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## ABOUT THE COVER: DIOPHANTUS'S *ARITHMETICA*

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The cover of this issue of the *Bulletin* is the frontispiece to a volume of Samuel de Fermat's 1670 edition of Bachet's Latin translation of Diophantus's *Arithmetica*.[1] This edition includes the marginalia of the editor's father, Pierre de Fermat. Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation $x^2 + y^2 = z^2$, the marginal comment that hints at the existence of a proof (a *demonstratio sane mirabilis*) of what has come to be known as *Fermat's Last Theorem.*

Diophantus's work had fired the imagination of the Italian Renaissance mathematician Rafael Bombelli,[2] as it inspired Fermat a century later. Six of the thirteen books of *Arithmetica* in Greek have come down to us,[3] and four more books, in Arabic, were discovered in 1968. But we have little idea what kind of audience Diophantus's *Arithmetica* was originally meant for. How were the problems in these volumes to be studied? Was it a teacher's manual with some answers provided for the instructor's convenience? Is this text the notes of a student who, sometimes diligently and sometimes less diligently, recorded the proceedings of class sessions?

[1]We thank the Niedersachsische Staats und Universitatsbibliothek, Göttingen, for permission to use this image.

[2]In the introduction to his *L'Algebra* Bombelli writes: "[I]n these last few years in the Library of our Lord in the Vatican, a Greek work of [algebra] had been found which had been written by a certain Diophantus of Alexandria, a Greek author living in the time of Antoninus Pius. Master Antonio Maria Pazzi from Reggio (public lecturer in mathematics in Rome) showed this work to me. Together we judged this author extremely intelligent regarding numbers (although he does not deal with irrational numbers). . . . Thus, in order to enrich the world with such a work, we started to translate it." (cf. *Reading Bombelli*, F. La Nave and B. Mazur, Mathematical Intelligencer **24** (2002), 12–21).

[3]Some manuscripts have six books and some seven, but the total number of problems is the same; only the way in which the problems are divided is different.

When was it written? Even the question of which century the work was written in has offered up a delicious subject for dispute among scholars.[4]

Diophantus's text is in the form of a series of problems. You will not find here an articulated, discursive, reasoned science, as in Euclid's *Elements*. No lemmas, definitions, propositions. You will not find lengthy explanations of method, as in the works of Archimedes. Despite scholarly arguments to the contrary,[5] to my mind there is hardly any discernible order to the list of problems in the *Arithmetica*. Try, for example, this Virgilian game: open the volume to any problem—Problem $N$, say—and read with attention the entire text up to and including Problem $N$. Then try to guess the nature of problem $N+1$, and guess what species of numbers might be considered by the author to be a legitimate solution to that problem.[6]

How to classify the branch of mathematics dealt with in these volumes is already an issue of some scholarly contention. Is it *Algebra* properly speaking? If so, where are the variables? Where are the unknowns, the *species*, such as inhabit Viète's treatise one and a half millennia later? Diophantus's text is taken, sometimes, as offering us at least one frame of reference to help in considering the fundamental question *What Is Algebra*? (cf. Jacob Klein's *Greek Mathematical Thought and the Origins of Algebra,* M.I.T. Press, 1968).

But however you classify the branch of mathematics it is concerned with, Diophantus's *Arithmetica* can claim the title of founding document, and inspiring muse, to modern number theory. This brings us back to the goddess with her lyre in the frontispiece, which is the cover of this issue. As is only fitting, given the passion of the subject, this goddess is surely Erato, muse of erotic poetry.

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[4]W. R. Knorr, *Arithmêtikê stoicheiôsis*: On Diophantus and Hero of Alexandria, Historia Math. **20** (2) (1993), 180–192.

[5]For example, *The Birth of Literal Algebra*, I. G. Bashmakova, G. S. Smirnova, Abe Shenitzer, American Mathematical Monthly **106**, no. 1 (1999), 57–66.

[6]The motivation behind the organization of problems in the *Arithmetica* deserves closer commentary than it seems to have gotten in the literature so far. But see the 2004 Harvard University undergraduate senior thesis *On the Problem of Representation in Diophantus* of Samuel Lipoff, where this issue, and the scholarly literature on it, is discussed. As for what "counts" as a solution to a given problem in *Arithmetica*, this varies from problem to problem. For example, despite the assertion of Bombelli (quoted in the first footnote) Diophantus sometimes is happy with surds as solutions to his problems and sometimes not.

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