HEURISTICS

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In my lecture last week, Yuval Flicker asked me, given that I have labeled the framework of my lecture as *"heuristic,"*

> *What is a heuristic?*

My response was that I should think more before answering (!) and that I'll take it up as (part of) my homework for today's lecture. As a help for this homework, Sam Shaki sent me some extremely interesting articles he wrote with collaborators on the psychology of—and on heuristics and biases related to—*mental arithmetic*[1]. They follow the spirit of the classical experiment of Kahneman and Tversky where the subjects for their experiment are separated into two groups: one group is asked to quickly estimate the number

> 10 times 9 times 8 times ... times 2 times 1

and the other group is asked to quickly estimate

> 1 times 2 times 3 times ... times 9 times 10.

The estimates differ significantly—the first group averaging a much higher guess than the second, but both 'averaged guesses,' if I remember right, were much lower than the true answer.

In reviewing the possible 'takes' on the concept of 'heuristic,' the word itself coming from Ancient Greek *heurisko* (I find, I discover) resonant with Archimedes' bathtub proclamation "Eureka," it seems to me that there are four *quite different* senses to it. 'Heuristic' appears as:

- a term in common usage
- a term labelling a type of cognitive behavior in Psychology
- a term labelling an approach in Science (Physics, especially)
- a term in mathematical discussions that labels a process of thought that offers a reason for believing statements that may therefore, with some degree of confidence, be conjectured to be true.

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[1]I very much appreciate this help. See the bibliography below.)

1. As for 'common usage'

we can go to Wikipedia to learn:

> A **heuristic** or **heuristic technique**, is any approach to problem solving or self-discovery that employs a practical method that is not guaranteed to be optimal, perfect, or rational, but is nevertheless sufficient for reaching an immediate, short-term goal or approximation. Where finding an optimal solution is impossible or impractical, heuristic methods can be used to speed up the process of finding a satisfactory solution. Heuristics can be mental shortcuts that ease the cognitive load of making a decision. Examples that employ heuristics include using *trial and error*, a *rule of thumb* or an *educated guess.*

2. As for the appearance of this concept in Cognitive Psychology

- The cognitive psychologist Herbert Simon introduced the concept of heuristics (in the 1950s) when he suggested that human judgment is subject to cognitive limitations. Purely rational decisions may take too much time, and require knowing more than one actually knows, so people—needing to make some kind of choice—depend on some intuitive guesswork.
- During the 1970s, Amos Tversky and Daniel Kahneman focused on 'biases' in cognitive judgments, and framed (and named) four types of biases:
    - **The availability heuristic** which involves making decisions based upon how easy it is to bring something to mind.
    - **The representativeness heuristic** which involves making a decision by comparing the present situation to the most representative mental prototype.
    - **The affect heuristic** which involves making choices that are influenced by the emotions that you are currently experiencing.
    - **The anchoring** (or **Framing**) **bias** which involves the tendency to be overly influenced by the first bit of information we hear or learn.

3. As for the term—or at least the concept—appearing in Science

It is often taken as a synonym for the word *Ansatz* that appears quite often in Physics: an initial estimate of the solution to a mathematical or technical problem that is used to guide work to a more precise answer.

4. Finally, as for the term in Mathematics

George Polya, in the Preface to his book *Mathematical Discovery: On Understanding Learning and Teaching Problem Solving* (1961) wrote:

> I wish to call heuristics... the study of means and methods of problem solving. The term heuristic, which was used by some philosophers in the

past, is half-forgotten and half-discredited nowadays, but I am not afraid
to use it.

This suggests that the word hadn't been in very common use for mathematics before
that. But since then it does often appear as meaning exactly what Polya suggested.

But that doesn't catch the meaning or spirit of "Cohen-Lenstra" heuristics or "Random
Matrix" heuristics, nor does it catch what I meant when I used the term. So here's a
natural suggestion:

> A *mathematical heuristic* is an organized intuitive 'take' on a mathematical
> question (or more generally, a mathematical setting) that—brings every-
> thing relevant (that is known about the concepts involved) into the game—
> where:
>
> - all this gives some (reasonable–but *not rationally demonstrated*) sup-
>   port for a viewpoint that predicts solutions to some problems formu-
>   lated in that mathematical setting;
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> - and, especially importantly, makes use of the assumption that every-
>   thing else relevant to the problem in question is *random.*

References

[1] Samuel Shaki, Michal Pinhas & Martin H. Fischer (2017): Heuristics and biases in mental arithmetic:
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[2] Samuel Shaki and Martin H. Fischer, Competing Biases in Mental Arithmetic: When Division Is
More and Multiplication Is Less, Front. Hum. Neurosci., 01 February 2017 `https://doi.org/10.3389/fnhum.2017.00037`

[3] Giovanna Mioni, Martin H Fischer and Samuel Shaki, Heuristics and biases in the mental manipula-
tion of magnitudes: Evidence from length and time production, Quarterly Journal of Experimental
Psychology (2020) 1-12

[4] Fischer MH and Shaki S (2018) Repeating Numbers Reduces Results: Violations of the Identity Axiom
in Mental Arithmetic. Front. Psychol. (2018)