

**Chalcogenide electronic materials research:
Computation, synthesis, characterization**

First principles calculations of electronic structure, defects, disorder

Stephan Lany, National Renewable Energy Laboratory, Golden, CO

Thin film growth

Rafael Jaramillo, Massachusetts Institute of Technology, Cambridge, MA

High-throughput computation and thermoelectrics

Prashun Gorai, Colorado School of Mines and NREL, Golden, CO

Thin film characterization

Rafael Jaramillo

Tutorial

First principles calculations: Electronic structure, defects, disorder

Stephan Lany

*National Renewable Energy Laboratory,
Golden, CO*

What do we want to predict by theory?

Properties of semiconductors

- **Structure (total-energy)**
 - Inter-atomic potentials (Lennard-Jones, Morse, VFF)
 - Electrostatic interactions
 - Quantum-mechanical (DFT)
- **Thermodynamic stability (total-energy)**
 - Quantum-mechanical (DFT)
- **Defects and doping (total-energy)**
 - Cavity model for vacancies
 - Model potentials (e.g., Morse)
 - Quantum-mechanical (DFT)
- **Band-structure (quasi-particle energies)**
 - Empirical (tight-binding, $k \cdot p$, EPM, NLEP)
 - Quantum-mechanical (Hartree-Fock \rightarrow CI, CC, MCSCF)
 - Quantum-mechanical (DFT \rightarrow GW)

Working horse of electronic structure theory: *Local Density Approximation (LDA)*

Schrödinger's equation:

$$\hat{H}\psi = E\psi$$

General Quantum mechanics:

$$[\hat{T} + \hat{V}]\psi(\mathbf{r}_1, \dots, \mathbf{r}_N) = E\psi(\mathbf{r}_1, \dots, \mathbf{r}_N)$$

Density Functional Theory:

$$[\hat{T}_0 + \sum_Z V_Z + \sum_e V_e + V_{\text{XC}}]\psi_i(\mathbf{r}) = \varepsilon_i \psi_i(\mathbf{r})$$

ΣV_e includes self-interaction

*V_{XC} corrects for SI,
many-particle effects*

$$E_{\text{DFT}} = T_0 + E_{ii} + E_{ei} + E_{ee} + E_{\text{XC}}$$

$$E_{\text{XC}} = \int n(\mathbf{r}) \varepsilon_{\text{XC}}[n, |\nabla n|] d\mathbf{r} \quad V_{\text{XC}}(\mathbf{r}) = \frac{\delta E_{\text{XC}}(n)}{\delta n(\mathbf{r})}$$

Hohenberg-Kohn 1964, applications ~70s, Perdew-Zunger LDA 1981

- High accuracy for electronic density and total-energies
- Usually not sufficiently accurate for single-particle energies

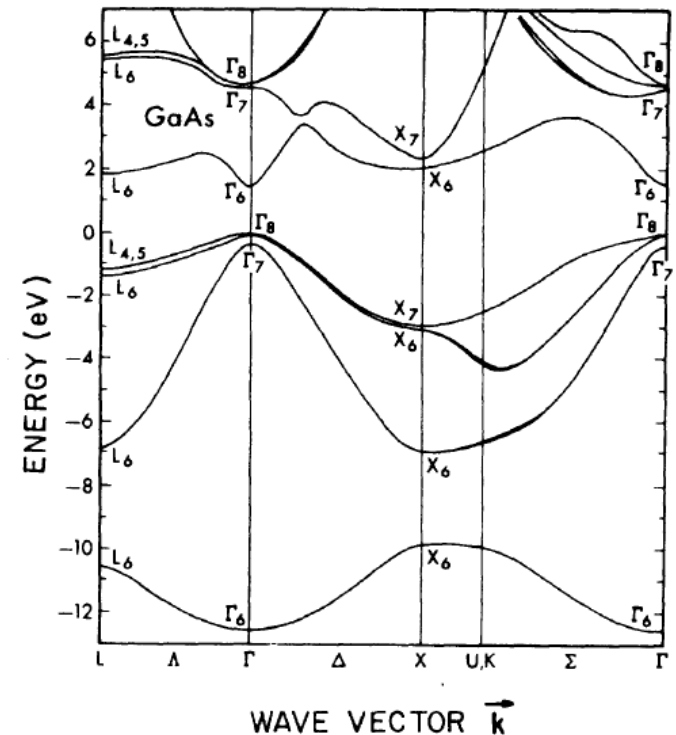
Electronic structure and optical properties

Empirical approaches

- Empirical approaches for band structure calculation
Tight-binding (LCAO), $k \cdot p$, **empirical pseudopotentials**

$$\left(-\frac{1}{2} \nabla^2 + \sum_{\alpha,n} v_{\alpha}(|\mathbf{r} - \mathbf{R}_{\alpha,n}|) \right) \psi_{i,\mathbf{k}}(\mathbf{r}) = e_{i,\mathbf{k}} \psi_{i,\mathbf{k}}(\mathbf{r})$$

- (1) Fit parameters to reference data
(experiment, accurate theory)
- (2) Assuming transferability,
apply to more complex cases
Full BZ, alloys, nanostructures, ...



Energy-band interpolation scheme based on a pseudopotential

J.C. Phillips, Phys. Rev. **112**, 685 (1958)

Nonlocal pseudopotential calculations for the electronic structure of eleven diamond and zinc-blende semiconductors

J.R. Chelikowsky, M.L. Cohen, Phys. Rev. B **14**, 556 (1976)

Properties for energy materials: Band structure calculation

Importance of electronic-structure

- Band-gaps, band-structure, optical properties (α , ϵ), transport: effective masses
- Energy materials: Photovoltaics, solar fuels, thermoelectrics, electronic materials, battery materials

Band-structures and band gaps with DFT starting point

- Starting from DFT, calculate electron self energy Σ , get quasiparticle energy correction to DFT

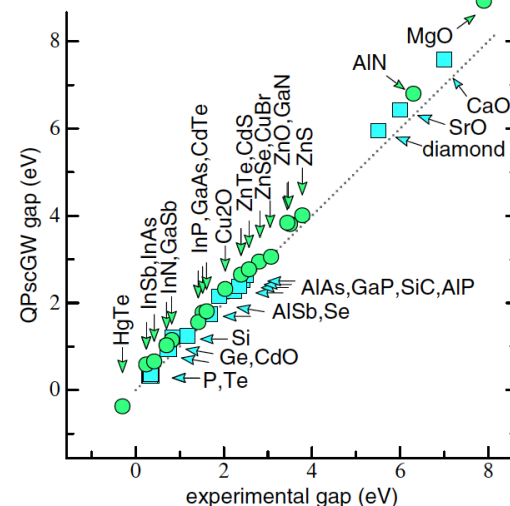
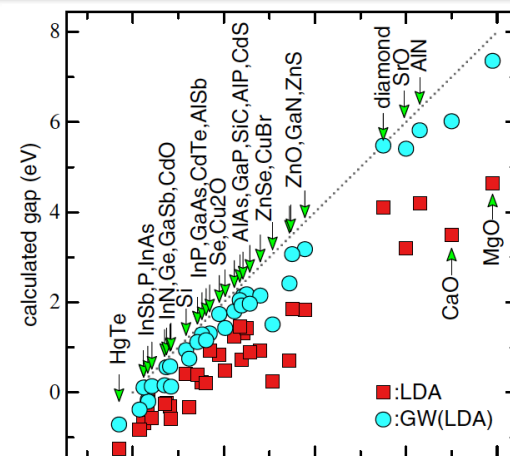
$$e_n^{GW} = e_n^{DFT} + \text{Re} \left\langle \psi_n^{DFT} \left| \Sigma(e_n^{GW}) - V_{xc}^{DFT} \right| \psi_n^{DFT} \right\rangle$$

GW approximation for self energy Σ

G: Green's function, W: screened Coulomb interaction

- GW uses the **screened** Coulomb potential (cf. "screened exchange")

$$\Sigma(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{4\pi} \int_{-\infty}^{\infty} e^{i\omega' \delta} G(\mathbf{r}, \mathbf{r}', \omega + \omega') W(\mathbf{r}, \mathbf{r}', \omega') d\omega'$$



For main group compounds, GW calculations give band gaps in very good agreement with experiment

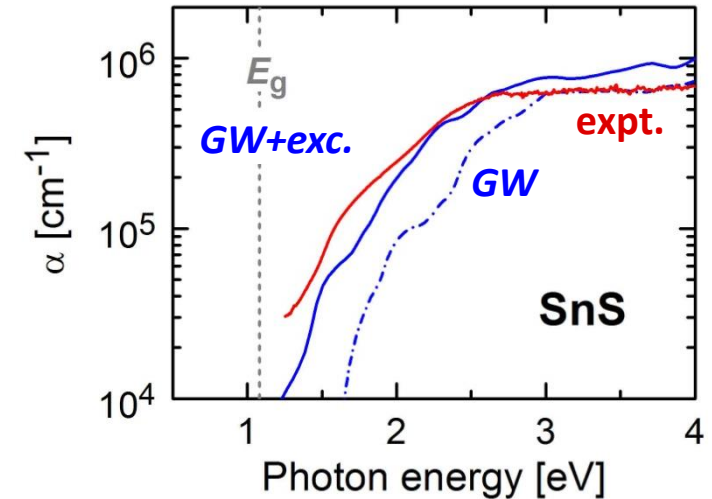
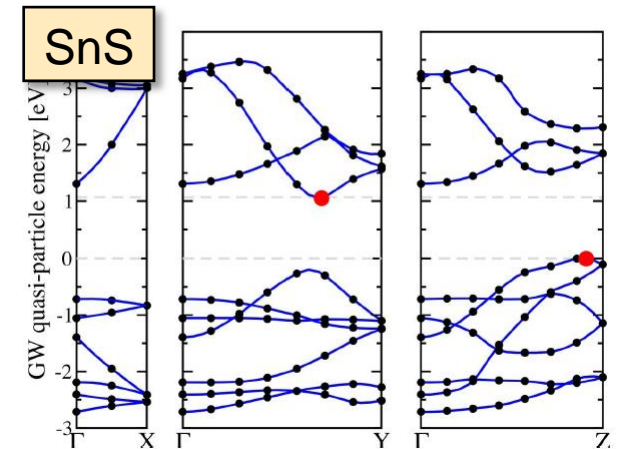
M. van Schilfgaarde *et al.*,
PRL **96**, 226402 (2006)

GW calculation of band-structure and optical properties

- Many-body perturbation theory in GW approximation implemented in numerous codes (VASP, BerkeleyGW, Questaal, Yambo)
- More demanding than DFT calculations:
 - CPU and memory requirements
 - Convergence, self-consistency, vertex-corrections
 - 3d transition metals
- Optical properties from dielectric function $\epsilon(\omega) + i\epsilon_2(\omega)$
 - independent particle approximation
 - Bethe-Salpeter equation (excitonic effects)

Implementation and performance of the frequency-dependent GW method within the PAW framework, M. Shishkin, G. Kresse, Phys. Rev. B **74**, 035101 (2006)

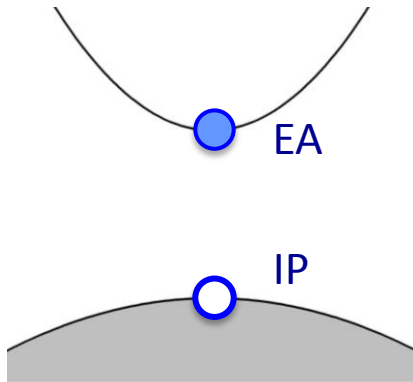
Dielectric properties and excitons for extended systems from hybrid functionals, J. Paier, M. Marsman, and G. Kresse, Phys. Rev. B **78**, 121201(R) (2008)



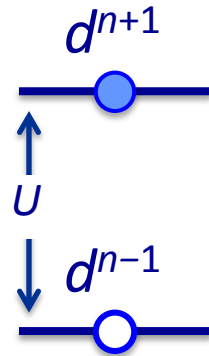
J. Vidal *et al.*, Appl. Phys. Lett. **100**, 032104 (2012)

Transition metal compounds: Band vs Mott insulator

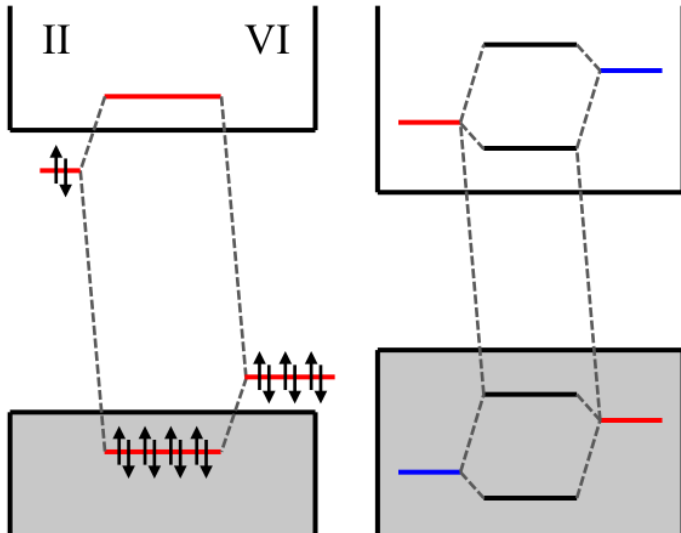
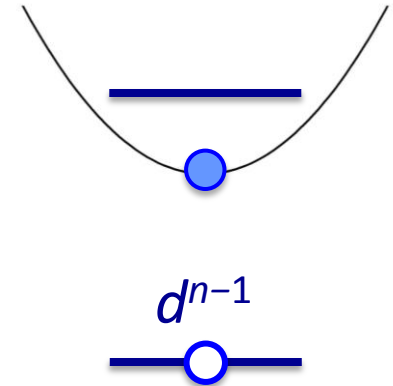
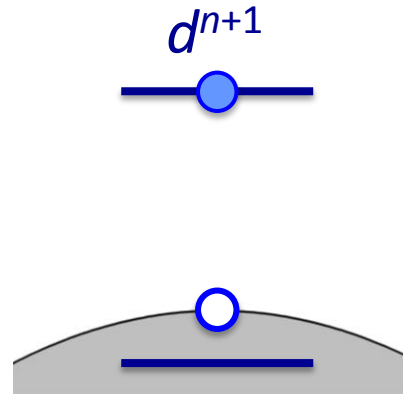
Band insulator



Mott insulator



Charge transfer insulator



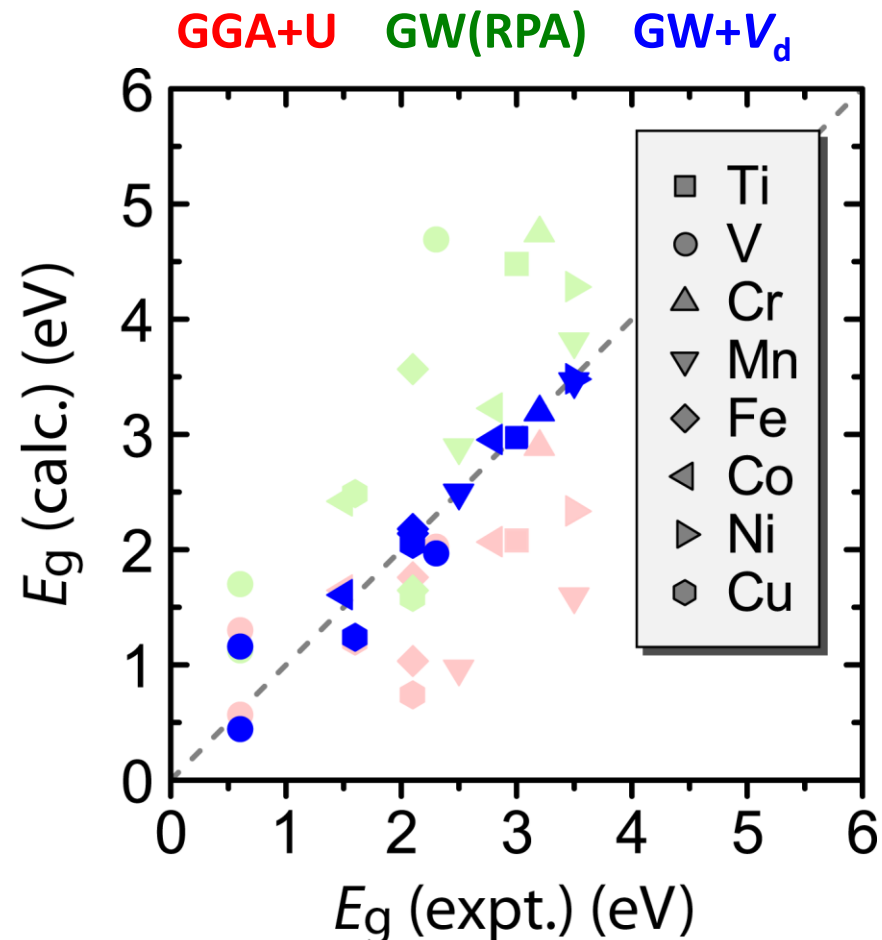
- Dispersion of d states
- Hybridization of d and sp
- d and sp states on same footing
- Band gap, effective masses
- Doping, defects, compensation
- Band vs small polaron transport

Band gap prediction: GW scheme for transition metal oxides

- GW method is computationally expensive and converges slowly
- Transition metal oxides are even more challenging due to localized d-electrons
- Standard GW(RPA) scheme that works well for main group compounds overestimates gaps for TM oxides (TiO_2 , ..., Cu_2O)
- Approach: GW with additional potential term for d -orbital energies ($\text{GW}+V_d$)

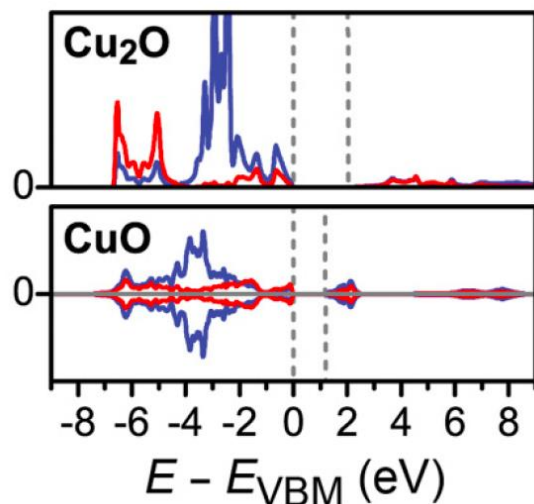
$$\hat{V} = \sum_{i,j} \left| p_i \right\rangle \left\langle \phi_i^{\text{AE}} \left[V_d \right] \phi_j^{\text{AE}} \right\rangle \left\langle p_j \right|$$

- One empirical parameter per TM atom, but applicable for (semi-) high-throughput band gap prediction for 100s of materials

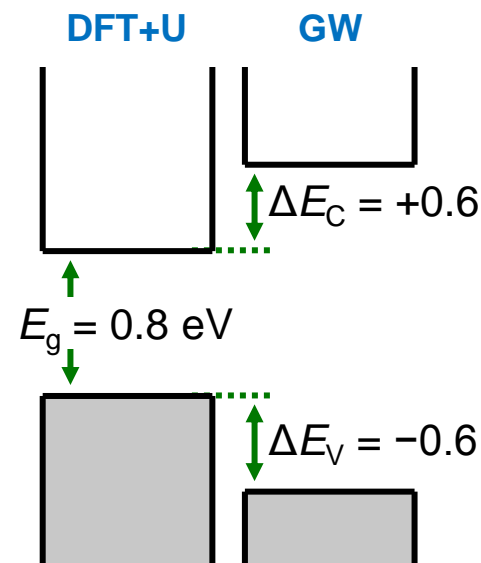
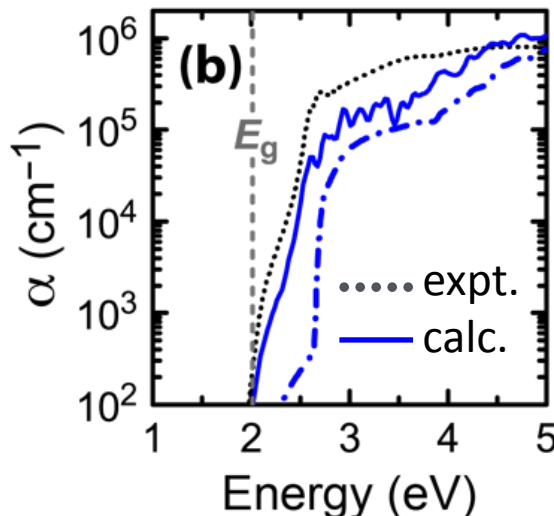
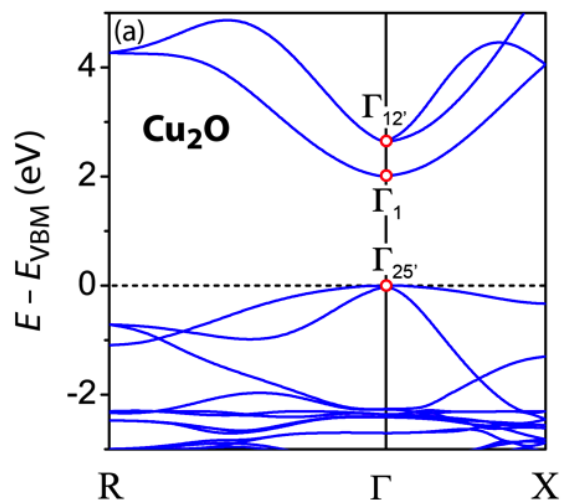


SL, Phys. Rev. B **87**, 085112 (2013)

Example: GW+ V_d for copper oxides

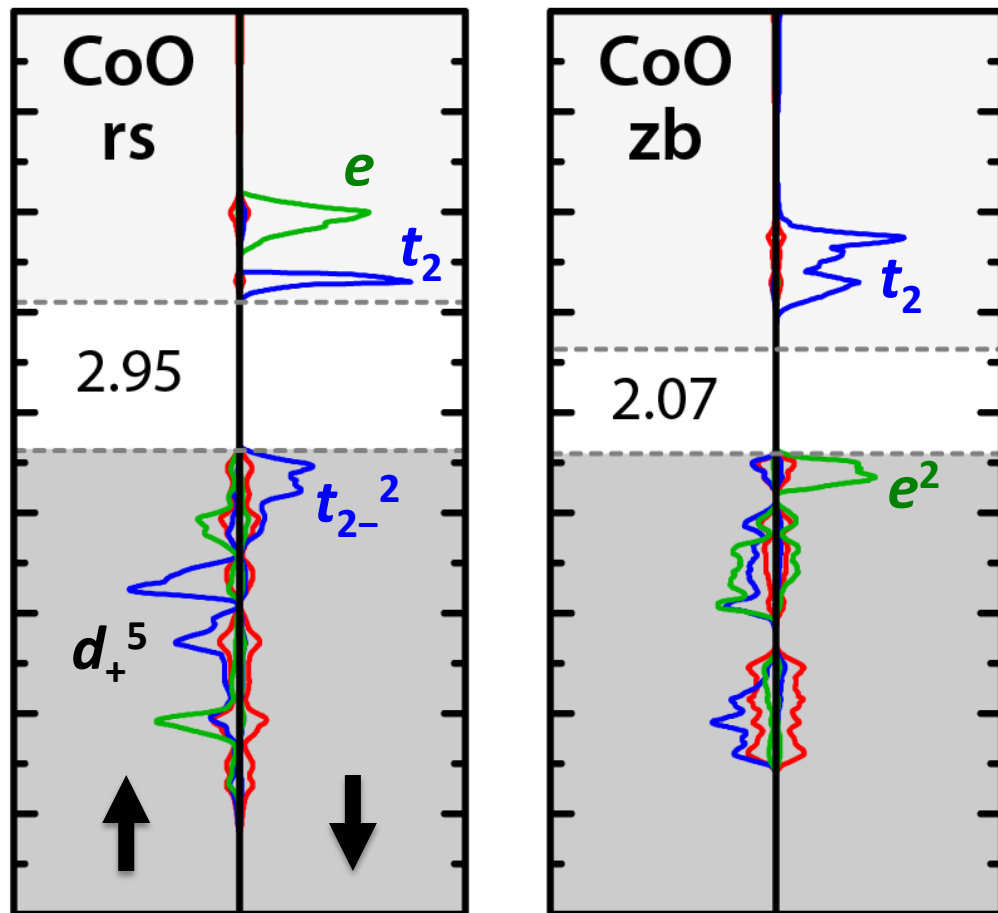


- Cu- d : $V_d = -2.4$ eV; good band gaps for both **Cu₂O** (direct-forbidden 2.04 eV) and **CuO** (indirect 1.24 eV) note: expt. $E_g = 1.6$ eV is direct
- Correct $\Gamma_1 - \Gamma_{12}$ band ordering in Cu₂O (wrong in GW-RPA)
- Absorption coefficient in near-quantitative agreement with experiment
- Band-edge shifts useful for defect calculations, band-offsets, IP/EA



Coulomb gap in CoO

- Mott insulators often defined by breakdown of band theory
- CoO has large gap between occupied and unoccupied t_2 spin-down crystal field levels (Coulomb gap)
- Quasi-closed shell configuration: crystal field splitting (t_2 , e) spin exchange splitting only occ. or empty spin/cf levels
- Tetrahedral Co^{+II}
QCS; Co_3O_4 , ZnO:Co
zinc-blende CoO lowest E in GGA+U
Co-s like CBM, $m_e^* = 0.5 m_0$
(DOS-effective mass)



Semiconducting Transition Metal Oxides (Topical Review)
SL, J. Phys. Cond. Mat. **27**, 283203 (2015)

NREL materials Database <http://materials.nrel.gov>



High Performance Computing Center
Materials Database

[HOME](#) [QUERY](#) [HELP](#)

NREL MatDB Query

[Disclaimer](#)

Required elements subset: ☐ At most these elements ☐ Exactly these elements ☒ At least all these elements ☐

Formula: exactly this formula

Required elements: Forbidden Elements:

Restrictions:

(Restriction info: **energy** (total energy, per atom, eV/Atom), **gap** (bandgap, eV), **gapd** (direct bandgap, eV), **netcharge** (net charge), **pressure** (final pressure, Kbar), **stable** (quill stability)
Example: gap > 0.2 and gap < 0.8 and netcharge = 0 and stable = true

Standards: (Examples: fere, gwvd, post-lopt) [Standards help](#)

Keywords: (Examples: bandgap, enthalpy, ggau, ggauwf, gwvd, hse, hsefere, hsewf, vexp)

View: ☐ General ☐ Thermochemistry ☒ GW_Bandgap (Changing the View and clicking Submit will reset the standards and columns.)

Show columns: ☒ id ☒ sorted formula ☐ ΔH_f (eV/atom) ☐ Stb ☐ Stb ☐ ΔH_{Gnd} ☐ ΔH_{Decomp}
☐ ICSD SG ☒ final SG ☐ E_{tot} (eV/Atom) ☐ minID ☒ ΔE_{cbm} ☒ ΔE_{vbm} ☐ ϵ_c (IP) ☒ ϵ_c (TDDFT)
☐ ϵ_c (post-lopt) ☒ E_g (eV) ☐ $E_{g,d}$ (eV) ☐ netCharge ☐ pressure ☐ mag mom ☒ numSpin ☐ icstdid
☐ lastName ☐ standards ☐ keywords ☒ parents

Sort order: sorted chemical formula, increasing

Hide dups: ☒ (Hide duplicate FERE entries for each formula and ICSD space group.)

Output format: ☒ HTML ☐ CSV [Submit Query](#)

Total matches: 238 Page length: 100 Page: [0](#) [1](#) [2](#)

id	sorted formula	final SG	ΔE_{cbm}	ΔE_{vbm}	ϵ_c (TDDFT)	E_g (eV)	$E_{g,d}$ (eV)	numSpin	parents	Family
8298	Al2 Cd O4	227	1.138	-1.420	2.960	5.716	5.716	1	[8982]	family 8027
9404	Al2 Co O4	227	1.602	-1.455	2.913	6.240	6.334	2	[7725]	family 8595
8686	Al2 Cu O4	141	-0.297	-1.422	3.308	2.291	2.810	2	[8699]	family 6992
7915	Al2 Mg O4	227	1.404	-2.004	2.530	8.896	8.905	1	[8837]	family 8983
8048	Al2 Mn O4	227	1.259	-1.719	2.831	6.157	6.157	2	[8622]	family 9022
8404	Al2 Ni O4	74	1.840	-1.205	3.047	5.676	5.676	2	[9310]	family 8217
9497	Al2 O3	167	1.619	-1.883	2.681	9.753	9.753	1	[7984]	family 8682
8146	Al2 O4 Zn	227	1.393	-1.696	2.767	7.554	7.554	1	[8767]	family 9110
8783	Al2 S3	167	0.248	-1.317	5.836	4.042	4.392	1	[8923]	family 7153
8038	Al2 Se3	9	0.504	-1.141	4.821	3.720	3.733	1	[9181]	family 9146
8192	Al As	216	-0.080	-0.674	7.764	2.029	3.237	1	[8876]	family 7982
9008	Al Cu O2	166	0.993	-1.079	3.898	4.385	5.077	1	[10357]	family 8973

DFT level (atomic structure and total energy)

V. Stevanovic

about 20,000 crystalline ordered materials

- Repository of atomic and magnetic structures
- Thermochemistry and stability

GW level (electronic structure)

about 300 semiconducting and insulating materials

- Limited up to 20-30 atoms/cell
- Mostly oxides, chalcogenides, nitrides
- Group 2-15 cations (plus few Li, Na, K)
- Including 3d transition metals and La (4d/5d planned)
- Low energy AFM magnetic configurations
- Polymorphism for II-VI

Supercell defect calculations

Defect equilibrium is determined by defect formation enthalpy

$$c_D = s \cdot \exp^{-\Delta H_D/kT}$$

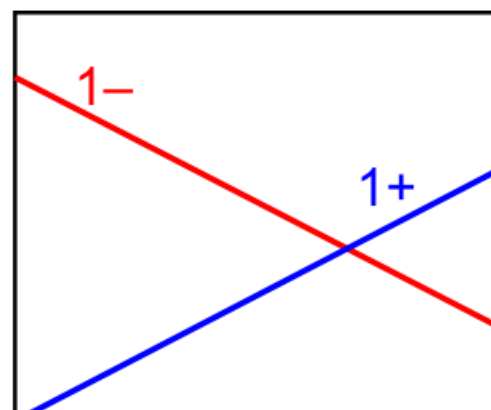
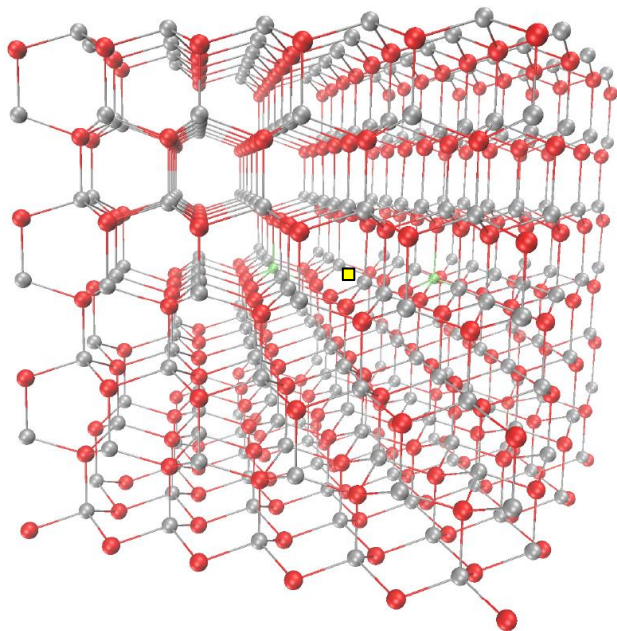
Compound stoichiometry
Doping and electrical properties
Defect levels, recombination centers

Defect formation energy

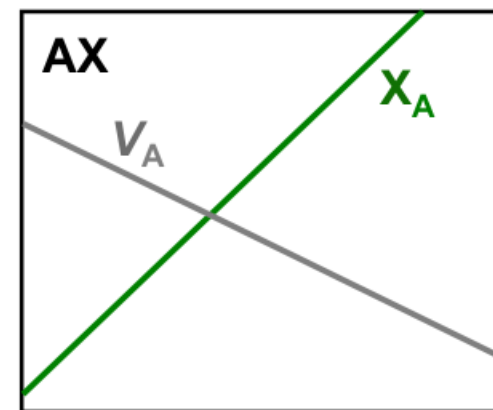
$$\Delta H_{\text{ref}} = [E_{\text{tot}}^{D,q} - E_{\text{tot}}^{\text{Host}}] + q(E_{\text{VBM}} + \Delta E_{\text{F}}) + \sum_a n_a (\mu_a^0 + \Delta\mu_a)$$

↑
↑
↑

supercell energies
electron reservoir
atomic reservoir

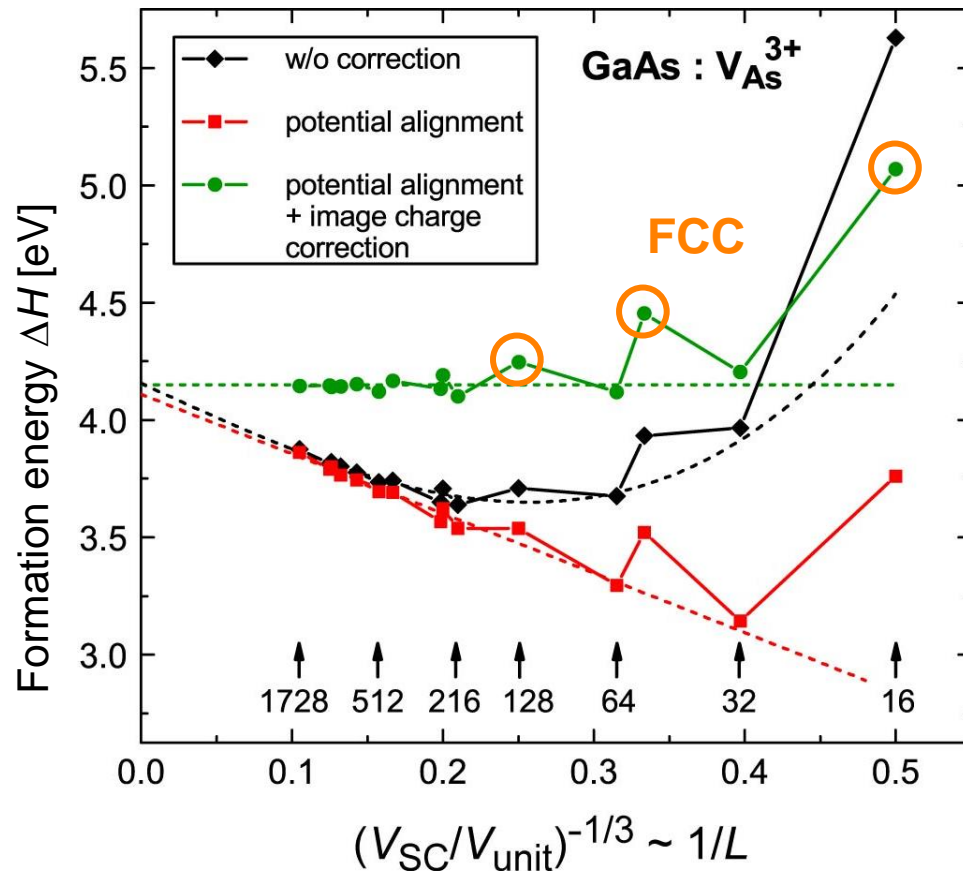


ΔE_{F}
 VBM CBM



$\Delta\mu_{\text{A}} = 0$ $\Delta\mu_{\text{X}}$ $\Delta\mu_{\text{X}} = 0$
 (A metal) (elem. X)

Supercell corrections vs. finite-size scaling



Finite-size scaling

$$\cdots \Delta H(L) = \Delta H(\infty) + \gamma_1/L + \gamma_3/L^3$$

$$\cdots \Delta H(L) = \Delta H(\infty) + \gamma_1/L$$

$$\cdots \Delta H(L) = \Delta H(\infty)$$

Conclusions

- image charge + pot. alignment correction give accurate ΔH
- Avoid FCC symmetries!
- Simple but accurate corrections available

Band edge shifts from GW quasi-particle calculations

- Semiconductor materials:

Cu_3N , Zn_3N_2 , AlN , Cu_2O , ZnO , Al_2O_3

- Projector augmented wave implementation of GW in VASP
[Shishkin & Kresse, PRB 74, 035101 (2006)]

- GW corrected defect formation energies for

GGA+U: $U_{\text{Cu-d}} = 5$, $U_{\text{Zn-d}} = 6$ eV

HSE06: standard parameter $\alpha = 0.25$

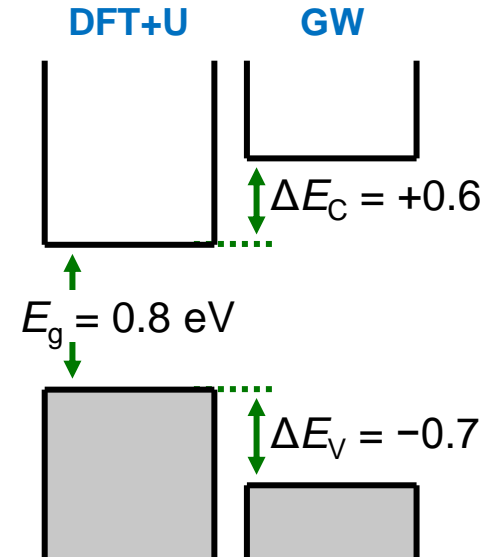
- Non-local Fock exchange in hybrid functional

$$V_{nl}(\mathbf{r}, \mathbf{r}') = -\alpha \sum_i \frac{\psi_i^*(\mathbf{r}')\psi_i(\mathbf{r})}{|\mathbf{r} - \mathbf{r}'|} \text{erfc}(\mu|\mathbf{r} - \mathbf{r}'|)$$

- GW self-energy

$$\Sigma(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{4\pi} \int_{-\infty}^{\infty} e^{i\omega' \delta} G(\mathbf{r}, \mathbf{r}', \omega + \omega') W(\mathbf{r}, \mathbf{r}', \omega') d\omega'$$

Example: Cu_2O



Reference energies for defect formation energies

Defect formation enthalpy for reference states ($E_F = E_{\text{VBM}}$; $\mu = \mu^{\text{elem}}$)

$$\Delta H_{\text{ref}} = [E_{\text{tot}}^{D,q} - E_{\text{tot}}^{\text{Host}}] + q \cdot E_{\text{VBM}}^{\text{ref}} + \sum_a n_a \mu_a^{\text{ref}}$$

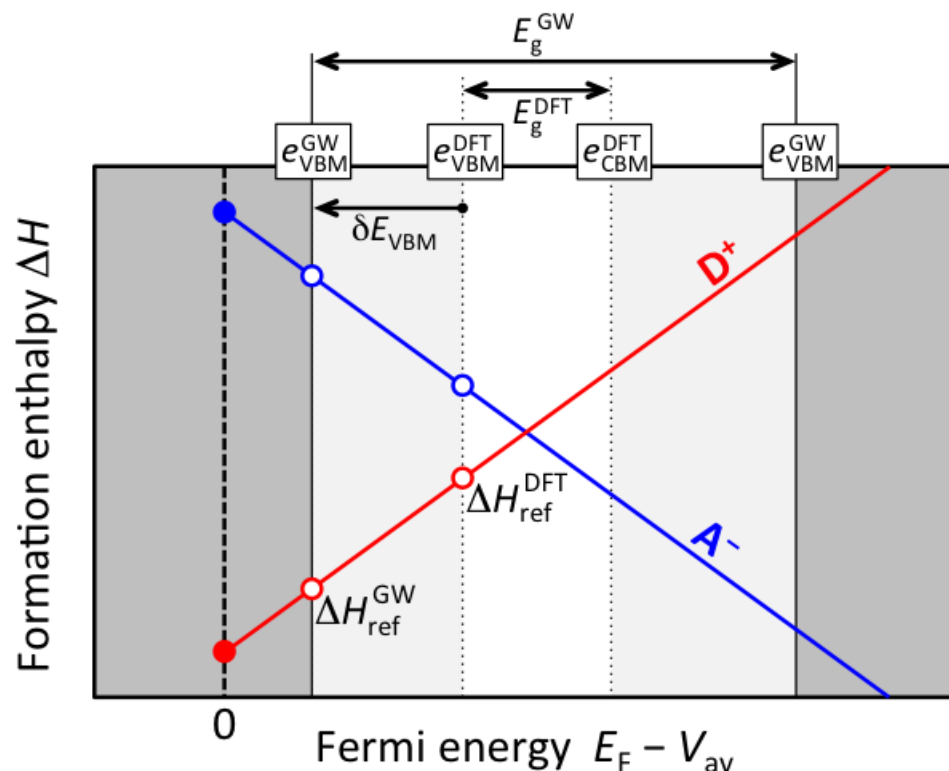
- “Band gap problem of DFT” affects E_{VBM}

- Proposed solution:**

Anchor ΔH_D at the average potential, use band edge shifts from GW

$$\delta E_{\text{VBM}} = e_{\text{VBM}}^{\text{GW}} - e_{\text{VBM}}^{\text{DFT}}$$

- Compare standard DFT (GGA+U) with hybrid functional (HSE06) calculations



Reference energies for defect formation energies

Defect formation enthalpy for reference states ($E_F = E_{\text{VBM}}$; $\mu = \mu^{\text{elem}}$)

$$\Delta H_{\text{ref}} = [E_{\text{tot}}^{D,q} - E_{\text{tot}}^{\text{Host}}] + q \cdot E_{\text{VBM}}^{\text{ref}} + \sum_a n_a \mu_a^{\text{ref}}$$

- Compound formation enthalpies not very accurate in DFT
- Origin: Incomplete error cancellation between compounds and elements
- **Proposed solution:**
Fitted elemental reference energies (FERE)

$$\delta\mu_a = \mu_a^{\text{FERE}} - \mu_a^{\text{DFT}}$$

- RMS error reduced in GGA from 0.24 to 0.07 eV/atom
- Consistency of DFT within cation-anion compounds
- Expected to improve thermodynamic boundary conditions for ΔH_{D} calculations

$$\Delta H_{\text{f}}^{\text{SC}} = E_{\text{tot}}^{\text{SC}} - \sum m_a \mu_a^{\text{ref}}$$

↑ Semiconductor compound ↑ Metals Molecules

$$\sum m_a \delta\mu_a = \Delta H_{\text{f}}^{\text{DFT}} - \Delta H_{\text{f}}^{\text{ex}}$$

least square fit for $\delta\mu_a$

Example:

$$\begin{aligned} \Delta H_{\text{f}}(\text{Al}_2\text{O}_3) &= E_{\text{tot}}(\text{Al}_2\text{O}_3) - 2\mu_{\text{Al}} - 3\mu_{\text{O}} \\ &= -14.9 \text{ eV (GGA)} \\ &= -17.4 \text{ eV (expt.)} \\ &= -17.3 \text{ eV (FERE)} \end{aligned}$$

Comparison of density and hybrid functionals

Band gaps

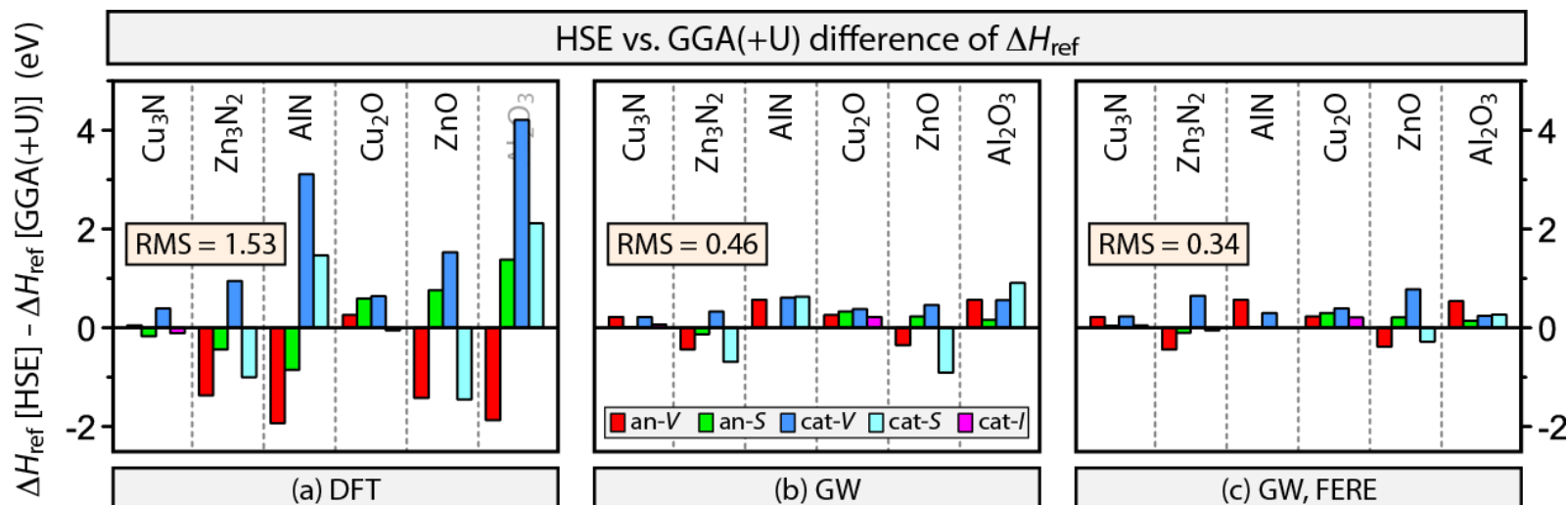
E_g	PBE+U eV	HSE eV	GW eV
Cu ₃ N	0.8	1.0	1.0
Zn ₃ N ₂	0.2	0.9	1.1
AlN	4.2	5.6	6.4
Cu ₂ O	0.8	2.0	2.2
ZnO	1.5	2.5	3.5
Al ₂ O ₃	6.3	8.1	9.9

VBM energy

ΔE_{VBM}	PBE+U eV	HSE eV
Cu ₃ N	-0.4	-0.3
Zn ₃ N ₂	-0.4	-0.1
AlN	-1.3	-0.5
Cu ₂ O	-0.7	-0.4
ZnO	-1.1	-0.6
Al ₂ O ₃	-1.9	-0.7

Elemental energy

FERE $\delta\mu$	PBE+U eV	HSE eV
Cu	-0.02	± 0.00
Zn	-0.08	+0.24
Al	+0.66	+0.35
N	-0.03	-0.04
O	+0.34	+0.32



Defect equilibria and “defect phase diagrams”

Thermodynamic modeling

Defect formation energy

$$\Delta H = \Delta H_{D,q}(\mu, E_F)$$

Defect concentration

$$c_D \approx N_{\text{site}} \times \exp(-\Delta H/kT)$$

Electron/hole density

$$c_e = \int f_{\text{FD}}(E - E_F) g(E) dE$$

Charge neutrality

$$-c_e + c_h + \sum [q \cdot c(D^q)] = 0$$

Self-consistent solution

$$\Delta H(E_F) \longrightarrow c_D(\Delta H) \longrightarrow E_F$$

$p\text{O}_2$ dependence of μ_{O}
(ideal gas)

$$\Delta\mu_{\text{O}}(T, P_0) = \frac{1}{2} [H_0 + \Delta H(T)] - \frac{1}{2} T \cdot [S_0 + \Delta S(T)]$$

$$\Delta\mu_{\text{O}}(T, P) = \Delta\mu_{\text{O}}(T, P_0) + \frac{1}{2} kT \ln(P/P_0)$$

Temperature dependence of band gap (CBM)

Dopant-defect association / dissociation within law of mass action

Example: *n*-type doping in ZnO

Formation energies of dopants, defects and defect pairs

192 atoms supercells, GGA+U, ΔE_V (GW), fitted elemental reference energies (FERE) [1]

$$\Delta H_{D,q}(\mu, E_F) = [E_{D,q} - E_{\text{host}}] + [\mu_{\text{host}} - \mu_D] + q \cdot E_F$$

Supercell total energies Atomic chemical potentials Electron chem. potential

Active players:

Isolated dopants/defects: $\text{Ga}_{\text{Zn}}^{1+}$, $\text{V}_{\text{Zn}}^{2-}$, $\text{V}_{\text{O}}^{0/2+}$

Pairs and complexes: $(\text{Ga}_{\text{Zn}} - \text{V}_{\text{Zn}})^{1-}$
 $(2\text{Ga}_{\text{Zn}} - \text{V}_{\text{Zn}})^0$

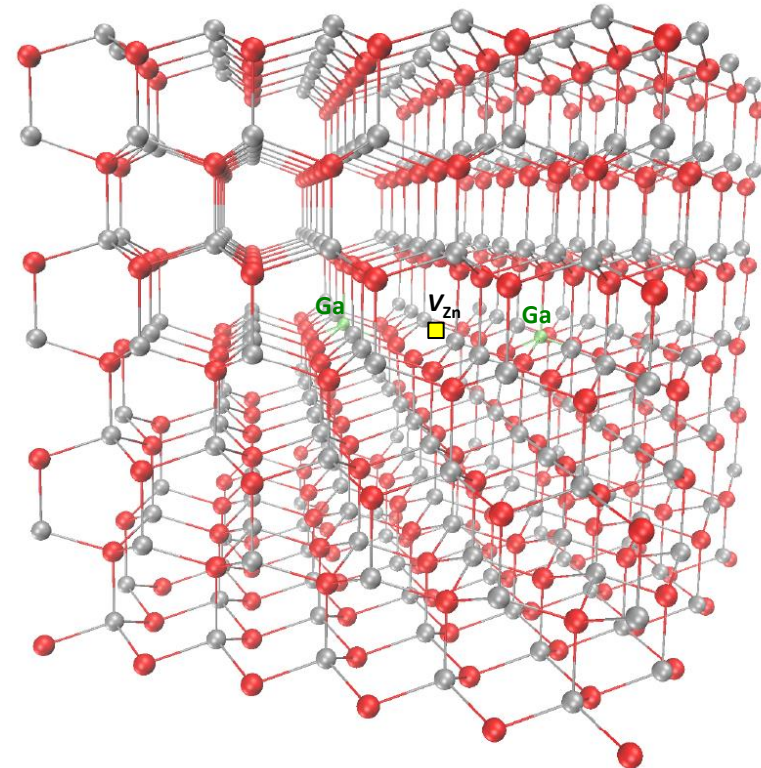
Law of mass action

$$[(\text{Ga}_{\text{Zn}} - \text{V}_{\text{Zn}})] = [\text{Ga}_{\text{Zn}}] \times [\text{V}_{\text{Zn}}] \times \exp(-E_{b1}/kT)$$

$$E_{b1} = -1.1 \text{ eV}$$

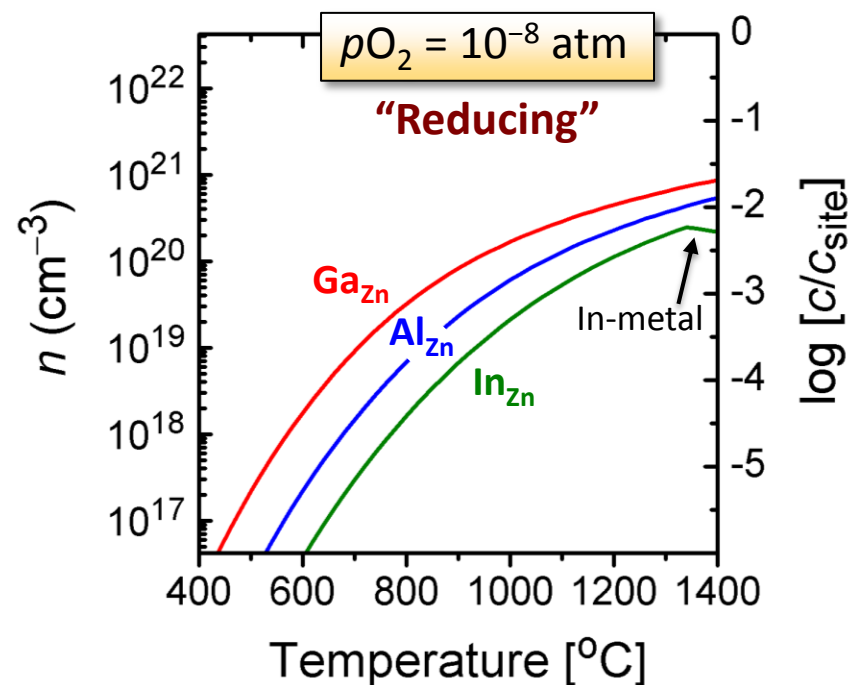
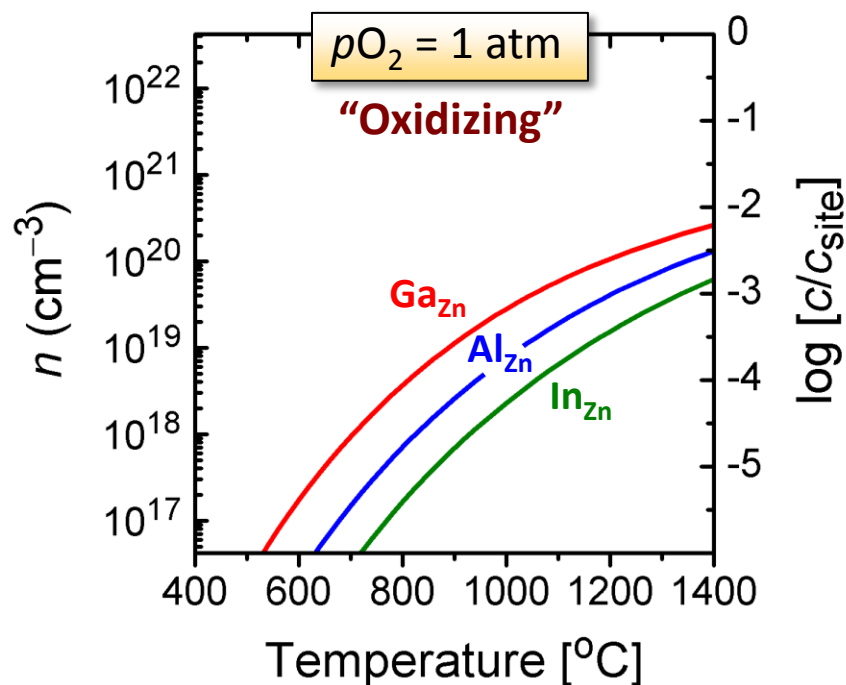
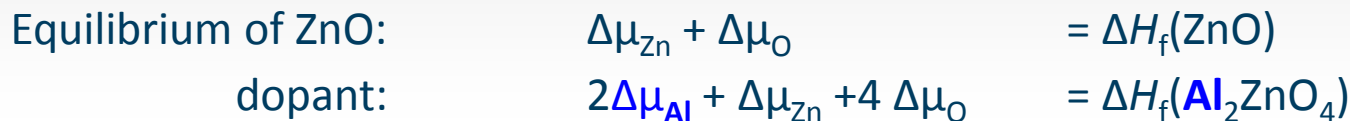
$$[(2\text{Ga}_{\text{Zn}} - \text{V}_{\text{Zn}})] = [\text{Ga}_{\text{Zn}}]^2 \times [\text{V}_{\text{Zn}}] \times \exp(-E_{b2}/kT)$$

$$E_{b2} = -1.8 \text{ eV}$$



Solubility limit of Al, Ga, In in ZnO

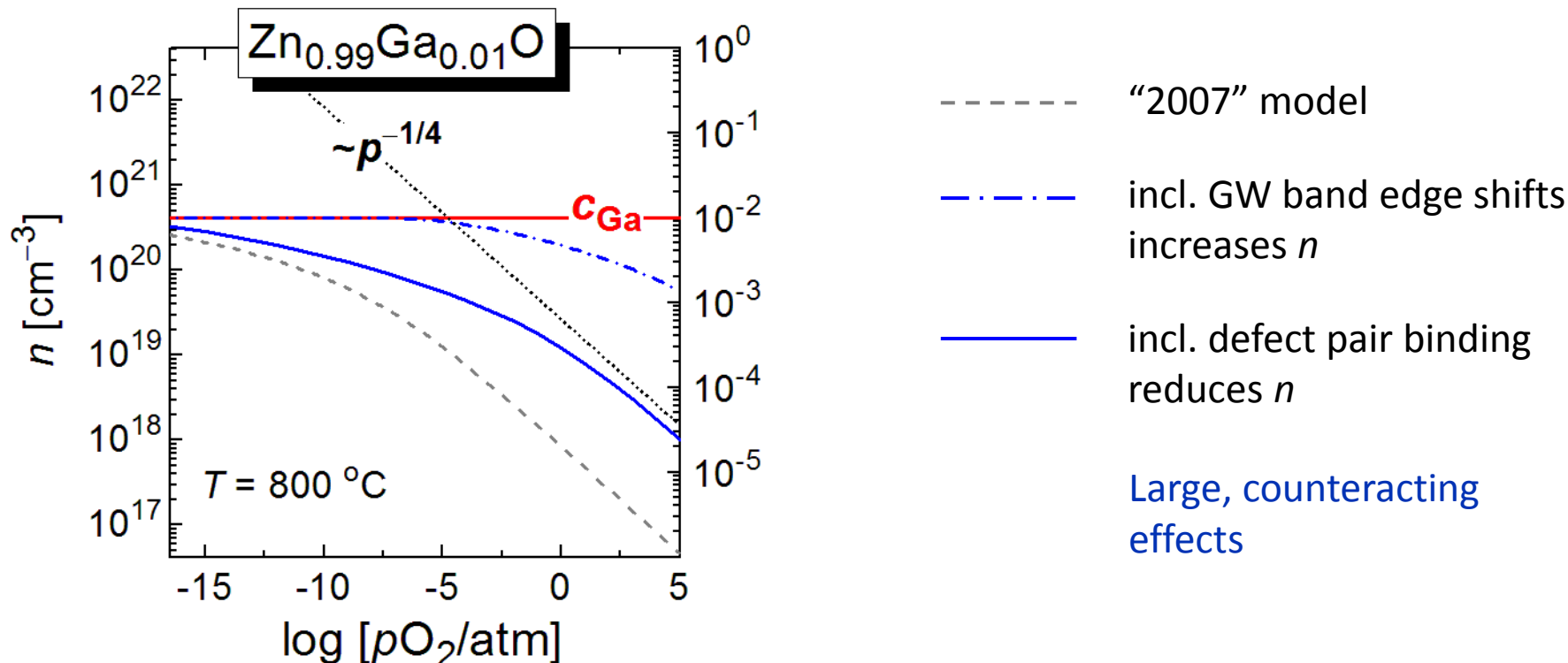
Limiting competing oxide-phases: Al_2ZnO_4 , Ga_2ZnO_4 , In_2O_3



Solubility in the per cent range only at high temperatures above 1000°C
At equilibrium conditions, moderate compensation (up to $\sim 10\%$)

Influence of band edge shift and defect pairs

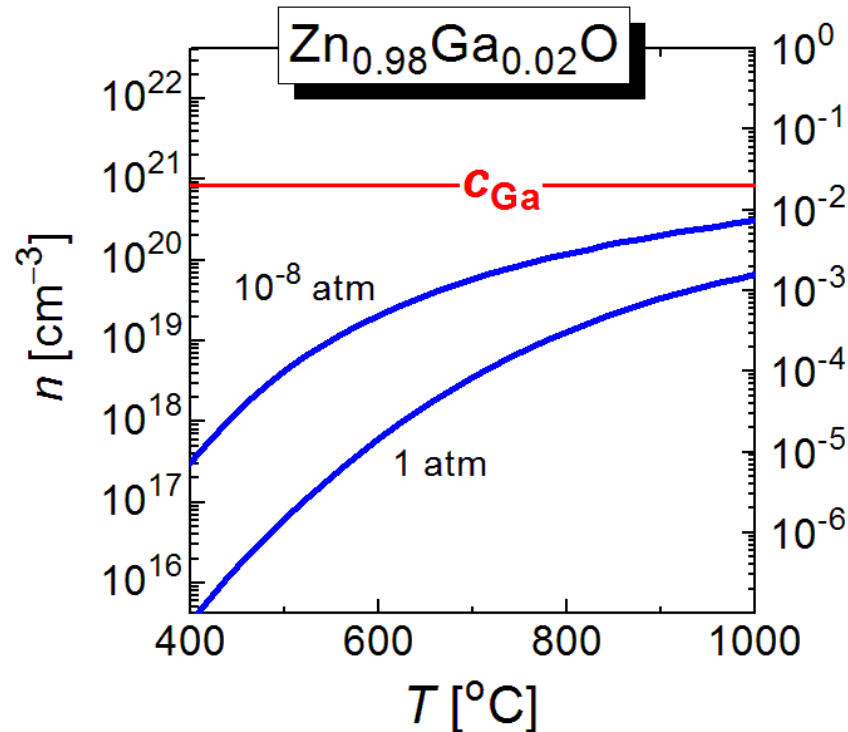
Constrained equilibrium: $c(\text{Ga}_{\text{Zn}})$ is constant, but compensation by V_{Zn} is equilibrated



Strong compensation at high (supersaturated) Ga dopant concentrations, most compensating defects (V_{Zn}) occur as fully passivated complexes ($2\text{Ga}_{\text{Zn}} - V_{\text{Zn}}$)

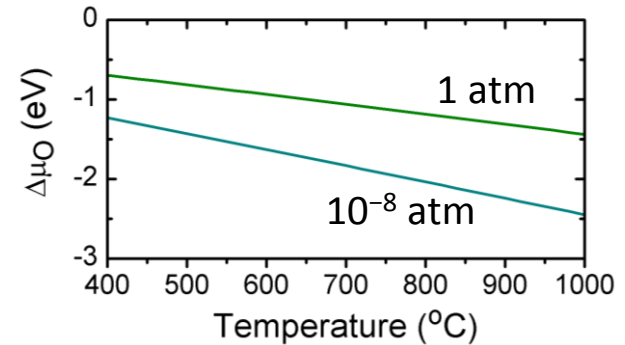
Temperature dependence of n

Constrained equilibrium
(2% cat. Ga)



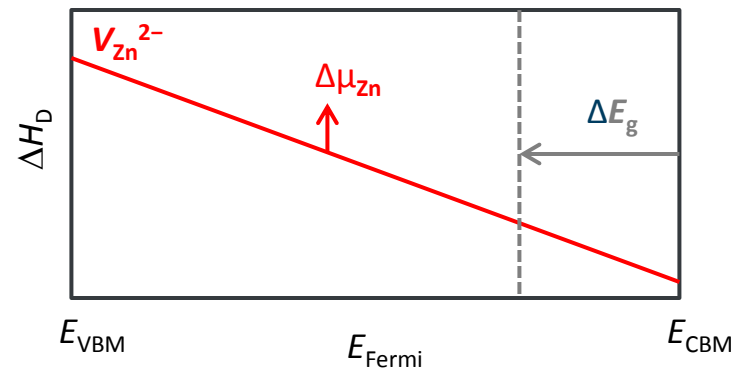
Why does n increase with T ?

- reduction of $\Delta\mu_{\text{O}}$ with T at given $p\text{O}_2$
respective increase of $\Delta\mu_{\text{Zn}}$ and $\Delta H_{\text{D}}(V_{\text{Zn}})$



- reduction of band gap (E_{CBM})
 $\Delta E_{\text{g}}/\Delta T = 0.6 \text{ meV/K}$

F.J. Manjon et al, Sol. Stat. Comm. **128**, 35 (2003)



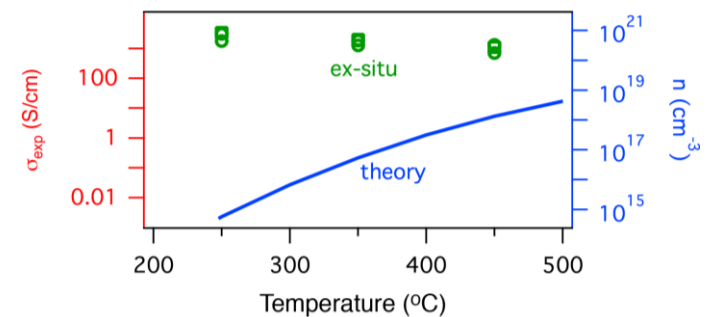
Comparison with as-grown doped ZnO

Highly doped ZnO:Al, ZnO:Ga only as thin-films

Typical literature growth methods and results

- PLD, Sputtering, MOCVD
- $T_{\text{sub}} \approx 200\text{-}300\text{ }^{\circ}\text{C}$
- $p\text{O}_2 \leq 10^{-6}\text{ atm}$
- $c(\text{Al}, \text{Ga})$ up to 5% cat.
- $n \approx 10^{21}\text{ cm}^{-3}$

ZnO:Ga, PLD ($p\text{O}_2 = 10^{-8}\text{ atm}$)

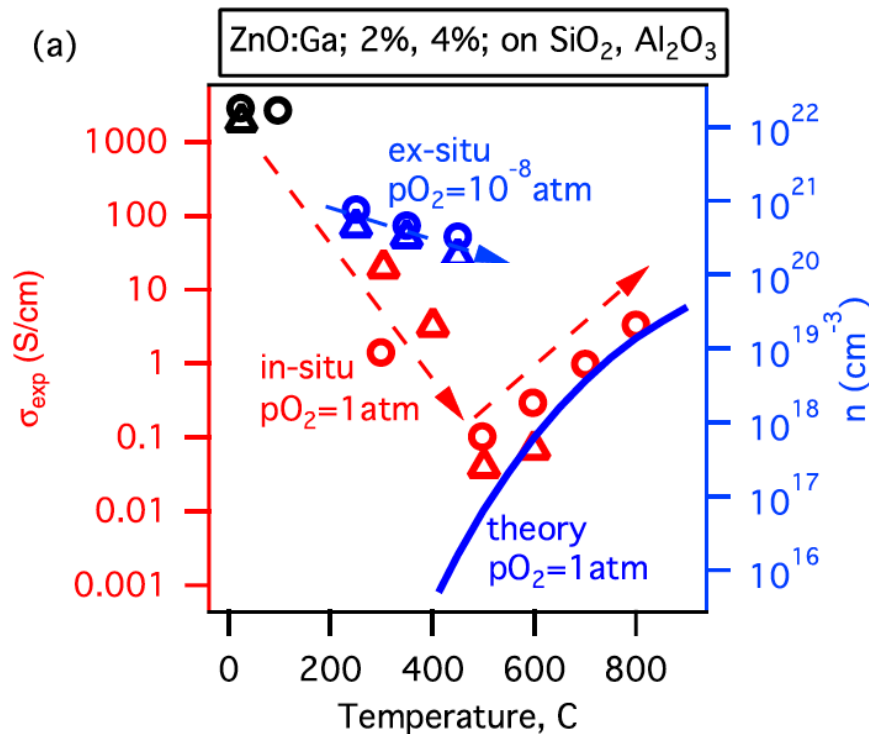


As-grown films have much higher **dopant** concentration than predicted **equilibrium solubility** limit
(not so surprising)

As-grown films have much higher **carrier** concentration than predicted for **constrained equilibrium** at moderate T
(surprising?)

Temperature dependence of n

In-situ measurement of conductivity (NU): Direct monitoring of the equilibration process



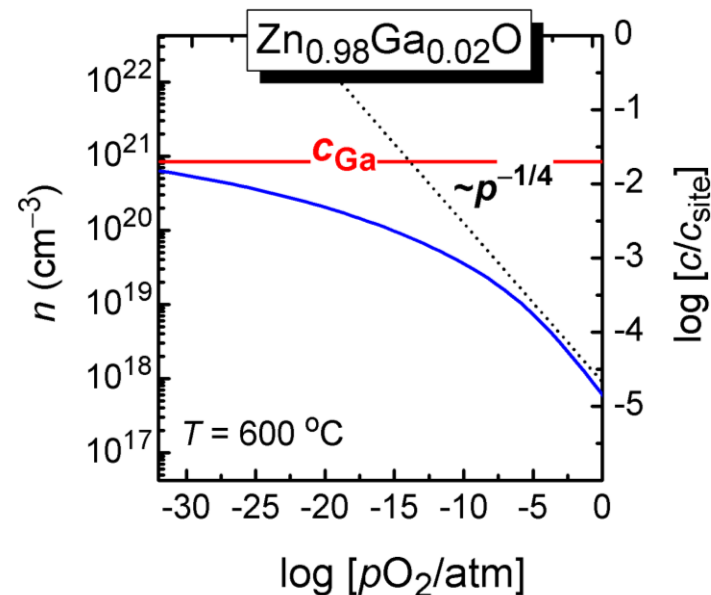
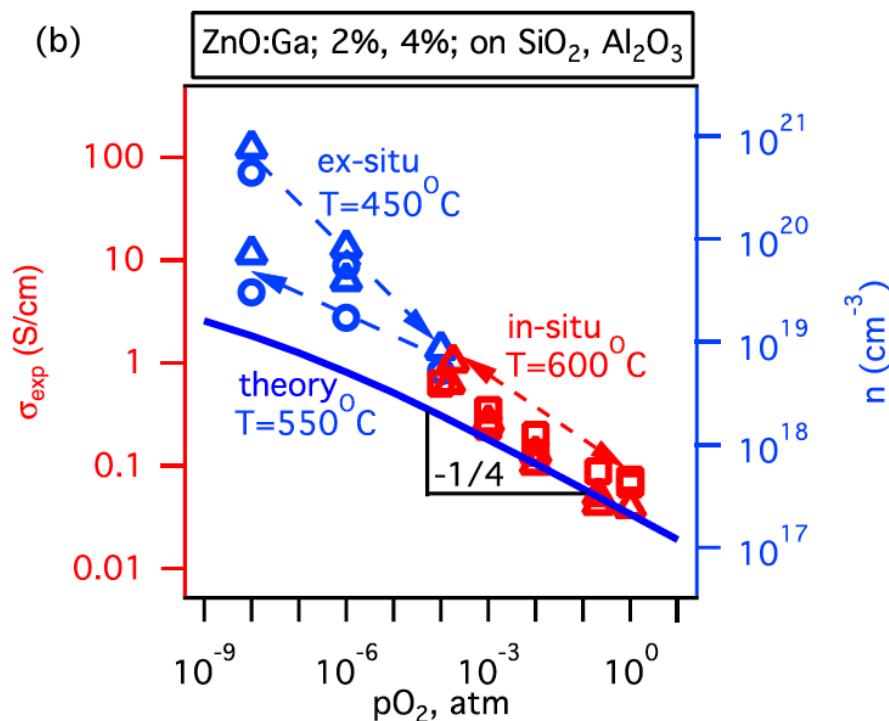
Mobility

initially $\mu = 20$ cm²/Vs
at room temperature

in-situ lower $\mu = 1$ cm²/Vs
(plot scaling)

- Conductivity decreases 3 orders of magnitude already at 300°C (1 atm $p\text{O}_2$)
- Minimum of σ around 500°C
- Conductivity increase at higher T in agreement with prediction

pO_2 dependence of n



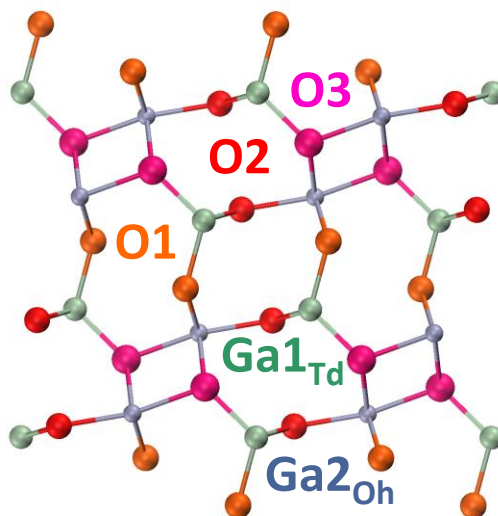
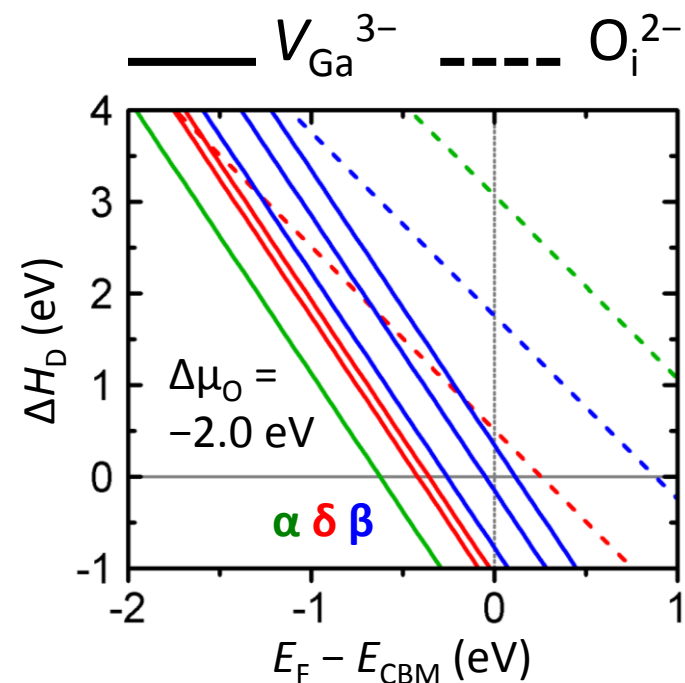
- Fast equilibration at high pO_2 (1 atm) around 500°C
- Slow equilibration at low pO_2 (10^{-8} atm)
- Observation of expected $p^{-1/4}$ dependence at high pO_2

A. Zakutayev, N.H. Perry, T.O. Mason, D.S. Ginley, S. Lany, *Non-equilibrium origin of high electrical conductivity in gallium zinc oxide thin films*, Appl. Phys. Lett. **103**, 232106 (2013).

Compensating defects in Ga₂O₃ polymorphs

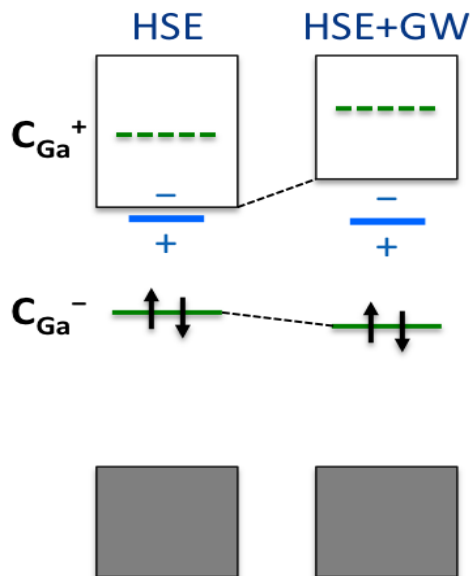
Ga ₂ O ₃ polymorphs	β (GS)	α (Al ₂ O ₃)	δ (In ₂ O ₃)
$E - E_{GS}$ (meV/at.) - DFT	0	+28	+30
E_g (eV) - GW	4.96	5.28	4.89

- Similar band gaps in all polymorphs
- Ga vacancies are the dominant defect (unlike O_i in In₂O₃)
- PAS experiments
[Korhonen et al. APL **106**, 242103 (20150)]
- Highest ΔH_D in ground state β-Ga₂O₃

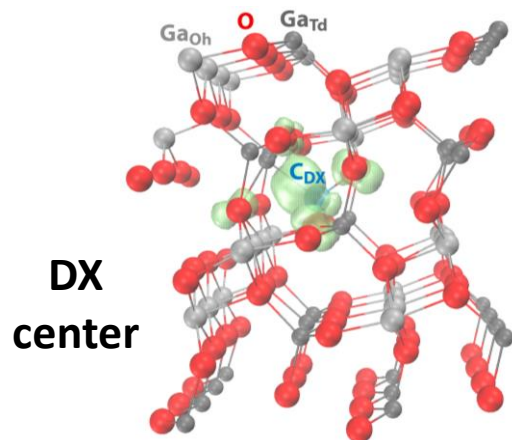


5 non-equivalent sites in β-Ga₂O₃

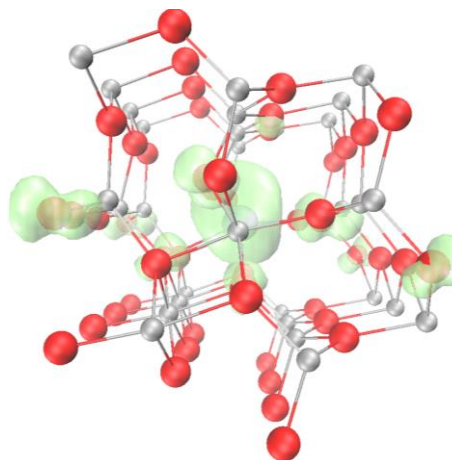
Which dopant releases free carriers?



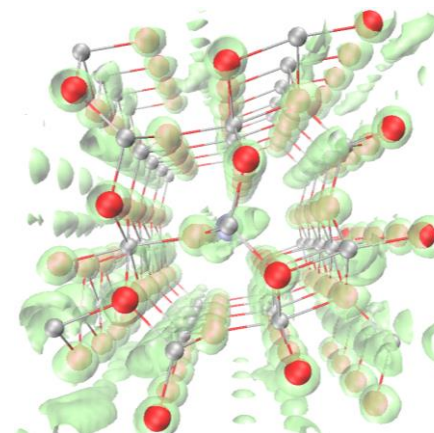
	E_{Oh-Td} (eV)	$\epsilon(+/-)$ HSE	$\epsilon(+/-)$ GW
C_{Td}	+1.15	-0.04	-0.81
Si_{Td}	+0.73	+1.19	+0.80
Ge_{Td}	+0.11	+0.56	-0.14
Sn_{Td}		+0.27	-0.41
Sn_{Oh}	-1.10	+0.58	-0.19



C_{Ga1} : deep level ☹️

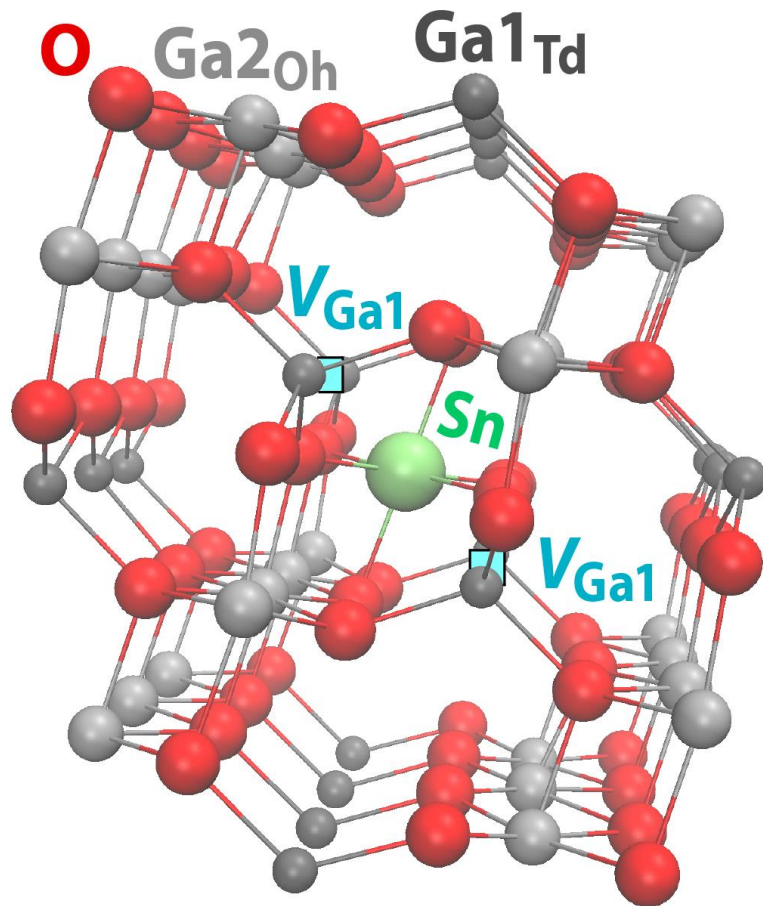


Sn_{Ga2} : so-so



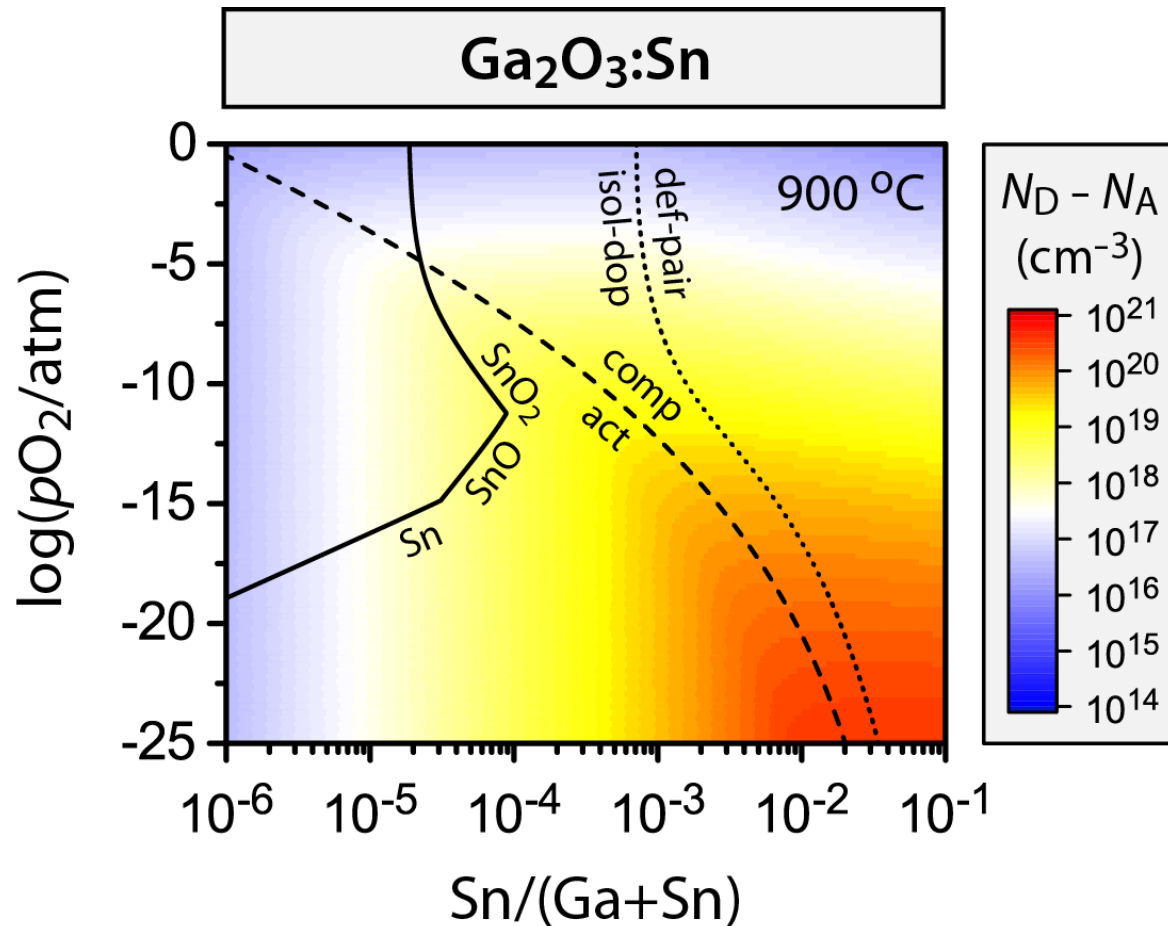
Si_{Ga1} : shallow level 😊

Dopant – defect interactions



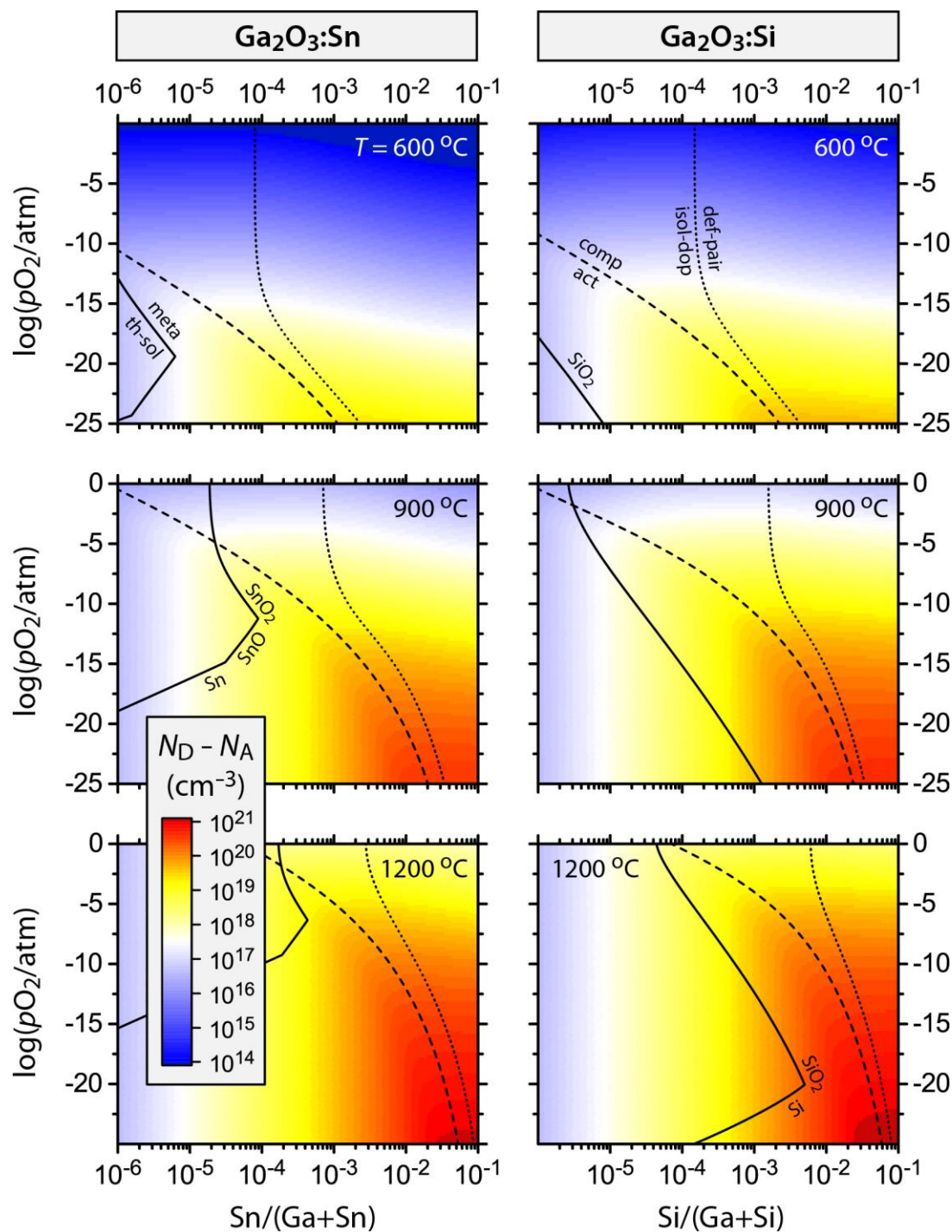
- **Supercells**
Standard DFT (GGA)
160 atom cells for defect pairs
- **Site-preference**
 $\text{Sn}_{\text{Ga}} @ \text{Ga2}_{\text{Oh}}$
- **“Split-vacancy”**
 $V_{\text{Ga}} \rightarrow (\text{Ga}_i + 2V_{\text{Ga1}})$
- **Defect pairs**
 $\text{Sn}_{\text{Ga1}} + V_{\text{Ga}} \rightarrow (\text{Sn}_i + 2V_{\text{Ga1}})$
 $\Delta E_b = -1.2 \text{ eV wrt } \text{Sn}_{\text{Ga2}}$
 $(\text{Sn}_i + 2V_{\text{Ga1}}) + \text{Sn}_{\text{Ga2}}$
 $\Delta E_b = -0.6 \text{ eV}$
 $(\text{Sn}_i + 2V_{\text{Ga1}}) + 2\text{Sn}_{\text{Ga2}}$
 $\Delta E_b = -0.2 \text{ eV}$

Defect equilibria



- Net doping ($N_D - N_A$) vs. $p\text{O}_2$ and $c(\text{Sn})$
- Solubility limits
- Activated vs. compensated doping
- Isolated defects vs. defect-pairs

Sn vs Si and temperature trends



Dopant choice

- Si slightly less compensated than Sn

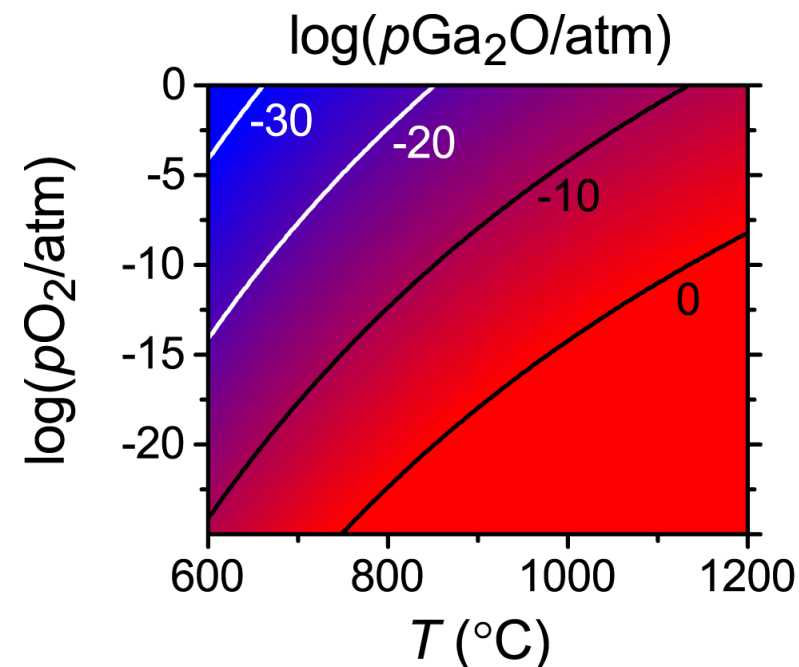
Increasing T causes

- Higher solubility limits
- Lower compensation ratio
- Higher limit of dopant conc.
- Higher $N_D - N_A$

Optimizing experimental conditions

DPD: High T , low $p\text{O}_2$, optimal dopant conc.

Thermodynamics: High $p\text{Ga}_2\text{O}$ causes material loss

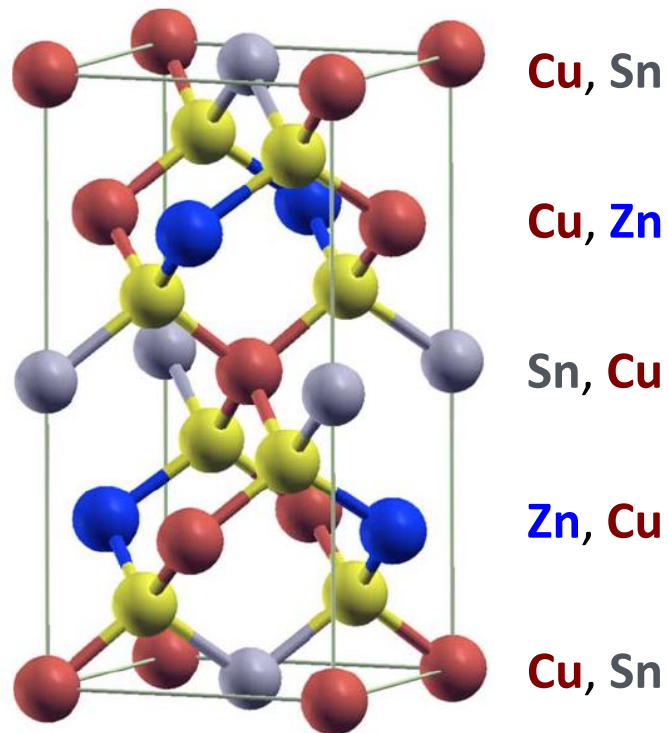


T ($^{\circ}\text{C}$)	$\log(p\text{O}_2)$ (atm)	$N_{\text{D}} - N_{\text{A}}$ (10^{20} cm^{-3})	Si/Si+Ga (10^{20} cm^{-3})	in Si cat%
600	-29.2	1.4	11.0	2.9%
700	-22.6	1.0	6.9	1.8%
800	-17.4	0.8	4.5	1.2%
900	-13.0	0.6	3.0	0.8%
1000	-9.2	0.4	2.2	0.6%

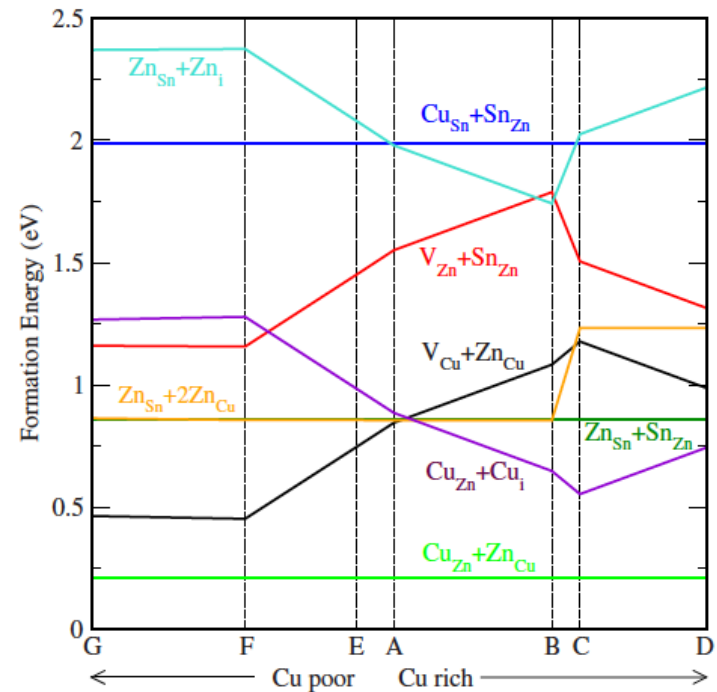
Maximal doping: Lower T , optimize $p\text{O}_2$ and Si conc.

Disordered multinary compounds

Multinary compounds: $\text{Cu}_2\text{ZnSnS}_4$ (CZTS) and Cu_2SnS_3 (CTS)



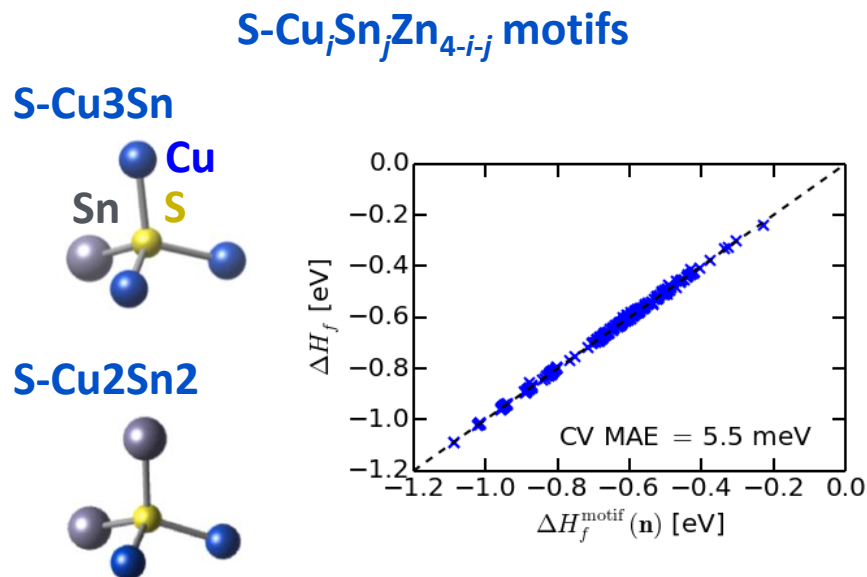
A. Nagoya, PRB **79**, 115126 (2009)



S. Chen et al., PRB **81**, 245204 (2010)

- Experiment: Order – disorder phenomena
- Theory: $\text{Cu}_{\text{Zn}} - \text{Zn}_{\text{Cu}}$ defect pair
- Description beyond the defect model?

Model Hamiltonian: CZTS and Cu₂SnS₃ (CTS)



$$D_f H = 2 \sum_{(i,j)} e_f(i,j) n(i,j)$$

$j \backslash i$	0	1	2	3	4
0	-1.083	-0.914	-0.679	-0.404	-0.238
1	-0.880	-0.790	-0.663	-0.492	
2	-0.565	-0.563	-0.463		
3	-0.379	-0.254			
4	-0.026				

S-CuSnZn₂
S-Cu₂SnZn
S-Cu₃Sn
S-Cu₂Sn₂

- Motif based Hamiltonian emphasizes local order and octet rule
- Good fit even though long range order effects are excluded
- Simple Hamiltonian allows efficient Monte-Carlo simulations

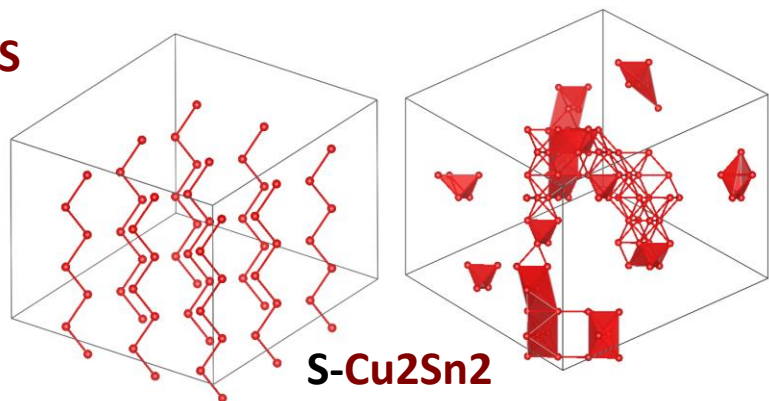
Entropy-driven clustering in CTS

Monte Carlo simulations

ordered

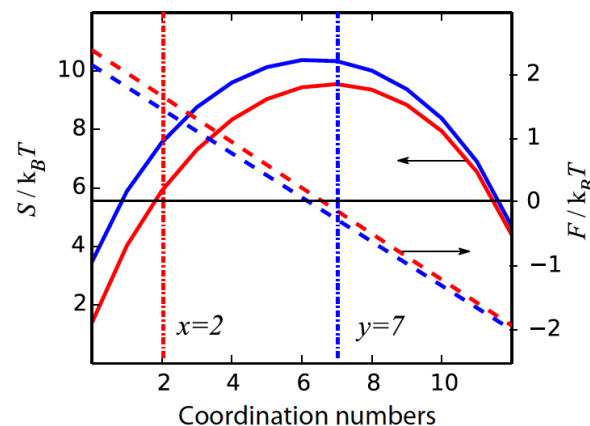
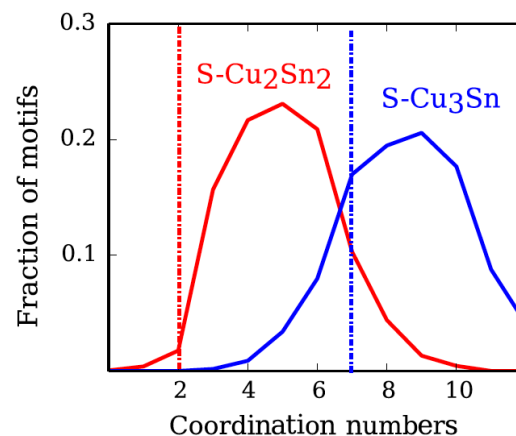
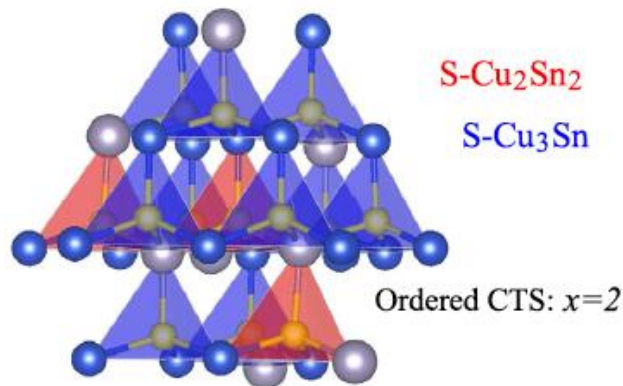
disordered

CTS



S-Cu₂Sn₂

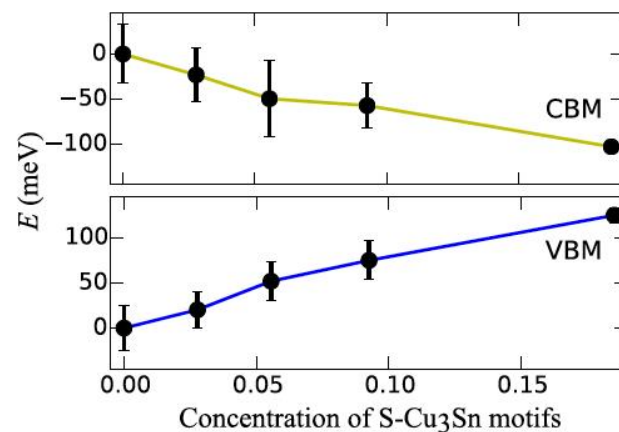
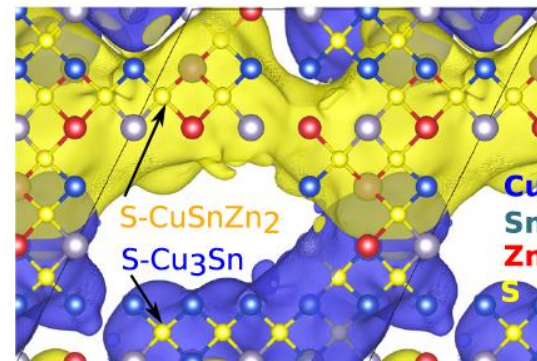
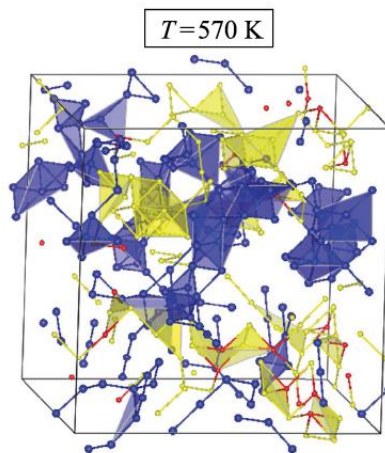
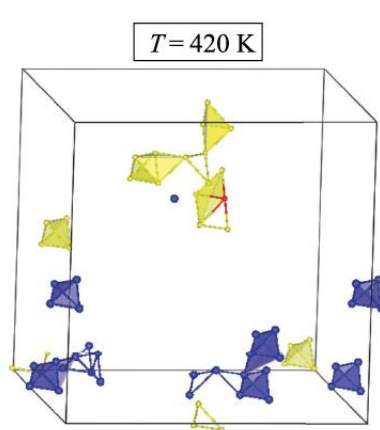
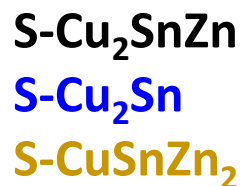
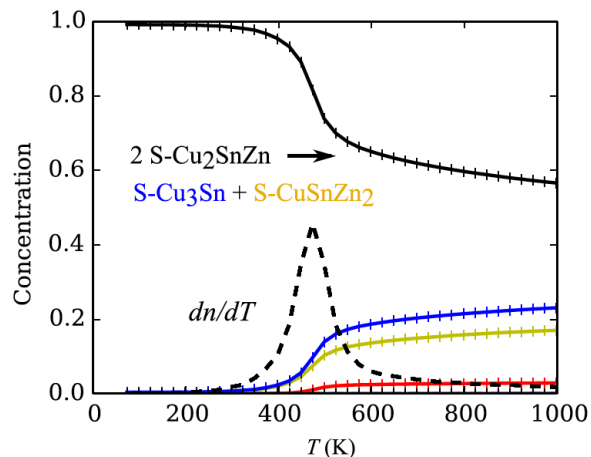
S-Cu₃Sn₁



- Disorder induced clustering
- Weakly temperature dependent

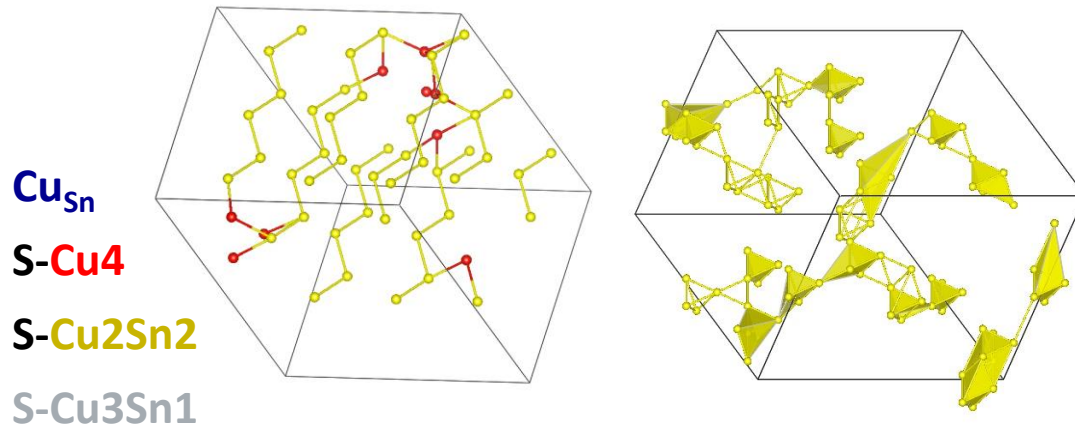
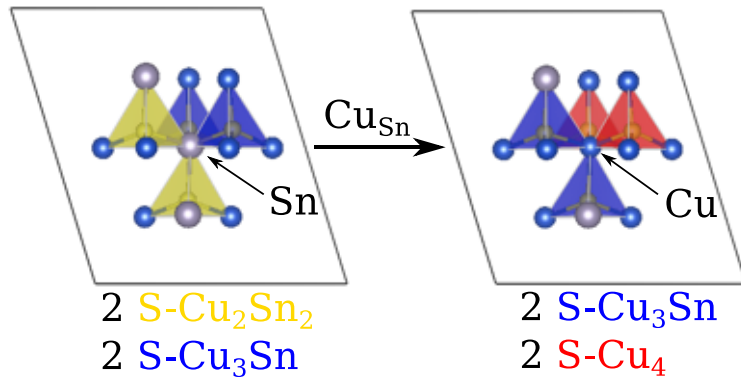
Order-disorder transition in CZTS

Monte Carlo simulations

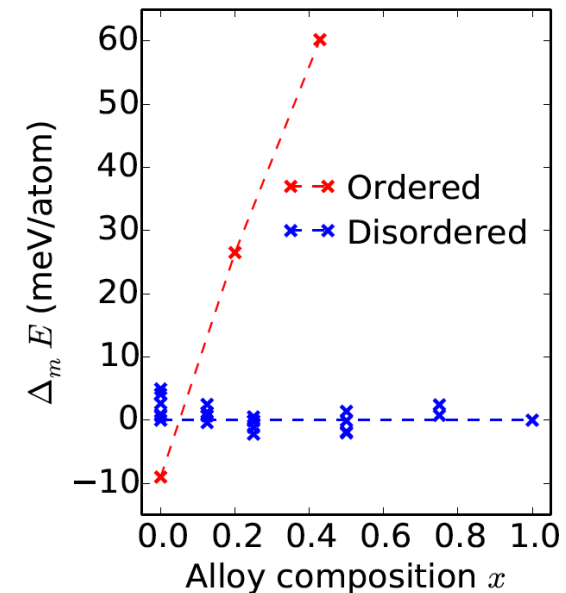
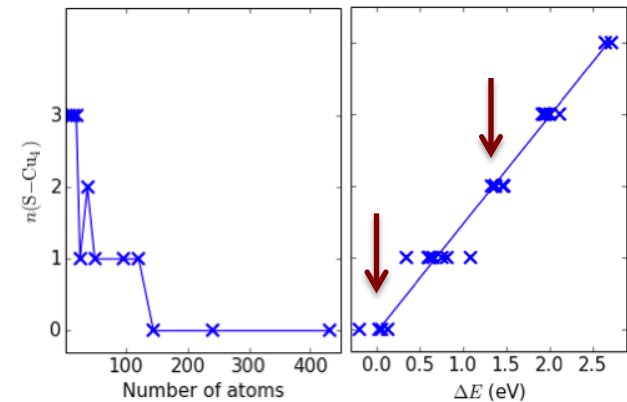


- Entropy driven clustering amplifies potential fluctuations

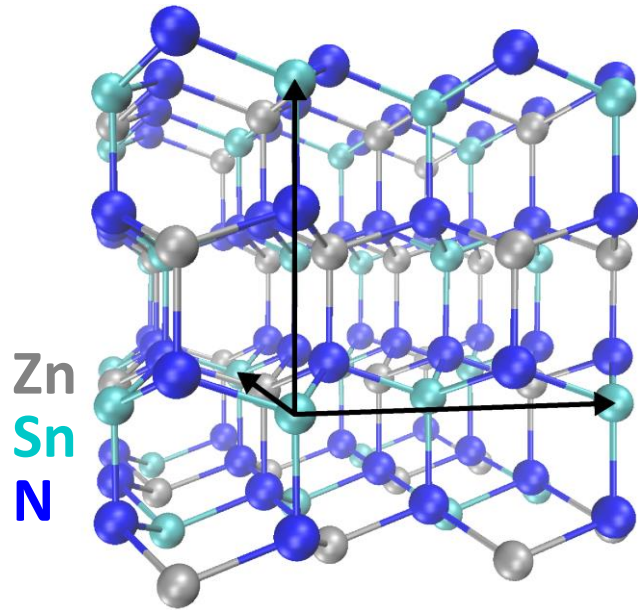
Implication for point defects: “Extended anti-sites”



- Formation energy of Cu_{Sn} reduced by 1.5 eV in disordered CTS
- Point defect model in ordered crystal does not capture energetics of off-stoichiometry



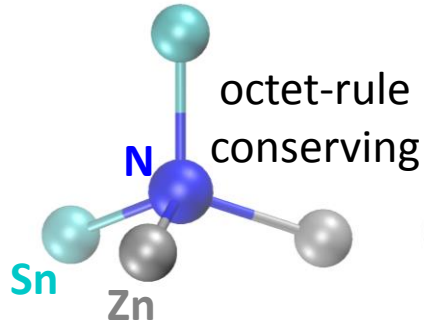
Cation ordering in ZnSnN_2



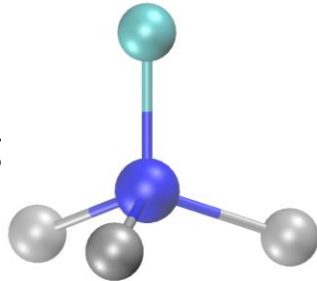
Crystal structure of ordered ZnSnN_2 (ZTN)

- Orthorhombic (ORC) cell is supercell of Wurtzite (WZ)
- All N atoms coordinated by 2 Zn and 2 Sn

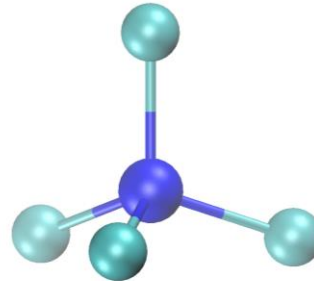
Zn_2Sn_2



Zn_3Sn_1



Sn_4

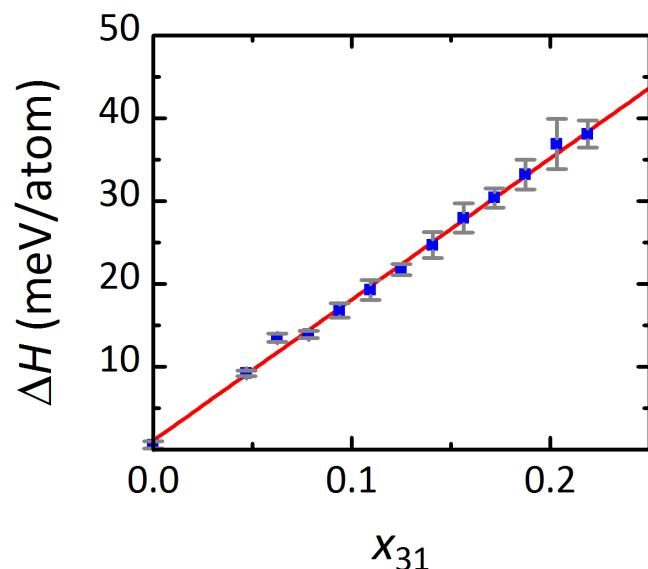


$$\Delta H = \sum_i n(i) e(i)$$

ZTN disorder: Phase transition and energies

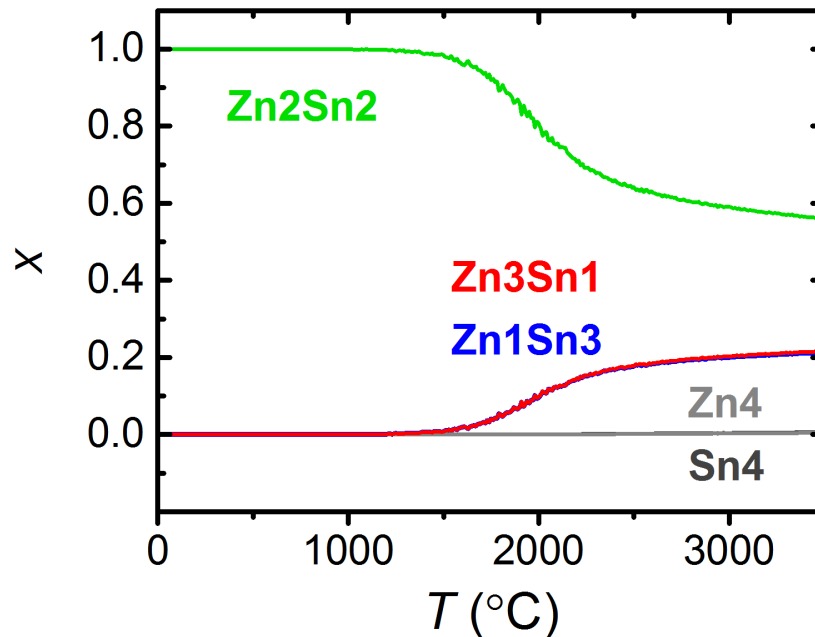
Energies (DFT)

- Disordered octet-conserving: Essentially degenerate, $\Delta H = 0.5$ meV/atom, occurs at any temperature
- Average energy is linear in concentration $x_{31} = x_{13}$ of Zn_3Sn_1 motifs



Disorder vs temperature (model H)

- Ordered ground state: only Zn_2Sn_2 motifs (octet)
- Around 2000°C transition to phase with 40% of $\text{Zn}_3\text{Sn}_1 + \text{Zn}_1\text{Sn}_3$ motifs
- Zn_4 and Sn_4 stay below 1%



Band gap calculations

Approach

- Need to avoid “DFT band gap error”
- Full-fledged GW calculations not feasible for supercells of disordered ZTN
- Use “single-shot” hybrid functional, plus on-site potential U for Zn-d
- Fitting to GW calculation for ZnSnN_2 , Zn_3N_2 , and Sn_3N_4

$$e_n^{GW} = e_n^{\text{DFT}} + \text{Re} \langle \psi_n^{\text{DFT}} | \Sigma(e_n^{GW}) - V_{\text{xc}}^{\text{DFT}} | \psi_n^{\text{DFT}} \rangle$$

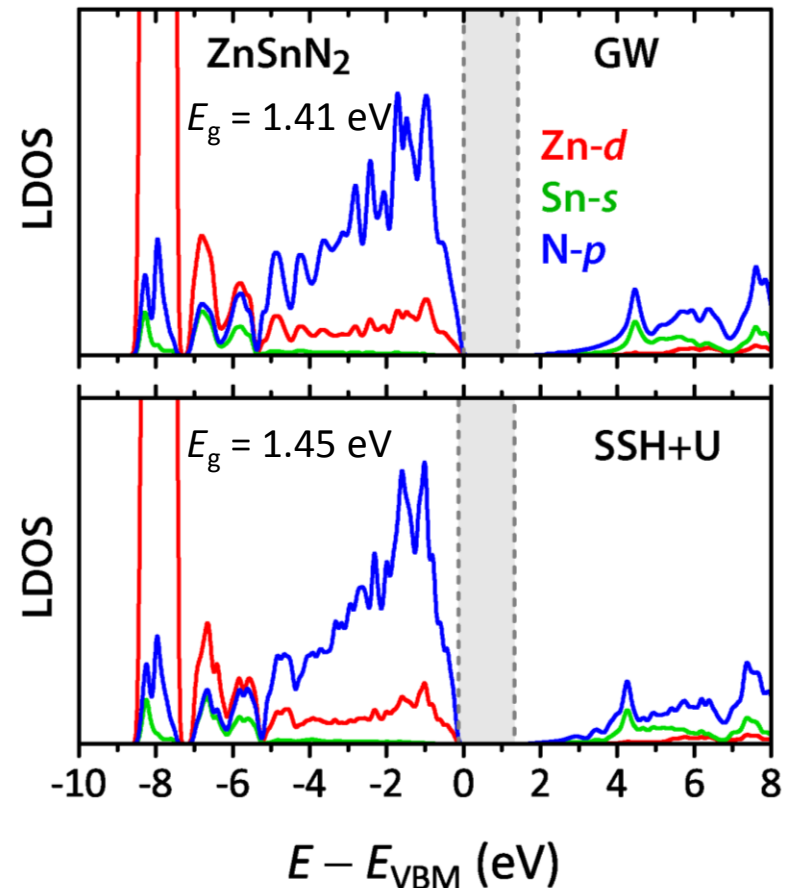
Screening in GW: $W = W(\mathbf{r}, \mathbf{r}', \omega)$

Hybrid functional: scaling by $\alpha < 1$

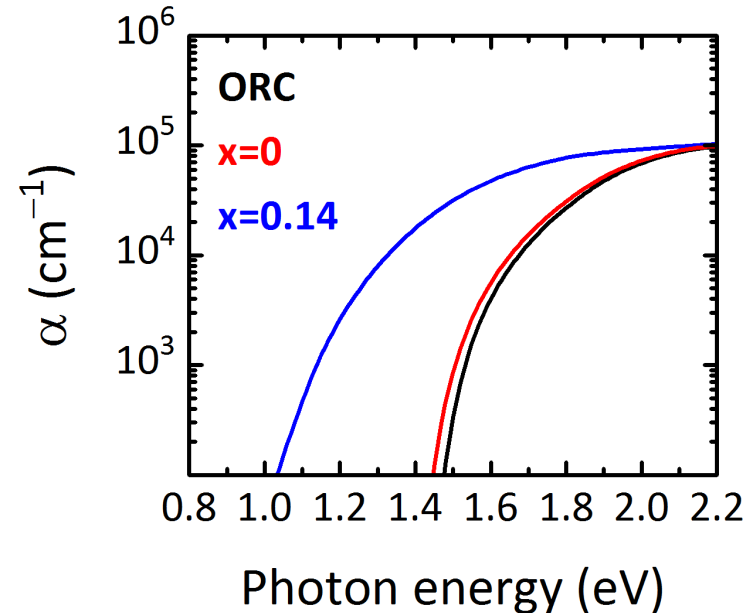
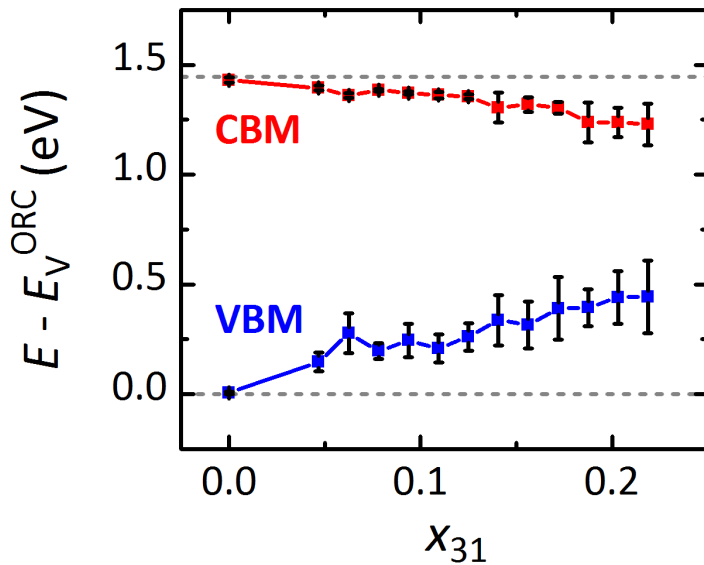
Fit result: $\alpha = 0.14$, $U_{\text{Zn-d}} = 4$ eV

E_g within 0.1 eV,

VBM, CBM, Zn-d within 0.2 eV



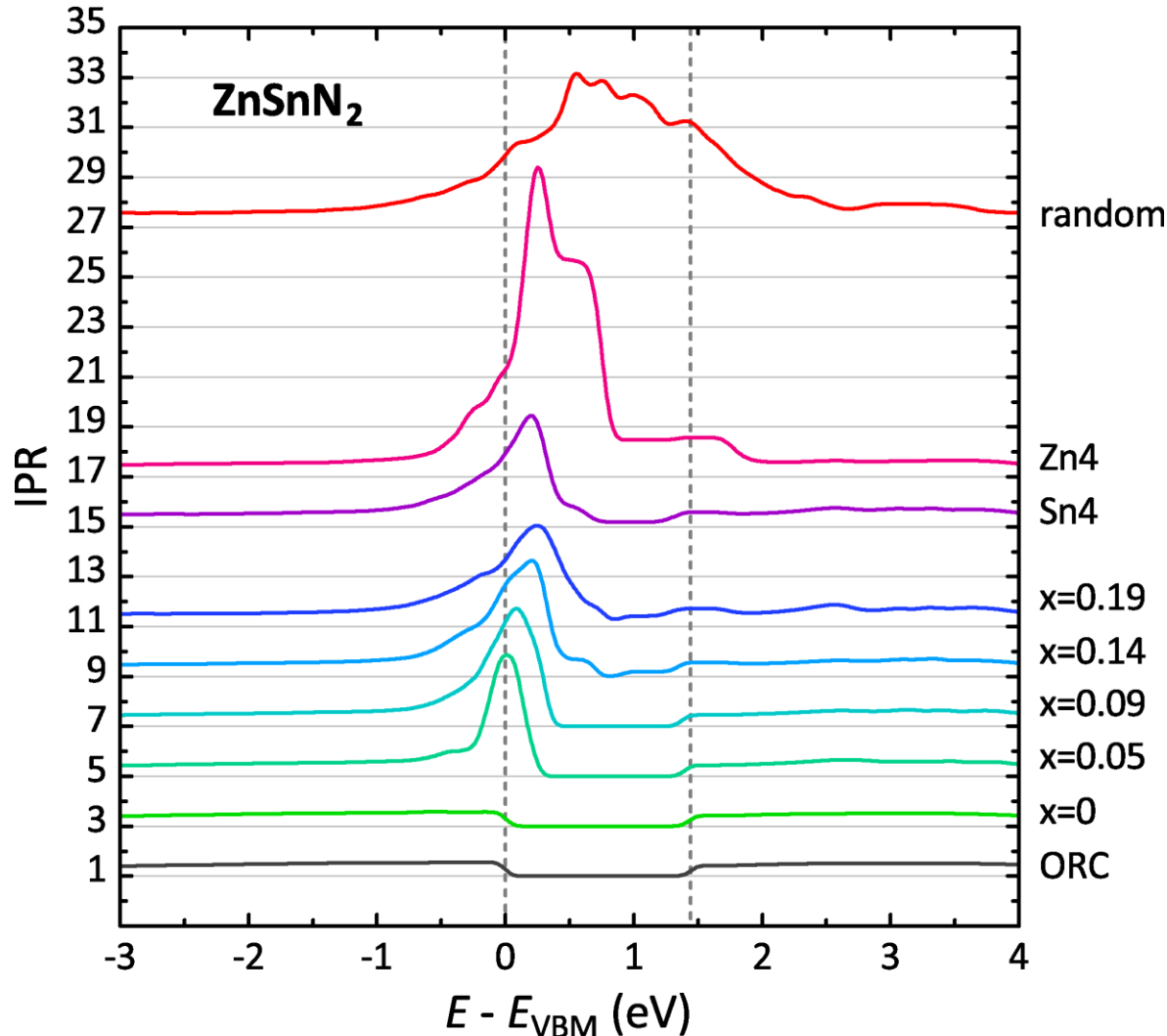
Disorder effects on electronic structure



Band gaps and optical properties

- Valence conserving disorder (only Zn₂Sn₂ motifs) has virtually no effect, as compared to the perfectly ordered ORC structure
- Band gap shrinks with increasing x_{31}
- Absorption edge becomes less steep

Inverse Participation Ratio: Measure of localization



$$IPR(E) = \frac{N \sum_i p_i(E)^2}{\left(\sum_i p_i(E)\right)^2}$$

$p_i(E)$: atomic density of states

For example, IPR=10 means DOS is localized on 1 out of 10 atoms

- Octet-rule conserving disorder is benign
- “31” disorder causes only moderate localization
- Zn4 motifs create localized VB derived gap states
- Total random disorder fills gap with localized defect states

Conclusions

- **Electronic structure**
Many-body perturbation theory in GW approximation Band-gaps, band-structure, optical properties
- **Defects**
Defect formation energies, dopant-defect pair interaction
Thermodynamic modeling, “defect phase diagrams”
- **Disorder**
Model-Hamiltonian for total energy: Motif expansion
Monte-Carlo simulations for atomic structure
Electronic structure for disordered compounds
- **<http://materials.nrel.gov>**
Database for DFT and GW calculations