

Usefulness

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December 7, 2017

Here is the collection of skeletal notes I wrote to initiate seminar discussions in the course on Utility (Phil 273a Fall 2017) that I taught with Eric Maskin and Amartya Sen.

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Part I

About our seminar-course “Utility”

Four years ago I had the pleasure of teaching a seminar on *Models* with my friends Eric Maskin and Amartya Sen. We covered mathematical models of all shapes and purposes; models in aid of statistics, or used in economics and in the theory of social choice. We dealt with models designed for a wide assortment of specific structures—auctions, voting procedures, for example.

In this up-coming seminar-course ‘Utility,’ (PHIL 273A) I look forward to as enjoyable an examination of the concept ‘Utility’ interpreted generously to include ideas about ‘usefulness,’ ‘purpose,’ and broad human desires (e.g., *happiness, justice*), as well as specific problems related to ‘strategy’ and ‘assessment of value’ in connection with the pursuit of particular goals.

‘Measurement of utility’, alone, has a wide-ranging literature related to

- behaviorial issues (e.g., as developed by Kahnemann and Tversky),
- mathematical issues (e.g., the—often impossible—task of optimizing two or more competing preferences at the same time—leading to its formulation in terms of Game Theory as done by von Neumann and Morgenstern), and
- issues central to Economics and the Theory of Social Choice (e.g., ‘utility functions’ have played an interesting role in shaping the format of models in those fields).

Here are five specific aspects that we will explore:

1. Personal questions of ethics and meaning:

- Greek origins of utility as in Aristotle’s trichotomy of utility, pleasure, and virtue as the three goods for friendship ([5] *Nichomachean Ethics* Book VIII) from which follows a broad discussion of the nature of friendship,
- or Epicurus’s linking of utility to happiness ([4]) and his discussion of justice: *Natural justice is a symbol or expression of usefulness, to prevent one person from harming or being harmed by another—* leading to his definition of justice: *useful for mutual association—*
- and continuing to Kant’s dictum of treating people as *ends* in themselves rather than as useful *means* to some other end. ([16]).

2. Social moral issues related to utilitarian ideas:

Pro:

- **Hume.** Although there is no suggestion of a strategy for, or even a taste for, ‘maximalization’ of utility in Hume’s writings, the concept itself plays an exceedingly important role in his thought. See Section V (*Why Utility pleases*) in the Enquiry concerning the Principles of Morals [14]. E.g.:

It seems so natural a thought to ascribe to their utility the praise, which we bestow on the social virtues, that one would expect to meet with this principle every where in moral writers, as the chief foundation of their reasoning and enquiry. In common life, we may observe, that the circumstance of utility is always appealed to; nor is it supposed, that a greater eulogy can be given to any man, than to display his usefulness to the public, and enumerate the services, which he has performed to mankind and society. What praise, even of an inanimate form, if the regularity and elegance of its parts destroy not its fitness for any useful purpose! And how satisfactory an apology for any disproportion or seeming deformity, if we can show the necessity of that particular construction for the use intended! A ship appears more beautiful to an artist, or one moderately skilled in navigation, where its prow is wide and swelling beyond its poop, than if it were framed with a precise geometrical regularity, in contradiction to all the laws of mechanics. A building, whose doors and windows were exact squares, would hurt the eye by that very proportion; as ill adapted to the figure of a human creature, for whose service the fabric was intended. What wonder then, that a man, whose habits and conduct are hurtful to society, and dangerous or pernicious to every one who has an intercourse with him, should, on that account, be an object of disapprobation, and communicate to every spectator the strongest sentiment of disgust and hatred.

- **J.S. Mill.** ([20]) His treatise *Utilitarianism* on the one hand, offers general guidelines (and formats) for rational argument about moral principles in general, and on the other, describes and defends utilitarianism as a viewpoint framed in broad terms to incorporate human desire and questions of happiness.

The utilitarian doctrine is that happiness is desirable as an end, and is the only thing that is so; anything else that is desirable is only desirable as means to that end.

- **Jeremy Bentham** and **J.S. Mill.** I found sections 2.1 and 2.2 of [14], the entry *Utilitarianism* in the *Stanford History of Philosophy* extremely useful for a discussion of the interplay of ideas (of Bentham and Mill) regarding utilitarianism.

Anti:

- **Kant** ([16]) Regarding the principle of “greatest happiness for the greatest number,” Kant is not wishy-washy:

... it is odd how it could have occurred to intelligent men, [merely] because the desire for happiness and hence also the maxim whereby everyone posits this happiness as the determining basis of his will is universal, to therefore pass this [maxim] off as a universal practical law. For although ordinarily a universal law of nature makes everything accordant, here, if one wanted to give to the maxim the universality of a law, precisely the extreme opposite of accordance would result: the gravest conflict, and the utter annihilation of the maxim itself and of its aim. For then the will of all does not have one and the same object, but each person has his [own] object (viz., his own well-being); and although contingently this object may indeed be compatible with the aims of other people as well, who likewise direct them at themselves, it is far from being sufficient for a law, because the exceptions that one is occasionally authorized to make are endless and cannot at all be encompassed determinately in a universal rule.

In this way there results a harmony similar to that depicted by a certain satirical poem on the concord of soul between a married couple who are [bent on] bringing themselves to ruin: “O marvelous harmony, what he wants she also wants,” etc.; or to what is reported about the pledge made by King Francis I against Emperor Charles V: “What my brother Charles wants to have (Milan) I also want to have. ”

Empirical determining bases are not suitable for any universal external legislation, but just as little also for an internal one; for each person lays at the basis of inclination his [own] subject, but another person another subject; and in each subject himself now this inclination and now another is superior in influence. Discovering a law that under this condition would govern them all [viz., with accordance on all sides] is absolutely impossible.

- **Smith** ([10] IV.2.4, IV 2.9): Here, Smith offers the eponymous example of a general species of reasoning that Nozick [21] nicely labeled *invisible hand arguments*:

Every individual is continually exerting himself to find out the most advantageous employment for whatever capital he can command. It is his own advantage, indeed, and not that of the society, which he has in view. But the study of his own advantage naturally, or rather necessarily, leads him to prefer that employment which is most advantageous to the society. . . he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value,

he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it.

- **Nozick.** (see especially pp. 28-30 in [21]). Nozick works out a (libertarian) defense of something like a 'minimal state.' This may not be exactly the 'Nightwatchman state,' which restricts its activities to bodily protection of its citizens and the enforcement of contracts, but it is close to that. In such a state, looking out for the 'happiness' of its citizens is not one of its primary goals—although, he allows—it might be a consequence. Rather, Nozick wishes to have states whose primary goals are the ('maximalization of the') nonviolation of individual's rights. He toys with the phrase 'utilitarian of rights.'
- **Rawls.** Rawls's objection to utilitarianism also has to do with its relationship to individual rights. He wants (see (xi) of [22]):

... to work out a conception of justice that provides a reasonably systematic alternative to utilitarianism, which in one form or another has long dominated the Anglo-Saxon tradition of political thought. The primary reason for wanting to find such an alternative is the weakness, so I think, of utilitarian doctrine as a basis for the institutions of constitutional democracy. In particular, I do not believe that utilitarianism can provide a satisfactory account of the basic rights and liberties of citizens as free and equal persons, a requirement of absolutely first importance for an account of democratic institutions.

He writes that the first objective of his take on justice (*justice as fairness*) is to emphasize the priority of basic rights and liberties. He "used a more general and abstract rendering of the idea of the social contract by means of the idea of the original position as a way to do that."

3. On the usefulness of 'theory'.

- Here is G.H. Hardy, a mathematician of the early twentieth century discussing this in his *A Mathematician's Apology*:

Is mathematics 'useful, directly useful, as other sciences such as chemistry and physiology are? This is not an altogether easy or uncontroversial question, and I shall ultimately say No, though some mathematicians, and some outsiders, would no doubt say Yes. And is mathematics 'harmless? Again the answer is not obvious, and the question is one

which I should have in some ways preferred to avoid, since it raises the whole problem of the effect of science on war. Is mathematics harmless, in the sense in which, for example, chemistry plainly is not?

The publication date of *A Mathematician's Apology*, 1940, is relevant for an appreciation of what underlies these sentiments.

- And here is an excerpt of Abraham Flexner's eloquent essay *The Usefulness of Useless Knowledge*. (Flexner was the founding director of the Institute of Advanced Study at Princeton.)

From a practical point of view, intellectual and spiritual life is, on the surface, a useless form of activity, in which men indulge because they procure for themselves greater satisfactions than are otherwise obtainable. In this paper I shall concern myself with the question of the extent to which the pursuit of these useless satisfactions proves unexpectedly the source from which undreamed-of utility is derived.

The publication date, 1939, is also significant.

- And here is Adam Smith's curious comment on the relationship between usefulness and beauty in the sciences (*Theory of Moral Sentiments*¹).

It is in the abstruser sciences, particularly in the higher parts of mathematics, that the greatest and most admired exertions of human reason have been displayed. But the utility of those sciences, either to the individual or to the public, is not very obvious, and to prove it, requires a discussion which is not always very easily comprehended. It was not, therefore, their utility which first recommended them to the public admiration. This quality was but little insisted upon, till it became necessary to make some reply to the reproaches of those, who, having themselves no taste for such sublime discoveries, endeavoured to depreciate them as useless.

4. **Aesthetic aspects:** As in Kant's discussion of *purposiveness*² as opposed to *purpose* and *usefulness* in *The Analytic of the Beautiful* ([17]; Introduction pp. 20-28; Book I Sections 10, 11 pp. 64-67).

... we do call objects, states of mind, or acts purposive even if their possibility does not necessarily presuppose the presentation of a purpose; we

¹This is from [9] Chapter II *Of the beauty which the appearance of Utility bestows upon the characters and actions of men...* in Part IV *Of the Effect of Utility upon the Sentiment of Approbation*. It was published in 1759.

²Zweckmässigkeit

do this merely because we can explain and grasp them only if we assume that they are based on a causality [that operates] according to purposes, i.e., on a will that would have so arranged them in accordance with the presentation of a certain rule. Hence there can be purposiveness without a purpose, insofar as we do not posit the causes of this form in a will, and yet can grasp the explanation of its possibility only by deriving it from a will. Now what we observe we do not always need to have insight into by reason (as to how it is possible). Hence we can at least observe a purposiveness as to form and take note of it in objects even if only by reflection-without basing it on a purpose.³

or the ‘form follows function’ dictum in modern architecture.

5. Formal and more mathematical tools.

- *Initial ideas of utility and utility-maximizing.*

Daniel Bernoulli (1738) ([5]⁴)

[T]he determination of the value of an item must not be based on its *price*, but rather on the *utility* it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount.

- *‘Betting behavior’ as a way of formalizing personal preferences of utility.*
Frank Ramsey, Bruno de Finetti; choice-based subjective probability: see [23]. The use of utility functions as representation of preferences.
- *Arguments against the use of a determinate utility function*
Lionel Robbins [24]:

Bailey pointed out over a hundred years ago, “As we cannot speak of the distance of any object without implying some other object between which and the former this relation exists, so we cannot speak of the value of a commodity, but in reference to another commodity compared with it. A thing cannot be valuable in itself without reference to another thing, any more than a thing can be distant in itself without reference to another thing.”

³All this hangs a bit on Kant’s rather curious notion of *the will* (cf. loc.cit.)

⁴And for modern discussions regarding the St. Petersburg Paradox, one of the focusses of Bernoulli’s thought, see [6], [7].

It follows from this that the term which, for the sake of continuity and to raise certain definite associations, we have used hitherto in this chapter, the term “economic quantity” is really very misleading. A price, it is true, expresses the quantity of money which it is necessary to give in exchange for a given commodity. But its significance is the relationship between this quantity of money and other similar quantities. And the valuations which the price system expresses are not quantities at all. They are *arrangements in a certain order*. To assume that the scale of relative prices measures any quantity at all save quantities of money is quite unnecessary. Value is a relation, not a measurement... Recognition of the ordinal nature of the valuations implied in price is fundamental. It is difficult to overstress its importance. With one slash of Occam’s razor, it extrudes for ever from economic analysis the last vestiges of psychological hedonism.

- *Game Theoretical format.*

von Neumann-Morgenstern. The starting-point of their classic text is

to obtain a real understanding of the problem of exchange by studying it from an altogether different angle; this is, from the perspective of a “game of strategy”⁵

leading the authors to consider models of social exchange economy that represent individuals exposed to a constellation of social influences exposed to multiple factors. They formalize this by stipulating, in their model, that each party attempts to maximize his ‘interests’ given that he does not control all variables.

This is certainly no maximum problem, but a peculiar and disconcerting mixture of several conflicting maximum problems. Every participant is guided by another principle and neither determines all variables which affect his interest.

See [27] and read especially pp. 1-48 as given in the on-line link in [27]. For a somewhat less technical text covering the mathematics involved, cf. [18].

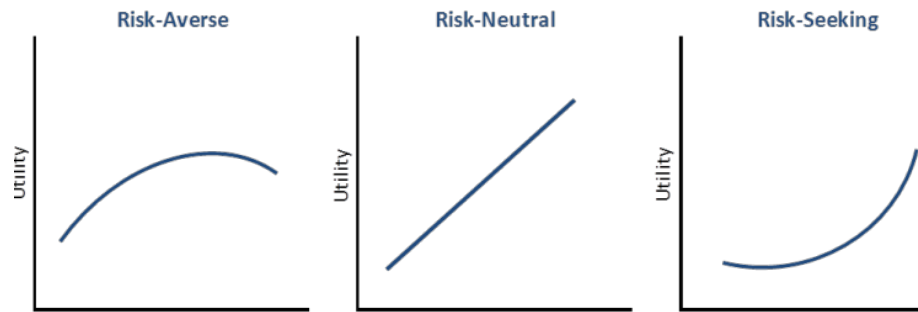
- *Axiomatic preference relations interpreted as the maximization of the expectations of a utility function on the set of consequences with respect to a probability measure on the set of all events.*

L.J. Savage (1954) [26]. Also: **Gerard Debreu** (1954) and **Ken Arrow** (1972); see [6].

6. ‘Behavioral’ issues:

⁵This is in contrast to earlier “Robinson Crusoe” models that involve only a single actor balancing his various preferences to make his next move.

- ‘Measurement of happiness.’
- The utility of money.
- Interpersonal comparisons—including the informational basis of comparability connected to things discussed above.
- Models to describe a participant’s re-evaluation of preferences related to marketplace issues —given changes of wealth—the nature of this re-evaluation depending on overall psychological temperament—ranging from the risk-averse to the thrill-seeking.



- ‘Behavioral challenges’ to expected utility:
 - See Lionel Robbins [25]:

My own attitude to problems of political action has always been one of what I might call provisional utilitarianism... But, as time went on, things occurred which began to shake my belief in the existence of so complete a continuity between politics and economic analysis. I began to feel that there were profound difficulties in a complete fusion between ... the economic and the hedonistic calculus. It did not take long to see that the “law” of diminishing marginal utility, assumed ... in the analysis of inequality, differed from the “law” of the same name invoked in the analysis of exchange and that the difference was precisely the introduction of [the assumption that all humans] have equal capacity for satisfaction.
 - The ‘Allais Paradox,’ [19]; see also the exposition in the Wikipedia entry [2].
 - The ‘Ellsberg Paradox’[4]; see also the exposition in the Wikipedia entry [3].
 - Kahneman-Tversky’s ‘Prospect Theory’ [15].

References

- [1] *The History of Utilitarianism* Stanford Encyclopedia of Philosophy
<https://plato.stanford.edu/entries/utilitarianism-history/>
- [2] The Allais Paradox (Wikipedia)
https://en.wikipedia.org/wiki/Allais_paradox
- [3] The Ellsberg Paradox (Wikipedia)
https://en.wikipedia.org/wiki/Ellsberg_paradox
- [4] Epicurus, *Principal Doctrines*—transl: Robert Drew Hicks
<http://classics.mit.edu/Epicurus/princdoc.html>
- [5] Aristotle *Nicomachean Ethics; Book VIII*—transl.: W.D. Ross
<http://classics.mit.edu/Aristotle/nicomachaen.8.viii.html>
- [6] Arrow, K.: (his essay in) *A volume in honor of Jacob Marschak* / Edited by C. B. McGuire and Roy Radner. Contributors Kenneth J. Arrow, [and others]. McGuire, Charles Bartlett (1972)
- [7] Bernoulli, D.: (transl.) *Exposition of a New Theory on the Measurement of Risk*, *Econometrica* 22 (1954): 23 - 26.
- [8] Ellsberg, D.: *Risk, Ambiguity, and the Savage Axioms*, *Quarterly Journal of Economics* 75: 643-659 (1961).
- [9] Adam Smith *Theory of Moral Sentiments*
<http://www.earlymoderntexts.com/assets/pdfs/smith1759.pdf>
- [10] Adam Smith *Wealth of Nations*
<http://www.econlib.org/library/Smith/smWN13.html>
- [11] Aumann, R. J. (1977) “The St. Petersburg Paradox: A Discussion of Some Recent Comments,” *Journal of Economic Theory* 14: 443-445,
- [12] Samuelson, P. (1977) “The St. Petersburg Paradox: Defanged, Dissected and Historically Described,” *Journal of Economic Literature* 15: 24-55.
- [13] Flexner, A.: *The Usefulness of Useless Knowledge*
<https://library.ias.edu/files/UsefulnessHarpers.pdf>
See also: <http://blog.press.princeton.edu/2017/02/28/robbert-dijkgraaf-on-the-usefulness-of-useless-knowledge/>
- [14] Hume, D.: *An Enquiry concerning the Principles of Morals*
<http://www.davidhume.org/texts/epm.html#M5dot1>

- [15] Kahneman, D.; Tversky, A.: ‘Prospect Theory’ :<http://socsci2.ucsd.edu/~aronatas/project/academic/tversky%20and%20kahnemann.pdf>
- [16] Kant, I.: *Critique of Practical Reason*—transl.: W.S. Pluhar
<http://www.morelightinmasonry.com/wp-content/uploads/2014/06/Kant-Critique-of-Practical-Reason-Cambridge.pdf>
- [17] Kant, I.: *Critique of Judgment*—transl.: W.S. Pluhar
https://monoskop.org/images/7/77/Kant_Immanuel_Critique_of_Judgment_1987.pdf
- [18] Luce, R. D.; Raiffa, H.: *Games and Decisions: Introduction and Critical Survey* Dover (1989)
- [19] Machina, M.: *Choice Under Uncertainty: Problems Solved and Unsolved*, The Journal of Economic Perspectives. 1 (1): 121-154 (1987)
- [20] Mill, J. S.: *Utilitarianism*
<http://www.earlymoderntexts.com/assets/pdfs/mill1863.pdf>
- [21] Nozick, R.: *Anarchy, State, and Utopia*
http://www.lib.cmb.ac.lk/wp-content/uploads/2014/01/0001_anarchy_state_and_utopia.pdf
- [22] Rawls, J.: *Theory of Justice*
<http://www.univpgri-palembang.ac.id/perpus-fkip/Perpustakaan/American%20Phylosophy/John%20Rawls%20-%20A%20Theory%20of%20Justice~%20Revised%20Edition.pdf>
- [23] Ramsey, F.P.: *Truth and Probability* In The Foundations of Mathematics and Other Logical Essays. R. B. Braithwaite and F. Plumpton (Eds.) London: K. Paul, Trench, Truber and Co. (1931).
- [24] Robbins, L.: *An Essay on the Nature and Significance of Economic Science* MacMillan (1945)
- [25] Robbins, L.: *Inter-personal comparisons of utility*. Economic Journal, 48: 635- 641 (1938).
- [26] Savage, L.J. 1954. *The Foundations of Statistics*. New York: John Wiley.
- [27] von Neumann, J.; Morgenstern, O.: *Theory of Games and Economic Behavior* Princeton University Press (1944)
For pp. 1-48 of this text, see:
<https://pdfs.semanticscholar.org/0375/379194a6f34b818962ea947bff153adf621c.pdf>

Part II

Before Utility

When we say that something—some type of ‘goods,’ or some set-up— is *useful*, we invoke—perhaps implicitly—the existence of some agent or agents (that can make use of it), of some type of situation (in which it can be put to use), of some mode of operation (i.e., way of using it) and some goal (the reason why it is useful).

1 Usefulness of goods/ friendship/ justice

At times Aristotle focuses his attention on ‘goods’ in the context of commerce, as in the Nichomachean Ethics Book V.5 ([1]):

All goods must therefore be measured by some one thing. . . this unit is in truth *demand*, which holds all things together (for if men did not need one another’s goods at all, or did not need them equally, there would be either no exchange or not the same exchange); but money has become by convention a sort of representative of demand.

but in Book VII of The *Politics* ([2]) the word has broader personal significance. The goal—writes Aristotle— of “external goods, goods of the body, and goods of the soul” is, in the end, happiness, and:

happiness whether consisting in pleasure or virtue, or both, is more often found with those who are most highly cultivated in their mind and in their character, and have only a moderate share of external goods, than among those who possess external goods to a useless extent but are deficient in higher qualities; and this is not only matter of experience, but, if reflected upon, will easily appear to be in accordance with reason. *For, whereas external goods have a limit, like any other instrument, and all things useful are of such a nature that where there is too much of them they must either do harm, or at any rate be of no use, to their possessors, every good of the soul, the greater it is, is also of greater use, if the epithet useful as well as noble is appropriate to such subjects.*

In the last quoted sentence one already sees what one might call the principle of concavity of the relationship between sheer quantity and assessment of usefulness—as will be a theme of later discussions⁶.

‘Usefulness’ itself has its limits in human interactions, as in love and friendship (Nichomachean Ethics Book VIII; [3]):

⁶The Greek word for useful *ophelimos* was ‘nominalized into English’ in the form of *ophelimity* (by the economist Arthur Cecil Pigou; 1877-1959), meaning the capacity to satisfy a need, desire, or want.

... friends whose affection is based on utility do not love each other in themselves, but in so far as some benefit accrues to them from each other. And similarly with those whose friendship is based on pleasure: for instance, we enjoy the society of witty people not because of what they are in themselves, but because they are agreeable to us. Hence in a friendship based on utility or on pleasure men love their friend for their own good or their own pleasure, and not as being the person loved, but as useful or agreeable. And therefore these friendships are based on an accident, since the friend is not loved for being what he is, but as affording some benefit or pleasure as the case may be. ... And utility is not a permanent quality; it differs at different times. Hence when the motive of the friendship has passed away, the friendship itself is dissolved, having existed merely as a means to that end.

In the view of Epicurus, the essential human pursuit is happiness or some Epicurian version of *eudaimonia*, so the notion of ‘usefulness’ in his writings is pointed toward that primary human goal, happiness. It is interesting to try to interpret, then, his treatment *Justice*,⁷ as being so strongly linked to ‘usefulness’ in his thought ([4] at least as formulated by Diogenes Laertius in the Principal Doctrines: *Kuriai Doxia*).

- (31) Natural justice is a symbol or expression of usefulness, to prevent one person from harming or being harmed by another.
- (36) Taken generally, justice is the same for all, to wit, something found useful in mutual association; but in its application to particular cases of locality or conditions of whatever kind, it varies under different circumstances.
- (37) Among the things accounted just by conventional law, whatever in the needs of mutual association is attested to be useful, is thereby stamped as just, whether or not it be the same for all; and in case any law is made and does not prove suitable to the usefulness of mutual association, then this is no longer just. And should the usefulness which is expressed by the law vary and only for a time correspond with the prior conception, nevertheless for the time being it was just, so long as we do not trouble ourselves about empty words, but look simply at the facts.

Curious definition of just: “useful for mutual association” (38) below simply repeats:

- (38) Where without any change in circumstances the conventional laws, when judged by their consequences, were seen not to correspond with the notion of justice, such laws were not really just; but wherever the laws have ceased to be useful in consequence of a change in circumstances, in that case the laws were for the time being just when they were useful for the mutual association of the citizens, and subsequently ceased to be just when they ceased to be useful.

⁷which Epicurus takes to be principally the promotion of social arrangements where no one is ‘harming or is being harmed by others’

2 Bernoulli

The innocent act of nominalizing this adjective *useful*, i.e., turning it into a noun (*usefulness, utility*)—a move that was made in quite early days of economic thought—has the effect of increasing the complexity and subtlety of the questions that you can ask about this concept. For example, can “it” be measured? And if so, what species of object can one measure it against? (*‘Itself’* turns out to be a possible answer.)

Utility, the curious (possibly measurable, but certainly subjective) concept, relates to the notions of *Value, Price* and *Money*. In the hands of Daniel Bernoulli (~1738) it makes specific connection with *Risk* and *Expectation* as well. The essay *Exposition of a New Theory on the Measurement of Risk* [5] radiates with the intensity of original ideas, and it also has the delicious crudeness of fresh thinking—the rough edges are not smoothed out in any way. In [5] Bernoulli dives into a qualitative description of the measurement—not only of someone’s—a pauper’s or millionaire’s— sense of the utility of money, per se, but also—as the title indicates—of the person’s assessment of expected outcome—an admittedly subjective assessment—of some venture that will be entered into with incomplete knowledge; i.e., in the context of *risk*. For example, in sections 3-5 he considers the prospects of

a very poor fellow [who] obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill-advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats. If I am not wrong then it seems clear that all men cannot use the same rule to evaluate the gamble.

...

[T]he determination of the *value* of an item must not be based on its *price*, but rather on the *utility* it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount.

At this point in Bernoulli’s essay the notion ‘utility’ has a qualitative—but not quantitative—status. The viewpoint changes abruptly though:

[L]et us use this as a fundamental rule: If the utility of each possible profit expectation is multiplied by the number of ways which it can occur, and we then divide the sum of these products by the total number of possible cases,

a **mean utility** [moral expectation] will be obtained, and the profit which corresponds to this utility will equal the value of the risk in question. Thus it becomes evident that no valid measurement of the value of a risk can be obtained without consideration being given to its utility, that is to say, the utility of whatever gain accrues to the individual or, conversely, how much profit is required to yield a given utility.

The utility resulting from any small increase in wealth will be inversely proportionate to the quantity of goods previously possessed. Considering the nature of man, it seems to me that the foregoing hypothesis is apt to be valid for many people to whom this sort of comparison, can be applied. Only a few do not spend their entire yearly incomes. But, if among these, one has a fortune worth a hundred thousand ducats and another a fortune worth the same number of semi-ducats and if the former receives from it a yearly income of, five thousand ducats while the latter obtains the same number of semi-ducats it is quite clear that to the former a ducat has exactly the same significance as a semi-ducat to the latter, and that, therefore, the gain of one ducat will have to the former no higher value than the gain of a semi-ducat to the latter. Accordingly, if each makes a gain of one ducat the latter receives twice as much utility from it, having been enriched by two semi- ducats.

Bernoulli then retreats from precise quantitative formulations:

However it hardly seems plausible to make any precise generalizations since the utility of an item may change with circumstances. Thus, though a poor man generally obtains more utility than does a rich man from an equal gain, it is nevertheless conceivable, for example, that a rich prisoner who possesses two thousand ducats but needs two thousand ducats more to repurchase his freedom, will place a higher value on a gain of two thousand ducats than does another man who has less money than he.

And then returns to quantitative precision (with *logarithmic curves* measuring utility against more objective markers).

... in order to perceive the problem more correctly we shall assume that there is an imperceptibly small growth in the individual's wealth which proceeds continuously by infinitesimal increments. Now it is highly probable that any increase in wealth, no matter how insignificant, *will always result in an increase in utility which is inversely proportionate to the quantity of goods already possessed.*

In plain English, the moral Bernoulli wants to draw is this. Let us be given that there is a measure of utility $u(t)$ that depends on $t :=$ the amount of goods, say, —or money—that

you have. (That there is a meaningful such function measuring the level of utility of the goods in your possession is, of course, already the big assumption.) And that the curve $u(t)$ representing utility as a function of t is *concave*. In a fair game, if you have x amount (of money) and you are willing to invest some of it, say y with a 50% probability of losing it or gaining $2y$, then—if you play the game—your mean utility will decrease, even if you are in this 'fair game.'

Her's why: you have begun the game with 'utility' $u(x)$. If you lose, you'd have utility $u(x - y)$; if you win it would be $u(x + y)$. Since it's 50/50 (if you play the game) your expected 'mean' utility—following Bernoulli—is $\frac{u(x-y)+u(x+y)}{2}$, which by the concavity of the function $u(t)$ is strictly less than $u(x)$, your starting utility level.

He then works with somewhat shocking explicitness, in effect inverting his utility function—specifically: *turning his logarithmic utility function into what one might call a 'multiplicative' function that describes perceived value*—to get a 'mean' expected value in various situations. He explains why there are occasions, therefore, where everyone—e.g., agents working on opposite sides of a deal should eagerly participate—given that different agents will have different amounts of wealth—hence the utility of a gain or loss will be significantly different. He applies this to *Insurance* and to dividing risk, spreading it over a number of situations (should you have all your cargo shipped in one ship, or spread it over a few ships?)

The basic arithmetic is simple enough. First the pre-Bernoulli analysis: if you have \$100 and invest in a project where you have an equal chance of losing \$50 or gaining \$50, the straightforward evaluation is that it is a totally fair (neutral, in a sense) game, in that you will end up with either a bank account of \$50 or \$150 with equal probability, so your 'Expected wealth' after the game is

$$\frac{1}{2}50 + \frac{1}{2}150 = 100 \quad (1)$$

dollars, exactly what it was before the game.

Now, given

- Bernoulli's hypothesis that expected utility has a logarithmic relationship to wealth⁸, and
- that the Expected utility of an event that will have one of a range of outcomes, each with specific probabilities, is the sum, over the range of outcomes, of the product of the Utility of the particular outcome times the probability that it will occur,

Bernoulli reckons your 'Expected wealth', after the game above, to be:

$$\$\sqrt{50 \cdot 150} = \$86.60 \quad (2)$$

So, an 'Expected loss' of \$13:40. He writes:

⁸which is a big assumption; far too explicit to be meaningful!

We must strongly emphasize this truth, although it be self evident: the imprudence of a gambler will be the greater the larger the part of his fortune which he exposes to a game of chance.

E.g., if—for example—the gambler has \$10000, the 'Expected wealth', after the game would be:

$$\$\sqrt{9950 \cdot 10050} = \$9999.87 \quad (3)$$

So, an 'Expected loss' of 13 cents.

Let's do same analysis with 'insurance': imagine that the probability of a certain event happening is 1/4 and if it happens, it will cost you \$100; if it doesn't, you have as much money as you had before—say \$1000. The insurance company, is worth \$100,000. Following the recipe $(900)^{1/4}1000^{3/4}$, your expected wealth after the event is \$973, so if your insurance premium is less than \$27, that sounds like it's worth it. But for the insurance company the recipe is

$$(99,900)^{1/4}100,000^{3/4} = 99984.28.$$

So if the insurance company charges more than $100,000 - 99,984.28 = \$15.72$, it may expect a profit. Any premium, then between \$16 and \$27 would—given Bernoulli's analysis(!) — be a reasonable risk to take for either the insurer or the insuree.

Bernoulli ends his essay discussing (what is now known as) the St. Petersburg Paradox: how much would you be willing to pay to be a participant of the game of the following sort. In this game you can only win, not lose. A coin is thrown, and if it is heads the first throw, you get \$1 (and the game ends). If it is tails, it gets thrown again and if—then—it lands on heads you get \$2 (and the game ends). . . . If it is tails the first n times and is heads the $(n + 1)$ st time, you get $\$2^n$ dollars (and the game ends).

How much money might you naively expect to get on average if you play this game in long runs, if—for example—this game is scheduled to be cut off, finished or not, after n (or fewer, if it is finished at fewer) times? And what if it is slated to go on indefinitely? *There's an underlying empirical/psychological question here, and it pays to think about it on that level as well, before closer analysis, worth taking some time to discuss.*

The issue here is that if you apply the naive notion of "Expectation" to this problem—you arrive at one—unbelievable—answer, but if you work with "Expected utility" you get an answer that (you might argue) conforms—at least qualitatively— more closely to what people would actually pay to get into this game.

A naive computation of "expectation" of the money you might expect to win is

$$\sum_{k=1}^{\infty} \text{Prob}(k) \cdot \text{Payoff}(k),$$

where $\text{Prob}(k)$ is the probability that you'll win at the k -th stage, and $\text{Payoff}(k)$ is the payoff if you win at the k -th stage. So,

$$\sum_{k=1}^{\infty} \text{Prob}(k) \cdot \text{Payoff}(k) = \sum_{k=1}^{\infty} \frac{1}{2^k} 2^{k-1} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots \quad (4)$$

which seems to recommend that you should be willing to pay absolutely any amount to get into that game. But (apparently) none of us would pay a very high figure to get into this game—so what’s wrong? Are we (somewhat irrationally) risk averse? Or is there already something simply wrong or paradoxical with this naive notion of Expectation? Or perhaps, does the utterly unreal nature of this strange game spook us? A neat discussion ensues, along with remarks about the parallel work of Gabriel Cramer (1728; in a letter to Nicholas Bernoulli).

To quote Bernoulli quoting Cramer:

The “paradox” consists in the infinite sum which calculation yields ... This seems absurd since no reasonable man would be willing to pay 20 ducats as equivalent. You ask for an explanation of the discrepancy between the mathematical calculation and the vulgar evaluation. I believe that it results from the fact that, in their theory, mathematicians evaluate money in proportion to its quantity while, in practice, people with common sense evaluate money in proportion to the utility they can obtain from it.

Cramer then explains how how unravels the paradox—not terribly different from the way Bernoulli does. Here’s how Bernoulli deals with it:

If your wealth is α and the game actually can plausibly be played for n times, then—as Bernoulli theorizes!—the value of the game *for you* is

$$V_n(\alpha) := \prod_{k=1}^n (\alpha + 2^{k-1})^{\frac{1}{2^k}} - \alpha.$$

And this converges for any given α (as $n \rightarrow \infty$). E.g., if you own nothing, i.e., if $\alpha = 0$, its value is in the limit as $n \rightarrow \infty$ is:

$$\begin{aligned} V_{\infty}(0) &= \prod_{k=1}^{\infty} (2^{k-1})^{\frac{1}{2^k}} = \prod_{k=2}^{\infty} (2^{k-1})^{\frac{1}{2^k}} \\ &= 2^{\sum_{j=1}^{\infty} \frac{j}{2^{j+1}}} = 2^1 = 2. \end{aligned}$$

This is a weird conclusion, *and an interesting discussion might be: does any of this make sense?*

Bernoulli ends his essay with an engaging discussion regarding the fact—paradox, in essence—that (as computed by Bernoulli) if your wealth α gets larger and larger, your expectation of gain gets larger as well. It’s left quite a bit unresolved, but see [6] and [7].

People offer specific (low) amounts to enter such a game so this becomes a behavioral issue, or curiosity: what does their behavior tell us about personal assessments of risk?

We could discuss this. Various suggestions are in the literature: a prospective player of this game might be aware that the casino has finite resources so the game, as presented is untenable, or bogus/ also things that happen with miniscule odds are neglectable even if a naive computation of Expectation seems to get us forget this/ etc.

For an experimental study, see [8] where, among other things the authors test people's willingness to participate in various versions of the game and offer bids; and also in inversions of the game where—if you enter, you can lose as well as win:

Recent experimental research has returned to the questions originally posed by Bernoulli: Is human choice behaviour in St. Petersburg lotteries (a) inconsistent with expected value theory and (b) consistent with risk aversion? Recent experiments (Cox, Sadiraj and Vogt 2009; Neugebauer 2010) have used real money payoffs and finite games in order, respectively, to provide the experiment subjects with economic motivation and the experimenters offers of the lottery with credibility. Data from these experiments are inconsistent with risk neutrality but consistent with risk aversion, in this way appearing to provide support for Bernoulli's general conclusion about risk aversion (but not his specific conclusion about log utility).

References

- [1] Aristotle *Nicomachean Ethics; Book V*—transl.: W.D. Ross
<http://classics.mit.edu/Aristotle/nicomachaen.5.v.html>
- [2] Aristotle *Politics; Book VII*—transl.: W.D. Ross
<http://classics.mit.edu/Aristotle/politics.7.seven.html>
- [3] Aristotle *Nicomachean Ethics; Book VIII*—transl.: W.D. Ross
<http://classics.mit.edu/Aristotle/nicomachaen.8.viii.html>
- [4] Epicurus, *Principal Doctrines*—transl: Robert Drew Hicks
<http://classics.mit.edu/Epicurus/princdoc.html>
- [5] Bernoulli, D.: (transl.) *Exposition of a New Theory on the Measurement of Risk*, *Econometrica* 22 (1954): 23-26.
- [6] Aumann, R. J. (1977) "The St. Petersburg Paradox: A Discussion of Some Recent Comments," *Journal of Economic Theory* 14: 443-445.
- [7] Samuelson, P. (1977) "The St. Petersburg Paradox: Defanged, Dissected and Historically Described," *Journal of Economic Literature* 15: 24-55.

- [8] James C. Coxa, J.C; Krollb, E.B.: Sadiraja, V.; Vogtb, B.: “The St. Petersburg Paradox despite risk-seeking preferences: An experimental study”
<http://excen.gsu.edu/fur2012/fullpapers/ekroll.pdf>

Part III

Early Utilitarians

That guiding phrase,

The greatest good for the greatest number

is not quite a literary topos such as “all must die,” i.e., a phrase on which narratives can be easily built. Nor is it a fragment of wisdom literature such as “Lazy hands make for poverty/but diligent hands bring wealth,” as in The Book of Proverbs.

And of course it is just a phrase, and isn’t yet precise enough to connect *directly* with von Neumann-Morgenstern’s formulation of “games of strategy” (as we have seen in Professor Maskin’s presentation), or yet to be much help modeling social exchange economies and dealing with the mixture of many conflicting maximin problems, where every participant is guided by another principle and neither determines all variables which affect his interest, and—on top of this—where it is unspecified how many participants are involved.

Nevertheless, that phrase—standing for Utilitarianism—is a powerful caption for longstanding—and ongoing—debate on theoretical (moral) and practical (political) levels, opening various types of questions. We’ll discuss seven such themes for discussion:

1. ‘Goods’ and happiness

E.g., as in our readings in Aristotle and Epicurus, questions about the word ‘good’ (whether it refers to “external goods, goods of the body, and goods of the soul” as Aristotle trichotomized) expand to the discussions about happiness as a primary goal for humanity, and what that actually means—

Here one of the big issues in conversation begun by Aristotle is: what is the moral hierarchy—if there is such a hierarchy—that relates, i.e., that ranks, different types of happiness (different ‘pleasures’)? How does such a hierarchy affect our moral calculus?

And what is the relationship between morality, utility, and happiness? David Hume, in [1] makes his view clear:

In all determinations of morality, this circumstance of public utility is ever principally in view.” . . . Upon the whole, then, it seems undeniable, THAT nothing can bestow more merit on any human creature than the sentiment of benevolence in an eminent degree; and THAT a PART, at least, of

its merit arises from its tendency to promote the interests of our species, and bestow happiness on human society. We carry our view into the salutary consequences of such a character and disposition; and whatever has so benign an influence, and forwards so desirable an end, is beheld with complacency and pleasure.

The conversation about goods (as in Aristotle's trichotomy) and hence happiness, addresses one of the major problematic ambiguities in any version of Utilitarianism. Jeremy Bentham seems to make fewer moral distinctions between pleasures than (certainly) Mill and Hume—and, as one would expect, was attacked for seeming to put animal pleasures and the pleasures of virtue in the same discussion. Related to this, J.S. Mill (1863) takes pains (in Chapter 2 of his essay *Utilitarianism*) to draw a distinction between *happiness* and *contentment*:

Someone with higher faculties

- requires more to make him happy,
- is probably capable of more acute suffering, and
- is certainly vulnerable to suffering at more points, than someone of an inferior type;

but in spite of these drawbacks he can't ever really wish to sink into what he feels to be a lower grade of existence. Explain this unwillingness how you please! We may attribute it to

- pride, a name that is given indiscriminately to some of the most and to some of the least admirable feelings of which human beings are capable;
- the love of liberty and personal independence (for the Stoics, that was one of the most effective means for getting people to value the higher pleasures); or
- the love of power, or the love of excitement, both of which really do play a part in it. But the most appropriate label is a sense of dignity. . . . anyone who denies that the superior being is, other things being anywhere near equal, happier than the inferior one—is confusing two very different ideas, those of happiness and of contentment.

As for the centrality of happiness in the doctrinal “-ism” of *Utilitarianism*, here's Mill:

The utilitarian doctrine is that happiness is desirable as an end, and is the only thing that is so; anything else that is desirable is only desirable as means to that end.

2. Utilitarianism as a moral dictum

Should (or can) one take the phrase—*The greatest good for the greatest number*— as normative guide to personal (e.g., *my*) moral behavior, as in: *I must act to effect the greatest good for the greatest number?* And/or, should it be taken (normatively) in a collective way: as a guide to *how the state—or any broad social plan—should so act?*

On the optimistic side—regarding whether this normative principle can be adopted—here's Mill:

As the human mind improves, there is a steady increase in the influences that tend to generate in each individual a feeling of unity with all the rest; a feeling which in its perfect state would make him never think of or want any benefit for himself if it didn't also involve benefits for all the rest.

Regarding the unlikelihood of it being universally taken up and adhered to as a personal moral principle, here's Kant [4]:

... it is odd how it could have occurred to intelligent men, [merely] because the desire for happiness and hence also the maxim whereby everyone posits this happiness as the determining basis of his will is universal, to therefore pass this [maxim] off as a universal practical law. For although ordinarily a universal law of nature makes everything accordant, here, if one wanted to give to the maxim the universality of a law, precisely the extreme opposite of accordance would result: the gravest conflict, and the utter annihilation of the maxim itself and of its aim. For then the will of all does not have one and the same object, but each person has his [own] object (viz., his own well-being); and although contingently this object may indeed be compatible with the aims of other people as well, who likewise direct them at themselves, it is far from being sufficient for a law, because the exceptions that one is occasionally authorized to make are endless and cannot at all be encompassed determinately in a universal rule.

...

Empirical determining bases are not suitable for any universal external legislation, but just as little also for an internal one; for each person lays at the basis of inclination his [own] subject, but another person another subject; and in each subject himself now this inclination and now another is superior in influence. Discovering a law that under this condition would govern them all [viz., with accordance on all sides] is absolutely impossible.

3. The issue of selflessness

If it is a personal moral dictate, how much altruism does it demand? I.e., what if pursuing this moral obligation would cause suffering for me, but establish the greatest good for the greatest number?... J.S. Mill (Utilitarianism 1.2):

The utilitarian morality does recognize that human beings can sacrifice their own greatest good for the good of others⁹ ; it merely refuses to admit that the sacrifice is itself a good. It regards as wasted any sacrifice that doesn't increase, or tend to increase, the sum total of happiness. The only self-renunciation that it applauds is devotion to the happiness, or to some of the means to happiness, of others...

and

Utilitarian moralists believe that actions and dispositions are virtuous only because they promote an end other than virtue; and that it is on this basis that we decide what is virtuous.

4. Intent (and/) or Consequences

Is it a purely consequentialist guide? (I.e., having little or nothing to do with inner intentions: only consequences matter.

Here's J.S. Mill on this issue:

... doing good acts (i.e., acting in a moral way) is, after all, different from having private motivations that are morally laudable. Utilitarianism dictates nothing about private motivations.

5. Good for whom?

Who are to be counted among the beneficiaries of this 'greatest good?' How do you count them? How do you measure 'greatest good'? How in the world might you effect such a plan without encountering 'unforeseen consequences'? (More pointedly: is there a danger, here, of hubris?)

Regarding the issue of measurement—e.g. representing 'happiness' as quantifiable—we have seen the somewhat audacious move of Daniel Bernoulli in that direction, in our reading of his essay *Exposition of a New Theory on the Measurement of Risk*. The context of that essay could be characterized largely (but not entirely) as “one-person games,” but we might think of it as foreshadowing the possibility of two- and many-person games as in more contemporary thinking.

⁹There are, of course, dicta that take such tit-for-tat equations on other levels: for example, Rabbi Israel Salanter a decade earlier had proclaimed (essentially) that *your neighbor's material needs are your spiritual needs*.

All the above questions already vibrate in the earlier (pre-) Utilitarian literature such as Francis Hutcheson's *An Inquiry into the Original of Our Ideas of Beauty and Virtue* (1726: twelve years earlier than the publication date of the essay of Bernoulli.).

VIII. In comparing the moral Qualities of Actions, in order to regulate our Election among various Actions propos'd, or to find which of them has the greatest moral Excellency, we are led by our moral Sense of Virtue to judge thus; that in equal Degrees of Happiness, expected to proceed from the Action, the Virtue is in proportion to the Number of Persons to whom the Happiness shall extend; (*and here the Dignity, or moral Importance of Persons, may compensate Numbers*) and in equal Numbers, the Virtue is as the Quantity of the Happiness, or natural Good; or that the Virtue is in a compound Ratio of the Quantity of Good, and Number of Enjoyers.

I italicized the phrase *and here the Dignity, or moral Importance of Persons, may compensate Numbers* to compare it with the insistence of Utilitarians such as Mill that—despite the hierarchy of *pleasures* he delineated—any *person* counts as equal to any other.

What Hutcheson is asserting here is—in effect—a formula. He would have it that there are three quantities involved: Virtue, Good, and People. (And as for *People*, he envisions a sort of algebraic sum, weighted by Dignity or Moral Importance.) He also allows these quantities to be negative:

In the same manner, the moral Evil, or Vice, is as the Degree of Misery, and Number of Sufferers; so that, that Action is best, which procures the greatest Happiness for the greatest Numbers; and that, worst, which, in like manner, occasions Misery.

IX. Again, when the Consequences of Actions are of a mix'd Nature, partly Advantageous, and partly Pernicious; that Action is good, whose good Effects preponderate the evil, by being useful to many, and pernicious to few; and that, evil, which is otherwise. *Here also the moral Importance of Characters, or Dignity of Persons may compensate Numbers*; as may also the Degrees of Happiness or Misery: for to procure an inconsiderable Good to many, but an immense Evil to few, may be Evil; and an immense Good to few, may preponderate a small Evil to many.

and then he addresses the fraught problem of unintended (or less intended) consequences:

But the Consequences which affect the Morality of Actions, are not only the direct and natural Effects of the Actions themselves; but also all those Events which otherwise would not have happend. For many Actions which

have no immediate or natural evil Effects, nay which actually produce good Effects, may be evil; if a man foresees that the evil Consequences, which will probably flow from the Folly of others, upon his doing of such Actions, are so great as to overbalance all the Good produced by those Actions, or all the Evils which would flow from the Omission of them: *And in such Cases the Probability is to be computed on both sides.* Thus if an Action of mine will probably, through the Mistakes or Corruption of others, be made a Precedent in unlike Cases, to very evil Actions; or when my Action, tho good in it self, will probably provoke Men to very evil Actions, upon some mistaken Notion of their Right; any of these Considerations foreseen by me, may make such an Action of mine evil, whenever the Evils which will probably be occasion'd by the Action, are greater than the Evils occasion'd by the Omission.

6. Justice

One further issue raised by any doctrinal assertion of Utilitarianism is the question of how it relates to—how it infringes, perhaps—individual rights. In a word, its relation to *justice* (hearkening back to Epicurus). Some contemporary critics, Robert Nozick and John Rawls, are wary of utilitarianism, in view of the possible curtailment of personal liberty that might go hand in hand with a utilitarian plan. Nozick (cf ([6]) works out a (libertarian) defense of something like a 'minimal state.' This may not be exactly the 'Nightwatchman state,' which restricts its activities to bodily protection of its citizens and the enforcement of contracts, but it is close to that. In such a state, looking out for the 'happiness' of its citizens is not one of its primary goals—although, he allows—it might be a consequence. Rather, Nozick wishes to have states whose primary goals are the ('maximalization of the') nonviolation of individual's rights. He toys with the phrase 'utilitarian of rights.' Rawls's objection to utilitarianism also has to do with its relationship to individual rights. He writes (see (xi) of [7]):

... I do not believe that utilitarianism can provide a satisfactory account of the basic rights and liberties of citizens as free and equal persons, a requirement of absolutely first importance for an account of democratic institutions.

It seems that one can also see traces of concerns regarding justice already in the writings of Francis Hutcheson, the early utilitarian that we have been considering:

And this is the Reason that many Laws prohibit Actions in general, even when some particular Instances of those Actions would be very useful; because an universal Allowance of them, considering the Mistakes Men would

probably fall into, would be more pernicious than an universal Prohibition; nor could there be any more special Boundaries fix'd between the right and wrong Cases. In such Cases, it is the Duty of Persons to comply with the generally useful Constitution; or if in some very important Instances, the Violation of the Law would be of less evil Consequence than Obedience to it, they must patiently resolve to undergo those Penalties, which the State has, for valuable Ends to the Whole, appointed: and this Disobedience will have nothing criminal in it.

7. Rational Foundations

One other issue—especially focused on in Mill's essay—is the question of whether one could formulate in some 'mathematical, or science-like, format rigorous foundations that might be put forward as rational grounds for believing the Utilitarian project.

It isn't unusual to offer a mathematical framework for ethics. Spinoza, for example, tries to give such a format to his discussion in *Ethics*—with his Definitions, Axioms, Propositions, Corollaries, Lemmas and Postulates—as does Kant in the Critique of Practical Reason—with his Theorems.

Here's how Mill begins his essay *Utilitarianism*—to my mind conceding how difficult it might be to set proper foundations (for Utilitarianism, as well as many other things!).

It is true that similar confusion and uncertainty, and in some cases similar disagreements, exist concerning the basic principles of all the sciences—even including the one that is thought to be the most certain of them, namely mathematics—without doing much harm, and usually without doing any harm, to the trustworthiness of the conclusions of those sciences. This seems odd, but it can be explained: the detailed doctrines of a science usually are not deduced from what are called its first principles and don't need those principles to make them evident. If this weren't so, there would be no science more precarious, and none whose conclusions were more weakly based, than algebra. This doesn't get any of its certainty from what are commonly taught to learners as its elements or first principles, because these, as laid down by some of its most eminent teachers, are as full of fictions as English law and as full of mysteries as theology. The truths that are ultimately accepted as the first principles of a science are really the last results of metaphysical analysis of the basic notions that are involved in the science in question. Their relation to the science is not that of foundations to a building but of roots to a tree, which can do their job equally well if they are never dug down to and exposed to light. But though in science the particular truths precede the general theory, the reverse of that might be expected with a practical art such as morals or legislation.

References

- [1] Paragraphs 1-15, and 42-47 in Section 5 (Why Utility pleases) in David Hume's *Enquiry Concerning the Principals of Morals*
<http://www.davidhume.org/texts/epm.html#M5dot1>
- [2] Mill, J. S.: *Utilitarianism* Especially Chapter 4 (pp. 24-28)
<http://www.earlymoderntexts.com/assets/pdfs/mill1863.pdf>
- [3] (Bentham and Mill) Sections 2.1 and 2.2 of [14], the entry *Utilitarianism* in the Stanford Encyclopedia of Philosophy
<https://plato.stanford.edu/entries/utilitarianism-history/>
- [4] Kant, I.: *Critique of Practical Reason*—transl.: W.S. Pluhar Section 4 (pp.40-42) and Sections 8 (pp.48-53)
<http://www.morelightinmasonry.com/wp-content/uploads/2014/06/Kant-Critique-of-Practical-Reason-Cambridge.pdf>
- [5] Adam Smith *Wealth of Nations* Paragraphs IV.24, IV.2.9-2.12
<http://www.econlib.org/library/Smith/smWN13.html>
- [6] Nozick, R.: *Anarchy, State, and Utopia* (pp. 28-30)
http://www.lib.cmb.ac.lk/wp-content/uploads/2014/01/0001_anarchy_state_and_utopia.pdf
- [7] Rawls, J.: *Theory of Justice*
<http://www.univpgri-palembang.ac.id/perpus-fkip/Perpustakaan/American%20Phylosophy/John%20Rawls%20-%20A%20Theory%20of%20Justice-%20Revised%20Edition.pdf>

Part IV

On axomatizing ‘fairness.’

The limits of axioms.

What do axioms for social choice help us understand? ‘Standards’ in the repertoire, i.e., the axioms of

- Anonymity,
- Independence of irrelevant alternatives,

- (Various versions of) the Pareto Principle
- Independence of Indifferent Individuals,
- Equity, and
- Non-dictatorship

are marks, pointing to ways of measuring ‘fairness’ of any Social Welfare Function that satisfies these axioms¹⁰. The discussion that arises from consideration of such axioms might be a helpful preliminary to any general discussion regarding fairness—and perhaps also justice. All this being perfectly good setting-up exercises for discussions more broadly, regarding consequentialist issues in morality in a social context, and perhaps even intentionalist issues in personal ethics.

One might imagine replacing the word *axioms*¹¹ by *desiderata* since they are usually formulated not as the armature of a formal system—such as in Euclid’s *Elements*—to be the ‘prime movers’ building up a system, but rather as a wish-list of features that any recipe for making fair social choices should enjoy. The great virtue of these axioms (or desiderata) is that they force us to focus. Focus on what? On the notion of *fairness of any social compromise*. This, I suppose, will never be a finished exercise. In contrast, any *traditional* axiomatic system is quintessentially a ‘closed system,’ necessarily then ‘artificial.’ It sets out, a priori, what we propose to be relevant. But the axiomatic set-up (for any discussion about utility) provides an incisive, undeniably useful, cartoon, which expresses each of us ‘individuals’ (in our economic or, more broadly, social interactions) not as *subjects* with a constellation of possible (very subjective) behavioral preferences¹² that can distort what we might call our naively ‘rational’ utility functions; but rather as totally described by our utility functions; i.e., with an a priori describable mode of formulating preferences.

Surely we, as subjects—not as rational agents—are acutely ‘menu-dependent/’social-situation-dependent’/ and also we often prefer a richness of choice (sometimes with no particular motivation for one choice rather than another: just variety for variety’s sake). So the distinction between what we actually ‘prefer’—at a given moment in a given context—and what it might naively be predicted that we would rationally choose (and in what vocabulary we express our choice) is the clash between us as *subjects* versus us as *axiom-describable agents*. The axioms then are ways of signaling what is cleanly capturable, and quantifiable—the uncapturable rest comprises the pure essence of our subjecthood. Thus

¹⁰I left out of this list the axiom of *Co-cardinality* in that I don’t see this as *promoting* fairness per se, but rather as offering a helpful structure—in a fair way. BUT, since addition commutes with linear affine transformations, Cocardinality strongly favors utilitarianism!

¹¹These are called *Conditions* in Kenneth Arrow’s *Social Choice and Individual Values*.

¹²—preferences that we are sometimes only vaguely conscious of having—

this exercise—formulating an axiomatic approach—, in passing, gives us an apophatic view of “us” as pure subjects. At the very least, it reminds us that—as subjects—we are, in the end, uncapturable by any axioms.

And once one allows our interestingly named “Social Welfare Functions” to take into account moral (or immoral) motives for individuals making their choices¹³, the game changes even more.

Some vocabulary—and a review of things discussed in our seminar

- Let X be a set of alternatives that people might want to rank.
- Let $\mathcal{R} = \mathcal{R}_X$ denote the set of all possible orderings (or rankings) of the objects in X (allowing for indifference).
- Let N be a set of individuals.
- If A, B are sets then let $F(A, B)$ denote the set of all functions from A to B .
- So $F(N, \mathcal{R})$ is the set of all possible rankings of the objects in X these N individuals might have.

A **Social Welfare Function** S is a recipe designed for

- a fixed community N of individuals and
- a roster of alternatives X and
- a format $\mathcal{P}(N; X)$ that describes all possible preferences that an individual in N can have regarding the roster X .

An ‘SWF’ S offers a ‘Social’ ranking for any possible collection of individual preferences. So, we may think of S as a mapping:

$$S : \mathcal{P}(N; X) \rightarrow \mathcal{R}_X.$$

Our task, then, is to understand the level of fairness of such recipes S . Before being able to do this, we must stipulate precisely what this set “of all possible preferences,” $\mathcal{P}(N; X)$, should actually consist of.

¹³as in Part VI (*Limitations of Welfarism even with rich utility information*)—in Professor Sen’s *Personal Utilities and Public Judgement: Or what’s wrong with Welfare Economics*

As Arrow showed, it is quixotic to hope that if we take $\mathcal{P}(N; X)$ to be merely $F(N, \mathcal{R}_X)$ —the set of all possible individual rankings of the alternatives in X —that any such Social Welfare Function,

$$S : F(N, \mathcal{R}_X) \rightarrow \mathcal{R}_X,$$

i.e., any mapping that, for every collection of individual rankings $\alpha \in F(N, \mathcal{R}_X)$, gives a single (social) ranking $S(\alpha) \in \mathcal{R}_X$, can reflect, totally satisfactorily and fairly, the communal judgment of the individuals in N .

Call a Social Welfare Function S a *Pure Social Welfare Function* if it only ‘sees’ the individual rankings, but no subtle personal assessment such as *utility functions*.

Since it seems to be impossible to provide Pure SWFs that are ‘fair,’ the next step is to refine these individual rankings $\alpha \in F(N, \mathcal{R})$ and associate to each individual in N more information reflecting his or her preference¹⁴—i.e., more information than just a ranking of the elements of X . E.g., for elements $a, b \in X$ beyond the mere knowledge that the individual “ i ” ranks a as more preferable than b we might consider some further information about the nature, or perhaps intensity, of this ranking. It is traditional to refine α (the mere ranking) replacing it by a real-valued function $u(x)$ for $x \in X$ where the ranking is given by the order relation on X as determined by the values of $u(x)$. This is in hopes that the actual values of $u(x)$ give us more precise information.

The Cocardinality Axiom, however, would say that if we translate and/or rescale the collection of utility functions $\{u_i(x) \text{ for } i \in N\}$ (i.e., by applying a *common* affine linear transformation¹⁵ $u'_i(x) = cu_i(x) + d$ where $c > 0$) the social ranking of that new (translated and/or rescaled) conglomerate of utility functions, $\{u'_i(x) \text{ for } i \in N\}$ doesn’t change. Structurally, it is saying that we may as well think of the *domain of values* of utility functions not (so much) real numbers, but rather elements in a directed straight (Euclidean) line L , without a chosen origin or unit. (And if you ever want, you can just arbitrarily give it an origin and unit, at which point you get back to—in effect—the *real number line*.)

We find ourselves, then, dealing with a mapping $N \rightarrow F(X, L)$ that associates to each individual $i \in N$ his or her utility function $u_i(X)$ from which one can ‘read off’ the ranking $\alpha_i \in \mathcal{R}$ that the individual i makes.

So we may hope to fashion a SWF making use of this extra personal information (i.e., utility functions: the mapping $N \rightarrow F(X, L)$), the fairness of which would be visible by its conforming to a list of axioms as in Professor Maskin’s essay *A Theorem on Utilitarianism*.

One thing to notice in the axioms of Aspremont and Gevers (that Professor Maskin uses; excluding ‘Equity’) is that there is no axiomatic request that one consider interpersonal

¹⁴Whether these preferences are normative (in the sense that they describe what rational agents would choose) or are descriptive is an issue taken up extensively in the work of Kahneman and Tversky (cf. Kahneman’s Nobel Prize speech *Maps of Bounded Rationality: Psychology for Behavioral Economics* delivered on December 8, 2002).

¹⁵An **affine linear transformation** is a mapping from the real number line to itself of the form $Y \mapsto cY + d$; the constant c we can call the **scaling constant** and d the **translation constant**. We will only be interested in such functions where $c > 0$.

comparison of the utility functions of two individuals in a *given* mapping $N \rightarrow F(X, L)$; e.g., the axioms never ask you to consider whether $u_i(x)$ is greater than $u_j(x)$. Let's call such an axiom system (that never asks you to 'combine' values of utility functions for different individuals) *utility discreet*¹⁶.

To be sure, any nondiscreet combination of the actual numerical values of different utility functions must depend on some understood yoking of the units in which the various individual's utility functions are formulated: you can't arbitrarily multiply $u_i(x)$ by 2, leaving $u_j(x)$ as it was for all $j \neq i$. After all, if the individual utility functions are—for example—offered by the individuals themselves, the more flamboyant expressions of enthusiastic rankings would rule. Some interpersonal calibration of the different individual utility functions is clearly necessary if one is thinking of actually numerically combining them in some way—e.g., as in the utilitarian format—to obtain a social conclusion. Here, for example, is (to my mind) a natural recipe for rescaling (which has its virtues and drawbacks, but is a simple rule that attempts to equalize the effect of each individual's preference in determining the social conclusion):

1. Assume given a collection of individual utility functions

$$\{u_i(x) \in F(X, \mathbf{R}) \mid i \in N\},$$

i.e., a utility function $u_i(x)$ for each individual $i \in N$. We ignore the utility functions $u_i(t)$ that are constant—i.e., corresponding to individuals who are indifferent, in terms of their preference for any of the alternatives. We rescale *each* of the non-constant functions $u_i(x)$ separately by an affine linear transformation¹⁷

$$u_i(x) \mapsto u'_i(x) := c_i u_i(x) + d_i,$$

where

2. the scaling constant $c_i > 0$ is chosen so that the **variance** of the values of $u'_i(x)$ about its mean is equal to 1, and
3. the translation constant b_i is chosen so that the mean is 0.

This recipe for recalibration has, at least,—in its favor—the virtue that the affine linear recalibrating transformations are uniquely characterized by properties (2) and (3) above.

¹⁶this, of course, is different from the mathematical term *discrete*.

¹⁷Any such recalibration preserves, of course, the pure ranking and preserves the *proportions* in preferences as recorded in the numerical values of these individual utility functions.

Non-discreet Social Welfare Functions

The social welfare recipe that most prominently ‘interpersonally combines’ utility functions is *utilitarianism* which directly produces a *social* utility function from the conglomerate of individual utility functions $\{u_i(x)\}_{i \in N}$:

$$U(x) := \frac{1}{|N|} \sum_{i \in N} u_i(x). \quad (5)$$

In fact consider any function $F(t_1, t_2, \dots, t_{|N|})$ that

- is sufficiently smooth¹⁸,
- satisfies **co-cardinality** in the sense that it commutes with any common rescaling of the $|N|$ -variables by an affine linear transformation, and
- satisfies **anonymity** in the sense that it is symmetric in the $|N|$ variables.

Then a nonconstant function $F(t_1, t_2, \dots, t_{|N|})$ is, after scaling by an appropriate affine linear transformation, equal to the summation function:

$$F(t_1, t_2, \dots, t_{|N|}) = \frac{1}{|N|} \sum_{i=1}^{|N|} t_i. \quad (6)$$

So, any (nonconstant) SWF that satisfies three axioms corresponding to the bullets above is Utilitarianism¹⁹.

¹⁸E.g., is C^∞ in the $|N|$ -variables

¹⁹Turning the tables it is amusing to note that no sufficiently smooth transformation $L \rightarrow L$ other than affine linear transformations commutes, in the above sense, with the summation function. Changes in the ‘utilitarian’ social ranking certainly may occur if you rescale utility functions, for example, by applying non-affine linear transformations—e.g., if you change by a logarithm ($u'_i(x) := \log u_i(x)$ for $i \in N$) thereby replacing addition with multiplication, making the new utilitarian social function

$$U'(x) := \left(\prod_{i \in N} u_i(x) \right)^{\frac{1}{|N|}}. \quad (7)$$

Here’s a simple example:

Let N consist of two individuals (“1” and “2”) and let X consist of two alternatives (a and b). Suppose the utility functions $u_1(x), u_2(x)$ are as follows:

$$u_1(a) = 5; \quad u_1(b) = 2 \qquad u_2(a) = 4; \quad u_2(b) = 8.$$

Then following Equation (1) the ‘social ranking’ has b beat a , while following Equation (3), a beats b . To sum up: ‘calibrating’ these different utility functions by affine linear transformations is the thing to do if you want to make a utilitarian option viable.

Each axiom is an invitation to a discussion

Regarding ‘rational models’ as exemplified by the axioms we are considering, Daniel Kahneman has written:

... psychological theories of intuitive thinking cannot match the elegance and precision of formal normative models of belief and choice, but this is just another way of saying that rational models are psychologically unrealistic. Furthermore, the alternative to simple and precise models is not chaos. Psychology offers integrative concepts and mid-level generalizations, which gain credibility from their ability to explain ostensibly different phenomena in diverse domains.

Every one of the axioms can be taken as an ‘opening move’ for a discussion. The named paradoxes (Ellsberg, Allais, St. Petersburg) are products of such. But all of the axioms deserve—if not paradoxes related to them, at least—‘para-doctrines’ to elaborate on our comprehension of how they may need tempering in the hurly-burly of actual social contexts.

Take *anonymity*, which raises the question: what constitutes an *individual*? For example, if one weighs the pros and cons of some town project, one might want to take into account the advantages and disadvantages at stake for each of the citizens of the town, but also for the schools (as personified, say, by the school board), for the fire department, for the combined commercial interests in the town’s shopping mall—each being considered an ‘interested party,’ an *agent*, and each with different, perhaps quite incommensurate needs and desires. Surely one wants some structure—other than crude blind anonymity—that weights the utility assessment of each of these agents, when one deliberates on the social choice to be enacted.

Given this sort of situation, the question then arises: who, exactly, is to do the weighting of the disparate utility functions? Of course our initial questions regarding the linkage between usefulness and happiness puts strong responsibility and authority on the individual as the decider of his-or-her own utility function: who is better than *me* to say what gives *me* happiness. The gauging of happiness—hence also usefulness—is ultimately a personal judgment, such as the judgment of beauty.

Part V

The reach of usefulness

On usefulness and beauty

Chapter I of Adam Smith's *The Theory of Moral Sentiments* (our reading for an earlier session) begins with quite a bold assertion:

That utility is one of the principal sources of beauty has been observed by every body, who has considered with any attention what constitutes the nature of beauty. The conveniency of a house gives pleasure to the spectator as well as its regularity, and he is as much hurt when he observes the contrary defect, as when he sees the correspondent windows of different forms, or the door not placed exactly in the middle of the building. That the fitness of any system or machine to produce the end for which it was intended, bestows a certain propriety and beauty upon the whole, and renders the very thought and contemplation of it agreeable, is so very obvious that nobody has overlooked it.

Smith's claim *That utility is one of the principal sources of beauty has been observed by every body* is countered forcefully in *The Analytic of Beauty* in Kant's *Critique of Judgment*. Kant's take on the relationship between *beauty* and *usefulness* is a good deal subtler than—and opposite to—Smith's view.

First, for Kant, the judgment that 'X is beautiful' is an activity on the level of what Kant refers to as the 'universal subjective.' That is, when we make the proclamation *X is beautiful* (in the midst of a genuine experience of beauty) we are not merely asserting that we think that this X has beauty, but also that—given our internal model of humanity and its sensibilities—we think that exactly this judgment would or should generally be made, by all who experience X. Our conception of 'all mankind' is somehow invoked by the force of the experience. And second, for Kant,—quite opposite to Smith's view—a true experience of the beauty of X has *nothing whatsoever* to do with any actual use to which X may be put; it has nothing to do with the purpose, or purposes, of X, if X had such.

Nevertheless, for us to be captured by the beauty of some object of art, there has to be the *feel* of something like purpose invoked by that work of art—not specific purpose—but some (effectively ungraspable) tincture of purpose, that beckons us to formulate what 'purpose-like quality' it has explicitly—but any such formulation will forever elude us, yet

keep our faculties and sensibilities continuously engaged²⁰.

Kant's term for that 'purpose-like quality' that is *not* purpose is: *Purposiveness*²¹.

... we do call objects, states of mind, or acts *purposive* even if their possibility does not necessarily presuppose the presentation of a purpose; we do this merely because we can explain and grasp them only if we assume that they are based on a causality [that operates] according to purposes, i.e., on a will that would have so arranged them in accordance with the presentation of a certain rule. Hence there can be purposiveness without a purpose, insofar as we do not posit the causes of this form in a will, and yet can grasp the explanation of its possibility only by deriving it from a will. Now what we observe we do not always need to have insight into by reason (as to how it is possible). Hence we can at least observe a purposiveness as to form and take note of it in objects even if only by reflection—without basing it on a purpose.²²

On usefulness and knowledge

The peculiar virtue of the type of knowledge we most value is that its 'utility,' if it has such, is rarely restricted to one time, or one circumstance, or one setting; but rather—like poetry—it shapes a viewpoint, or predilection, or a sensibility. The attribution of 'utility' to such knowledge then requires something of a generous stretch in that knowledge's force is as much (or more) in its ever-present potential, rather than its pinpoint actualization. Regarding poetry, the poet Jane Hirschfield once commented:

... one way poems may be useful is by showing how thin usefulness is. Animal joy, the babbling of babies to their stuffed bears, limericks, Gerard Manley Hopkins sonnets of praise and despair— sometimes the dismantling of rational response is the most needed thing. So much of the illness of the contemporary world comes from living in silo-mind, fixed inside received concepts and purpose. Good poems take down the silos. They are windows flung open.

. And the line containing the often-quoted phrase "For poetry makes nothing happen:" (in Auden's elegy for Yeats²³) goes on to emphasize poetry's patience:

²⁰E.g., the self-portrait of Van Gogh in the Fogg museum.

²¹Zweckmäßigkeit; cf. Kant, I.: *Critique of Judgment* transl.: W.S. Pluhar
https://monoskop.org/images/7/77/Kant_Immanuel_Critique_of_Judgment_1987.pdf

²²All this hangs a bit on Kant's rather curious notion of *the will* (cf. loc.cit.)

²³I am thankful to Kristen McCormack, who brought attention to the fact that early drafts of this poem are very much worth considering as well, in view of our seminar's discussion.

For poetry makes nothing happen: it survives
In the valley of its making. . .
. . . it survives,
A way of happening, a mouth.

Given that (for weeks) we have been considering the subtler issues regarding the qualitative assessment—and/or quantitative measurement—of utility in a broad context, it is interesting to turn to the conundrum that *usefulness of knowledge* presents to various people, G.H. Hardy among them. I have excerpted²⁴ the relevant parts of his essay *A Mathematician's Apology* which can be thought of as an extended meditation on the personal and universal aspects of the usefulness of mathematics. I hope that it sets the stage appropriately for some discussion about usefulness of knowledge in our seminar.

Hardy writes in something of a nervous style—anxiety about the possible blame that might be attached to mathematics in its responsibility for increasing the possibilities of devastation in wartime (it was indeed wartime when his essay was published!) and, on a personal level, sensitivity to criticism—as in the footnote:

I once said that ‘a science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life’, and this sentence, written in 1915, has been quoted (for or against me) several times. It was of course a conscious rhetorical flourish, though one perhaps excusable at the time when it was written.

Nevertheless, he has quite pointed opinions, such as:

If useful knowledge is, as we agreed provisionally to say, knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless.

. . .

It will probably be plain by now to what conclusions I am coming ; so I will state them at once dogmatically and then elaborate them a little. It is undeniable that a good deal of elementary mathematics and I use the word elementary in the sense in which professional mathematicians use it, in which it includes, for example, a fair working knowledge of the differential and integral calculus—has considerable practical utility. These parts of mathematics are, on the whole, rather dull; they are just the parts which have least aesthetic value. The ‘real’

²⁴in the reading for today's session

mathematics of the ‘real’ mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly ‘useless’ (and this is as true of ‘applied’ as of ‘pure’ mathematics). It is not possible to justify the life of any genuine professional mathematician on the ground of the ‘utility’ of his work.

...

There is one comforting conclusion which is easy for a real mathematician. Real mathematics has no effects on war. No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years.

...

So a real mathematician has his conscience clear; there is nothing to be set against any value his work may have; mathematics is, as I said at Oxford, a ‘harm- less and innocent’ occupation.

Nevertheless, what emerges from Hardy’s essay is something of a Linnaean classification of aspects of usefulness of mathematics—and in our discussion we might think of widening the application of these rough categories to other types of knowledge. Briefly, here—in increasing order of breadth, if not of importance—is the list of facets of utility (of mathematics)—or of what is claimed to be utility—and about which one gets glimmers of a discussion in Hardy’s essay:

- capability of performing very specific practical tasks,
- proficiency in routine mathematical skills,
- acquiring general mathematical experience,
- achieving a powerful mathematical viewpoint or sensibility,
- capturing the type of ‘goods of the soul’ as we’ve seen in Aristotle’s hierarchy—i.e. the beauty of mathematics.

Hardy sums things up (a few times, with somewhat contradictory *summations*):

One rather curious conclusion emerges, that pure mathematics is on the whole distinctly more useful than applied. A pure mathematician seems to have the advantage on the practical as well as on the aesthetic side. For what is useful above all is technique, and mathematical technique is taught mainly through pure mathematics.

In Abraham Flexner's *The usefulness of useless knowledge* one finds quite a different attitude toward utility and the pursuit of knowledge. He feels that it is the kick-start of sheer curiosity rather than the end-magnet of usefulness that accounts for the awakening of imagination, offering the reward of the deepest knowledge²⁵. Utility was merely the bonus, and not the primary sought-for aim:

...throughout the whole history of science most of the really great discoveries which had ultimately proved to be beneficial to mankind had been made by men and women who were driven not by the desire to be useful but merely the desire to satisfy their curiosity.

I shall concern myself with the question of the extent to which the pursuit of these useless satisfactions proves unexpectedly the source from which undreamed-of utility is derived.

And, bracketing utility, he writes:

I sometimes wonder whether that current has not become too strong and whether there would be sufficient opportunity for a full life if the world were emptied of some of the useless things that give it spiritual significance; in other words, whether our conception of what is useful may not have become too narrow to be adequate to the roaming and capricious possibilities of the human spirit.

The perplexing distinction between 'useful' knowledge and 'pure' knowledge has pervaded discussions about the sciences and mathematics. The general view of what specific directions of research has—or may have—application to the social good, or perhaps, social detriment, influences much (including funding). The terms 'mixed' and 'pure' used in the early seventeenth century by Francis Bacon were meant to characterize types of mathematics²⁶. Here's Bacon's 'tree of mathematical knowledge'.

²⁵This is a view in harmony with his role as director of the Institute for Advanced Study, attracting theoretical scientists (such as Einstein) immigrating to the United States just before World War II.

²⁶For a very clear discussion of the history of these terms, see Gary Brown's "The evolution of the Term "Mixed Mathematics" (Journal of the History of Ideas Vol. 52 (1991) 81-102. <http://www.jstor.org/stable/2709583> (I thank Alma Steingart for recommending this article.).

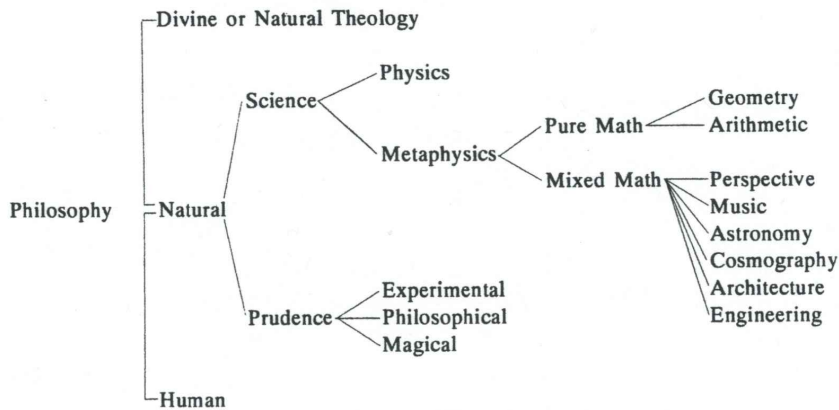


Figure 1: Bacon's Tree of Knowledge
(Human Learning)

This influenced a small branch of d'Alembert's tree configuring the entire 'system of human understanding' that appeared in the *Encyclopedie* a century later. There, mathematics appeared as a part of 'The Science of Nature' and consisted of three sub-pieces: *Pure*, *Mixed*, and *Physico-Mathematics*; the *Pure* consisting of *Arithmetic and Geometry*; and *Geometry* separating into—of all things: (*Elementary (Military Architecture Tactics)* and *Transcendental (Theory of curves)*).

The curious entanglement of applied and pure in the mapping of knowledge—e.g., as in D'Alembert's attempt—might make for interesting discussion in connection with the general difficulty of pinning down 'utility.'

After-note: There was indeed an interesting discussion during the session devoted to this material. It was pointed out that the cultivation of and respect for *curiosity* itself, and the following of its impetus—this being a virtue—is already a proper end for— and utility of—the pursuit of knowledge. It was mentioned (by a number of people) that certain scientific programs—such as landing men on the moon—irrespective of the possible longterm benefits that might ensue—have the 'utility' of expressing the extent of our—i.e., humanity's—range of interests; that certain programs not immediately helpful in the protection of humanity have the utility of making it clear that humanity is worth being protected. To put it in vaguer terms, knowledge, insofar as it is yoked to our sense of values and meaning, just comes along with its own usefulness.