On time (in mathematics and literature)

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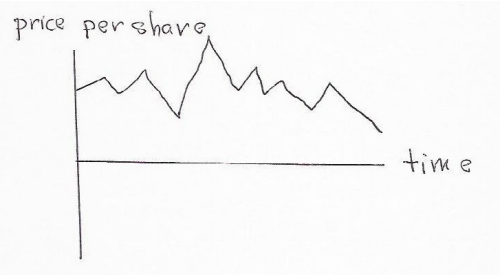
"Far in the future" or the "distant past" are curious phrases: we hear them often and think nothing of their peculiarity. But for me—a mathematician—the ubiquitous use of such phrases has a poignant significance. For mathematicians often are told how arcane, how unapproachable, their subject appears in the eyes of so many people. How difficult are the leaps of thought that mathematics invites us to make!

And yet, almost everyone, it seems, is at home with the truly strange—and essentially mathematical—representation: time *as* distance; distance *as* time.

How lucky! If the metaphorical bridge between time and distance were not utterly familiar to us, we would be made even more uncomfortable than we currently are, when some poet asks us to reflect on the fact that *yonder all before us lie/ Deserts of vast eternity,* or, more prosaically—when we are required to appreciate some graphical representations of stock market

fluctuations:



Given what I have just said, my aim in this talk might be considered perverse, for I want to spend these twenty minutes savoring, and working up, the real complexity of the metaphorical relationship of time and distance—to defamiliarize it for us. And then I will give a few examples of how imaginative literature makes use of the inherent strangeness in this relationship:

$$\text{Time} \leftrightarrow \text{Distance}.$$

And finally I will offer my opinion (which I think must be everyone's opinion) about why we derive significant—but not total—comfort from this equation.

Let's start with the dictionary definition of a *year*. One reference source has it that a *year* is

> n. The time required for one complete revolution of the earth about the sun, relative to the fixed stars.

In celebration of the fact that 2009 has been conferred an extra "leap" second, let us pause to think about the pre-suppositions in this seemingly simple definition of a *year*. There are three bodies at work in the quoted definition: the earth, the sun, and the substrate of the fixed stars. If you have never thought about this before, you might wonder what the fixed stars have to do with it. The point is that we wish to track the relative position of two bodies (the earth and the sun, both of them actually moving) within an *anchored* background, a fixed plane: the "fixed stars" serve to fix that plane for us[1].

[1]Draw an imaginary line between the earth and the sun and consider *that line's motion* relative to the background of the "fixed stars:" every time *that line* has effected a 360 degree rotation, we chalk up "one complete revolution

This definition of this basic unit of time—the year—has these general features:

1. It is given by the time-duration of a specific event (convenient to measure) which is expected to repeat (to all practical purposes: ad infinitum).
2. Each repetition of this event is expected to (more or less) "take the same time."

The first of these features let's refer to as *cyclicity*: our time-measurement is given by recording the number of repetitions of some identifiable occurrence—call it a *cycle*— that happens again and again. The second feature we will call *uniformity:* each cycle takes the same time to complete.

Might you be concerned that the feature of *uniformity* (i.e., that each rep take "the same time") requires yet some other time-measurement criterion to guarantee it, or even to give it a meaning? If so, here are three attitudes you might adopt, to lessen your worries, and to justify the claim that (a) your time-measurement is uniform and (b) that this is a meaningful claim. But none of these justifications are entirely satisfactory:

- *Justification by mere fiat:* *We* are the definers of time, and by our definition, we proclaim that each repetition (of the cyclic event we use to define time) takes "the same time."
- *Justification by consistency of definition:* We have an impressive variety of precise devices—i.e., clocks of various sorts— for measuring time; we require only that these time-pieces all give consistent readings. And, largely, they do.
- *Justification by theoretical model:* Newton's laws, to take an example, model this phenomenon *time*; within the format of these laws time is represented by a numerical variable t. As one can compute from this

of the earth about the sun." In all this we happily don't have to pay too much attention to the fact that the "fixed stars" are creatures in an expanding universe...

model, given no outside influences and idealizing the situation appropriately, each repetition of any of our designated time-pieces does take "the same time."

The source of the discomfort about *uniformity* is that if we are hellbent in checking that this half-hour of time of my lecture is *the same "length" of time* as yesterday's half-hour that we spent in the dentist's chair, we would be undertaking a procedure that is far less direct than what would be needed to check that, say, this arrow on the page:

is the same length as this one:

for we need only cut and superimpose these arrows to make the comparison; we have no similarly direct way of calling forth two distinct "half-hours" and comparing them; indirect strategies are our only recourse.

I've ordered the three bullets above (*Fiat, Consistency, Model*) in what I'm imagining is order of appearance on the world stage. The first human to fashion some unit of time measurement—to record events, or to plan for events—for example, could not have justified its uniformity either by appeal to consistency, or to theory. Surely—if he or she were at all motivated to justify the uniformity of units of time measurement—this original thinker would have proclaimed it simply by fiat.

The cyclicity of the days, the lunar months, the years, or other—more arcane—astronomical events might have provided this inventor the repetitive element necessary. Or other, more personal "events" such as heartbeats, might have done the trick.

You might wonder whether our inventive ancestor cared all that much about the touchy issue *uniformity* in all its metaphysical complexity. Perhaps just a smattering of uniformity would have been enough, so as to know that a certain number of lunar cycles corresponds—invariably—to the time between sowing and reaping. For other purposes, perhaps mere sequentiality was enough of

a record to keep, such as the habit of recording events of Chinese history in terms of placement in the sequence of dynasties (Xia, Shang, Zhou, Qin, Early Han, etc.).

As readers of imaginative literature we all are very open to whatever instruction our authors wish to give us about the flow of time. Any dialogue between two characters in a book will be paced, in the reader's imagination, in the "real time" it would take for the characters to do their talking. But other than that, our flexibility for accepting—and emotionally responding to—changes of time-speed in the flow of events in fiction is extraordinary. The John Cheever story "The Swimmer" with the uncanny effect that it achieves, of a gradually increasing speed of the passage of time in a languorous afternoon-cum-lifetime, explores this propensity in its readers.

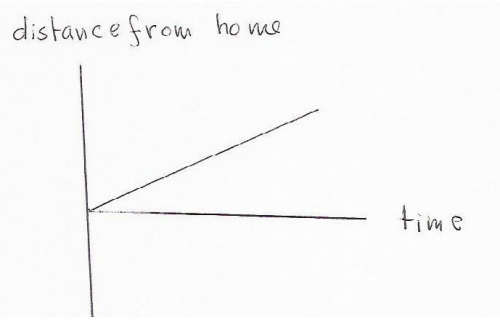
The format of counting cyclic events gives a discrete measure: you can record by a notch each cycle as they occur, getting a clean count $(1, 2, 3, \ldots N)$ of the number of palpable distinct events. Given the progress of technology we can find ever more rapid cyclic events to count, refining the mesh of our time-units (to nanoseconds nowadays). But such measurements remain quintessentially discrete, i.e., they depend, in the end, upon counting something. They offer us a ratchet-like expression for time moments, elegantly digital, even if so fast-clicking that they achieve a smooth flipbook feel.

Still, one might yearn for a representation of time—if even only on a metaphorical level—that expresses our felt experience of the smooth continuity of time. No matter whether Time corresponds to anything real or not, its legato *is* viscerally experienced: one feels the continuity of a tennis stroke, one sees the continuity of an eagle swoop, one hears the continuity of a sustained musical note.

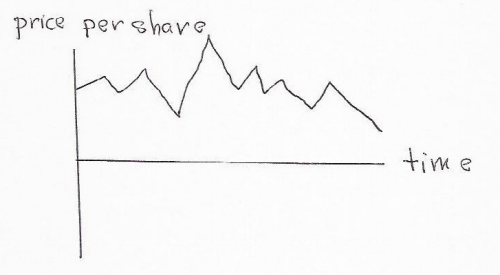
For this sense of continuity, we naturally invoke Geometry (more concretely, we compare durations of time to lengths of things). We happily use measuring-tape vocabulary (e.g., *a long span of time*) to assess time, and we invoke *light-years* to assess distance; and we all seem at home with such metaphorical leaps, despite the fact that there is a vast experiential gulf between the manner by which we perceive a "length of time" and a "length of something

in space."

Among the first people to explicitly make such connections was Nicholas of Oresme (1323-1382) in his treatise[2] "The Geometry of Qualities and Motions" where to visualize the variation, say, of the distance of an object to a fixed point as the object moves in time—Oresme initiated the clever idea of graphs. So, for example his picture of the progression of someone traveling with constant speed might look like



and his picture of the stock market fluctuations might look like



Two things are happening at once when we look at graphs like these: we are re-enacting the trajectory of the "recorded object" as we read the graph from left to right [3]: we inch along the graph. But we are also taking in the entire trajectory-record all at once: we perceive the whole graph—e.g. the life story of the stock market—as a "shape" with its crests and valleys; experiencing, one might say, the concept *Time* as imbedded in *Space*[4].

[2] *Tractatus de configuratione qualitatum et motuum; English translation by Marshall Claggett, Univ. of Wisconsin Press (1968).*

[3] We seem always to read graphs from left to right although no one has told us to.

[4] Mathematicians know that intuitions can go in the reverse direction as well, where at least one space dimension

Let us call the first manner of looking at the graph (as a progression, rising and falling as we trace along it, from left to right) the *road map* way of reading it; call the second (where we take it in, all-at-once, as a curve in the plane) the *god-like* way.

The *road map way* views the curve with its kinks and crannies as an itinerary to be travelled: our eyes move along it, re-enacting, or pre-enacting, the route taken or to be taken. Nowadays, when we get an itinerary from MapQuest or the AAA, it is an elegant curve traced through a surround of "general map," designating geographic features that are helpful to triangulate our positions, with mileage and estimated driving times all given. The early road maps, apparently, were called Auto-Photo-maps, and were presented to us in far less abstraction. Here is an example from 1907:

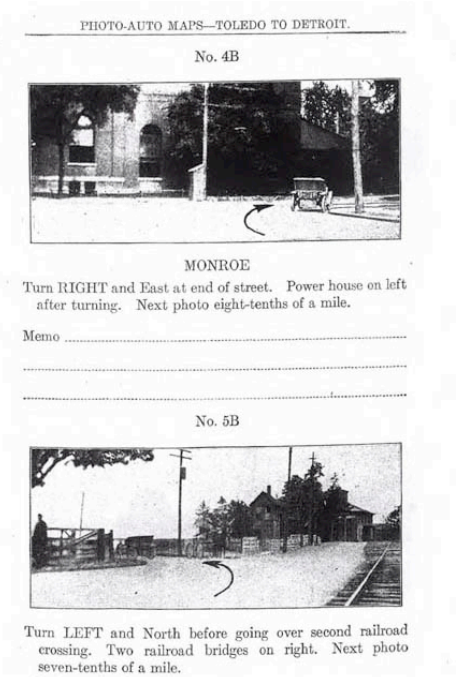


PHOTO-AUTO MAPS—TOLEDO TO DETROIT.

No. 4B

MONROE

Turn RIGHT and East at end of street. Power house on left after turning. Next photo eight-tenths of a mile.

Memo .....

No. 5B

Turn LEFT and North before going over second railroad crossing. Two railroad bridges on right. Next photo seven-tenths of a mile.

Earlier itineraries had a variety of ways of showing progression through territory, as in this map of the migration of Abraham and his tribe from Ur in the Chaldees to the land of Canaan, where the progression is depicted as a sequence of circular "snapshots" that circumnavigate the border of the map:

can be reconfigured so that Geometry then plays out as a motion picture through time. Physicists, of course, know that one can—and perhaps should—model a conglomerate of the two concepts Space and Time forming a unified structure that makes more sense than either one alone.

We have a variety of ways to visualize Time, and watch it unfold in paths—as in the graphs above—along which its denizens can travel. It's not a surprise, then, that our imaginative literature ascribes to those travelers—and even to Time itself—a broad assortment of vivid affects. The curious:

> To-morrow, and to-morrow, and to-morrow,
> Creeps in this petty pace from day to day[5]

has each mournful *Tomorrow* inhabiting—in turn, Dybbuk-like—each trudger in the dirge procession of Days. But when the *golden days glide by* there seems to be a glissando attributed to Time itself. And that doesn't stop the *whirligig of Time* from, scythe-like, bringing in—not the sheaves, but—revenges.

In contrast, the *god-like way* views The Whole—every syllable of recorded time all in the blink of an eye—as a static piece of geometry. There is something appealing about this: surely everyone must be brushed at least once, and at least in some way, with the yearning to render time static; either by transcending it as in these graphs (seeing it all in the blink of an eye) or transfixing it Joshua-like, or holding in an urn a precious second of it. This primordial urge to bid time stand still is—as Mephistopheles well knew—at times irresistible. In any event, it permeates imaginative literature. In Thomas Wolfe, one finds this impulse to possess and freeze time the very definition of an artist whose spirit is

> tortured by the anguish of possession–the intolerable desire to fix eternally in the patterns of an indestructible form a single moment

[5]To the last syllable of recorded time.

> of man's living, a single moment of life's beauty, passion, and unutterable eloquence, that passes, flames and goes, slipping for ever through our fingers with time's sanded drop, flowing for ever from our desperate grasp even as a river flows and never can be held. This is the artist, then–life's hungry man, the glutton of eternity, beauty's miser...

An unassailable discrepancy complicates the passage between *time* and *space* which the metaphorical equation *time*↔*space* lures us to make: space has–for us, if not for the string theorists—*three* dimensions while impoverished time has only one. A lopsided competition. As Susan Sontag puts it:

> Time does not give one much leeway: it thrusts us forward from behind, blows us through the narrow funnel of the present into the future. But space is broad, teeming with possibilities, positions, intersections, passages, detours, U-turns, dead ends, one-way streets. Too many possibilities indeed.

Or as John Hollander does:

> At home, at noon, I am located by three *where*
> Coordinates and one for *when* but none
> For *late* or *soon* which seems
> Unfair: the realm of here and there
> Scorns the immense expanse of now and then
> With its symmetrical shape ...

As illustrated in these quotations, once Time is thought of as represented in Space, the multi-dimensionality of Space opens some imaginative terrain to be explored. The Jorge Luis Borges short story "The Garden of Forking Paths" does this exploration with a vengeance, and as is only fitting, there is now (on the web) a hypertext version of this story[6].

[6]This famous story was original published in 1941 (El jardn de senderos que se bifurcan)and in it, Stephen Albert envisions an "infinite series of times, a growing, dizzying web of divergent, convergent, and parallel times." The hypertext website is http://www.geocities.com/papanagnou/

But there lurks another type of imaginative multi-dimensionality, or at least multi-strandedness, of time that can ensnare—I would guess—any of us: it is the *had-I-only* or the *thank-providence-that-I-didn't* type of musing, where we split the yarn of time into parallel paths[7], only one of which being the road that—in reality—we took. Although our rational selves tell us that there is no way to gauge what would have happened on the path not traveled, and no way to even recognize the person we would have become, if we travelled that way, our imaginative selves gambol down those byways just as Nabokov's character Pnin does: Pnin seems–at times—to be his own doppelgänger as he travels shoulder to shoulder with the person who he would have been, had he never emigrated, entangling himself in the difference between the Julian and the western calendar, and getting—for example—to never celebrating his birthday because "after his departure from Russia it sidled by in a Gregorian disguise (thirteen, no twelve, days late)."

The broader expanse of paths not taken is captured by the biologist Waddington's phrase *epigenetic landscape* of possible routes of development from embryo to adult. This can be thought of as a—metaphorical, of course—rough, grooved, and uneven sloping terrain down which each of us toboggans in our embryonic development to achieve adulthood, most of us careening down the "usual groove" but with possible pathological swerves off the healthy track at every turn of our morphology.

The epigenetic landscape, then, is a god-like way of viewing not only what has happened and will happen through time, but—more majestically—of viewing everything that *could possibly happen, or have happened* and viewing it all-at-once.

We are perpetually thwarted from this all-at-once-ness in our actual experience of Time, and we yearn, therefore, for some sort of objective correlative to stand for Time, to deploy it in front of us. Our attachment to *Time as Distance* is natural, given this yearning. As luck would have it, the success of this metaphor is astounding, what with its scientific utility, and splendid daily usefulness. And yet it can carry us only so far, for the metaphor in no way catches—and indeed is trying to work against—the main quality of our

[7]The physicists go one better with their Feynman diagrams where the poor elementary particles follow —in some probabilistic measure—*all* the paths at once.

relationship to time, our sense of its fleetingness. In the end it is a restless equation, and therefore marvelous fuel for the imagination.