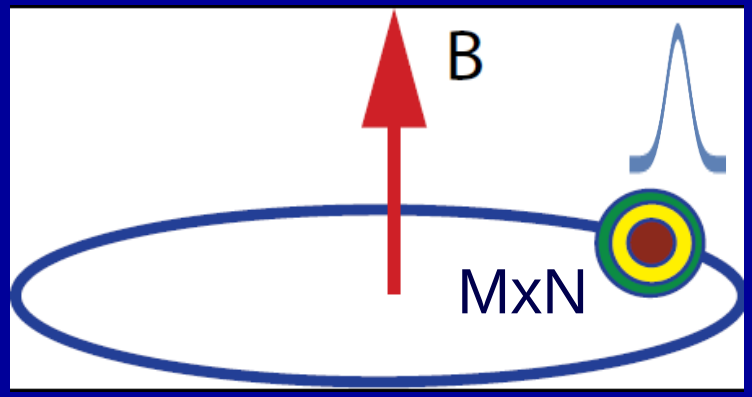
THE CONCEPT/ INTUITION/ TWO PROPOSALS/ STARTING POINT BASED ON SYMMETRY BREAKING



T. Li et al., PRL 109, 163001 (2012)



sion



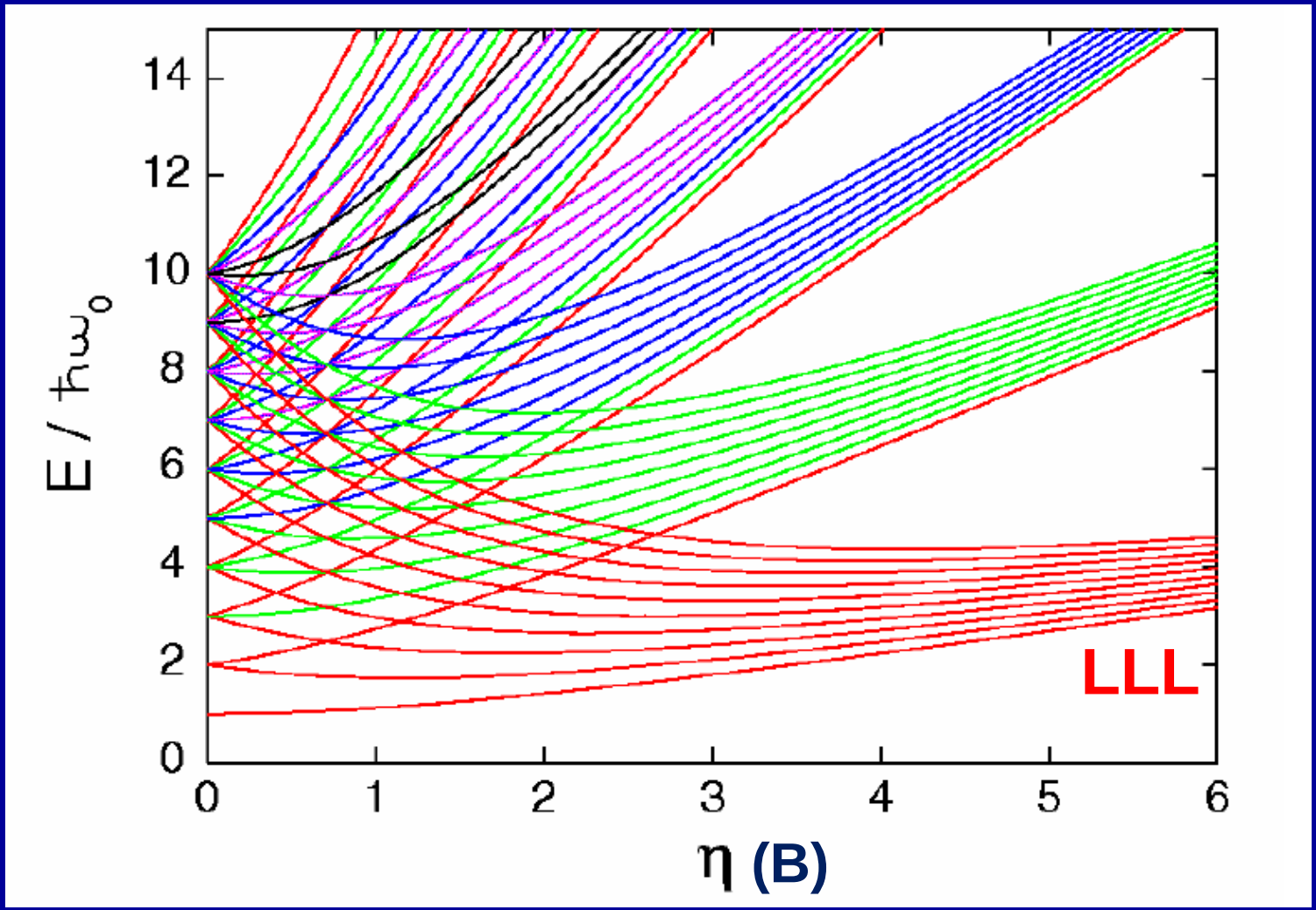
F. Wilczek, PRL 109, 160401 (2012)

Lump/ Bose-Einstein soliton

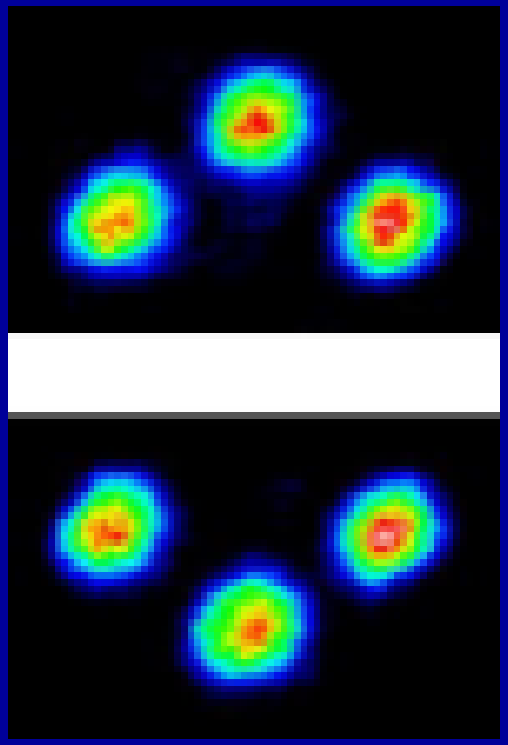
Ultracold neutral atoms

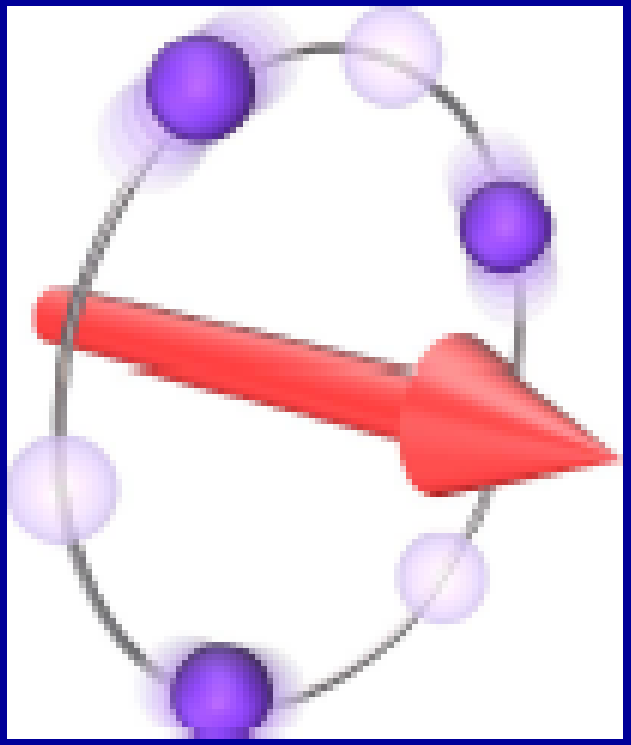
Attractive contact interaction

Bosons (^{87}Rb or ^{85}Rb)



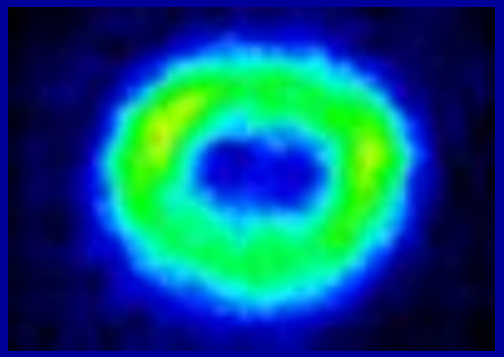
Effect of breaking time reversal symmetry
2D oscillator under a magnetic field
Darwin-Fock spectrum

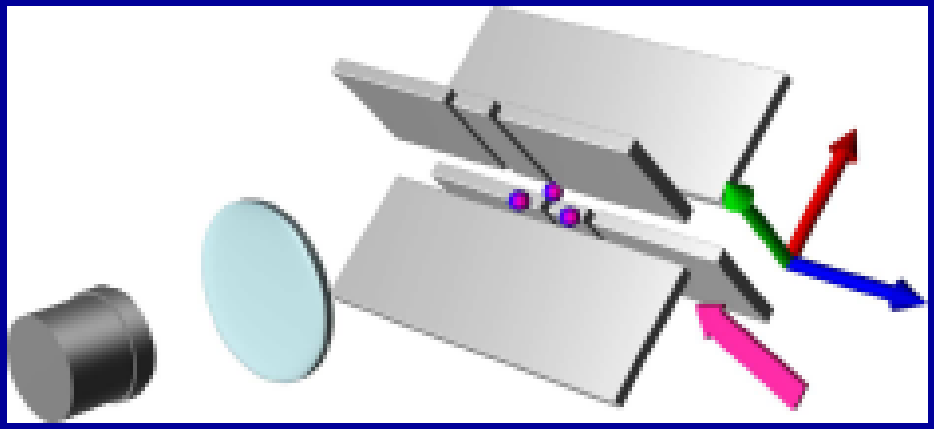




$^{40}Ca^{+}$

B

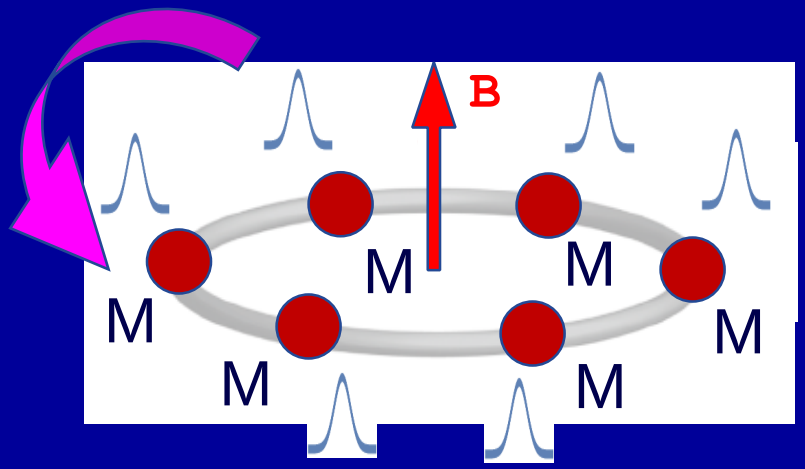




Noguchi, A. et al., Nat. Commun. 5:3868 (2014).

"ROTATING" QUANTUM-MECHANICAL SP DENSITY SHOULD EXHIBIT PERIODICITY IN BOTH SPACE AND TIME

BREAKING OF TIME TRANSLATIONAL SYMMETRY



T. Li et al., PRL 109, 163001 (2012)

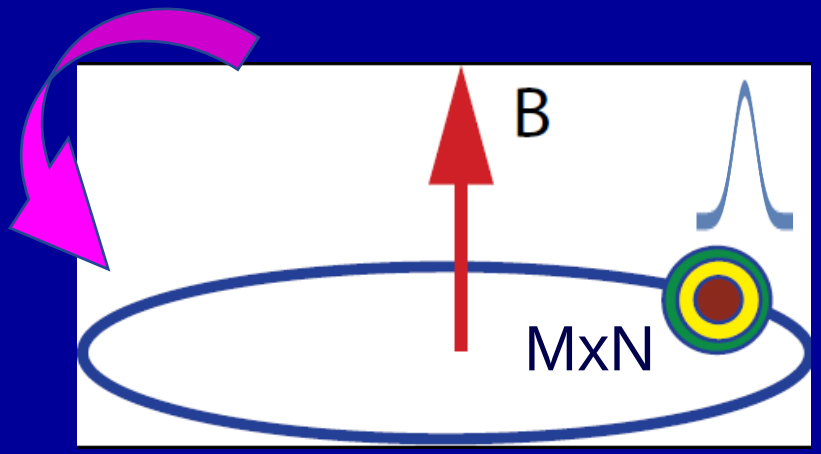
Ion crystal

Ultracold ions/ Coulomb repulsion

Both fermions ($^{24}Mg^{+}$) and

Bosons ($^{9}Be^{+}$)

A different orbital for each particle



F. Wilczek, PRL 109, 160401 (2012)

Lump/ Bose-Einstein soliton

Ultracold neutral atoms

Attractive contact interaction

Bosons (^{87}Rb or ^{85}Rb)

The same orbital for all particles

LOWERING OF ENERGY AND SYMMETRY BREAKING (SB) IN SELF-CONSISTENT MEAN-FIELD APPROACHES
Restricted vs unrestricted and fermions vs bosons

Attraction
Restricted approach (RHF)

Same orbital and symmetry breaking (SB):

Nilsson potential
(fermions, nucleons)

Gross-Pitaevskii (GP)
(attractive bosons,
Lump of ultracold neutral atoms)

GP for repulsive bosons:
SB raises the energy !

Repulsion
Unrestricted approach (UHF)

Different orbitals and symmetry breaking (SB):

Hydrogen molecule, dissociation
(electrons, chemistry)

Wigner molecules
(electrons, quantum dots)

Space-time crystals on rings
(fermion or bosons, ultracold ions)

UNRESTRICTED HF FOR REPELLING FERMIONS

(Self-consistent Pople-Nesbet Eqs.)

Different orbitals for different spins

Two coupled equations/ Spin-up coupled to spin-down

Self-consistent solution → orbital localization

[example from chemistry:
dissociation of Hydrogen molecule (next slide)]

UNRESTRICTED HF FOR REPELLING BOSONS?

A different orbital for each particle

Self-consistent set of Eqs. is not practical

Reason: orbitals for bosons are not orthogonal/
spin-orbitals for fermions are orthogonal

In case of crystals, employ ansatz = permanent of orbitals approximated as displaced Gaussians

The Hydrogen molecule

Szabo and Ostlund, Modern Quantum Chemistry

3.8.7 The Dissociation Problem and Its Unrestricted Solution

At very large bond lengths, however, one is really trying to describe two individual hydrogen atoms. A proper description will have one electron on one H atom and the other electron on the other H atom, i.e., the two electrons will have quite different spatial distributions. They should not have identical

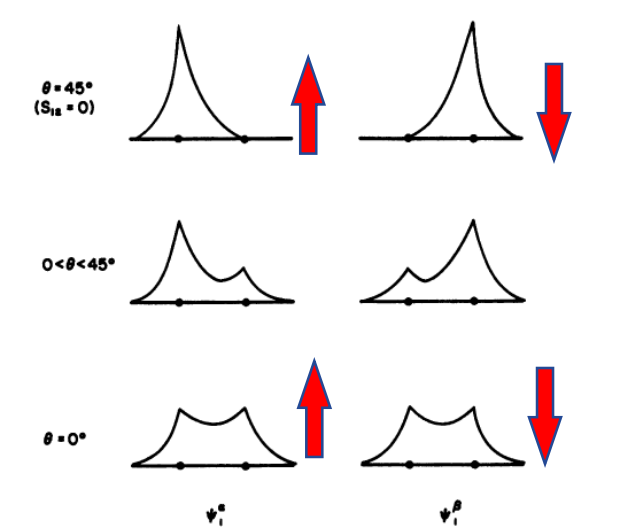
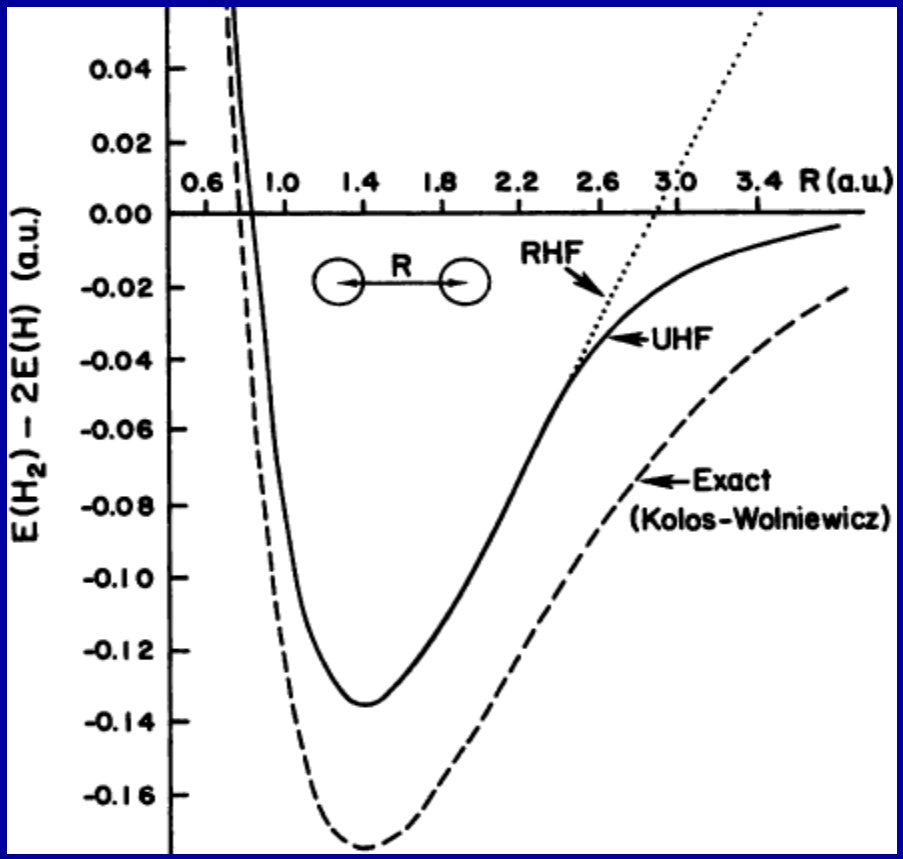


Figure 3.16 Qualitative behavior of unrestricted molecular orbitals ψ_1^α ar function of θ.



Next step: Heitler-London

RESOLUTION OF SYMMETRY DILEMMA:

RESTORATION OF BROKEN SYMMETRY BEYOND MEAN FIELD (Projection)!

- Per-Olov Löwdin
  (Chemistry - Spin)
- R.E. Peierls and J. Yoccoz
  (Nuclear Physics – *L, rotations*)

Ch. 11 in the book by P. Ring and P. Schuck

CONTROL PARAMETERS FOR SYMMETRY BREAKING

IN SINGLE QD'S: WIGNER CRYSTALLIZATION

- Essential Parameter at B=0: (parabolic confinement)

$$R_W = (e^2/\kappa l_0)/\hbar\omega_0 \sim 1/(\hbar^3\omega_0)^{1/2}$$

e-e Coulomb repulsion — kinetic energy

$$l_0 = (\hbar/m^*\omega_0)^{1/2}$$ } Spatial Extent of 1s s.p. state

κ : dielectric const. (12.9)

m^* : e effective mass (0.067 m_e) GaAS

$\hbar\omega_0$ (5 - 1 meV) => R_W (1.48 - 3.31)

- In a magnetic field, essential parameter is B itself

IN QDM'S: DISSOCIATION (Electron puddles, Mott transition)

Essential parameters: Separation (d)
Potential barrier (V_b)
Magnetic field (B)

$$R_\delta = gm/(2\pi\hbar^2)$$

Neutral bosons

A HIERARCHY OF APPROXIMATIONS

TOTAL ENERGY

Restricted Hartree-Fock (RHF)

All spin and space symmetries are preserved
Double occupancy / e-densities: circularly symmetric
Single Slater determinant (central mean field)

↓

Unrestricted Hartree-Fock (UHF)

Total-spin and space symmetries (rotational or parity) are broken / Different orbitals for different spins
Solutions with lower symmetry (point-group symmetry)
Lower symmetry explicit in electron densities
Single Slater determinant (non-central mean field)

Implementation of UHF: Pople-Nesbet Eqs.
2D harmonic-oscillator basis set
Two coupled matrix Eqs. (for up and down spins)

↓

Restoration of symmetry via projection techniques

Superposition of UHF Slater det.'s (beyond mean field)
e-densities: circularly symmetric
Good total spin and angular momenta
Lower symmetry is INTRINSIC (or HIDDEN)
Detection of broken symmetry:
CPDs and rovibrational excitations of quantum dots
CPDs and dissociation of quantum dot molecules

Correlations

Non-linear equations
Bifurcations

EMERGENT
PHENOMENA

Restoration of linearity
of many-body equatons

TIME EVOLUTION

HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$\mathcal{H} = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i} g\,\delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^{\,2}}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

Two-center oscillator with V_b control } QDM

Ω

$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^{\,2}]/2m^* + g^* \mu_B \vec{B}\cdot\vec{S}_i/\hbar$$

$$\vec{A}_i = B\,(-y_i, x_i, 0)/2$$

Zeeman

$\mathcal{H}$ can be generalized to Multi-component syste

Neutral Bosonic systems

External Confinement

$$\frac{(r_i - R)^2}{2l_0^2/(\hbar\omega_0)}$$

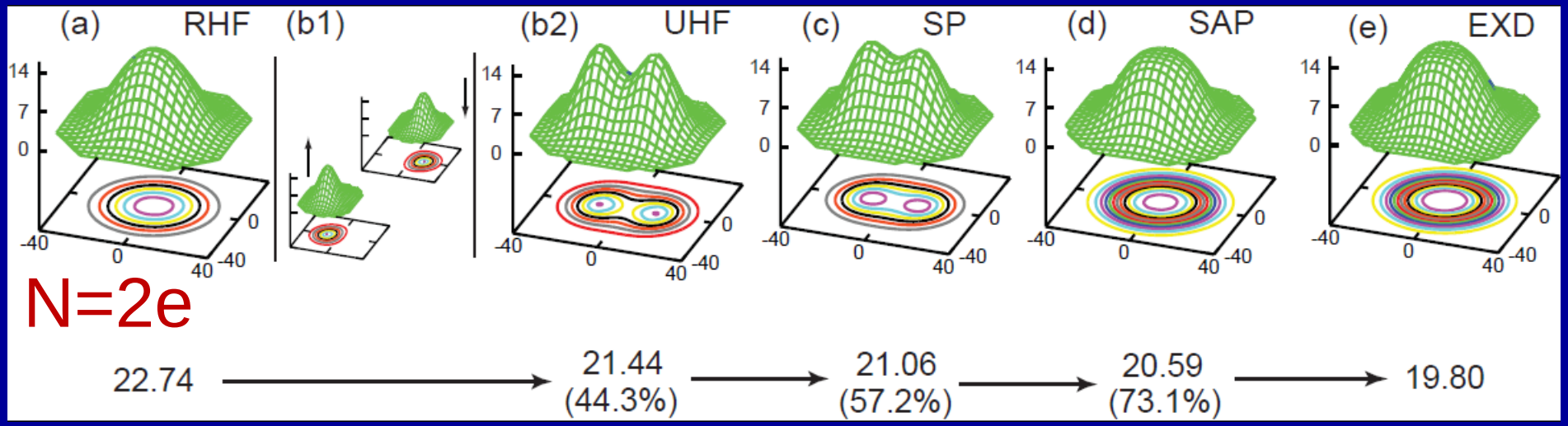
2D RING TRAP

2D TRAP/ 2D QD

1 hertz [Hz] = 4.135665538535 99E-15 electron-volt [eV]

Self-consistent HF → Exact

$R_W = 2.40$ $B = 0$

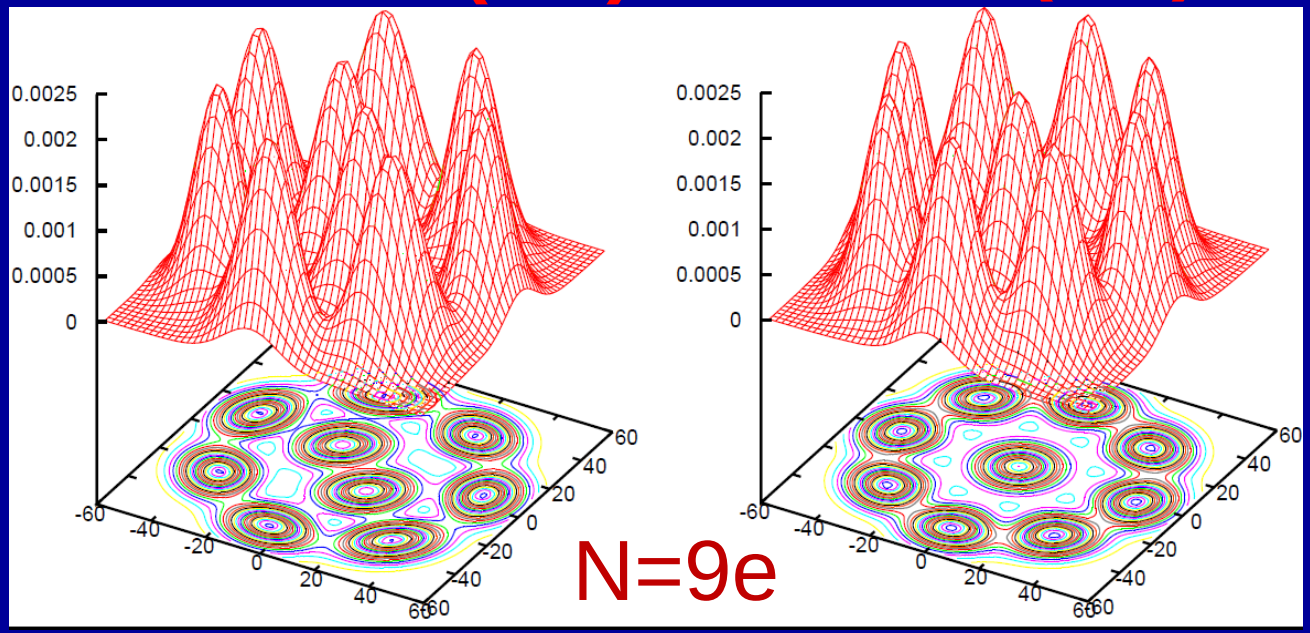


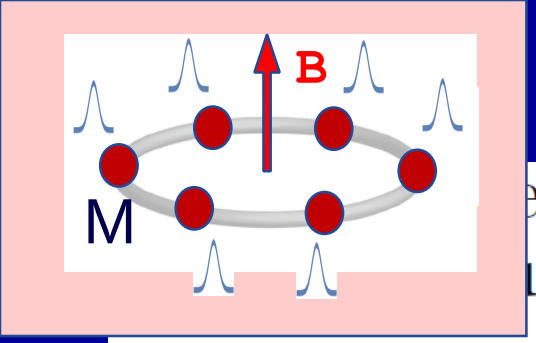
Self-consistent UHF

$S_z = 9/2$

$R_W = 6.365$

$B = 0$





e each particle localized at position $\mathbf{R}_j$ as a ussian function

$$u(\mathbf{r},\mathbf{R}_j)=\frac{1}{\sqrt{\pi}\lambda}\exp\left(-\frac{(\mathbf{r}-\mathbf{R}_j)^2}{2\lambda^2}-i\varphi(\mathbf{r},\mathbf{R}_j;B)\right), \quad (3)$$

with $\lambda=\sqrt{\hbar/(M\Omega)}$; $\Omega=\sqrt{\omega_0^2+\omega_c^2/4}$ where $\omega_c=\eta B/M$ is the cyclotron frequency. The phase in Eq. (3) is due to the gauge invariance of magnetic translations [57, 58]) and is given by $\varphi(\mathbf{r},\mathbf{R}_j;B)=(xY_j-yX_j)/(2l_B^2)$, with $l_B=\sqrt{\hbar/(\eta B)}$ being the magnetic length. For

$$\varphi(\mathbf{r},\mathbf{R}_j;B)=(xY_j-yX_j)/(2l_B^2)$$

Construct determinant/ permanent Ψ^{SB}
(MF symmetry-breaking ansatz)

PROJECTION/ ANGULAR MOMENTUM

$$\mathcal{P}_L = \frac{1}{2\pi}\int_0^{2\pi} e^{i\gamma(L-\hat{L})}d\gamma$$

$$\Phi_L^{\mathrm{PROJ}} = \frac{1}{2\pi}\int_0^{2\pi} d\gamma \Psi^{\mathrm{SB}}(\gamma)e^{i\gamma L}$$

$$E^{\mathrm{PROJ}}(L) = \int_0^{2\pi} h(\gamma)e^{i\gamma L}d\gamma \Big/ \int_0^{2\pi} n(\gamma)e^{i\gamma L}d\gamma,$$

where

$$h(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\mathcal{H}|\Psi^{\mathrm{SB}}(\gamma)\rangle,$$

and the norm overlap

$$n(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\Psi^{\mathrm{SB}}(\gamma)\rangle$$

Rotational spectrum: quantum rigid rotor

$R_W = 1000$

$R_\delta = 50$

$$E^{\mathrm{PROJ}}(L) \approx V_{\mathrm{int}} + C_R(L - N\Phi/\Phi_0)^2$$

V_{int} Band head/ where interaction and Correlations show up

Φ Magnetic flux

$$C_R \approx C_R^{\mathrm{cl}} = \hbar^2/[2\mathcal{I}(R_{\mathrm{eq}})] \qquad \mathcal{I}(R_{\mathrm{eq}}) = NMR_{\mathrm{eq}}^2$$

L => magic (fermions spin polarized)

Repelling, polygon crystal:

$L_m = kN; \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots$ Fermions, N odd; bosons

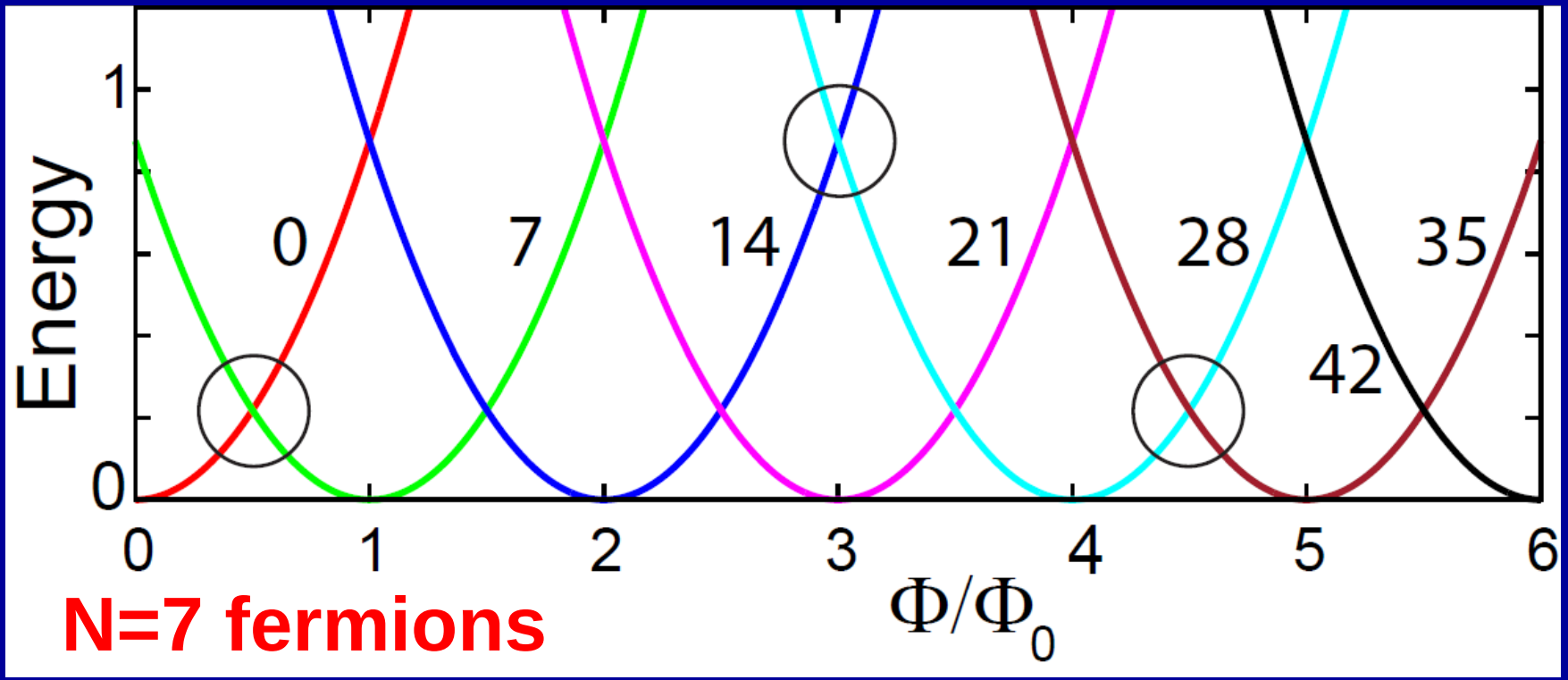
$L_m = (k + \frac{1}{2})N; \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots$ Fermions, N even

$L_m = 0, \pm 1, \pm 2, \ldots$ Attractive bosons, lump

Rotational spectrum: quantum rigid rotor

$$E^{\mathrm{PROJ}}(L) \approx V_{\mathrm{int}} + C_R(L - N\Phi/\Phi_0)^2$$

SECOND TERM/ AHARONOV-BOHM TYPE SPECTRUM



N=7 fermions

GOA/ GCM

Norm overlap/ Time-reversal

KAMLAH expansion

where in such a way that the quotient $h(\alpha)/n(\alpha)$ is a rather smooth tion. This is a general property of many-body wave functions (which discussed in great detail in Sec. 10.7). Following the arguments of ion 10.7.4, we obtain*

$$n(\alpha) \simeq \exp\left(i\langle \hat{J} \rangle \cdot \alpha - \tfrac{1}{2}\langle \Delta\hat{J}^2 \rangle \alpha^2\right), \tag{11.79}$$

$$\Delta\hat{J} = \hat{J} - \langle \hat{J} \rangle,$$

nly difference now being that we also allow for time odd components e wave functions, which does not give a pure Gaussian, but an ional phase in Eq. (11.79) ($\langle \hat{J} \rangle \neq 0$). The idea of Kamlah is now to he operator

$$\hat{I} := -\langle \hat{J} \rangle + \frac{1}{i}\frac{\partial}{\partial \alpha},$$

projected energy now has the form:

$$E^I_{\text{proj}} = \langle H \rangle - \frac{\langle \Delta\hat{J}^2 \rangle}{2\vartheta_Y} + \frac{\langle \hat{J} \rangle}{\vartheta_{sc}}(I - \langle \hat{J} \rangle) + \frac{1}{2\vartheta_Y}(I - \langle \hat{J} \rangle)^2. \tag{11.91}$$

t us first study the method of *variation before projection*, which was nally proposed by Peierls and Yoccoz [PY 57, Yo 57]. Here the wave tion $|\Phi\rangle$ is obtained from a minimization of H without constraint. It erefore, time-reversal invariant and has vanishing expectation values and $\langle H\hat{J} \rangle$. The spectrum then has the form

$$E^I_{\text{proj}} = \langle H - \frac{\hat{J}^2}{2\vartheta_Y} \rangle + \frac{I^2}{2\vartheta_Y}. \tag{11.92}$$

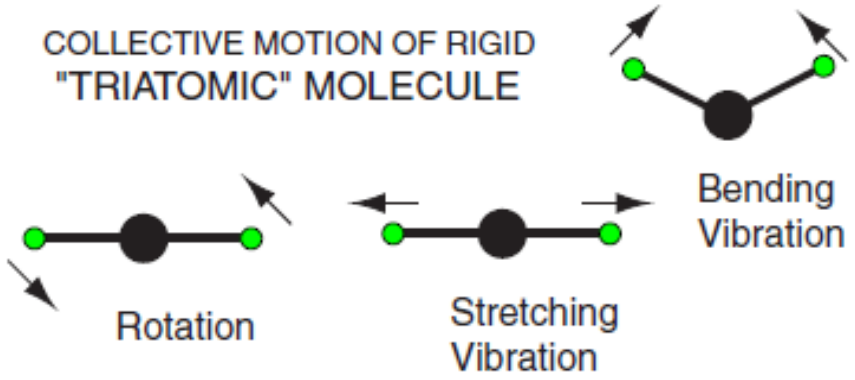
the spectrum of a one-dimensional rotor with the Yoccoz value ϑ_Y for noment of inertia (see Sec. 11.4.5).† The band head is obtained from

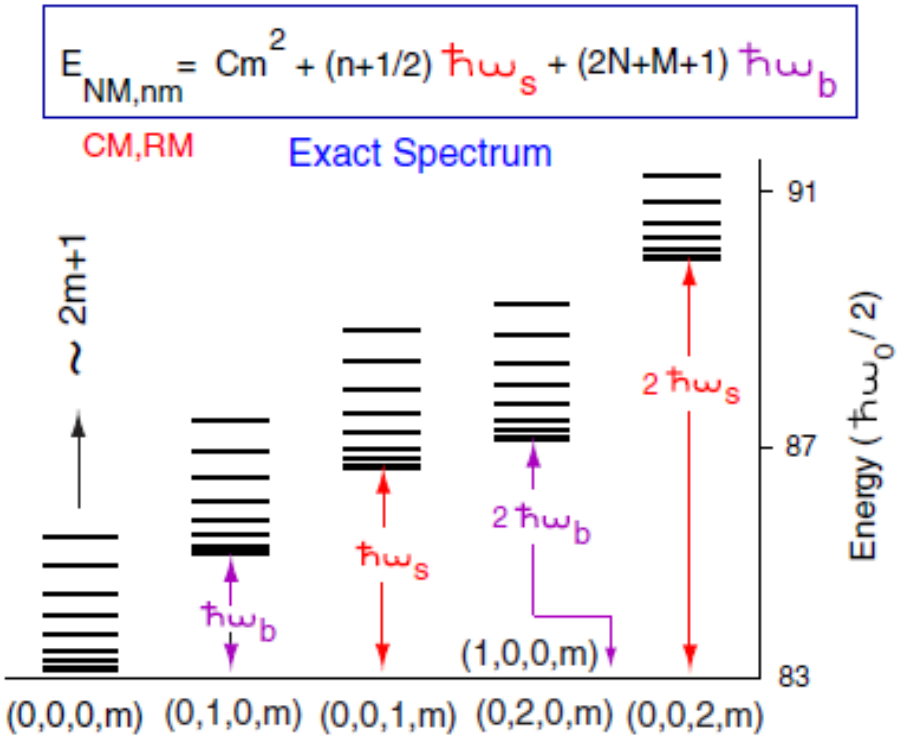
$\hat{}$
<J> ➡
Φ/Φ_0

Ring & Schuck, Ch. 11

N=2e QD, R_W=200; EXACT

RIGID ROTOR

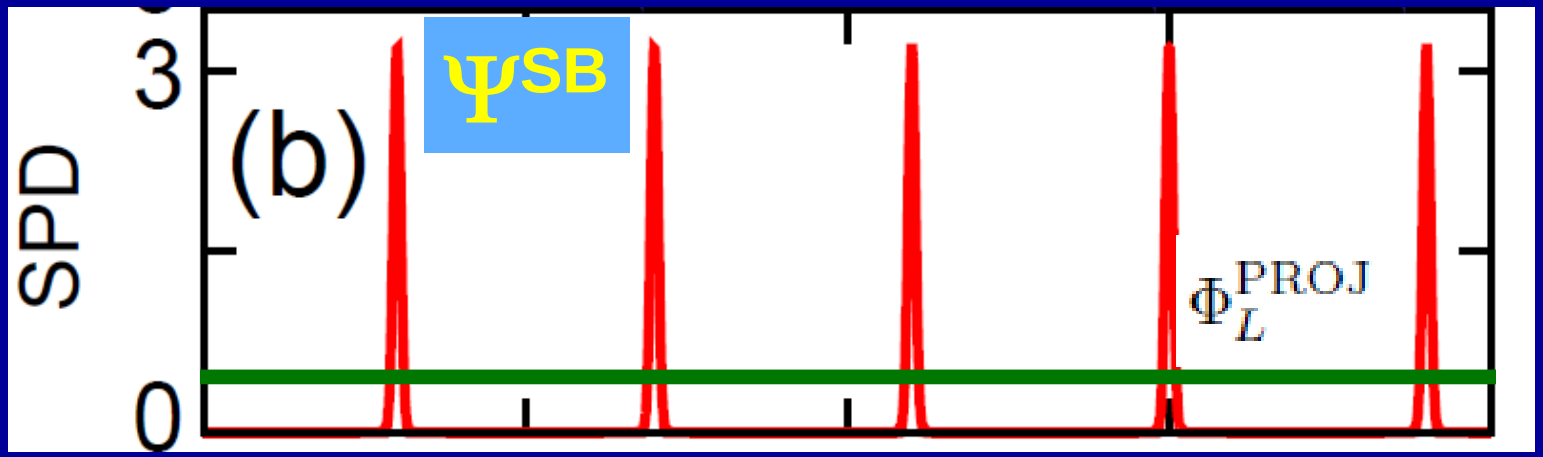


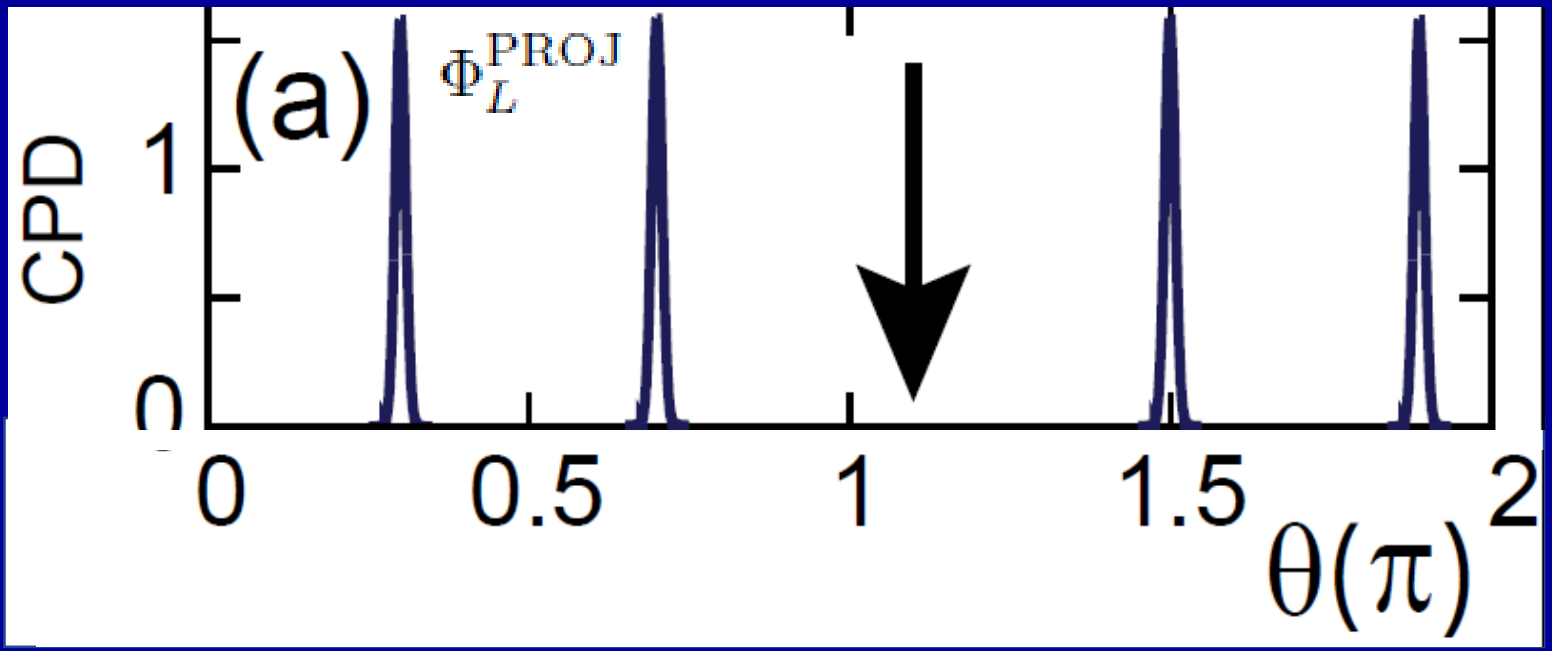


Y&L, PRL **85**, 1726 (2000)

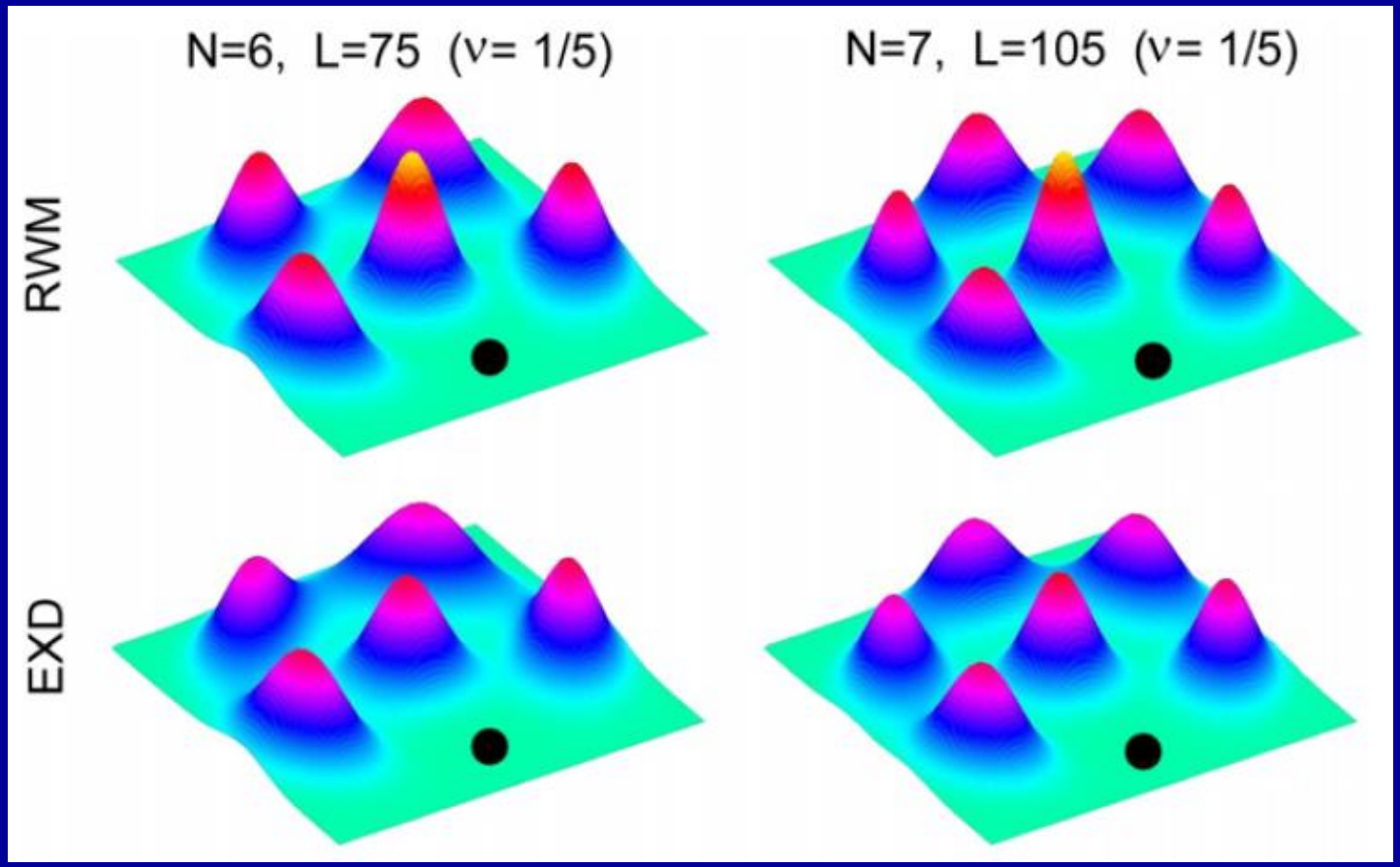
Structure of many-body wave functions on ring

N=5 ultracold ions (fermions)





CPDs for QDs, fully spin polarized



Lowest Landau level B → Infinity

Wave packets/ Time evolution

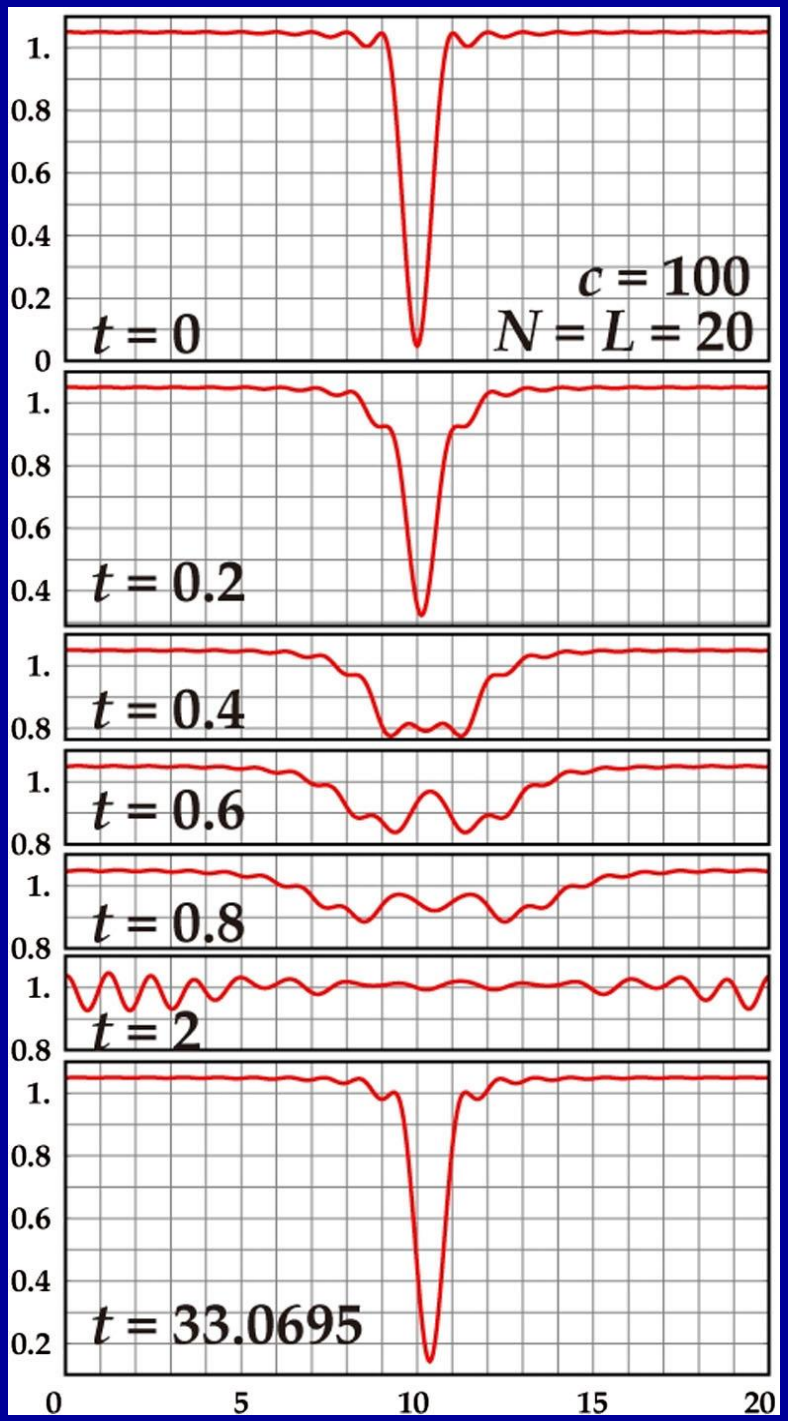
$$\Psi^{\mathrm{SB}} = \sum_L \mathcal{C}_L \Phi_L^{\mathrm{PROJ}}$$

Invert the projection

$$\mathcal{C}_L = \frac{1}{2\pi} \int_0^{2\pi} d\gamma e^{-i\gamma L} n(\gamma)$$

Many frequencies, terms e^{-iE_L}

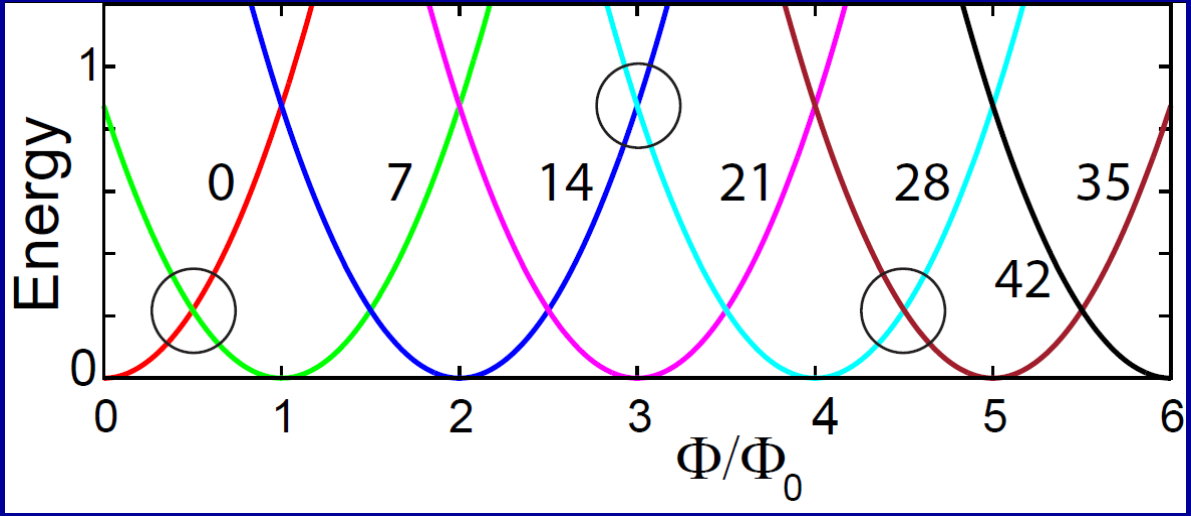
Diffusion and Revival



J. Sato et al.,
PRL **108**, 110401 (2012)

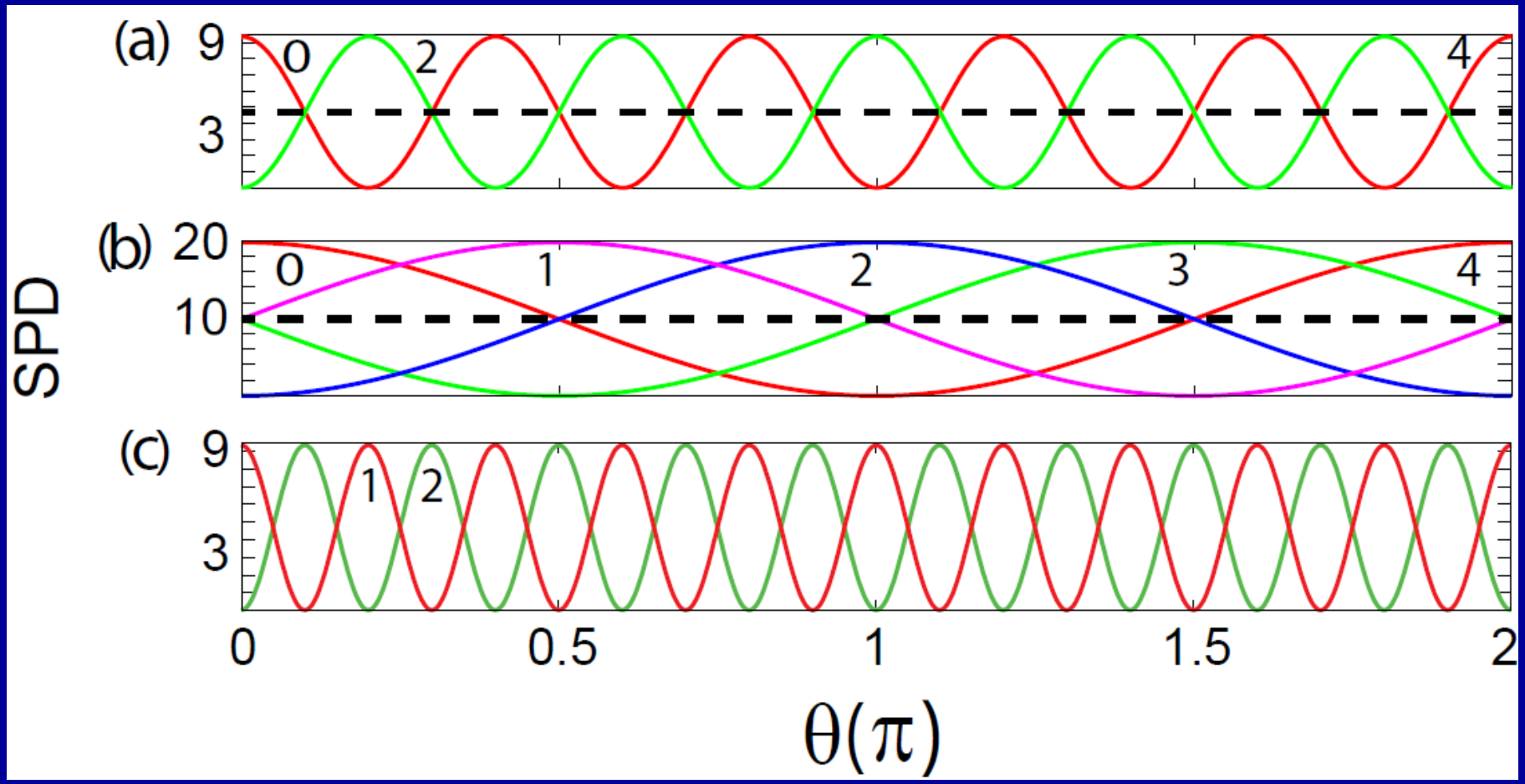
PINNED WIGNER MOLECULE (PWM)/ TWO-PROJECTED-STATE SUPERPOSITION

$$\Phi^{\mathrm{PWM}}(L_1, L_2; t = 0) = \alpha \Phi^{\mathrm{PROJ}}_{L_1} + \beta e^{i\phi(t=0)} \Phi^{\mathrm{PROJ}}_{L_2}$$



PWM: TIME EVOLUTION OF SPD (snapshots)

$$\tau = 2\pi\hbar/|E_1 - E_2|$$



BOTH SPACE AND TIME TRANSLATIONAL SYMMETRY ARE BROKEN

CONCLUSIONS:

SYMMETRY RESTORATION IS A NATURAL METHOD TO BE USED IN ADDRESSING THE PHYSICS IN MESOSCOPIC SYSTEMS OTHER THAN NUCLEI