

ECE 6115 / CS 8803 - ICN

**Interconnection Networks for
High Performance Systems**

Spring 2020

INTRODUCTION

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INTRODUCTIONS!



Background

- PhD from MIT in EECS (2013)
- Researcher at Intel (2014-15)
- Georgia Tech (2015 - present)

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Georgia Tech  **School of Electrical and Computer Engineering**
College of Engineering

TUSHAR KRISHNA
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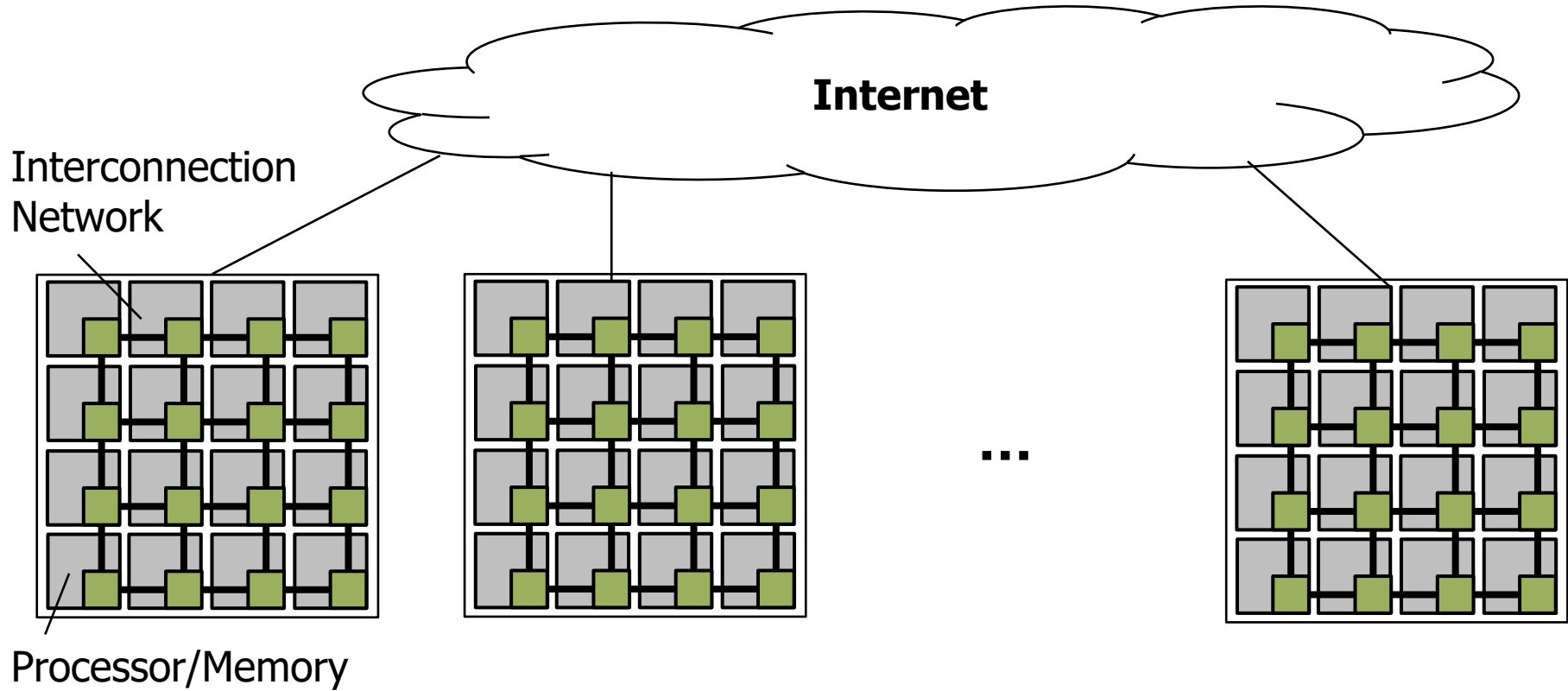
EMAIL: tushar@ece.gatech.edu
WEB: <http://tusharkrishna.ece.gatech.edu>

Research Interests

- Computer Architecture
- Interconnection Networks
- Network-on-Chip
- Deep Learning Accelerators
- Continuous Learning Systems

WHAT IS AN INTERCONNECTION NETWORK?

- Networks **within** a system (collaboration)
- Not the Internet: network **between** systems



WHY INTERCONNECTION NETWORKS MATTER?

News

Chinese 260-core processors ShenWei SW26010 enabled supercomputer Sunway TaihuLight be the most productive in the world

June 21, 2016 331 0

Technology

IBM reveals 'brain-like' chip with 4,096 cores

NEWS

Meet KiloCore, a 1,000-core processor so efficient it could run on a AA battery

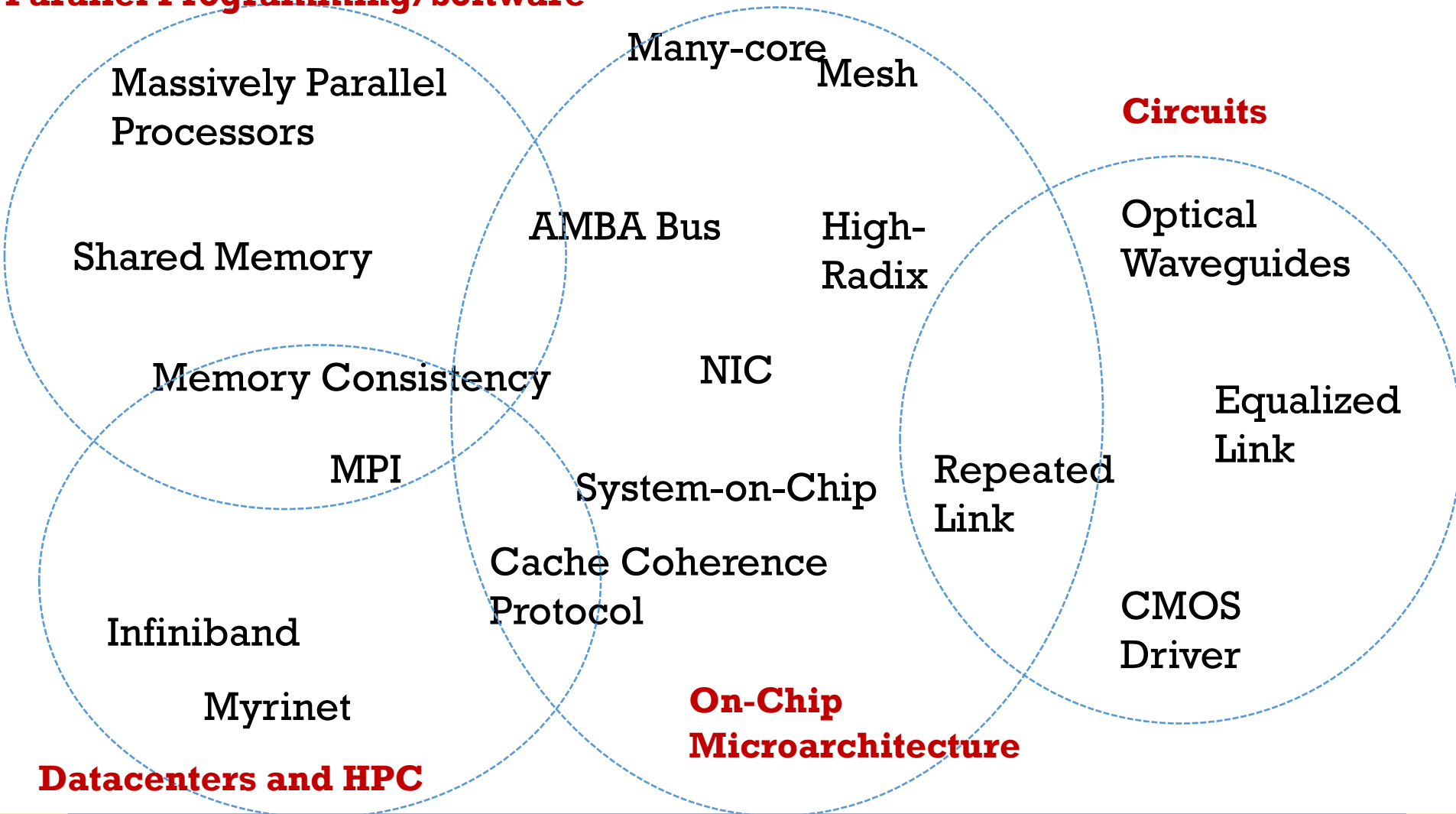
This monstrous CPU is 100 times more power-efficient than today's laptops.

IBM pushes silicon photonics with on-chip optics

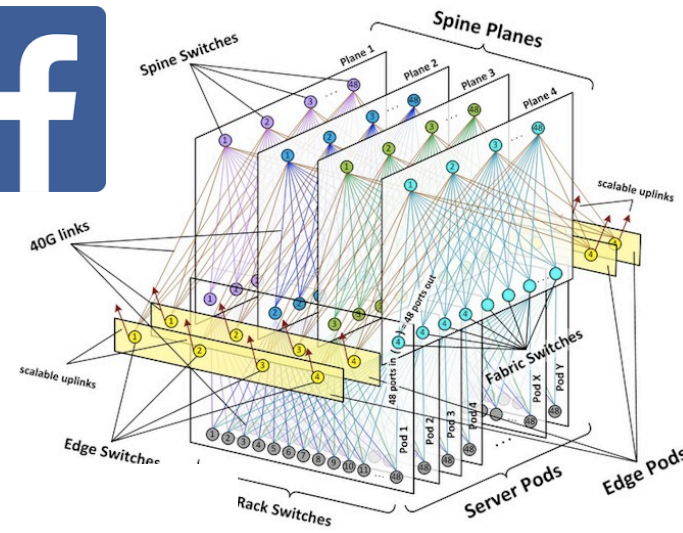
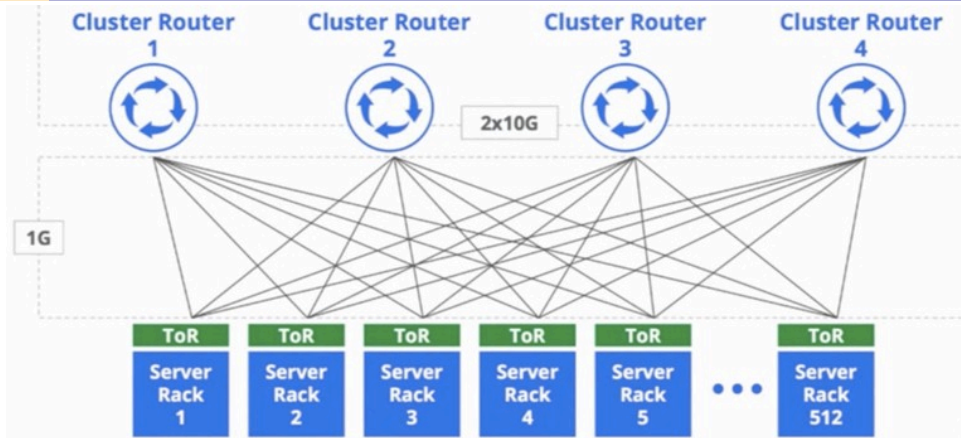
Big Blue researchers have figured out how to use standard manufacturing processes to make chips with built-in optical links that can transfer 25 gigabits of data per second.

WHAT IS AN INTERCONNECTION NETWORK?

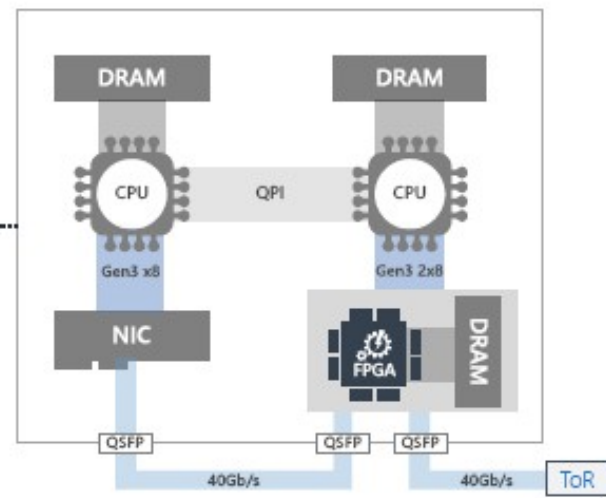
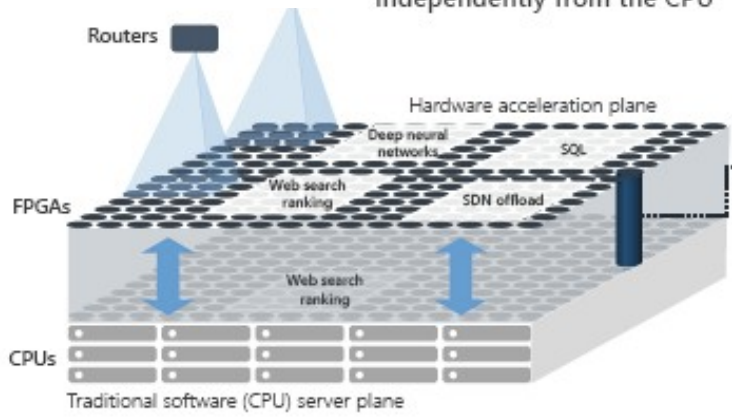
Parallel Programming/Software



DATACENTER NETWORKS



Interconnected FPGAs form a separate plane of computation
 Can be managed and used independently from the CPU

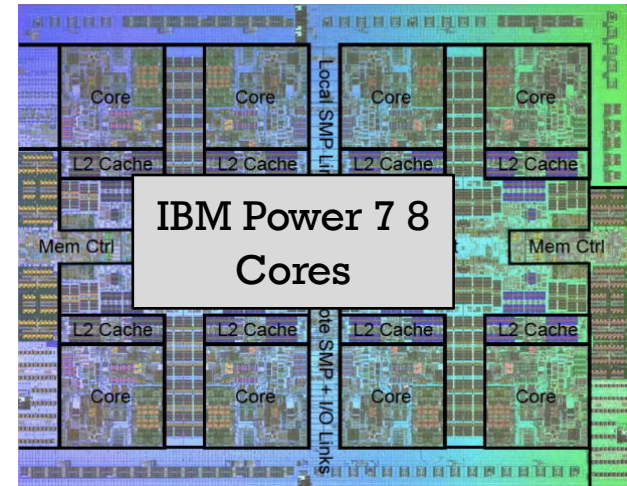
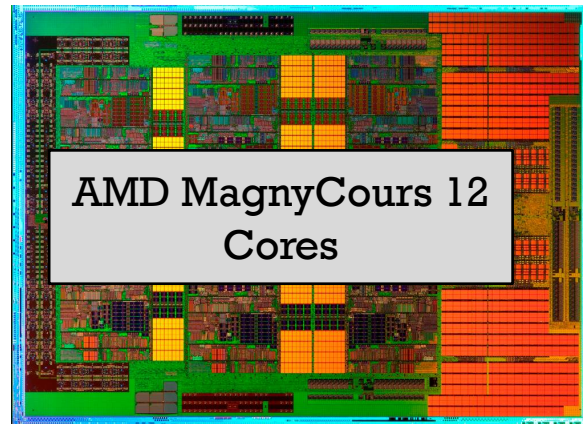
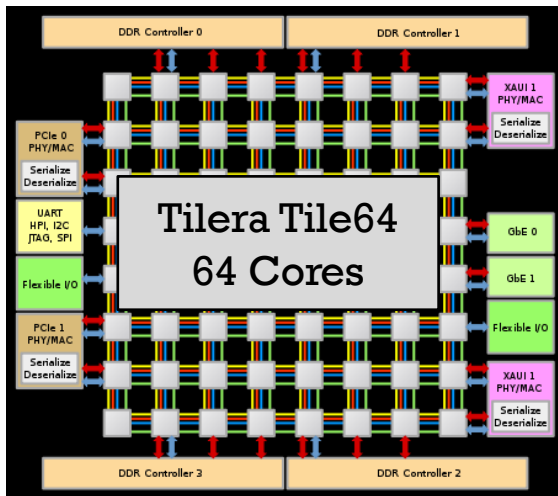
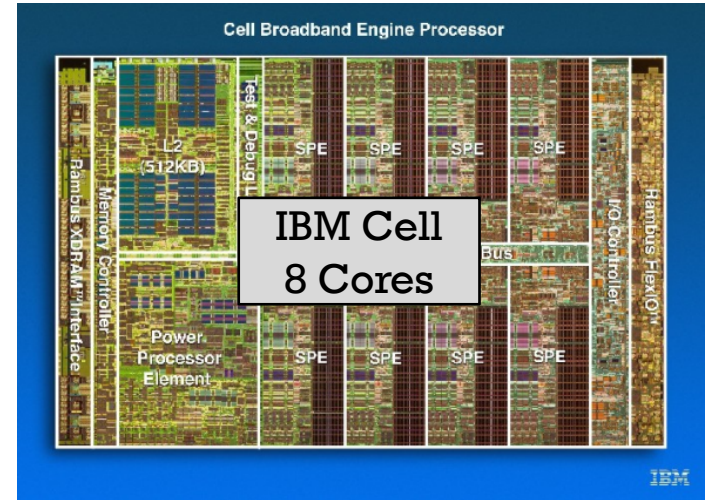
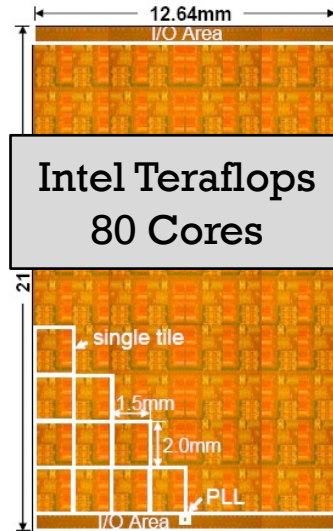
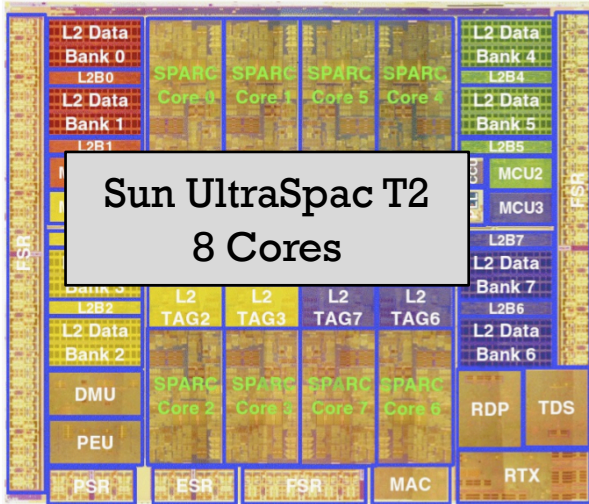


HPC NETWORKS

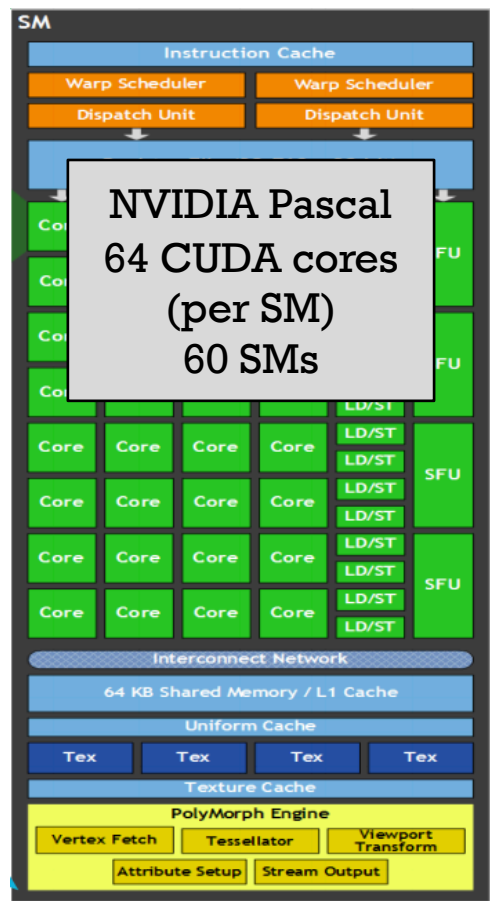
Top 10 positions of the 50th TOP500 in November 2017^[15]

Rank ↕	Rmax Rpeak (PFLOPS) ↕	Name ↕	Model ↕	Processor ↕	Interconnect ↕	Vendor ↕	Site country, year ↕	Operating system ↕
1	93.015 125.436	<i>Sunway TaihuLight</i>	Sunway MPP	SW26010	Sunway ^[16]	NRCPC	National Supercomputing Center in Wuxi 🇨🇳 China, 2016 ^[16]	Linux (Raise)
2	33.863 54.902	<i>Tianhe-2</i>	TH-IVB-FEP	Xeon E5-2692, Xeon Phi 31S1P	TH Express-2	NUDT	National Supercomputing Center in Guangzhou 🇨🇳 China, 2013	Linux (Kylin)
3	19.590 25.326	<i>Piz Daint</i>	Cray XC50	Xeon E5-2690v3, Tesla P100	Aries	Cray	Swiss National Supercomputing Centre 🇨🇭 Switzerland, 2016	Linux (CLE)
4	19.136 28.192	<i>Gyokou</i>	ZettaScaler-2.2 HPC system	Xeon D-1571, PEZY-SC2	Infiniband EDR	ExaScaler	Japan Agency for Marine-Earth Science and Technology 🇯🇵 Japan, 2017	Linux (CentOS)
5	17.590 27.113	<i>Titan</i>	Cray XK7	Opteron 6274, Tesla K20X	Gemini	Cray	Oak Ridge National Laboratory 🇺🇸 United States, 2012	Linux (CLE, SLES based)
6	17.173 20.133	<i>Sequoia</i>	Blue Gene/Q	A2	Custom	IBM	Lawrence Livermore National Laboratory 🇺🇸 United States, 2013	Linux (RHEL and CNK)
7	14.137 43.902	<i>Trinity</i>	Cray XC40	Xeon E5-2698v3, Xeon Phi	Aries	Cray	Los Alamos National Laboratory 🇺🇸 United States, 2015	Linux (CLE)
8	14.015 27.881	<i>Cori</i>	Cray XC40	Xeon Phi 7250	Aries	Cray	National Energy Research Scientific Computing Center 🇺🇸 United States, 2016	Linux (CLE)
9	13.555 24.914	<i>Oakforest- PACS</i>	Fujitsu	Xeon Phi 7250	Intel Omni-Path	Fujitsu	Kashiwa, Joint Center for Advanced High Performance Computing 🇯🇵 Japan, 2016	Linux
10	10.510 11.280	<i>K computer</i>	Fujitsu	SPARC64 VIIIfx	Tofu	Fujitsu	Riken, Advanced Institute for Computational Science (AICS) 🇯🇵 Japan, 2011	Linux

MANY-CORE ON-CHIP NETWORKS

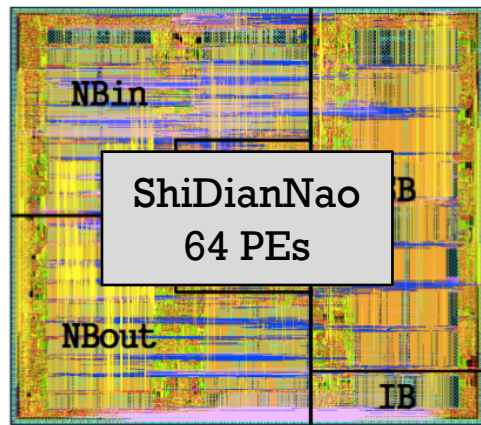


ACCELERATOR NETWORKS

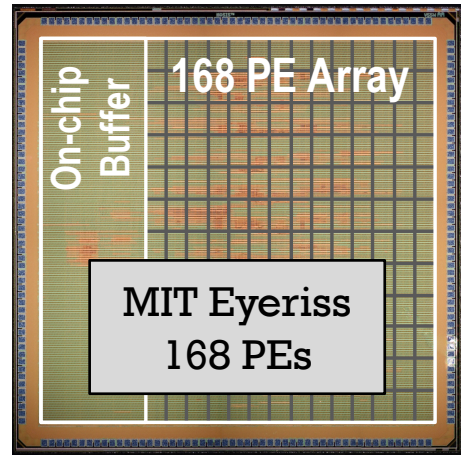


NVIDIA Pascal
 64 CUDA cores
 (per SM)
 60 SMs

GPUs

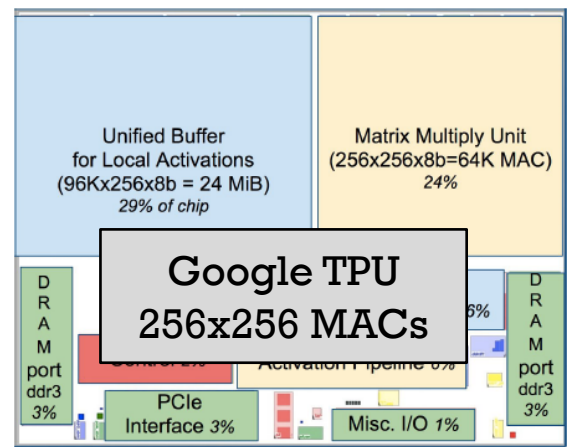


ShiDianNao
 64 PEs

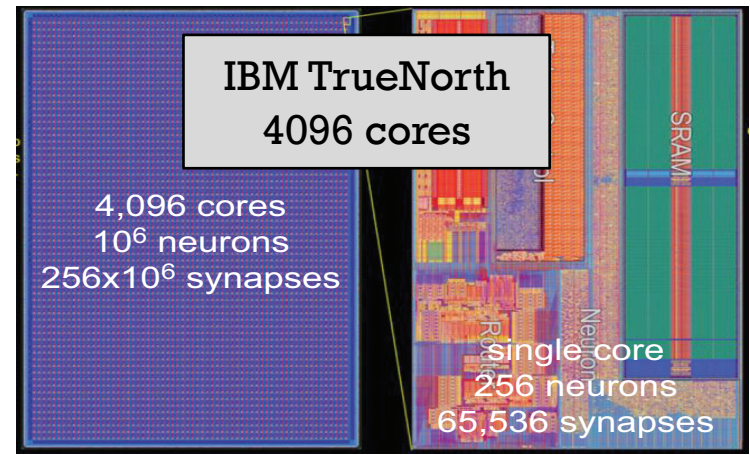


MIT Eyeriss
 168 PEs

Deep Learning Accelerators



Google TPU
 256x256 MACs

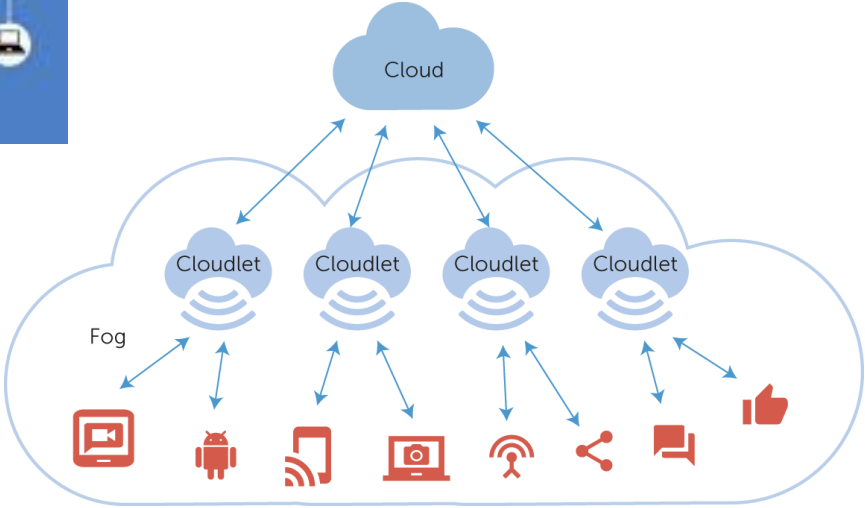
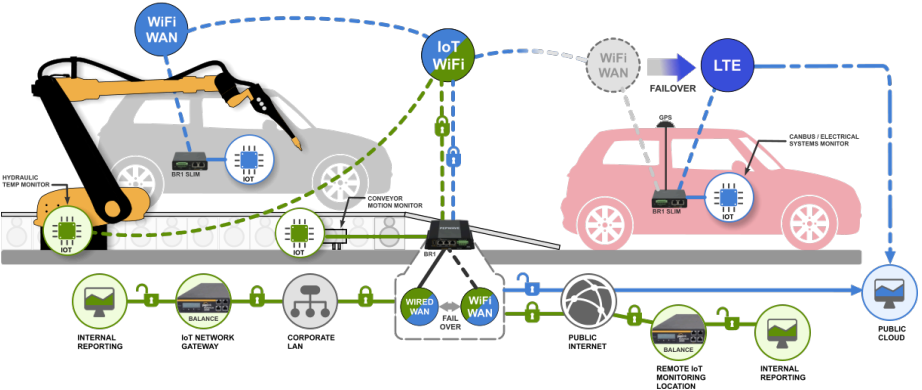


IBM TrueNorth
 4096 cores

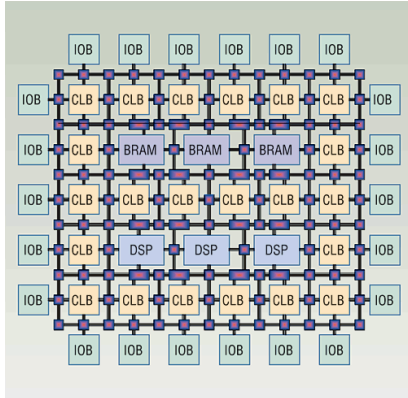
4,096 cores
 10⁶ neurons
 256x10⁶ synapses

single core
 256 neurons
 65,536 synapses

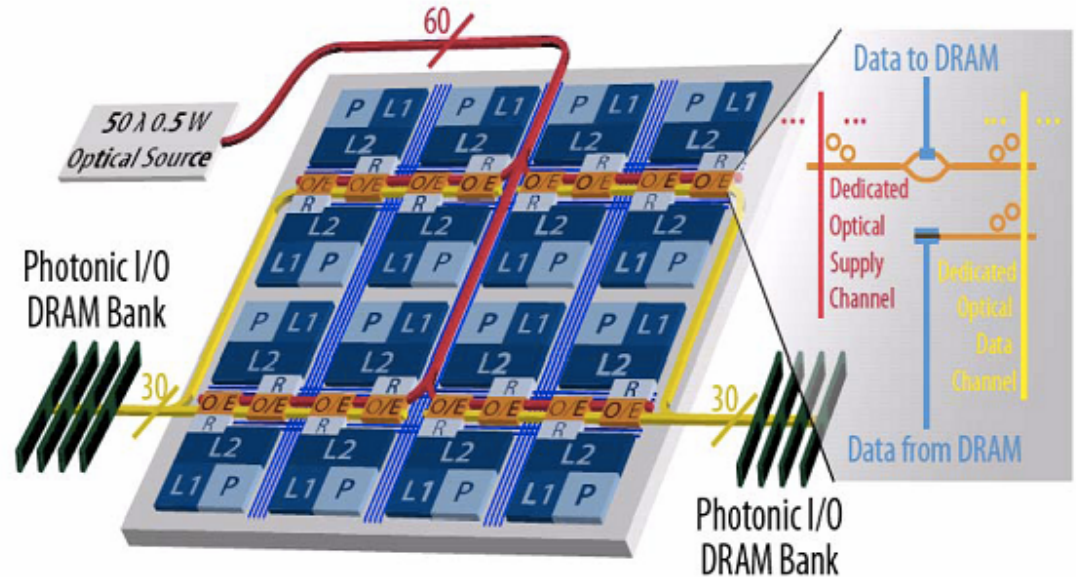
IoT NETWORKS



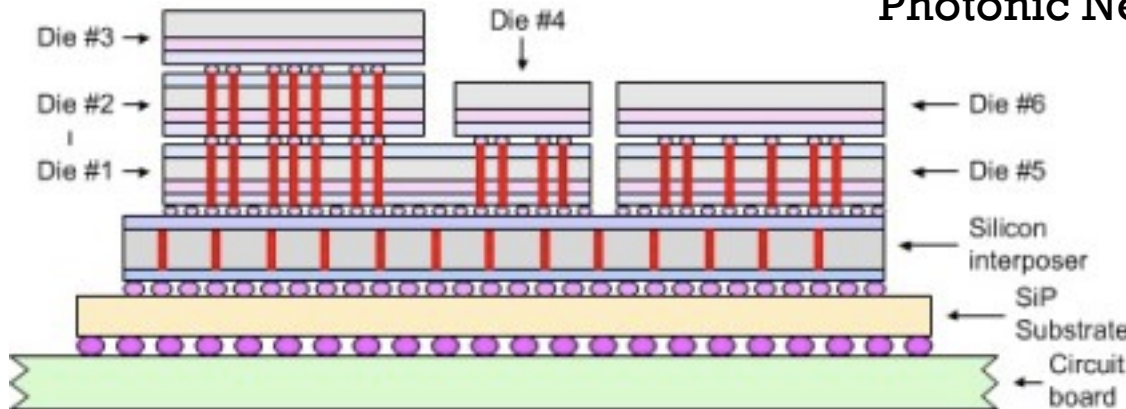
AND MANY MORE ...



FPGA Networks

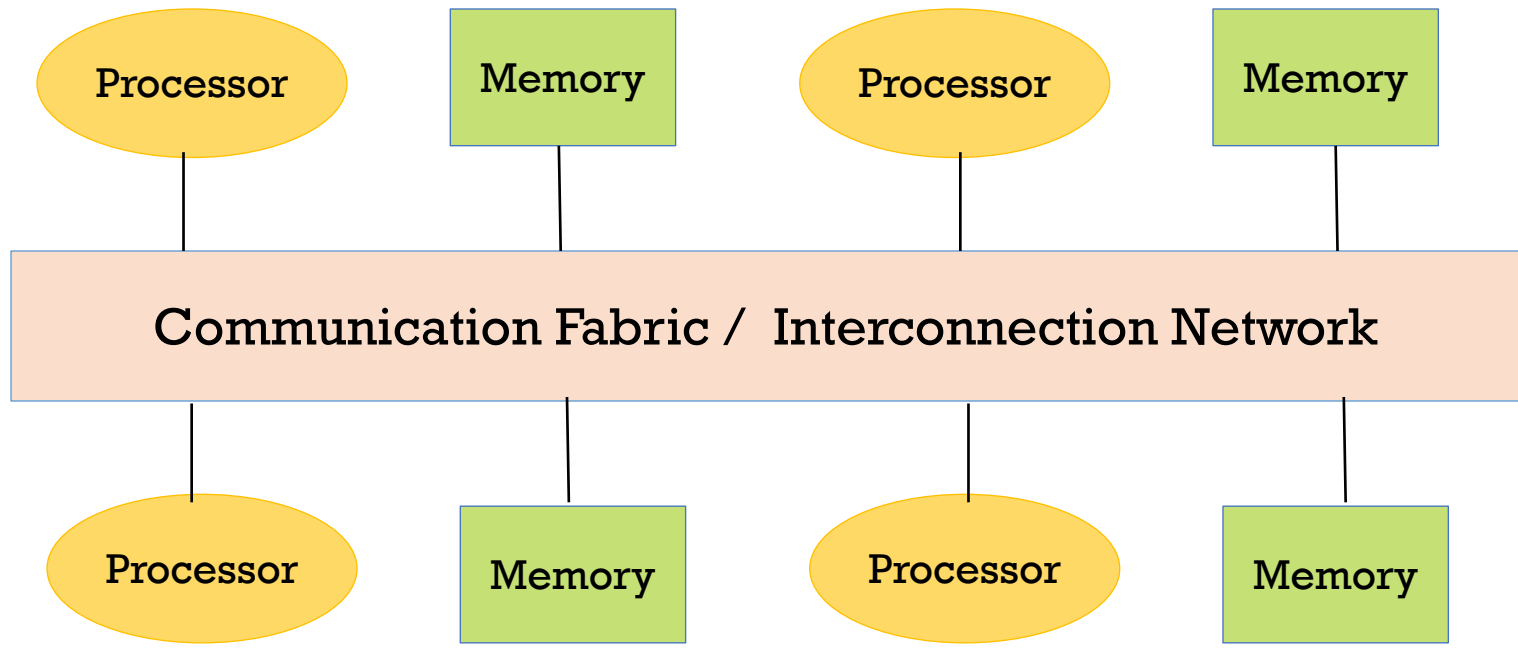


Photonic Networks



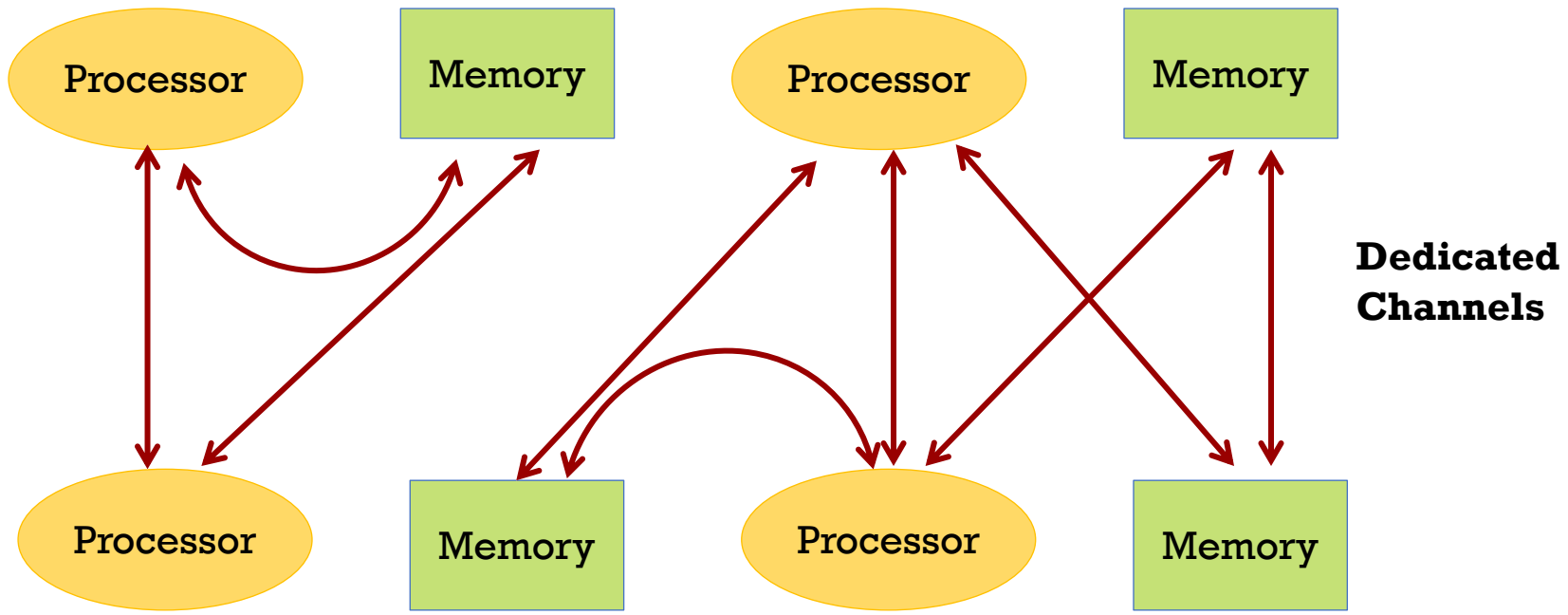
2.5D and 3D Networks

WHAT IS AN INTERCONNECTION NETWORK?



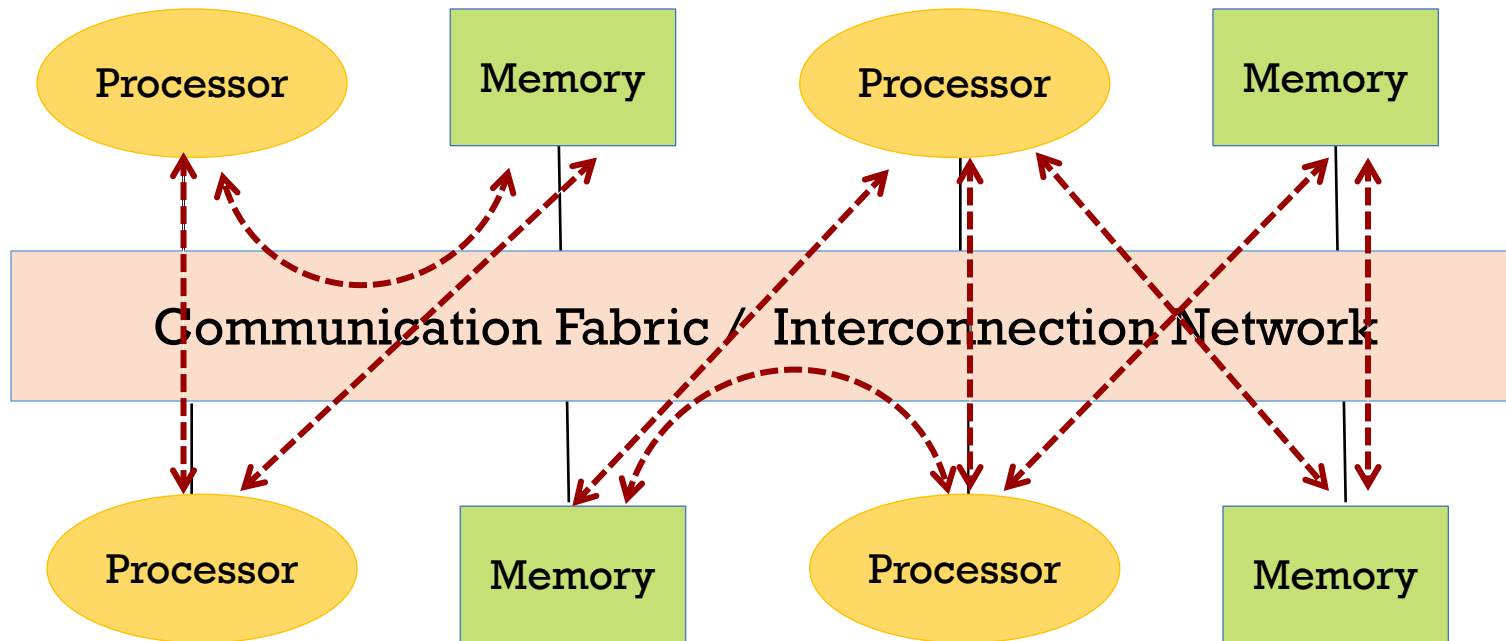
- Interconnection Networks connect processors and memory elements within and across computers

WHAT IS AN INTERCONNECTION NETWORK?



- **Application:** Ideally wants low-latency, high-bandwidth, dedicated channels between processors and memory
- **Technology:** Dedicated channels too expensive in terms of area and power

WHAT IS AN INTERCONNECTION NETWORK?



- **Interconnection Network:** A programmable system that transports data between terminals over a set of shared physical channels

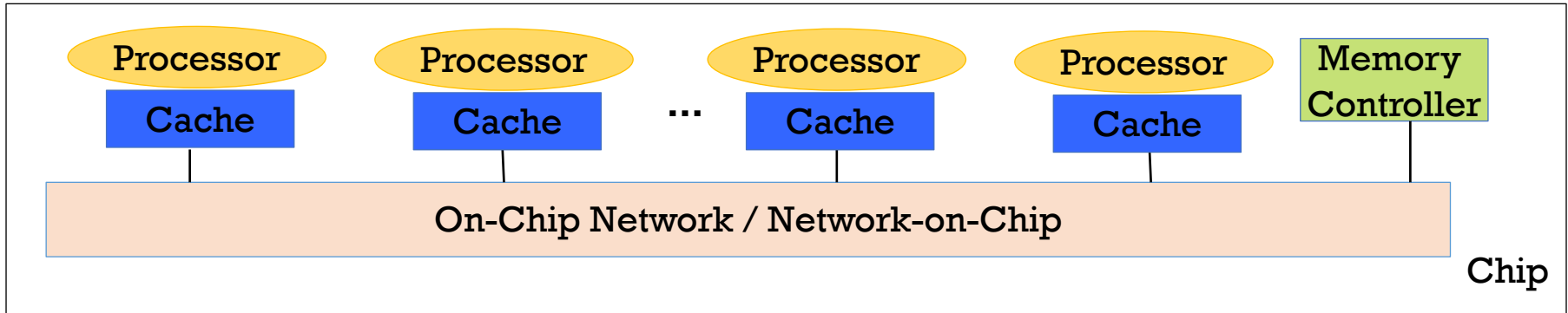
INTERCONNECTION NETWORKS

- Key Design Principles
 - Transfer maximum amount of information (**high bandwidth**) within the least amount of time (**low latency**) so as to not bottleneck the system
 - Efficiently utilize *shared* but *scarce resources* (buffers, links, logic) to **reduce area and power**

TYPES OF INTERCONNECTION NETWORKS

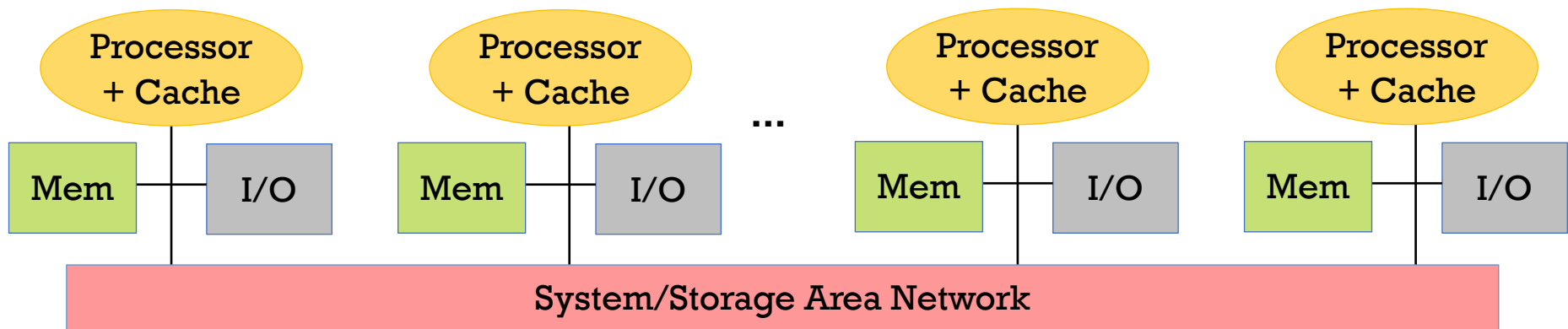
- Interconnection Networks can be grouped into four domains
 - Depending on the type and proximity of devices to be connected
- On-Chip Networks (OCNs or NoCs)
- System/Storage Area Networks (SANs)
- Local Area Networks (LANs)
- Wide Area Networks (WANs)

ON-CHIP NETWORK (OCN OR NOC)



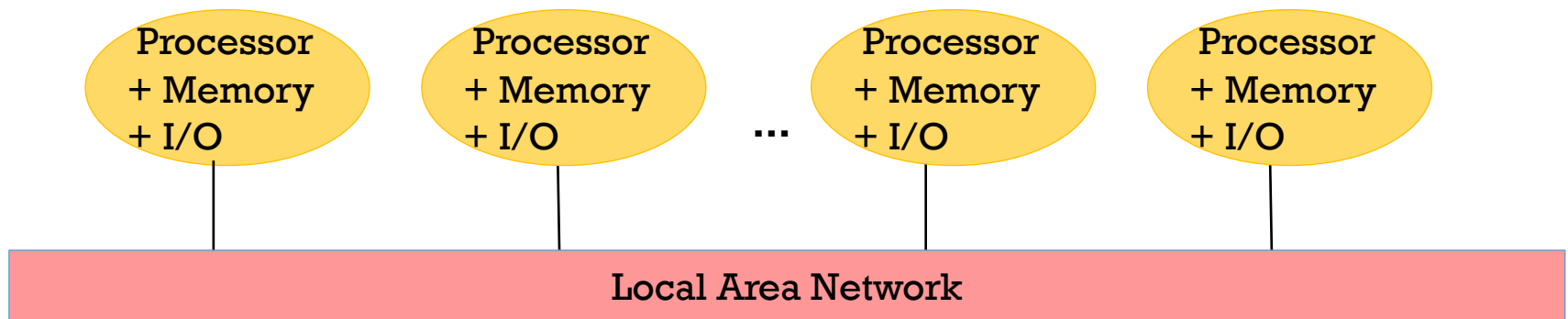
- Networks on multicore/MPSoC chips
 - Devices include micro architectural functional units, register files, processor/IP cores, caches, directories, memory controllers
- Current/Future Systems: tens to hundreds of devices
 - Intel Single-Chip Cloud Computer – 48 Cores
 - Tiler TILE64 – 64 Cores
- Tightly coupled with on-chip links
 - Proximity: milli meters
 - Delay: pico seconds

SYSTEM/STORAGE AREA NETWORK (SAN)



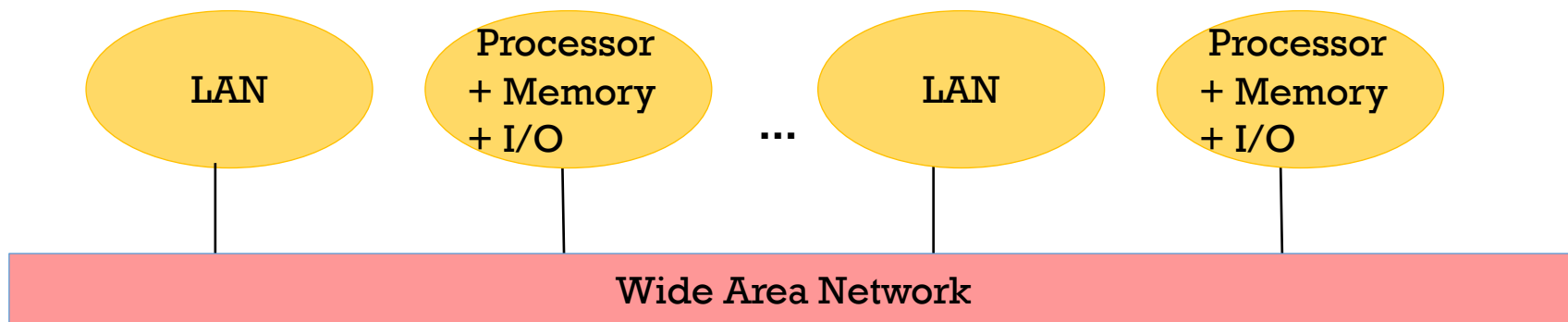
- Multi-processor and multi-computer Networks
 - Inter-processor and processor-memory interactions
- Server and Datacenter Networks
 - Storage and I/O components
- Hundreds to thousands of devices interconnected
 - IBM Blue Gene/L Supercomputer (64K nodes, each with 2 processors)
- Tightly-coupled with proprietary interconnects
 - Proximity: tens of meters (typical) to a few hundred meters
 - Link Delay: nano seconds
 - Examples: Infiniiband, Myrinet, Quadrics, Advanced Switching Interconnect

LOCAL AREA NETWORK (LAN)



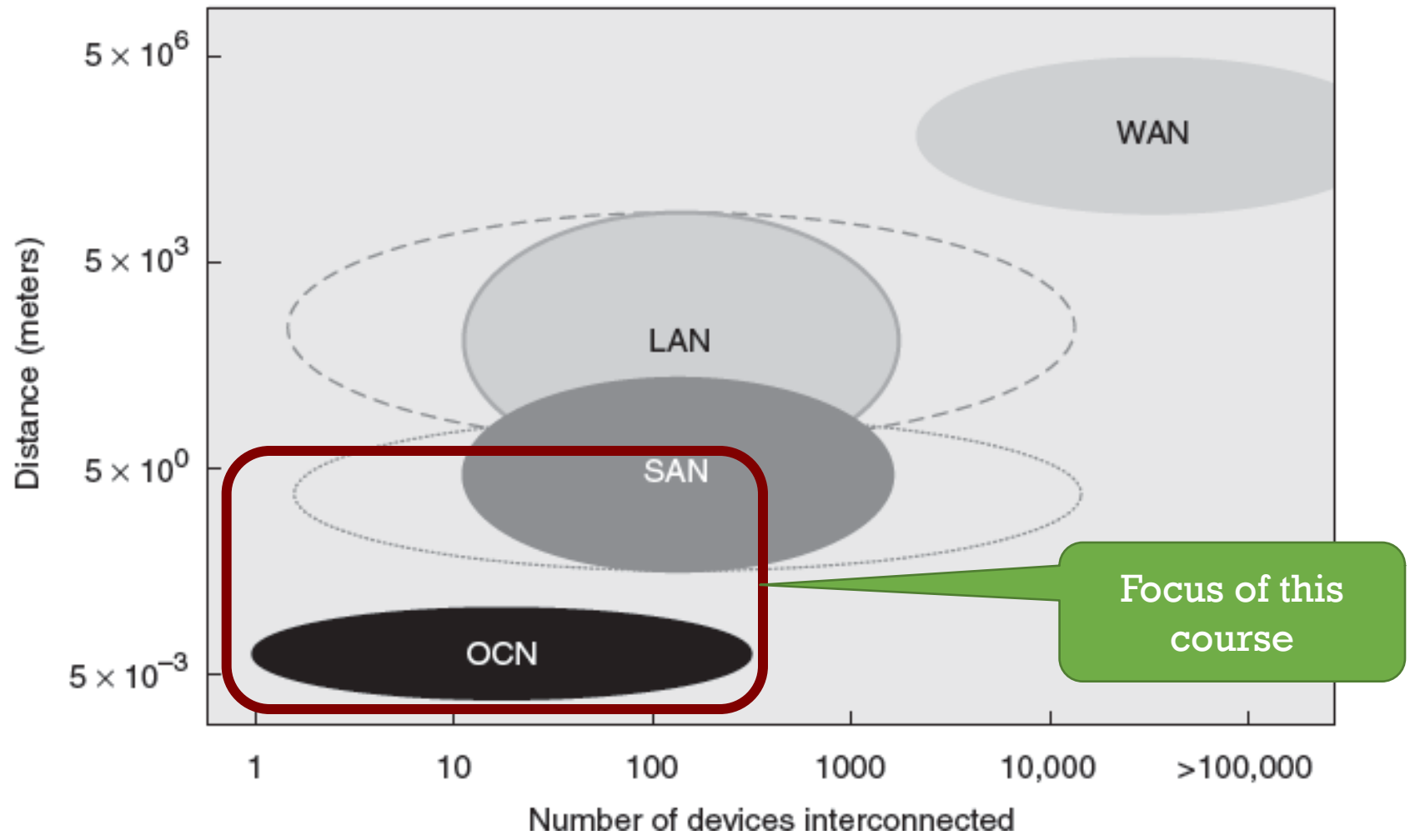
- Networks between autonomous computer systems
 - Example: Machine room or throughout a building or campus
 - “Clusters”
- Hundreds of devices
 - thousands with bridging
- Loosely coupled with commodity interconnects (e.g., Ethernet)
 - Proximity: few kilometers to few tens of kilometers
 - Delay: micro seconds

WIDE AREA NETWORK (WAN)



- Networks between LANs and autonomous computer systems distributed across the world
- Millions of devices
- Loosely coupled with electrical and optical interconnects
 - Proximity: many thousands of kilometers
 - Delay: milli seconds
- Largest WAN is the Internet

INTERCONNECTION NETWORK DOMAINS

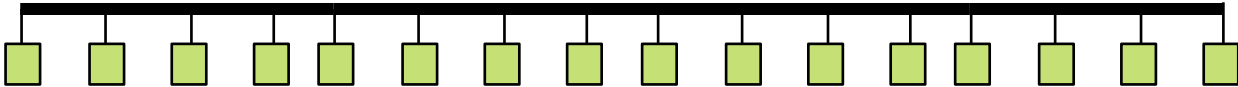
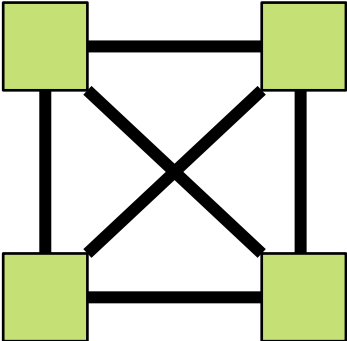


Source: Hennessy and Patterson, 5th Edition, Appendix F

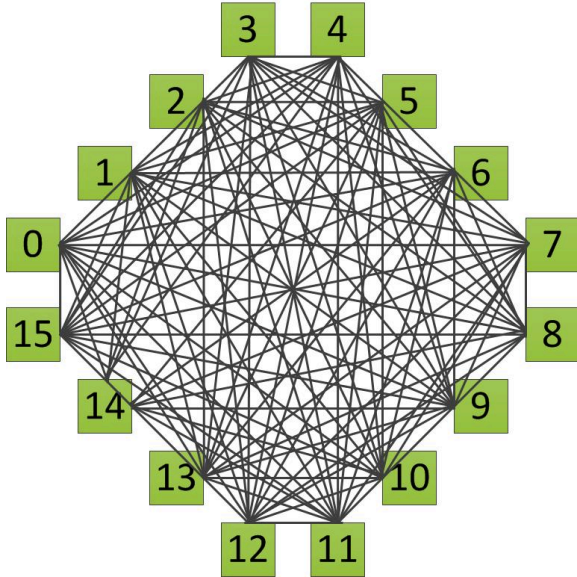
NETWORKS ARE CRITICAL FOR PERFORMANCE



Early designs were buses and point-to-point



DOES NOT SCALE!!!



DIFFERENCES BETWEEN OFF-CHIP (SANS) AND ON-CHIP NETWORKS

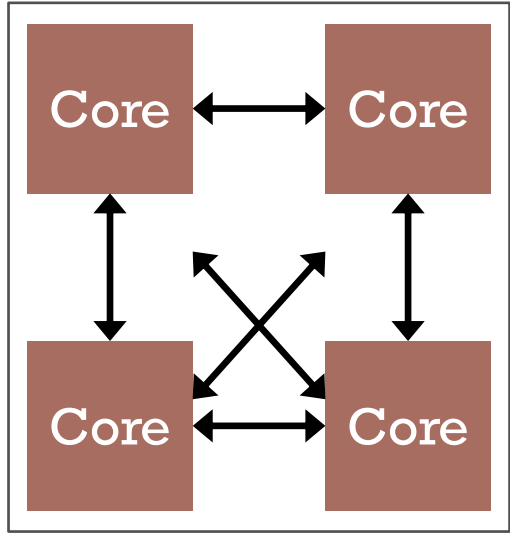
- Significant research in multi-chassis interconnection networks (off-chip) since the 90s
 - Supercomputers
 - Clusters of Workstations
 - Internet Routers

- We can leverage research insights, but ...
 - constraints are different
 - new opportunities

OFF-CHIP VS. ON-CHIP

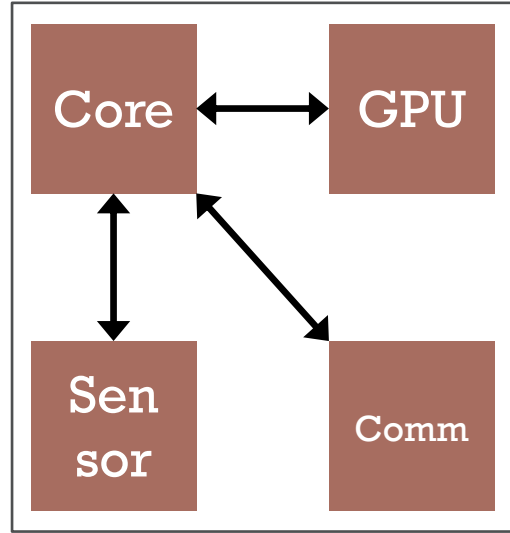
- Off-Chip Networks
 - Bandwidth limited by chip pin-bandwidth
 - Latency limited by long off-chip cables
- On-Chip Networks
 - Very high on-chip bandwidth
 - Abundant metal layers and wiring
 - Much lower latency due to short wires
- The key concepts remain the same across SANs and NoCs
 - the constraints are different → the design decisions are different
 - E.g., an off-chip topology may not be feasible on-chip due to on-chip layout constraints
 - Or an on-chip link circuit may not be feasible off-chip due to technology constraints
- *We will mostly focus on on-chip networks when discussing concepts, but will periodically look at implications in the off-chip space*

GENERAL-PURPOSE VS. SPECIALIZED



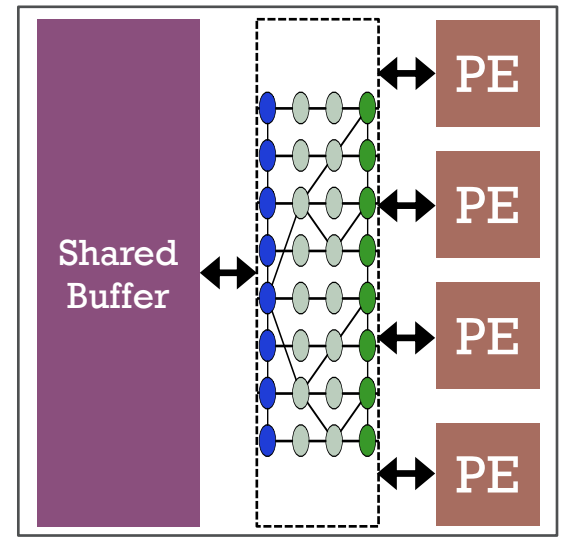
CMPs

**Dynamic
all-to-all traffic**



MPSoCs

**Static fixed
traffic**



DNN Accelerators

**Collective
Communication**

AGENDA

- Course Motivation
- **Course Logistics**
- Introduction to NoCs
- Simulation Infrastructure

WHAT'S UNIQUE ABOUT THIS COURSE

- Not taught as a standard course in any university
 - Sits at the intersection of Computer Architecture, Parallel and Distributed Architectures, Distributed Systems, Computer Networks, and VLSI Design
- Traditional Course Structure
 - **Computer Architecture** courses focus on processor pipeline and memory hierarchy, skimming through the system interconnection
 - **Networking** courses focus on protocols, ignoring router hardware
 - **Communications** courses focus on signal processing algorithms, skimming through hardware implementations
 - **Optics/electronic link** courses focus on link circuitry, skimming through how these links are composed as networks
- Sister courses
 - **Active:** University of Toronto ECE 1749H (N Jerger)
 - **Inactive:** Past offerings in MIT (Peh), Stanford (Dally), Penn State (Das), Utah (Balasubramonian), Cornell (Batten)

WHAT'S UNIQUE ABOUT THIS COURSE

- Handful of top researchers in NoCs across academia and industry
 - opportunity to gain expertise in a niche area
 - aka internships/jobs of students who take this class
 - manycore architectures (Intel, AMD, IBM)
 - GPUs and Accelerators (NVIDIA, ARM, Intel, Samsung)
 - supercomputers (IBM, NVIDIA, Cray)
 - datacenters (Google, Amazon, Facebook, Microsoft)
 - internet routers (Cisco, Juniper)
- Projects will be open-research questions
 - **Very high chance of publication!**
 - 2016 Version: HPCA, ISPASS, ICCAD, NOCS
 - 2017 Version: ASPLOS, ISPASS, MICRO
 - 2018 Version: NOCS, ICRC, ISPASS*
 - 2019 Version: NOCS, ISPASS*
 - (*under submission)

COURSE STRUCTURE

- Phase I [Jan-Feb]
 - ~7-8 Instructor Lectures
 - 4 Programming Lab Assignments
 - One Midterm Quiz

- Phase II [Mar-Apr]
 - Paper Readings, Critiques, and Discussions
 - Each student presents one paper in class and leads its discussion

- Phase III [End of Feb – Apr]
 - Research Project

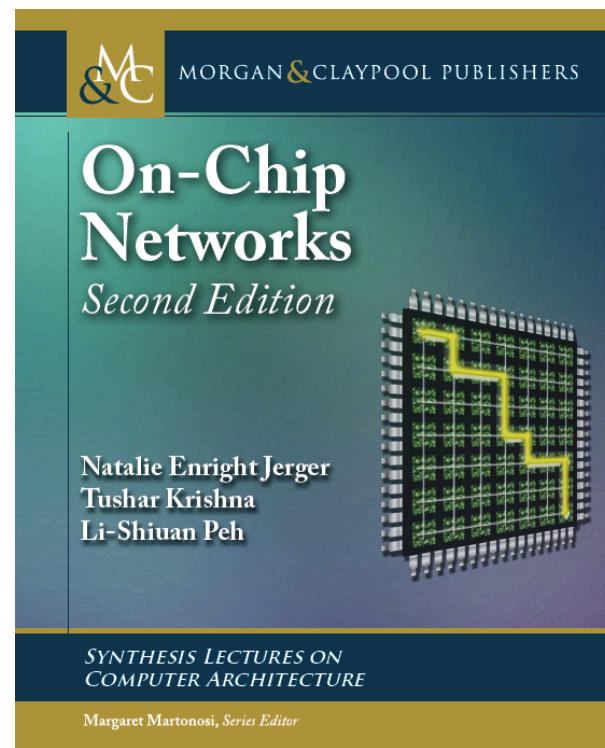
GRADING

Item	Percentage
Lab1	3%
Lab2	10%
Lab3	10%
Lab4	10%
Midterm Quiz	10%
Paper Critiques	10% [Best of 10]
Paper Presentation	10%
Project - Proposal	5%
Project - Milestones	10%
Project - Presentation	10%
Project - Final Report	12%
Total	100%

Phase I
Phase II
Phase III

PHASE I

- Lectures
 - ~7-8 weeks of instructor lectures
- Required Textbook
 - N. E. Jerger, T. Krishna, and L-S Peh, “On-Chip Networks”, 2nd Edition, Synthesis Lectures in Computer Architecture, Morgan & Claypool Publishers, 2017
 - Available for free download (within Georgia Tech)
 - Also added to Canvas
 - Supplemental reading material/papers will be posted
- Optional Textbooks:
 - W. Dally and B. Towles, “Principles and Practices of Interconnection Networks,” Morgan Kauffman Publishers, 2004.
 - J. Duato, S. Yalamanchili, L.Ni, “Interconnection Networks: An Engineering Approach,” Morgan Kauffman Publishers, 2002.



PHASE I

▪ Lab Assignments

- C++ based programming assignments
 - Build various components of a network, run traffic, analyze performance and costs
- Familiarize yourself with a network simulator called Garnet2.0
 - Part of gem5 (www.gem5.org), one of the leading open-source simulators for computer architecture research in industry and academia [Over 2000 citations in 6 years]
- Setup inside GT:
 - http://tusharkrishna.ece.gatech.edu/teaching/garnet_gt/

PHASE I

- **Lab 1 is due Friday at 1:00 pm**
 - **Hard Deadline**
 - **Already up on Canvas**
 - Gives you time to drop course if you don't have right background
 - **Goal:** run synthetic traffic simulations for 4 traffic patterns and report the latency vs. injection rate results
 - ***Setup Garnet right away and get started!***

PHASE I

- **Midterm Quiz**
 - Short in-class Quiz on the topics discussed in class
 - Aim is to make sure you go back and read the slides after each lecture

PHASE II

- Writing, Presenting, and Discussing research ideas will be key thrusts
- In every class, we will discuss 2-3 papers
 - State-of-the-art research across a breadth of domains
 - *Datacenters, HPC, On-Chip, GPU, TPU, FPGAs, Circuits, Novel Technologies*
 - Everyone (including presenter) has to submit a 1 page write-up *before the beginning of class* on one of the papers
 - **Short Summary + 2 strengths + 2 weaknesses + 1 suggested improvement**
- One student will present on one of the papers/topics for 15-20 minutes and lead the class discussion
 - Make your own slides
 - The presenter should create a Piazza post for the paper when it is assigned.
 - Might have 2-3 presentations per class on different papers taking opposing views, leading to a healthy debate

PHASE III

- Research Project
 - Propose a solution for a research problem in the Network-on-Chip/System-Area Network space; implement and evaluate it
 - Implement an idea from a paper and propose an extension, or implement a completely novel idea
 - Start thinking of project ideas as I present topics in class
 - I will also periodically provide a list of potential ideas
 - Groups of 2

PROJECT IDEAS

- **Deep Learning**
 - Performance evaluation of Google's TPU systolic array (released in 2017)
 - Performance characterization of NVIDIA's NVDLA Network (released in 2017)
 - NoC Topologies for efficient mapping strategies
 - NoC for scale-out DNN accelerators
 - NoCs for FPGA-based Deep Learning Accelerators
- **Microarchitecture**
 - SMART NoC emulating High-Radix Topology
 - NoCs for heterogeneous CPU-GPU systems
 - NoCs inside GPUs
 - Approximation-aware NoC
 - 2.5D Networks on a package using different packaging technologies
- **Open-source Hardware (in RTL/Chisel)**
 - Network-on-Chip in Chisel
 - NoC in Verilog [build upon prior work]
- **Cloud and Edge Networks**
 - Networks on a Rack
 - Wireless Networks between Raspberry Pis

PHASE III

- Research Project
 - Infrastructure
 - The project can be implemented in Garnet, or any other tool (C++/RTL)
 - Garnet/gem5 is useful since the plumbing related to other parts of the system (and even real apps) are provided.
 - We will introduce you to Chisel and Bluespec System Verilog (C-like abstractions for generating Verilog)
 - Projects related to your own Special Problem / MS / PhD research work will be encouraged as long as they have a networks component
 - Projects can be done individually (preferred) or in groups of two (if scope is larger, clearly defining each member's role)

PHASE III

- Research Project Milestones
 - Proposal Presentation
 - Progress Milestone #1
 - Progress Milestone #2
 - Final Presentation
 - No Final Exam!
 - Final Report

HOW PROJECTS WILL BE EVALUATED

- Looking for thorough understanding, implementation, evaluation, and presenting of idea
 - the idea might not lead to any improvement over the state-of-the-art
 - that is OK
 - rather that is research!
- If the idea leads to novel insights and there is a chance for publication, I am excited to work with you on polishing the work over summer and submit it to a conference or journal
 - I will fund your travel for presenting at the conference if it gets accepted 😊
 - If you want to work on a novel publishable idea or an extension to your own MS/PhD research, contact me – part of it can be used for the course

SCHEDULE (TENTATIVE)

Week	Dates	Monday	Wednesday	Due [Friday]
1	(Jan 6 -)			Lab 1
2	(Jan 11 -)			
3	(Jan 20 -)	MLK Day		Lab 2
4	(Jan 27 -)			
5	(Feb 3 -)			
6	(Feb 10 -)			Lab 3
7	(Feb 17 -)			Project Proposals
8	(Feb 24 -)		Midterm	
9	(Mar 2 -)			Lab 4
10	(Mar 9 -)		Proposal Ppt	
11	(Mar 16 -)	Spring Break		
12	(Mar 23 -)			
13	(Mar 30 -)			Milestone 1
14	(Apr 6 -)			
15	(Apr 13 -)			Milestone 2
16	(Apr 20 -)			
17	(Apr 27 -)	Final Presentations		Final Report

GT Holiday
Instructor Travel
Phase I
Phase II
Phase III

COURSE INFORMATION

▪ Course Website

- http://tusharkrishna.ece.gatech.edu/teaching/icn_s20/
- All additional readings will be posted here!

▪ Canvas

- Lecture Slides will be posted here
- Lab Assignments will be posted and submitted here
 - Lab 1 is also posted on website for wait-listed students
- Project Reports will be submitted here

▪ Piazza

- Access via Canvas
- Use for questions related to the lab assignments
 - Try to answer each other's questions
 - **There are no TAs in this course**
 - Do not upload code!
 - If no one is able to answer within a day I will respond
- Discussions on paper readings also encouraged

WHAT IS EXPECTED FROM YOU?

- Required Background
 - This is an **advanced** graduate-level class
 - Graduate-level Computer Architecture (ECE 6100 / CS 6290) background expected
 - C++ Programming Knowledge
 - Labs + Project
- Willingness to do explore open research problems!
 - Heavy paper reading [~15 papers]
 - Lot of Writing
 - Critiques, Reviews, and Report
 - Identify, Implement, Evaluate and Present a new idea
 - 3 presentations (Paper + Proposal + Final Report)

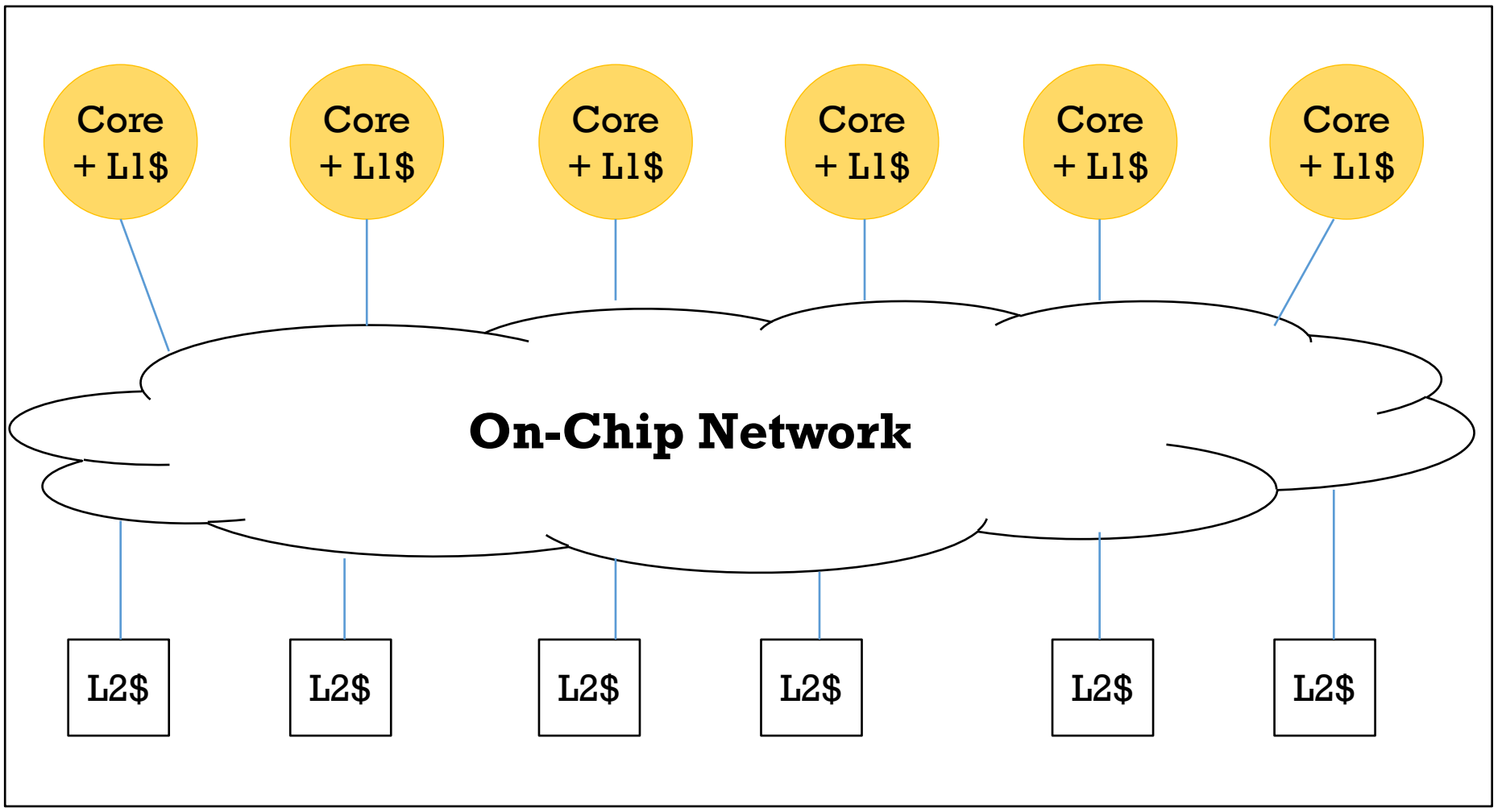
HOW TO CONTACT ME

- Piazza
- Email for questions not suitable for Piazza
 - Why did I lose points?
 - Is this an acceptable project proposal
- Office Hours
 - Friday 1:00 – 2 PM in Klaus 2318

AGENDA

- Course Motivation
- Course Logistics
- **Introduction to NoCs**
- Simulation Infrastructure

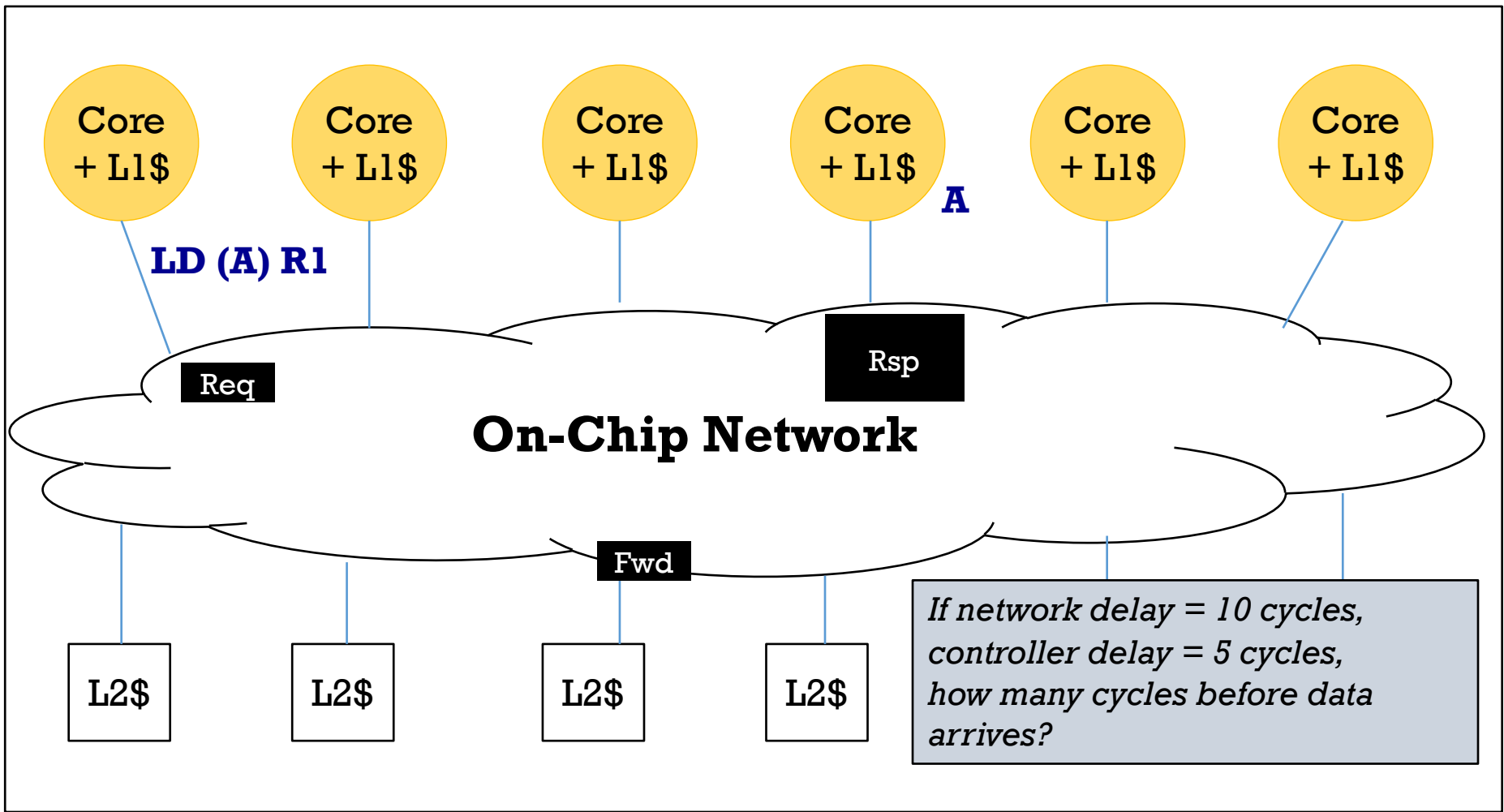
INTRODUCTION TO NOCS



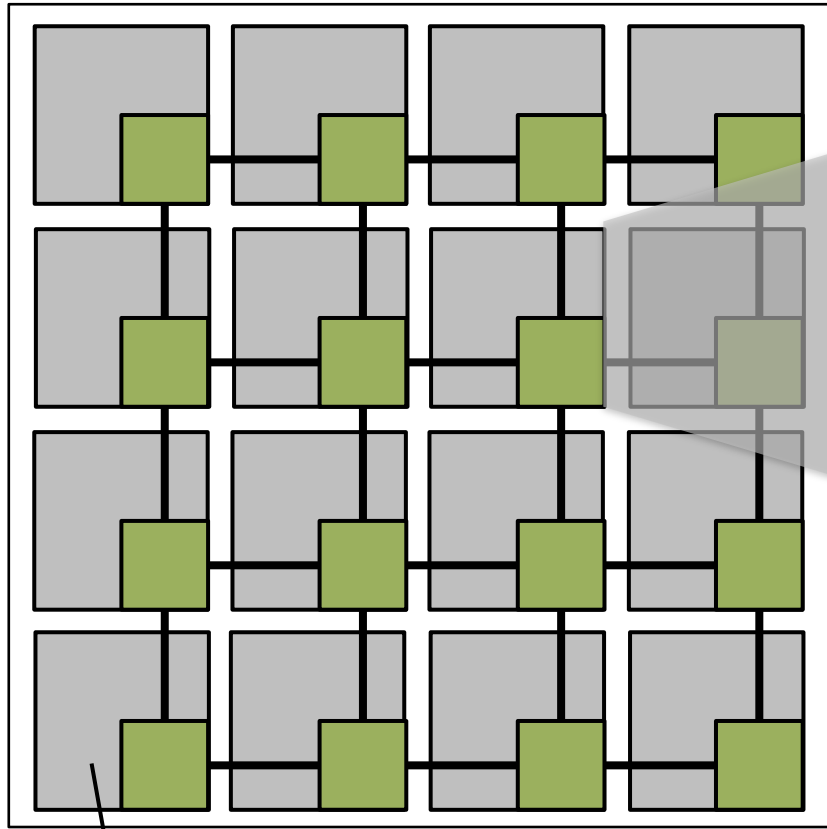
ROLE OF THE NETWORK-ON-CHIP

- “Shared Memory” Systems
 - Transport Cache Lines and Cache Coherence Messages between caches and memory controller(s) in shared memory CMPs
- “Message Passing” Systems
 - Transfer data between IP blocks in MPSoCs (Multi-Processor System on Chip)

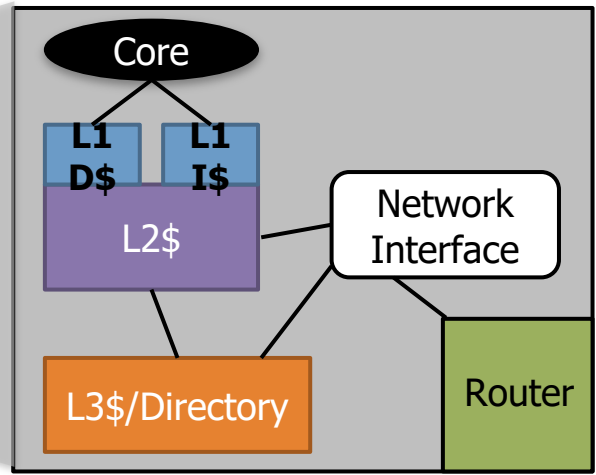
EXAMPLE OF TRAFFIC FLOWS



MODERN NOCS



"Tile"



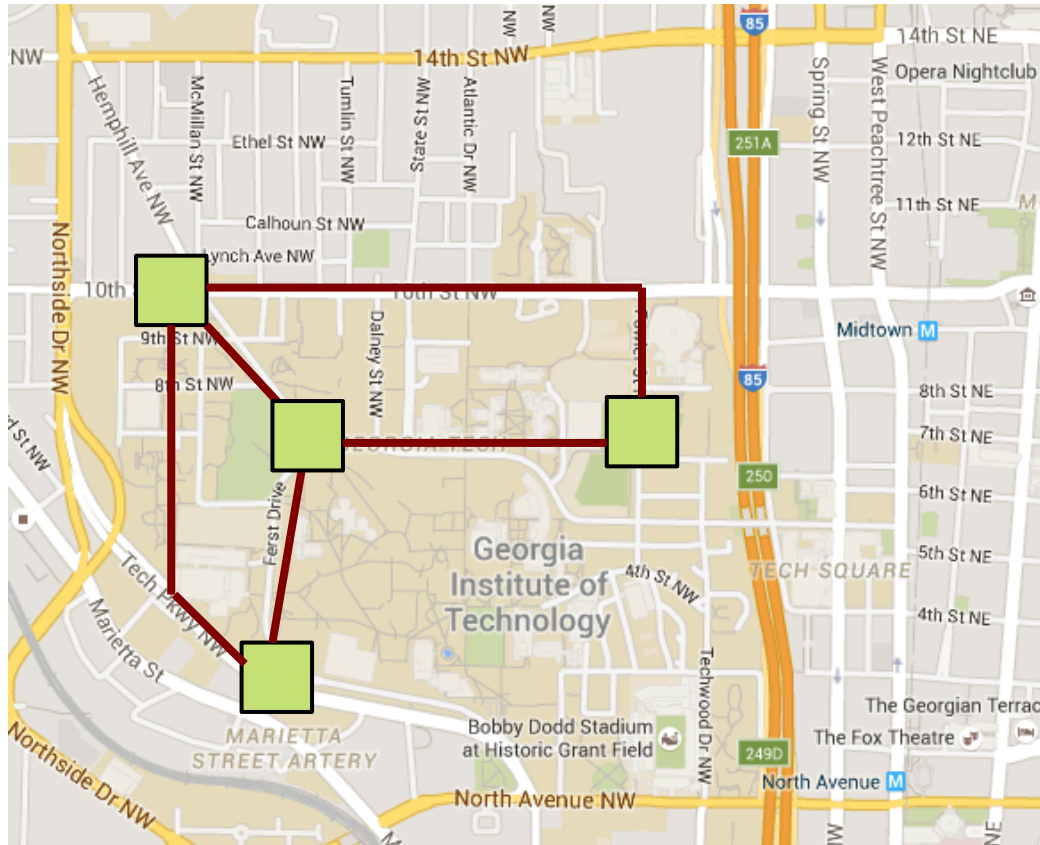
Core will not be shown explicitly in the most of the slides. Only the routers will be.

NETWORK ARCHITECTURE

- Topology
- Routing
- Flow Control
- Router Microarchitecture

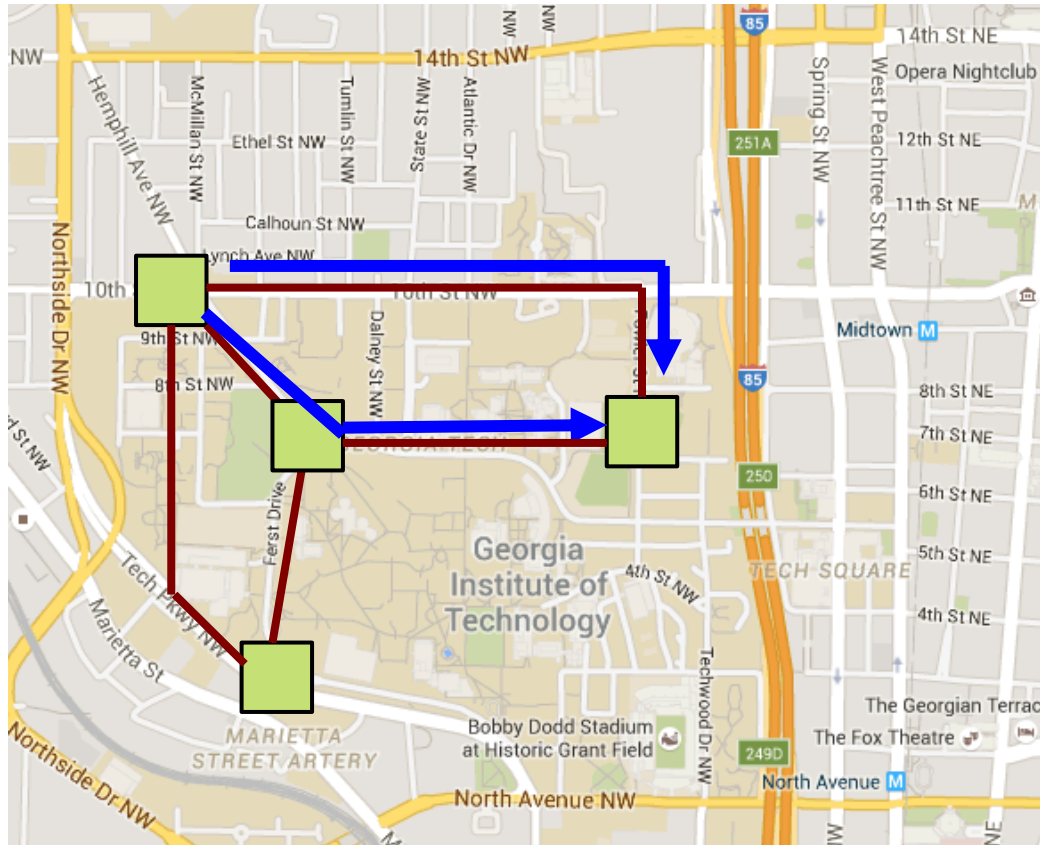
TOPOLOGY: HOW TO CONNECT THE NODES WITH LINKS

~Road Network



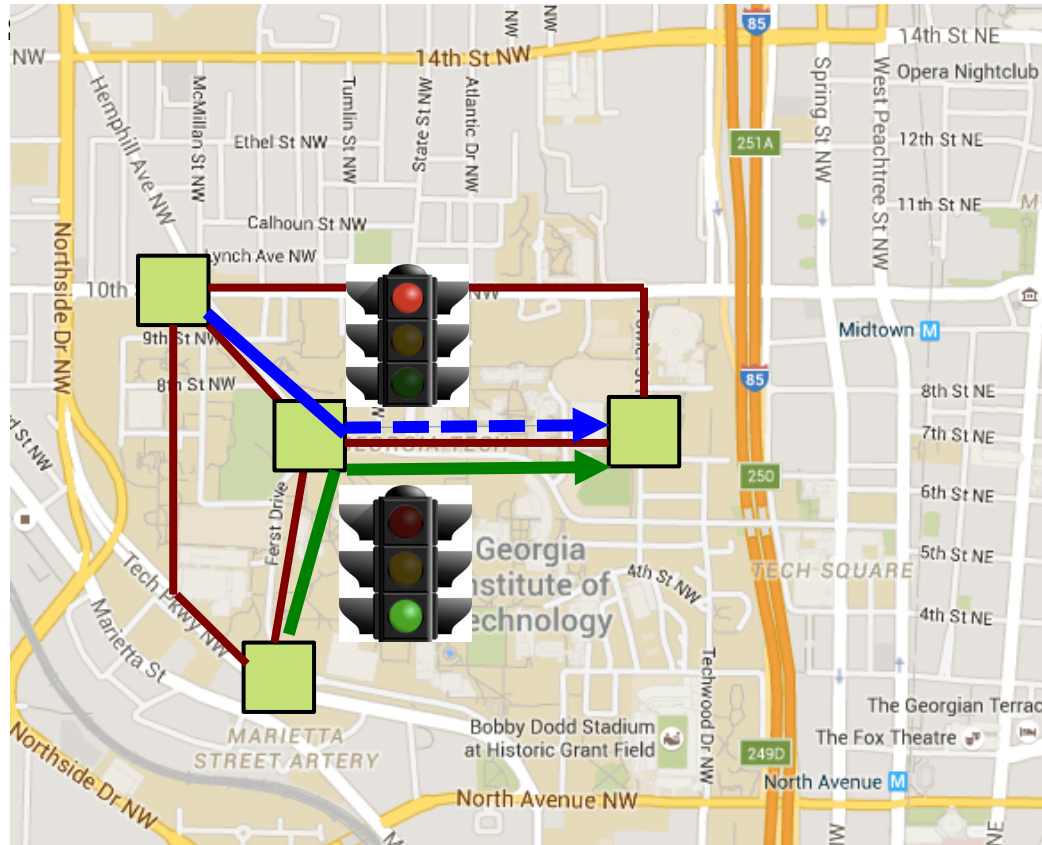
ROUTING: WHICH PATH SHOULD A MESSAGE TAKE

~Series of road segments from source to destination



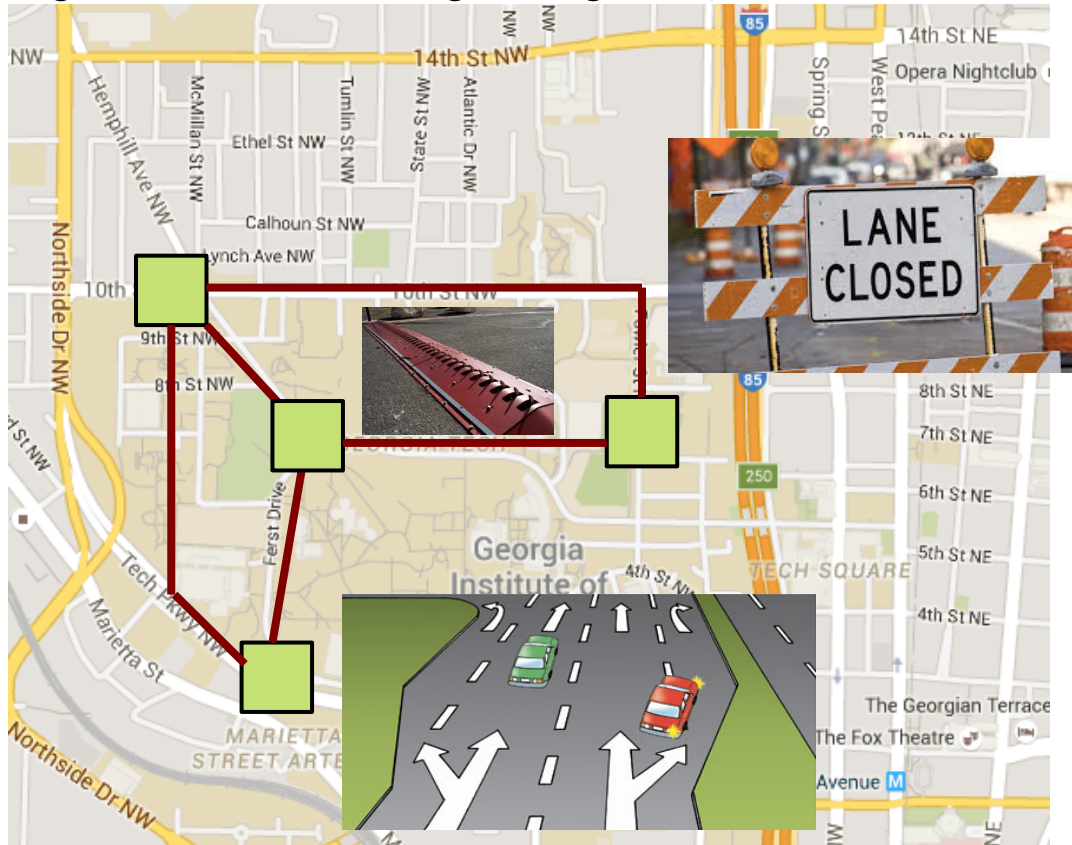
FLOW CONTROL: WHEN DOES THE MESSAGE STOP/PROCEED

~Traffic Signals / Stop signs at end of each road

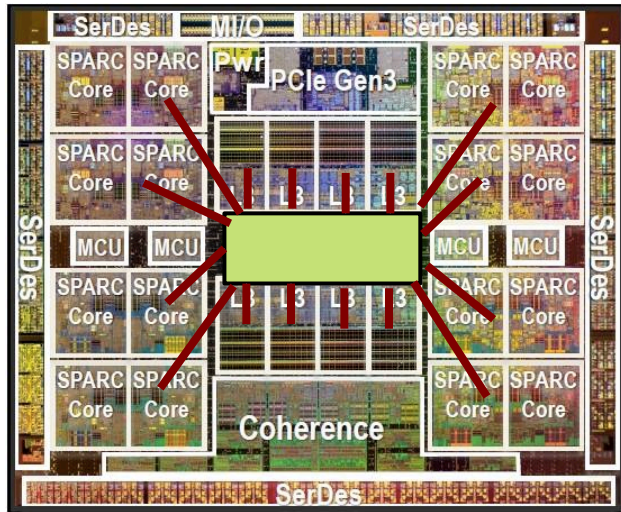


ROUTER MICROARCHITECTURE: HOW TO BUILD THE ROUTERS

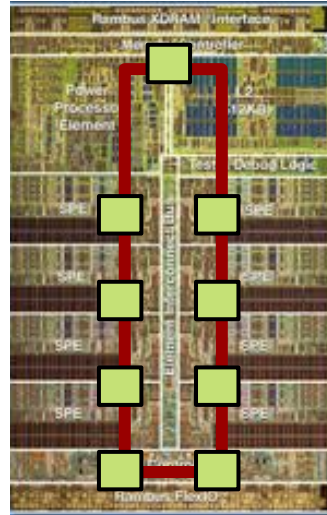
~Design of traffic intersection (number of lanes, algorithm for turning red/green)



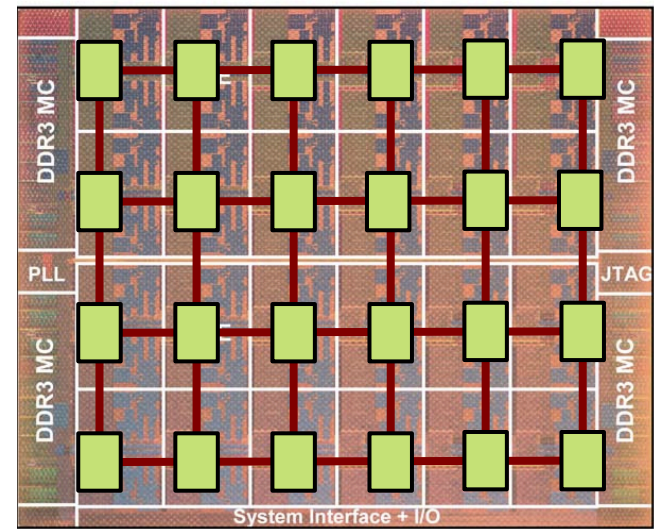
EXAMPLES



Oracle SPARC T5 (2013)
 16 multi-threaded cores and 8 L2 banks connected by a **Crossbar NoC**



IBM Cell (2005)
 1 general purpose and 8 special purpose engines connected by a **Ring NoC**



Intel SCC (2009)
 24 tiles with 2 cores each connected by a **Mesh NoC**

WHAT MAKES NETWORK DESIGN CHALLENGING (AND INTERESTING)

- Network resources are distributed with almost no centralized control
- Traffic is (often) unpredictable
 - Why?
 - Mapping of tasks to cores
 - Memory layout of data
 - Cache sizes, policies
 - Data sharing
 - ...
- Surprisingly easy for the network to bottleneck



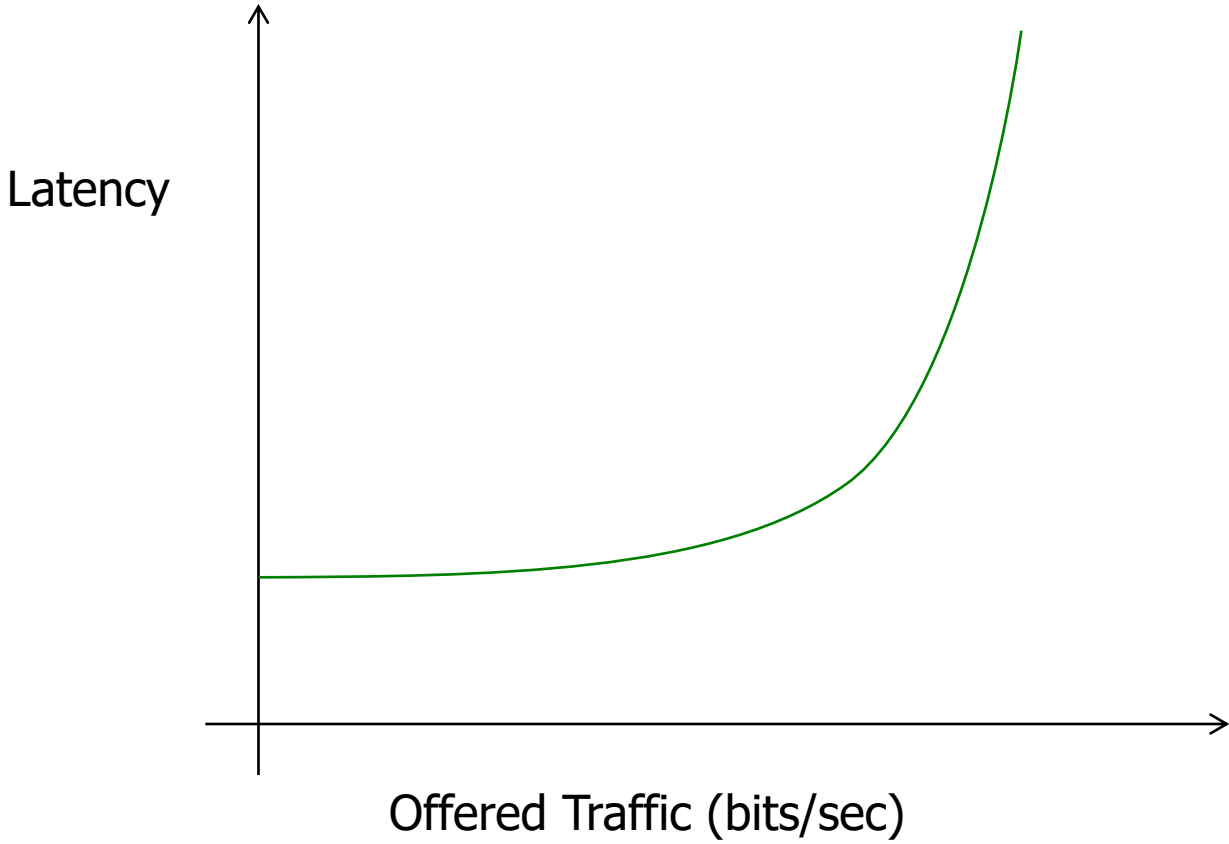
NOC METRICS

- Performance
 - Latency
 - Bandwidth

- Power
 - Energy Consumption in Links
 - Energy Consumption in Routers

- Area

HOW TO EVALUATE A NETWORK?



AGENDA

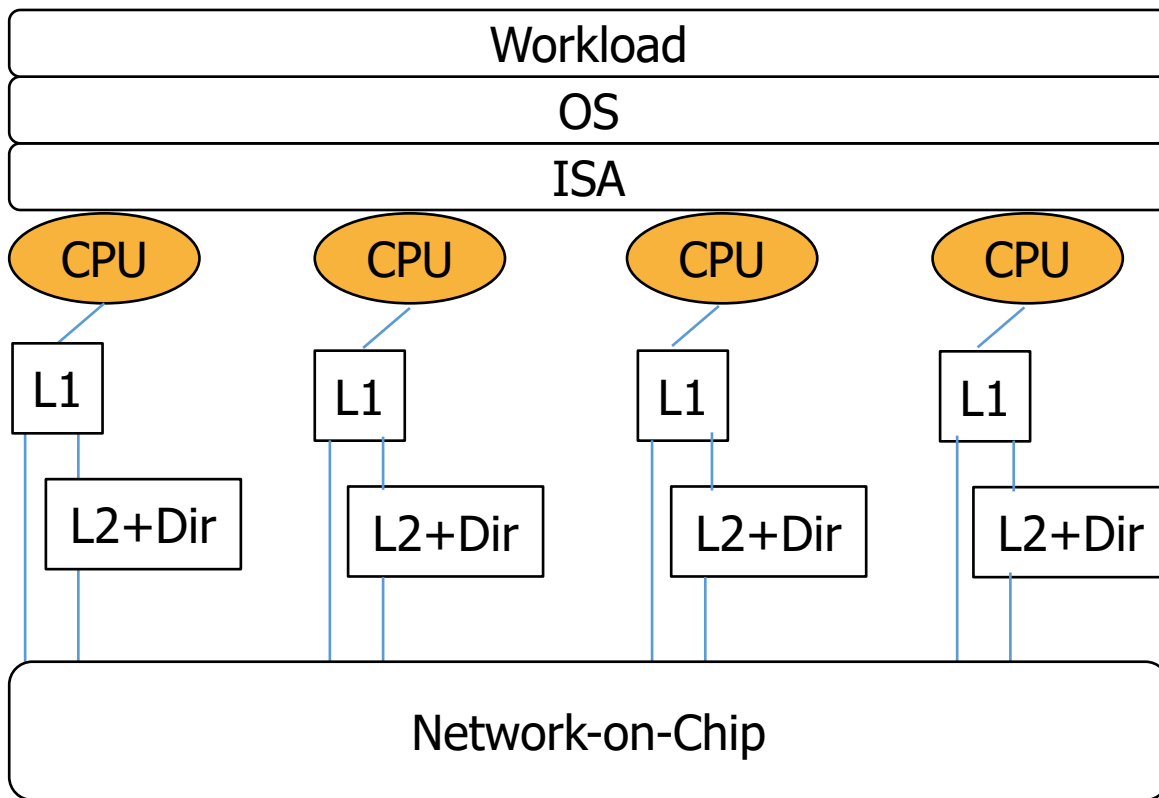
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THE GEM5 FULL-SYSTEM SIMULATOR

<http://www.gem5.org> Join the mailing list!

http://tusharkrishna.ece.gatech.edu/teaching/garnet_gt/

has instructions on setup for this class



	Simple (Fast)	Detailed (Slow)
OS	System Emulation	Full-System
CPU	AtomicSimple TimingSimple	InOrder OutOfOrder
Memory System	Classic	Ruby * Caches * Coherence protocols
NoC	Simple	Garnet2.0

GARNET IN *STANDALONE* MODE

User can inject any traffic pattern

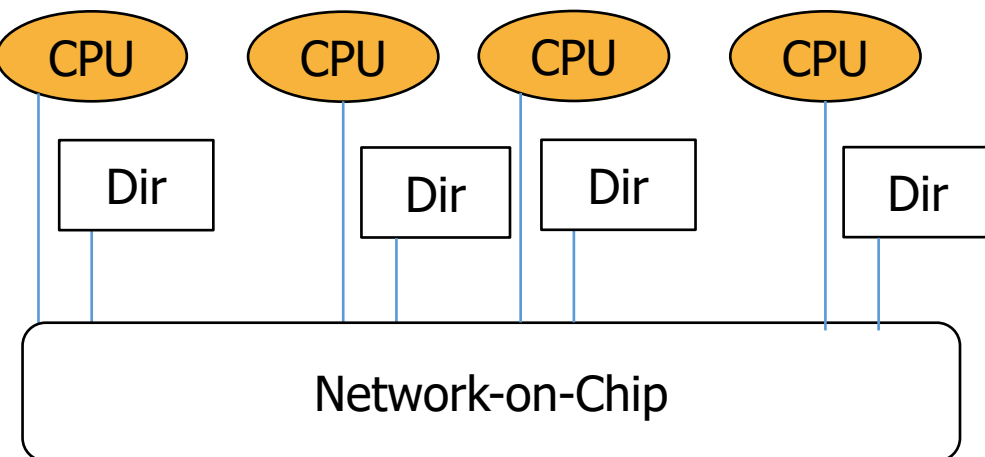
Sources: CPUs

Destinations: Directories

Traffic Pattern determines which CPU sends to which Directory.

Injection Rate is user specified

Directories consume any packet that is received.

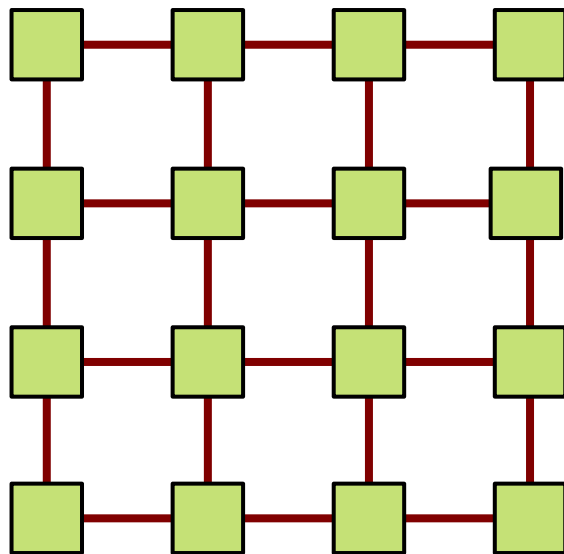


Source (binary coordinates):

$(Y_{k-1}, Y_{k-2}, \dots, Y_1, Y_0, X_{k-1}, X_{k-2}, \dots, X_1, X_0)$

Traffic Pattern	Destination (binary coordinates)
Bit-Complement	$(\bar{y}_{k-1}, \bar{y}_{k-2}, \dots, \bar{y}_1, \bar{y}_0, \bar{x}_{k-1}, \bar{x}_{k-2}, \dots, \bar{x}_1, \bar{x}_0)$
Bit-Reverse	$(x_0, x_1, \dots, x_{k-2}, x_{k-1}, y_0, y_1, \dots, y_{k-2}, y_{k-1})$
Shuffle	$(y_{k-2}, y_{k-3}, \dots, y_0, x_{k-1}, x_{k-2}, x_{k-3}, \dots, x_0, y_{k-1})$
Tornado	$(y_{k-1}, y_{k-2}, \dots, y_1, y_0, x_{k-1+\lceil \frac{k}{2} \rceil - 1}, \dots, x_{\lceil \frac{k}{2} \rceil - 1})$
Transpose	$(x_{k-1}, x_{k-2}, \dots, x_1, x_0, y_{k-1}, y_{k-2}, \dots, y_1, y_0)$
Uniform Random	$random()$

THAT'S ALL FOR TODAY!



If you have random traffic, how many hops does each message take on average on this 4x4 mesh topology?

Can you generalize this to a $k \times k$ mesh?