LECTURE 1 Wed 9/3

apost@math.mit.edu
course page: math.mit.edu/18.217
Office Hours: M 12-1pm

Most general: Algebraic Combinatorics

- Coxeter groups
- root systems
- rep theory

interactions between rep theory, Lie theory and Combinatorics

↳ tentative list of topics on website (will not cover them all)

GRADING...

★ ~3 problem sets (hard problems, start early!) 25%
★ Participating in class 80%

(Admin got worried about people skipping all lectures, doing problem sets and maybe using chat GPT and changed the requirements)

Ways to demonstrate class participitation
★ short pset solution presentation (NOT everyone, volunteer only)
★ mini quizzes ~ easy questions from lecture that also solicit feedback about the class
(should not need to prepare or worry about the quizes, just show up)

Games on Graphs

Game #1 Bert Kostant's game (secretly find the highest root game)

$G=(V,E)$ simple connected graph
↖ no loops/multiple edges

$V=[n]:=\{1,2,\dots,n\}$

$G =$ 

Vertex $v\in V$ has c_v chips $c_v\in\mathbb{Z}_{\geq 0}$

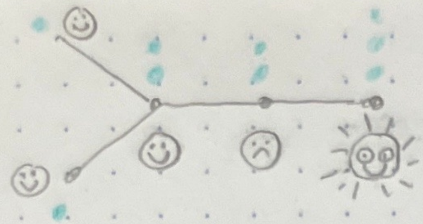
Chip configuration $(c_1,\dots,c_n)$

Vertex v is happy if $c_v \geq \frac{1}{2}\left(\sum_{u \text{ a neighbor of } v} c_u\right)$

unhappy if $c_v <$ ——〃——

excited if $c_v >$ ——〃——

Ex.



GOAL: Make everybody happy

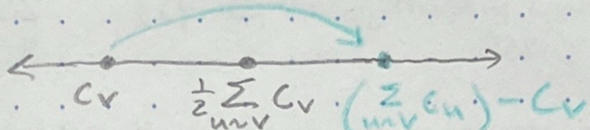
Start $c = (0, 0, \dots, 0)$

"Nobody have chips, everybody's happy — it's like communism" - Alex

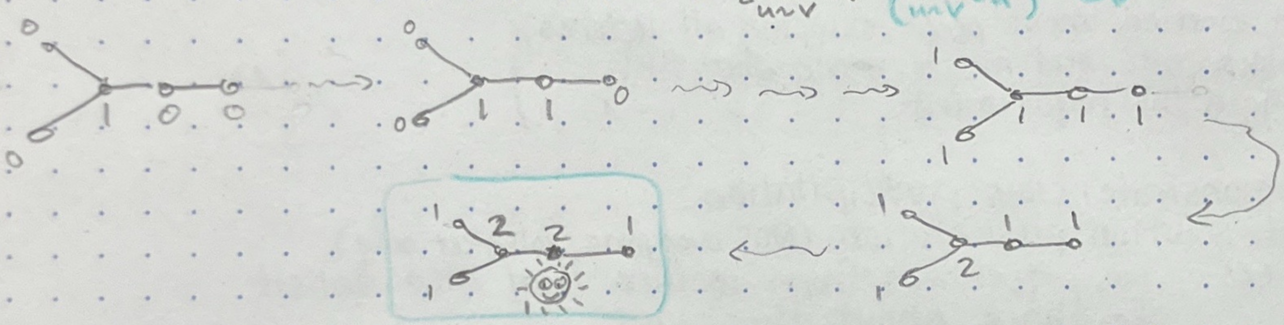
Pick any $w \in V$ and give one chip to w (i.e. $c_w \rightsquigarrow 1$)

Step: If v is unhappy $\left(c_v < \frac{1}{2} \sum_{\substack{u \text{ adjacent} \\ \text{to } v}} c_v\right)$ then reset $c_v = \left(\sum_{u \sim v} c_u\right) - c_v$

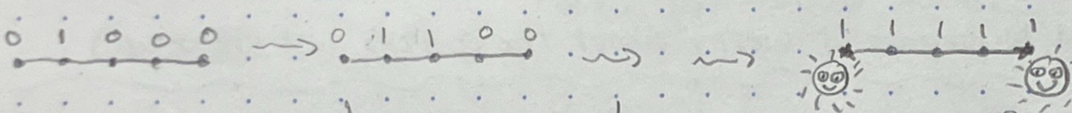
i.e. toggle around value



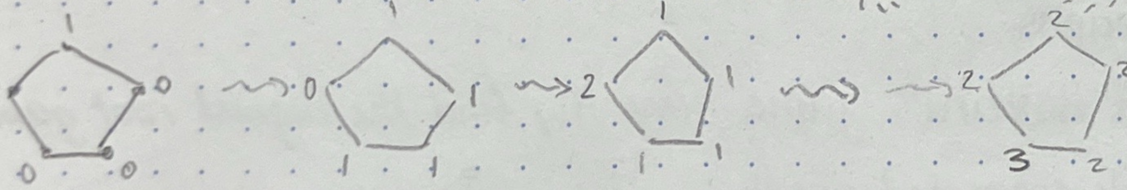
Ex. 1



Ex. 2



Ex. 3



game never stops...

Questions

1.) Does the result depend on how we play the game (which vertices we choose) or just the graph?

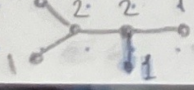
Thrm 1: Finiteness or infiniteness & end configuration if finite depend only on graph
(Proof in later lecture)

2.) How to tell from graph if game is finite or infinite?

Optional final step (if game is finite)

Add extra vertex V_0 with $c_{v_0} = 1$ chip and connect to all excited vertices to get the extended graph $\tilde{G}$.

Ex. 1



Ex.

Thrm: In extended graph, all vertices are happy but none are excited.

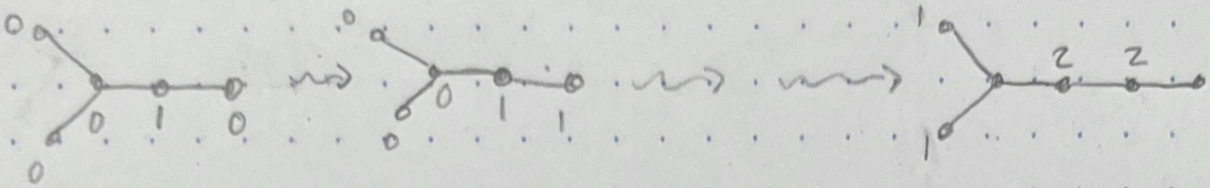
☆ ☆ ☆ ☆ ☆ ☆

GAME #2: Sponsor Game

Same setup (vertices, chip configuration initial step)
But now step of the game changes.

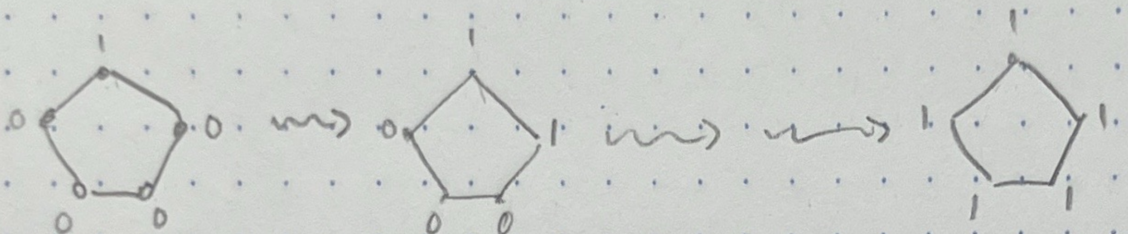
Step: If v is unhappy, replace $c_v \rightsquigarrow c_v' = c_v + 1$

Ex. 1



Ex. 2:

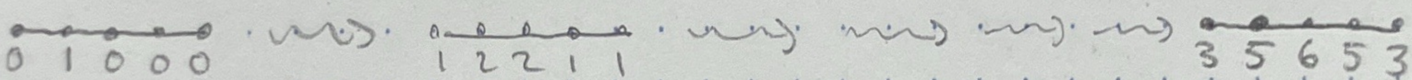
Ex. 3



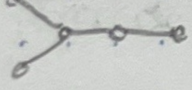
GAME #3: Excited sponsor game

Same setup, now give 1 to any chip that is not excited.

Ex. 1

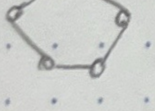


Ex. 2



(exercise: show it's also finite)

Ex. 3



infinite: no configuration where everything is excited

Thrm: Excited sponsor game is finite iff Kostant game is

Thrm: Sponsor game is finite iff
- graph has finite Kostant game OR
- graph is extended graph of finite Kostant game