A strategic model of public–private partnerships in transportation: Effect of taxes and cost structure on investment viability

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ABSTRACT

We formulate a game-theoretic model of a concession agreement between a government and a private party, a concessionaire, who has to engage a set of service providers as part of the operating responsibilities. We use the model to examine the importance of a government’s tax policy to induce private investments in transportation infrastructure. Our analysis brings to fore insights that are useful in the design of partnership agreements, such as the importance of early and binding government commitments to ensure stable partnerships, and thus, successful projects. Our analysis shows that strong commitments are even more critical in situations where the success of the partnership requires participation of additional, self-interested parties, such as specialized service providers. Finally, we consider variations of the model where government preferences are explicitly captured, and where the returns from the fixed cost portion of the concessionaire’s investment are exempt from taxes. We show that both variations can lead to outcomes where the concessionaire’s tax burden is shifted to the service providers. This flexibility can be critical in the design of partnership agreements for (high-risk or highly specialized transportation) projects where additional incentives may be needed to induce private party participation.

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1. Introduction

As described by Macario (2010) and the references therein, there is an increasing, global trend in the adoption of public–private partnerships (PPPs) for the provision of transportation infrastructure and services. For example, Kappeler and Nemoz (2010) indicate that there have been more than 1400 PPPs signed in the European Union in the past two decades, valued at more than €260 billion. In the last 5 years, transportation projects accounted for approximately 40% of the total number, and 75% of the total value. In the United States there are over 40 active, large-scale PPPs in transportation (Rall, Reed, & Farber, 2010). In PPPs, private parties willingly agree to share responsibility for (a subset of the following activities) investment, finance, design, construction, renovation, maintenance, management, or operation of transportation facilities, e.g., roads, bridges, airports, parking facilities. From a government’s perspective, these arrangements can provide direct benefits such as (additional) funding for projects, efficiencies in management and operations, as well as significant, though sometimes ignored, indirect benefits such as employment, which among other things, increases the tax base. As evidence of the potential benefits that can realize from private administration of transportation services, we note that a recent report prepared by the US Department of Transportation for Congress estimates that PPPs can result in savings of 6–40% (U.S.DOT, 2004).

The responsibilities of the different parties vary depending on the specific type of partnership agreement; however, as described by Garba (2009), it is common for the public sector to provide capital subsidies in the form of one-time grants to defray the initial investment, as well as tax credits to increase the returns on investment. This, in turn, serves as motivation for the work presented here in, where we model a concession agreement between a government (with limited resources) and a private party. The concessionaire decides to participate in the partnership by making an initial investment, and by engaging a set of service providers, e.g., a labor force, that will contribute to the operating phase of the project. The inclusion of service providers in the model constitutes a novel feature in this type of analysis, and is intended to capture the broad impact of transportation projects. From the government’s perspective, the viability of the project is tied to its ability to raise revenue from taxes levied both on the concessionaire’s investment return, and on the service providers’ income. Our analysis of the proposed model provides a number of insights about the stability of
PPPs, and consequently, the success of projects. Specifically, our analysis highlights the importance of timing and strength of the government’s commitments to avoid time inconsistent behavior leading to unstable partnerships (Chari, 1988; Chari, Kehoe, & Prescott, 1989). These characteristics are even more critical in agreements that require commitments from additional parties, such as the service providers. Our analysis also shows that government preferences and tax policies that rely on a project’s cost structure, can lead to partnerships where the concessionaire and the service providers bear different proportions of the tax burden. The latter can be especially useful in the design of mechanisms to support PPPs because additional (or fewer) incentives may be necessary to motivate private party participation in certain agreements.

Our model builds on a large body of work in Economics (too vast to be reviewed here in a meaningful way),1 with the seminal work of Ramsey (1927), on taxation issues in the context of public finances, providing, perhaps, a reasonable starting point. He posed the question of raising revenue by imposing taxes on some or all uses of income, the taxes on different uses being possibly at different rates. In this context the decision being to select rates so as to minimize the utility reduction. In this static, representative consumer economy with many goods, he investigated the decision that the government should make in a competitive equilibrium when choosing tax rates to maximize the welfare of the representative consumer, given a set of determined taxes, prices and quantities. Other relevant work includes the Fischer (1980), who presented a capital taxation model where the circumstances under which the problem of dynamic inconsistency arises, and discussed its implications for control theory and optimal policy-making. He explained that the problem arises when the government does not have commitment instruments, and when expectations of future variables are relevant to current private sector decisions. Chari (1988) and Chari et al. (1989) extended the discussion of the time consistency problem and the ensuing optimal policy design in detail with the illustration of taxation model and government debt model, where they focused on sustainable equilibria based on certain sequential rationality conditions. Further, Atkeson, Chari, and Kehoe (1999) argue that taxing capital investments may deter private parties from making investments. This, in turn, serves as motivation for our analysis of how an investment’s cost structure can be used as part of the tax regime to engineer a successful partnership. This, of course, makes the work related to the literature on Mechanism Design (see Myerson (1982, 1983) and the references therein). Finally, we mention the work of Baron and Myerson (1982), who discussed the case of regulating a monopolist with unknown costs, and showed the effects of treating consumers and firm differently by maximizing a weighted sum of utility functions. This work inspired our analysis of government preferences on the proportion of taxes that the other two parties pay.

Our model adds to the growing literature in transportation economics, where conceptual models such as ours, are being used to understand the interactions between self-interested parties, and specifically how the alignment, or lack thereof, of these interests influences project viability, and parameters such as capacity, service quality, etc. Small and Verhoef (2007) provide a seminal treatment of this literature. These papers are complements to recent quantitative and qualitative literature, see e.g., Abdul-Aziz (2006), Central-Guidelines (2003), Evenhuis and Vickerman (2010), Karlaftis (2007), Lopez-Lambas and Monzon (2010) documenting outcomes of PPPs, and lessons learned from their adoption.

The remainder of the paper is organized as follows: Section 2 provides a description of our model and the technical assumptions. Our analysis of the model is presented in Section 3, where in Section 3.1 we analyze the effect of the timing of the government’s commitments to tax rates on the stability of the partnership. We show that weak (or non-binding) commitments, i.e., reactive governments, can lead to unstable partnerships. In this analysis we, initially assume that the concessionaire and the service providers are represented by a single entity. In Section 3.2 we relax this assumption, and argue based on the structure, i.e., the sequence of utility maximization problems, that securing commitments from additional parties also motivates the need for early and binding government commitments. We then turn our attention to the design of tax policies that can lead to different splits in the proportion of taxes paid by the concessionaire and the service providers. In Section 3.3, we analyze the effect of government preferences, and in Section 3.4, we consider tax incentives that the concessionaire receives based on the cost structure of the initial investment. We conclude in Section 4 by summarizing some of the main insights drawn from our analysis.

2. Model formulation

We consider a situation where a government wants to execute a project, with associated capital costs, \( SW \), for which it does not have sufficient funding or willingness to invest/borrow. This, in turn, motivates the government to seek a partnership with a private party, i.e., a concessionaire, that will cover a portion of the capital costs, and will be responsible for the project’s operation. As part of the operating responsibilities, the concessionaire has to engage a set of service providers, e.g., a labor force. Building on Chari (1988), we formulate a two-stage, game-theoretic model to analyze the conditions that lead to stable partnerships between the aforementioned parties, and successful project executions. In the remainder of the section, we document the elements and technical assumptions in our model.

As stated, we formulate a three-player, two-stage game of complete and perfect information. The stages correspond to the project’s investment and operating phases. In the investment phase, a concessionaire with a budget of \( SC \), and a government decide to participate in the project by investing amounts, \( i \) and \( f \), respectively, and where \( 0 \leq i \leq C \) and \( f = W - i \). In order to operate the project, the concessionaire engages a set of service providers, who in turn decide to participate in the project by committing effort level \( l \), where \( l \geq 0 \) and might be measured in hours per year. The service providers are compensated at a rate of \( SW \), e.g., per hour.2 Successful execution of the project leads to operating returns, \( IR \), where \( R > 1 \).3 In our model, the government’s willingness to commit to the project by investing \( W - i \), depends on its ability to generate revenues of at least \( 5G \) during the operating phase. The government generates revenue by levying taxes on the concessionaire’s operating returns at a rate \( \theta \), and on the service providers’ income at a rate \( \tau \). We assume \( \theta \leq 1 \) and \( \tau \leq 1 \).4 Thus, the total tax revenue is given by \( 5R + \tau wL \).

1 For consistency, the specification of \( w \) may include a present value factor.
2 This amounts to assuming that, pre-taxes, the present value of the investment, \( IR - i - (R - 1)i \) is non-negative, i.e., the investment is viable. Importantly, we assume that the returns are proportional to the investment, \( l \). This setup constitutes a small variation to the model presented in Fischer (1980).
3 \( \theta < 0 \) or \( \tau < 0 \) corresponds to tax credits.
Having specified the decisions that each of the parties make, we proceed to formulate the respective utility maximization problems:

### 2.1. Concessionaire

The concessionaire’s utility, \( U^c(e^c_1, e^c_2) \), is a function of the wealth that realizes from the investment opportunity. This wealth consists of two components, referred to as expenditures, and denoted \( e^c_1 \) and \( e^c_2 \). \( e^c_1 = C - i \) is the capital withheld from the project, and \( e^c_2 = (1 - \theta)iR \) is the after-tax investment return. Notice that the expenditures realize in the stages denoted by the sub-indices. For simplicity, we assume the concessionaire’s utility is \( U^c(e^c_1, e^c_2) = e^c_1 + e^c_2 \), and thus the utility maximization problem is as follows:

\[
\max_{\theta \leq 1; l} e^c_1 + e^c_2 \quad (1)
\]

subject to:

\[
e^c_1 + i = C \quad (2)
\]

\[
e^c_2 = (1 - \theta)iR \quad (3)
\]

For a given value of \( \theta \), use \( \ell(\theta) \) to denote the optimal solution to the problem (1)–(3), \( e^c_1(\theta), e^c_2(\theta) \) are the optimal expenditures, and \( U^c(e^c_1(\theta), e^c_2(\theta)) \) is the associated maximum utility.

### 2.2. Service providers

The service providers’ utility is a function of the income they earn from the project, denoted \( e^w_l \), defined in (5), as well as the effort level that is committed, \( l \). In particular, we assume \( U^w(e^w_l, l) = e^w_l - \frac{1}{2}l^2 \), and that the utility maximization problem is as follows:

\[
\max_{l \geq 0} e^w_2 - \frac{1}{2}l^2 \quad (4)
\]

subject to:

\[
e^w_2 = (1 - \tau)wl \quad (5)
\]

By construction, the service providers’ utility is concave in the effort level, \( l \). This implies saturation, and in fact can decrease if \( l > (1 - \tau)w \). The \( \frac{1}{2} \) is merely a scaling constant. As before, \( \ell(\tau) \) represents the optimal solution to (4) and (5) for a given \( \tau \), \( e^w_2(\tau) \) is the optimal expenditure, and \( U^w(e^w_l(\tau), l(\tau)) \) is the associated optimal utility.

### 2.3. Government

We assume that the government has a vested interest in the success of the partnership, i.e., in the project’s execution, which we assume has a positive effect on social welfare, and in the success of its partners. From the latter we specify a utility function for the government that consists of the (weighted) sum of the utilities accrued by the concessionaire and the service providers, i.e.,

\[
U^g(U^c(\theta), U^w(\tau)) = \lambda U^c(e^c_1(\theta), e^c_2(\theta)) + U^w(e^w_l(\tau), l(\tau)), \quad \lambda > 0,
\]

where \( \lambda = 1 \). In subsequent sections we analyze the effect of assigning unequal weights to either of the terms, but initially we assume that the government weighs the utilities of the concessionaire and the service providers equally, i.e., \( \lambda = 1 \). Thus, the government’s utility maximization problem is:

\[
\max_{\theta \leq 1; \tau \leq 1} U^g(e^c_1(\theta), e^c_2(\theta)) + U^w(e^w_l(\tau), l(\tau)) \quad (6)
\]

subject to:

\[
\theta iR + \tau wl \geq G \quad (7)
\]

Equation (7) imposes the revenue generation constraint. Generally, we let \( \theta^*, \tau^* \) represent optimal solutions to the government’s utility maximization problem. From the structure of the optimization problems presented above, we notice the links between the decisions that the parties make. In particular, the concessionaire and service providers solve utility maximization problems that depend on the government’s choice of tax rates. The government’s utility, in turn, is a function of the utility of the other two parties.

### 3. Economic analysis

In this section, we analyze the model presented in Section 2. As stated, our focus is to identify conditions that lead to stable partnerships and successful project executions. Specifically:

1. In Subsection 3.1, we consider the effect of the timing of the government’s announcement/commitment to the tax rates. We introduce the notions of proactive and reactive governments. We show that proactive governments, that commit earlier (to \( \theta \)), can lead to successful partnerships; whereas reactive governments, cannot – except in cases where the revenue generation requirement, \( G \), is small. To simplify the analysis, we assume that the concessionaire negotiates on behalf of the service providers, i.e., that they are one party.

2. In Subsection 3.2, we assume that all parties are independent and self-interested. The main insight that stems from relaxing the assumption used in Subsection 3.1 is that, not-surprisingly, coordination between additional parties requires earlier commitments (to the income tax rate \( \tau \)) by the government.

3. In Subsection 3.3, we analyze the impact of government’s preferences on the stability of partnerships. Our analysis shows that both benevolent and non-benevolent governments can lead successful partnerships. Benevolent governments weigh the service providers’ utility, e.g., labor’s utility, more heavily than the concessionaire’s utility, i.e., \( \lambda < 1 \).

4. Finally, in Subsection 3.4, we consider a different tax regime where the government only levies taxes on the returns corresponding to the variable costs, i.e., there is a tax subsidy/credit to encourage the concessionaire to invest in the project. We show that as in the case of government preferences, stable partnerships can be designed where tax burden can be shared by the concessionaire and the service providers in different proportions.

#### 3.1. On the timing of the government’s commitments

Following Chari et al. (1989), we discuss the effect of the timing of the government’s announcement/commitment to the tax rates. Specifically, we consider two cases: the case of a proactive government – type 1, and the case of a reactive government – type 2. The proactive government commits to \( \theta \) at the start of the investment stage, i.e., stage 1, and to \( \tau \) (no later than) at the start of the operation stage, i.e., stage 2. In contrast, the reactive government commits to the tax rates at the start of stage 2. Both cases are illustrated in Fig. 1.

In terms of the utility maximization problems, the difference between the two cases is that in the case of a proactive government,
the concessionaire knows $\theta$ in advance of selecting an investment, $i$. In the case of a reactive government, the concessionaire selects $i$ based on expectations about $\theta$. Where relevant, we denote the contingent decision variable in the latter case $i(\theta)$. Ultimately, we show that the reactive government cannot support a stable partnership – except when $G$ is small. Before we present the details of our analysis, we begin by making the following simplification.

**Assumption 3.1.** Concessionaire represents the Service Providers

We assume that the concessionaire negotiates on behalf of the service providers, i.e., the two parties are replaced by a single entity. Under this assumption, the utility of the concessionaire is given by

$$ U^C(e_1, e_2, l) = U^C(e_1^C, e_2^C) + U^W(l, l), $$

where $e_1 = e_1^C$ and $e_2 = e_2^C + e_2^W$.

The ensuing utility maximization problem is as follows:

$$ \max_{0 \leq i \leq C, l \geq 0} e_1 + e_2 - \frac{1}{2} l^2 $$

subject to:

$$ e_1 + l = C $$

$$ e_2 = (1 - \theta) i R + (1 - \tau) w l $$

We note that under **Assumption 3.1**, the concessionaire’s utility maximization problem is now specified for given tax rates $\theta$ and $\tau$. To be consistent with our earlier description, and to differentiate the setup under this assumption from the earlier one, we now denote the corresponding optimal solution $i^*(\theta, \tau)$, $l^*(\theta, \tau)$. The corresponding optimal expenditures are also denoted as functions of the two tax rates, $e_1(\theta, \tau)$, $e_2(\theta, \tau)$. We also note that under this assumption the government’s utility, Equation (6), corresponds to the concessionaire’s utility. The following subsections contain our analysis for both types of government.

### 3.1.1. Proactive government problem

As described above, the proactive government commits to $\theta$ at the start of the investment stage, and to $\tau$ no later than at the start of the operation stage. Because under **Assumption 3.1**, the government interacts with a single-party, we conduct our analysis for the case of $\theta$ and $\tau$ agreed to simultaneously, at the start of the investment stage. The case of $\tau$ announced later is considered in Section 3.2.

To find an equilibrium solution for the case of a proactive government, relying on the assumption of perfect information, we begin by specifying the concessionaire’s optimal response function to the government’s choices of $\theta$ and $\tau$. Relying on the assumption of complete information, we proceed to specify the government’s optimal policy taking into account the concessionaire’s best response. Combining both parts of the analysis yields an equilibrium solution, referred to as a Ramsey Policy.

**Lemma 3.1.** For $(R - 1)C \leq G \leq (R - 1)C + \frac{1}{4} w^2$, there exists an equilibrium solution for the two-player, proactive government problem given by the following policies:

**Government:**

$$ \theta^* = \frac{R - 1}{R} \quad \text{and} \quad \tau^* = \tau_1 = \frac{1}{2} - \frac{1}{2w} $$

**Concessionaire:**

$$ i^*(\theta^*, \tau^*) = C \quad \text{and} \quad l^*(\theta^*, \tau^*) = (1 - \tau^*) w l $$

**Proof.** To find the concessionaire’s optimal response function, we begin by rewriting the utility maximization problem for given values of $\theta$ and $\tau$ as follows:

$$ \max_{0 \leq i \leq C, l \geq 0} C - i + (1 - \theta) i R + (1 - \tau) w l - \frac{1}{2} l^2 $$

which we observe is separable in the decision variables $i$ and $l$, and thus, the concessionaire’s optimal response function can be obtained by solving two optimization problems. To find $i^*(\theta, \tau)$ we consider:

$$ \max_{0 \leq i \leq C, l \geq 0} C - i + (1 - \theta) i R \Rightarrow \max_{0 \leq i \leq C} C + (R - \theta R - 1) i $$

Noticing that $(R - \theta R - 1) i$ is nondecreasing in $i$ for $\theta \leq \frac{R - 1}{R}$ and decreasing for $\theta > \frac{R - 1}{R}$, we obtain

$$ i^*(\theta, \tau) = \begin{cases} C; & \theta \leq \frac{R - 1}{R} \\ 0; & \frac{R - 1}{R} < \theta \leq 1 \end{cases} $$

The first case corresponds to a viable project for the concessionaire. The optimization problem to find $i^*(\theta, \tau)$ is

$$ \max_{l \geq 0} (1 - \tau) w l - \frac{1}{2} l^2 $$

which, in turn yields

$$ i^*(\theta, \tau) = (1 - \tau) w $$

Having specified the optimal response function for the concessionaire, as shown in Equations (11) and (12), we now turn our attention to the government’s utility maximization problem:

$$ \max_{0 \leq i \leq 1} e_1(\theta, \tau) + e_2(\theta, \tau) - \frac{1}{2} l^2(\theta, \tau)^2 $$

subject to:

$$ \theta^* i^*(\theta, \tau) + \tau^* l^*(\theta, \tau) \geq G $$

which we can rewrite as follows for the case of $\theta \leq \frac{R - 1}{R}$, i.e., the case of a viable project for the concessionaire.

$$ \max_{0 \leq i \leq 1} (1 - \theta) i R + \frac{1}{2} (1 - \tau)^2 w^2 $$

subject to:

$$ \theta R + \tau(1 - \tau) w \geq G $$

$$ \theta \leq \frac{R - 1}{R} $$

Other cases of $G$ are discussed below.

The result follows from evaluating the first-order optimality condition, and noticing that the function is (strictly) concave in $l$. 

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To complete the proof, we show in Appendix A.1 that an optimal policy for the government is given by

$$\theta^* = \frac{R - 1}{R}$$ and $$\tau^* = \frac{1}{2} - \frac{1}{2w} \sqrt{w^2 - 4(R - 1)C}$$  \hspace{1cm} (16)

Having specified a Ramsey Policy resulting in a successful partnership, we discuss the role of the government’s choice of $$G$$ on the project’s outcome. Substituting the concessionaire’s optimal response (in the case of a viable project, i.e., $$\theta \leq \frac{R - 1}{R}$$) into the government’s revenue generation constraint gives

$$\theta^* CR + \tau^*(1 - \tau^*)w^2 \geq G$$  \hspace{1cm} (17)

As discussed in Appendix A.1, $$\theta^* CR$$ is the revenue generated by taxing the concessionaire’s return. We note that setting $$\theta = \theta^*$$ maximizes the revenue collected from such taxes. $$\tau^*(1 - \tau^*)w^2$$ is the revenue generated by taxing the service providers’ income. It is maximized by setting $$\tau = 1/2$$, and the associated revenue is $$\frac{1}{4}w^2$$.

Thus, $$G \leq (R - 1)C - \frac{1}{4}w^2$$ is needed to ensure feasibility of the government’s utility maximization problem, viability of the project from the concessionaire’s perspective, and consequently, a successful partnership. Additional insights stem from evaluating (17) at $$\theta^*$$ and $$\tau^*$$ given by the Ramsey Policy, and noting that

$$\theta^* CR + \tau^* (1 - \tau^*)w^2 = G \Rightarrow \tau^* (1 - \tau^*)w^2 = G - (R - 1)C$$  \hspace{1cm} (18)

It follows that $$\theta^*$$ is set to maximize revenue generated from taxing the concessionaire’s return. As discussed in Appendix A.1, $$\tau^*$$ is set at $$\tau_2$$, the smaller of the two roots, $$\tau_1$$ and $$\tau_2$$, of (18). The intuition is illustrated in Fig. 2.

The two curves correspond to the government’s utility, as well as the tax revenue it generates as a function of $$\tau$$ (when evaluated at $$\theta^*$$). When $$\tau \leq 1$$, we see that the utility function is convex and decreasing, which is why

$$\ell^C (\ell_1 (\theta^*, \tau_1), \ell_2 (\theta^*, \tau_1), \ell^G (\theta^*, \tau_1)) \geq \ell^C (\ell_1 (\theta^*, \tau_2), \ell_2 (\theta^*, \tau_2), \ell^G (\theta^*, \tau_2)).$$

We note that Fig. 2 is for the case of $$(R - 1)C \leq G \leq (R - 1)C + \frac{1}{4}w^2$$.

As shown in Appendix A.1, when $$G < (R - 1)C$$, the solution is obtained by setting $$\theta^* = \frac{G}{R - 1}, \tau^* = 0$$. In this case, the revenue requirement, $$G$$, can be satisfied by taxing the concessionaire below the critical rate $$\frac{R - 1}{R}$$, and the service providers are not taxed. From a practical standpoint, this case may be less interesting than the one considered in Lemma 3.1.

### 3.1.2. Reactive government problem

In this section, we argue that, depending on $$G$$, the assumption of complete information may prevent reactive governments from supporting stable partnerships, and therefore from achieving the objective of executing a project with a positive effect on social welfare. Recall that a reactive government commits to $$\theta$$ and $$\tau$$ simultaneously at the start of the operation stage. In this case, the concessionaire’s investment decision in the first stage is based on expectations about $$\theta$$ (denoted $$\theta^R$$). From Lemma 3.1, it follows that

$$i^R (\theta^R, \tau) = \left\{ \begin{array}{ll} C; & \theta^R \leq \frac{R - 1}{R} \\ 0; & \frac{R - 1}{R} < \theta^R \leq 1 \end{array} \right.$$ and $$i^R (\theta, \tau) = (1 - \tau)w$$

To find the optimal tax rates, as in the proof of Lemma 3.1, we consider the government’s utility maximization problem

$$\max_{\theta^R \leq 1; \tau \geq 0} C - i^R (\theta^R, \tau) + (1 - \theta) i^R (\theta^R, \tau) \cdot R + (1 - \tau)w i^R (\theta, \tau) - \frac{1}{2} \ell^R (\theta, \tau)^2$$ subject to:

$$\theta^R + \tau(1 - \tau)w^2 \geq G$$

which can be reduced, as shown below, in the case of a viable project, $$\theta^R \leq \frac{R - 1}{R}$$.

$$\max_{\theta^R \leq \frac{R - 1}{R}; \tau \geq 0} (1 - \theta) CR + \frac{1}{2} \tau^2 w^2$$ subject to:

$$\theta^R + \tau(1 - \tau)w^2 \geq G$$

The constraint that ensures the viability of the project from the concessionaire’s perspective, $$\theta^R \leq \frac{R - 1}{R}$$, is irrelevant to the government’s utility maximization problem. As in the case of the proactive government problem, equilibrium solutions depend on the parameter $$G$$. The details are presented in Appendix A.2. We show that when $$RC \leq G \leq (R - 1)C + \frac{1}{4}w^2$$, the government’s optimal policy is to set $$\theta = 1$$, and $$\tau = \frac{1}{2} - \frac{1}{2w} \sqrt{w^2 - 4(R - 1)C}$$. The assumption of complete information implies that the concessionaire sets $$\theta^R = 1$$, which contradicts the assumption of a viable project, and shows that, in this case, the reactive government cannot support a stable partnership, i.e., an equilibrium solution, leading to a successful project. As explained in Chari (1988) and Chari et al. (1989), the government’s behavior is referred to as time inconsistent because it has no incentive to stick to any advance negotiations regarding $$\theta^R$$. In turn, the assumption of complete information means that commitments of $$\theta^R < 1$$ are not credible.

As shown in Appendix A.2, the case of $$(R - 1)C \leq G < RC$$ leads to a similar inconsistency. An equilibrium solution does exist for the less interesting case of $$G < (R - 1)C$$.

### 3.2. On the effect of representing the service providers as an independent, self-interested party

In this section, we discuss the effect of modeling the interactions between a proactive government and two independent parties representing the concessionaire and the service providers. In this case, the government commits to $$\theta$$ (with the concessionaire) at the
start of the investment stage, and to \( \tau \) (with the service providers) no later than the start of the operation stage. Because in Section 3.1, the government interacts with a single-party, we assume that \( \theta \) and \( \tau \) are agreed upon simultaneously, at the start of either the first or second stage. In this section, we consider a (more general) situation where the government negotiates with the other two parties separately. The main result is shown in Lemma 3.2.

**Lemma 3.2.** There exists an equilibrium solution for the three-player, proactive government problem given by the following policies:

- **Government:** \( \theta^* = \frac{R - 1}{R} \) and \( \tau^* = \frac{1}{2} \frac{R}{2w} \)
- **Concessionaire:** \( \Gamma(\theta^*) = C \) and \( \Gamma(\tau^*) = (1 - \tau^*)w. \)
- **Service Providers:** \( \Gamma^*(\tau^*) = (1 - \tau^*)w. \)

**Proof.** The optimal response functions for the concessionaire and the service providers follow from the analysis presented in the proof of Lemma 3.1. That is,

\[
\Gamma^*(\tau^*) = \frac{\theta}{C}; \quad \theta \leq \frac{R - 1}{R}; \quad 0, \frac{R - 1}{R} < \theta \leq 1
\]

Now, we consider the case of sequential commitments to \( \theta \) and \( \tau \)—the commitment to \( \theta \) takes place after the start of the investment stage and no later than the start of the operation stage.\(^7\) First, we have that for a given \( \theta \) and given the optimal response functions, the utility maximization problem that the government solves to set \( \tau \) is

\[
\max \frac{1}{2} \left( 1 - \tau \right)^2 w^2 \quad (19)
\]

subject to:

\[
\tau (1 - \tau)w^2 \geq G - \theta \Gamma(\theta) \cdot R
\]

The optimal \( \tau \), denoted \( \tau(\theta) \), corresponds to the smaller of the two roots of \( \tau (1 - \tau)w^2 = G - \theta \Gamma(\theta) \cdot R. \) The intuition is similar to the discussion of Fig. 2. In turn, the utility maximization problem that the government solves to set \( \theta \) is

\[
\max C - \Gamma^*(\theta) + (1 - \theta) \Gamma(\theta) \cdot R + \frac{1}{2} (1 - \tau(\theta))^2 w^2
\]

subject to:

\[
\theta \Gamma(\theta) \cdot R + \tau (1 - \tau)w^2 \geq G
\]

which can be reduced as follows for the case of a viable project

\[
\max (1 - \theta)CR + \frac{1}{2} (1 - \tau(\theta))^2 w^2
\]

subject to:

\[
\theta CR + \tau (1 - \tau)w^2 \geq G
\]

\[
\theta \leq \frac{R - 1}{R}
\]

The optimality of \( \theta = \theta^* \) and \( \tau = \tau^* \) follows from the observation that the reduced optimization problem corresponds to (13)–(15) with the additional constraint that \( \tau \) solve (19) and (20), i.e., that \( \tau \) be restricted to a point on the (response) function \( \tau(\theta) \). The fact that the optimal solution to (13)–(15), \( \theta^* \) and \( \tau^* \), is feasible for the reduced problem completes the proof.

While we show that there is a Ramsey Policy identical to the one given in Lemma 3.1 for the two-player, proactive government problem, the sequence of optimization problems that are solved reveals the potential for time inconsistencies because the government sets \( \tau \), i.e., solves (19) and (20), after the concessionaire responds. As discussed in the context of a reactive government, correcting these inconsistencies requires earlier and binding commitments.

### 3.3. On the effect of government preferences

In this section, we discuss the effect of government preferences on the stability of the partnership, and on the success of the project. As described in Section 2, we capture government preferences with a general form of the utility function, \( UC(U^C(\theta), U^W(\tau)) = \lambda UC(e^C_1(\theta), e^C_2(\theta)) + U^W(e^W_1(\tau), l(\tau)), \) consisting of the weighted sum of the utilities of the concessionaire and the service providers. The parameter \( \lambda \geq 0 \) is used to set the weight assigned to each of the terms in the utility function, an idea inspired by Baron and Myerson (1982). We refer to the case of \( \lambda < 1 \) as the case of a benevolent government, i.e., one who favors the service providers. The main result in this section, Lemma 3.3, is that a sufficiently large \( \lambda \) gives rise to a Ramsey Policy where the tax burden on the concessionaire is shifted to the service providers. To simplify the analysis we revert back to Assumption 3.1, the two-player game, and a proactive government that commits to \( \theta \) and \( \tau \) at the start of the investment stage.

**Lemma 3.3.** A utility function \( UC(U^C(\theta), U^W(\tau)) = \lambda UC(e^C_1(\theta), e^C_2(\theta)) + U^W(e^W_1(\tau), l(\tau)), \) with \( \lambda \geq 0 \), gives rise to equilibrium solutions for the two-player, proactive government problem given by the following policies:

- Case \( \lambda \leq \frac{1 - \tau_1}{1 - 2\tau_1} \):
  - **Government:** \( \hat{\theta} = \frac{R - 1}{R} \) and \( \hat{\tau} = \frac{1}{2} \frac{R}{2w} \sqrt{w^2 - 4G - (R - 1)C}; \)
  - **Concessionaire:** \( \Gamma(\hat{\theta}, \hat{\tau}) = C \) and \( \Gamma(\hat{\theta}, \hat{\tau}) = (1 - \hat{\tau})w. \)

- Case \( \lambda > \frac{1 - \tau_1}{1 - 2\tau_1} \):
  - **Government:** \( \frac{1 - \lambda}{1 - 2\lambda} \) and \( \hat{\theta} = \frac{G - (1 - \tau)w^2}{CR} \); and
  - **Concessionaire:** \( \Gamma(\hat{\theta}, \hat{\tau}) = C \) and \( \Gamma(\hat{\theta}, \hat{\tau}) = (1 - \hat{\tau})w. \)

### Tax Revenue

- If \( G < 0.5w^2 \)
  - **Government:** \( \hat{\tau} \), \( \hat{\theta} \)
  - **Concessionaire:** \( \hat{\tau} \)

![Fig. 3. Effect of \( \lambda \) on the tax revenue that each party contributes.](image-url)
From Lemma 3.1, the condition that ensures viability of the project from the concessionaire’s perspective, expressed as function of $\alpha$, becomes $\alpha \leq \frac{R - 1 F + kz}{kz}$. While the two tax regimes can be mapped, the assumption that only variable costs are taxed suggests that the government should restrict $\alpha \leq 1$. With this restriction, the government can impose equivalent tax rates as long as the ratio of fixed and variable costs satisfies Equation (21).

$$\frac{R - 1 F + kz}{kz} \leq 1 \Leftrightarrow \frac{F}{kz} \leq \frac{1}{R - 1} \quad (21)$$

To provide some insight, Fig. 4 presents the contributions to tax revenue by each of the parties for projects with increasing fixed-to-variable-cost ratios. Given the assumption of a constant $C$, we assume that the additional fixed costs are included in the one-time grant, $f$. The government, in turn, includes its additional contribution in the tax revenue requirement, $C$. We observe that for ratios $\frac{F}{kz} \leq \frac{1}{R - 1}$, i.e., when the two policies are equivalent, the concessionaire is taxed at a rate of $\alpha = \frac{R - 1 F + kz}{kz}$, leading after-tax profits of 0. As $\tau(1 - \tau)w^2 \geq G - \theta^s(\theta) \cdot R$ increases beyond $\frac{1}{R - 1}$, the government sets $\alpha = 1$, the concessionaire earns positive profits (from the returns based on the fixed cost portion of the investment), and the burden of covering the additional costs falls on the service providers. Again, we emphasize that such a mechanism may be needed to induce the concessionaire to participate.

4. Conclusions

We present a simple, two-stage, game-theoretic model of a partnership agreement between a government and a concessionaire. In the first stage, the concessionaire agrees to participate in the partnership by committing to a capital investment. The concessionaire’s responsibilities include engaging a set of service providers who contribute to the project’s operation in the second stage. The inclusion of a third party is, in our assessment, a novel feature that allows our model to capture the broad impact of investments in transportation infrastructure and services. In the model, the government’s decision to participate in the partnership depends on the portion of the initial investment that it has to cover, as well as on its ability to generate revenue from taxes, in the operating stage.

We use the model to analyze the effect of the government’s tax policy on the stability of the partnership, and on the success of the project. Our analysis reveals the following insights/lessons that we believe, can be useful in the design of PPP agreements:

- The timing and strength of the government’s commitments is an important element in the stability of PPPs. Specifically, our model shows that proactive governments lead to successful partnerships; whereas reactive governments, who postpone or are not bound by their commitments to tax rates, do not. Exceptions arise in cases where the government’s revenue generation requirement is (relatively) small.
- From the structure, i.e., the sequence, of utility maximization problems that need to be solved in the instances where the service providers as independent, self-interested parties, we argue that early and binding commitments become even more important to support stable and successful partnerships. These observations are manifestations of time inconsistent behavior (Chari, 1988; Chari et al., 1989), a widely-researched...
phenomenon in Economics, that needs to be accounted for in the design of effective mechanisms, including PPPs in transportation.

- We consider a variation of the benchmark model where the government can assign different relative weights to the utilities that the concessionaire and the service providers realize, thereby capturing preferences. The main result is that the inclusion of preferences can yield outcomes where the tax burden is shifted between the parties.

- Finally, we consider a situation where the concessionaire’s initial investment consists of a fixed and a variable cost component, and analyze the effect of a tax policy where the returns on the fixed cost portion are exempt. As in the case of preferences, we show that this tax regime can also shift the tax burden from the concessionaire to the service providers. These results can be critical for (high-risk or highly specialized transportation) projects where additional incentives may be needed to induce private party participation.

- In terms of possible directions for research, we believe that additional insights can be obtained by relaxing (restrictive or unappealing) assumptions that we used to formulate the (benchmark) model presented here in. Modeling the interactions between the parties as a game of complete and perfect information is an example of an assumption that is generally too strong in practice. Examples of restrictive assumptions include the proportional relationship (i.e.: constant returns-to-scale) between the concessionaire’s investment and return, and the specification of the utility maximization problems for each of the parties, e.g., capturing social welfare in the government’s utility function.

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Appendix A. Technical results

Appendix A1. Proactive government’s optimal policy (2-player game)

From Lemma 3.1, the government’s utility maximization problem for the case of \( \theta \leq \frac{R - 1}{R} \) is:

\[
\max_{\tau \leq 1, \, \tau \leq 1} \left( 1 - \theta \right) CR + \frac{1}{2} \left( 1 - \tau \right)^2 w^2 \quad (A.1)
\]

subject to:

\[
\theta CR + \tau \left( 1 - \tau \right) w^2 \geq G \quad (A.2)
\]

\[
\theta \leq \frac{R - 1}{R} \quad (A.3)
\]

To find the optimal tax rates, we begin by writing the Lagrangean for the above optimization problem:

\[
\mathcal{L}(\theta, \tau, \mu_1, \mu_2, s_1, s_2) = (1 - \theta) CR + \frac{1}{2} \left( 1 - \tau \right)^2 w^2 + \mu_1 \left[ \theta CR \\
+ \tau \left( 1 - \tau \right) w^2 - s_1^2 - G \right] + \mu_2 \left[ - \theta - s_2^2 \\
+ \frac{R - 1}{R} \right]
\]

where \( \mu_1, \mu_2 \geq 0 \).

The stationary points are found by considering the first-order conditions:

\[
\frac{\partial \mathcal{L}}{\partial \theta} = 0 = -CR + \mu_1 CR - \mu_2 \quad (A.4)
\]

\[
\frac{\partial \mathcal{L}}{\partial \tau} = 0 = -\left( 1 - \tau \right) w^2 + \mu_1 w^2 - 2 \tau \mu_1 w^2 \quad (A.5)
\]

\[
\frac{\partial \mathcal{L}}{\partial \mu_1} = 0 = \theta CR + \tau \left( 1 - \tau \right) w^2 - s_1^2 - G \quad (A.6)
\]

\[
\frac{\partial \mathcal{L}}{\partial \mu_2} = 0 = -\theta - s_2^2 + \frac{R - 1}{R} \quad (A.7)
\]

\[
\frac{\partial \mathcal{L}}{\partial s_1} = 0 \Rightarrow \mu_1 \left[ \theta CR + \tau \left( 1 - \tau \right) w^2 - G \right] = 0 \quad (A.8)
\]

\[
\frac{\partial \mathcal{L}}{\partial s_2} = 0 \Rightarrow \mu_2 \left[ - \theta + \frac{R - 1}{R} \right] = 0 \quad (A.9)
\]

Equations (A.4) and (A.5) are referred to as the optimality conditions, Equations (A.6) and (A.7) are the feasibility conditions, and Equations (A.8) and (A.9) are the complementary slackness conditions. In addition, by construction the Lagrange multipliers \( \mu_1 \) and \( \mu_2 \) are non-negative. To find the optimal tax rates, and thus complete the proof of Lemma 3.1, we construct a solution to the system of equations \( \frac{\partial \mathcal{L}}{\partial \theta} = 0, \frac{\partial \mathcal{L}}{\partial \tau} = 0, \frac{\partial \mathcal{L}}{\partial \mu_1} = 0, \frac{\partial \mathcal{L}}{\partial \mu_2} = 0 \).

\[
\tau \left( 1 - \tau \right) w^2 = G - C (R - 1) \quad (A.10)
\]

with solutions:

\[
\tau_1 = \frac{1}{2} - \frac{1}{2w} \sqrt{w^2 - 4G - (R - 1)C} \quad (A.11)
\]

\[
\tau_2 = \frac{1}{2} + \frac{1}{2w} \sqrt{w^2 - 4G - (R - 1)C} \quad (A.12)
\]

\[
C (R - 1) \text{ is the revenue generated by taxing the concessionaire’s return, } \tau \left( 1 - \tau \right) w^2 \text{ is the revenue generated by taxing the service providers. It is maximized by setting } \tau = 1/2, \text{ and the associated revenue is } \frac{1}{4} w^2. \text{ Thus, for problem } (A.1)-(A.3) \text{ to be feasible, the following relationship obtained by substituting } \theta^* \text{ into } (A.2):
\]

\[
\frac{1}{4} w^2 \geq G - C (R - 1) \Rightarrow w^2 \geq 4G - C (R - 1) \quad (A.13)
\]

This result implies \( \tau_1 \) and \( \tau_2 \) are real numbers (with \( \tau_1 \leq \frac{1}{2} \leq \tau_2 \)). We set \( \tau^* = \tau_1 \) because \( \mathcal{L}^* (e_1(\theta^*, \tau_1), e_2(\theta^*, \tau_1), \mathcal{L}^* (e_1(\theta^*, \tau_2), e_2(\theta^*, \tau_2)), \mathcal{(\theta^*, \tau_2)}) \). Finally, from (A.5) and (A.4), we get that

\[
\theta \geq \frac{R - 1}{R} \quad (A.14)
\]

\[\text{(The result follows from the fact that (A.1) is nonincreasing in } \tau.\]
\[
\mu_1 = \frac{1 - \tau_1}{1 - 2\tau_1} \text{ and } \mu_2 = \frac{\tau_1}{1 - 2\tau_1} C, \text{ both non-negative, which completes our solution to (A.4)–(A.9).}
\]

Finally, we note that when \( G < (R - 1)C \), the condition

\[
\mu_2 = \frac{\tau_1}{1 - 2\tau_1} \text{ cannot be satisfied with a } \tau_1 < 0, \text{ which means that the solution to (A.4)–(A.9) follows a different structure than the one constructed above. In particular, we now have }
\]

\[\theta^* = \frac{G}{RC} < \frac{R - 1}{R}, \quad \tau^* = 0, \quad \mu_1^* = 1, \quad \mu_2^* = 0, \quad s_1^* = 2, \quad s_2^* > 0. \text{ This is a solution where the revenue generation requirement is met by taxing the concessionaire exclusively.} \]

**Appendix A.2. Reactive government’s optimal policy (2-player game)**

As with the proactive government’s utility maximization problem, we have:

\[
\max_{\theta \in \Theta, \; \tau \geq 1} (1 - \theta)CR + \frac{1}{2}(1 - \tau)^2 w^2 \tag{A.12}
\]

subject to:

\[\theta CR + \tau (1 - \tau) w^2 \geq G \tag{A.13}\]

\[\theta \leq 1 \tag{A.14}\]

To find the optimal tax rates, we begin by writing the Lagrangean for the above optimization problem:

\[L(\theta, \; \tau, \; \mu_1, \; \mu_2, \; s_1, \; s_2) = (1 - \theta)CR + \frac{1}{2}(1 - \tau)^2 w^2 + \mu_1 [\theta CR + \tau (1 - \tau) w^2 - s_1^2 - G] + \mu_2 [-\theta - s_2^2 + 1], \]

where \( \mu_1, \mu_2 \geq 0 \)

The stationary points are found by considering the first-order conditions:

\[
\frac{\partial L}{\partial \theta} = 0 = -CR + \mu_1 CR - \mu_2 \tag{A.15}
\]

\[
\frac{\partial L}{\partial \tau} = 0 = -(1 - \tau)w^2 + \mu_1 w^2 - 2\tau \mu_1 w^2 \tag{A.16}
\]

\[
\frac{\partial L}{\partial \mu_1} = 0 = \theta CR + \tau (1 - \tau) w^2 - s_1^2 - G \tag{A.17}
\]

\[
\frac{\partial L}{\partial \mu_2} = 0 = -\theta - s_2^2 + 1 \tag{A.18}
\]

\[
\frac{\partial L}{\partial s_1} = 0 \Leftrightarrow \mu_1 [\theta CR + \tau (1 - \tau) w^2 - G] = 0 \tag{A.19}
\]

\[
\frac{\partial L}{\partial s_2} = 0 \Leftrightarrow \mu_2 [-\theta + 1] = 0 \tag{A.20}
\]

**Appendix A.2.1. Case RC \( \leq G \leq (R - 1)C + \frac{1}{4}w^2 \)**

To find the optimal tax rates, we again construct a solution to the system of equations (A.15)–(A.20), \( \theta^*, \tau^*, \mu_1, \mu_2, s_1, s_2 \). Setting \( \theta^* = 1 \), means that \( s_2 = 0 \) from (A.18). Assuming (A.17) is binding, i.e., \( s_1 = 0 \), and substituting \( \theta^* \) into (A.17) leads to

\[\tau (1 - \tau) w^2 = G - CR \tag{A.21}\]

with solutions:

\[
\tau_1 = \frac{1}{2} \sqrt{\frac{w^2 - 4(G - RC)}{2w}}; \quad \tau_2 = \frac{1}{2} + \sqrt{\frac{w^2 - 4(G - RC)}{2w}} \tag{A.22}
\]

Thus, \( \theta^* = 1, \tau = \tau_1 \) is the optimal policy in this case; and again, we need to check the validity of the system of equations (A.15)–(A.20). From (A.16) and (A.15), we know \( \mu_1 = \frac{1 - \tau_1}{1 - 2\tau_1} \), and \( \mu_2 = RC \frac{\tau_1}{1 - 2\tau_1} \). The requirements of \( \mu_1, \mu_2 \geq 0 \) mean that \( \tau \leq \frac{1}{2} \) which holds in the current case.\(^9\)

**Appendix A.2.2. Case \((R - 1)C < G < RC\)**

In this case, solving the system of equations (A.15)–(A.20) gives us another set of optimal taxes where \( \theta^* = \frac{G}{RC} \), \( \tau = 0 \), and \( \mu_1 = 1 \), \( \mu_2 = 0 \).

**Appendix A.2.3. Case \(G < (R - 1)C\)**

In this situation, the structure of the previous case still applies. In this case though \( \theta^* = \frac{G}{RC} < \frac{R - 1}{R} \), which means that the investment is viable from the concessionaire’s standpoint, and that there is an equilibrium solution where the project is executed. Due to the restriction on \( G \), the case may be less interesting from a practical standpoint.

**Appendix A.3. Effect of \( \lambda \) on optimal policy**

For \( \theta \leq \frac{R - 1}{R} \), and given the optimal response functions, the government’s utility maximization problem is

\[
\max_{\theta \geq 1, \; \tau \geq 1} (1 - \theta)CR + \frac{1}{2}(1 - \tau)^2 w^2 \tag{A.22}
\]

subject to:

\[\theta CR + \tau (1 - \tau) w^2 \geq G \tag{A.23}\]

\[\theta \leq \frac{R - 1}{R} \tag{A.24}\]

The Lagrangean for the above problem is

\[L_\lambda(\theta, \; \tau, \; \mu_1, \; \mu_2, \; s_1, \; s_2) = \lambda (1 - \theta)CR + \frac{1}{2}(1 - \tau)^2 w^2 + \mu_1 [\theta CR + \tau (1 - \tau) w^2 - s_1^2 - G] + \mu_2 [-\theta - s_2^2 + \frac{R - 1}{R}], \]

where \( \mu_1, \mu_2 \geq 0 \)

The first-order conditions are

\[
\frac{\partial L_\lambda}{\partial \theta} = 0 = -\lambda CR + \mu_1 CR - \mu_2 \tag{A.25}
\]

\[
\frac{\partial L_\lambda}{\partial \tau} = 0 = -(1 - \tau)w^2 + \mu_1 w^2 - 2\tau \mu_1 w^2 \tag{A.26}
\]

\(^9\) This also requires \( \frac{1}{4}w^2 \geq C \). Otherwise, the case of \( G < RC \) applies.
\[ \frac{\partial L_1}{\partial \mu_1} = 0 = \theta CR + \tau (1 - \tau) w^2 - s_1^2 - G \]  \hspace{1cm} (A.27)

\[ \frac{\partial L_1}{\partial \mu_2} = 0 = -\theta - s_2^2 + \frac{R - 1}{R} \]  \hspace{1cm} (A.28)

\[ \frac{\partial L_1}{\partial \delta_1} = 0 = \mu_1 \left[ \theta CR + \tau (1 - \tau) w^2 - G \right] = 0 \]  \hspace{1cm} (A.29)

\[ \frac{\partial L_1}{\partial \delta_2} = 0 = \mu_2 \left[ -\theta + \frac{R - 1}{R} \right] = 0 \]  \hspace{1cm} (A.30)

We use \( \hat{\theta}, \hat{\tau}, \hat{\mu}_1, \hat{\mu}_2, \hat{s}_1, \hat{s}_2 \) to represent a solution to the above system of equations. Noticing that Equations (A.26)–(A.30) are identical to (A.5)–(A.9), we construct a solution similar in structure to the one presented in Appendix A.1. Specifically, \( \theta = \theta^*, \hat{\tau} = \hat{\tau}^*, \hat{\mu}_1 = \mu_1^*, \hat{\mu}_2 = (\mu_1^* - \lambda) CR, \hat{s}_1 = s_1^*, \hat{s}_2 = s_2^* \) satisfies the above system of equations when

\[ \mu_2 > 0 \iff \lambda \leq \mu_1^* = \frac{1 - \tau_1}{1 - 2\tau_1} \]

For the case of \( \lambda > \frac{1 - \tau_1}{1 - 2\tau_1} \), i.e., when the government assigns sufficient weight to the concessionaire’s utility, we surmise that Constraint (A.24) is no longer binding, leading to a solution where \( \mu_2 = 0, s_2^* > 0 \). From (A.25), \( \hat{\lambda} = \lambda \), which, in turn, reduces (A.26) to

\[ -(1 - \tau) w^2 + \lambda w^2 - 2\tau \lambda w^2 = 0 \iff \hat{\tau} = \frac{1 - \lambda}{1 - 2\lambda} \]

Now, assuming (A.27) is binding leads to \( \hat{s}_1 = 0 \) and that \( \hat{\theta} = G - \hat{\tau} (1 - \hat{\tau}) w^2 \)

In this case, we specify additional requirements to achieve \( \hat{\theta} = \frac{R - 1}{R} \) with a sufficient large \( \lambda \). Notice that \( \lambda \geq 1 \), and thus, \( 0 \leq \hat{\tau} \leq \frac{1}{2} \) is needed to satisfy \( \hat{\mu}_1 \geq 0 \). Another observation is that when \( \lambda > \frac{1 - \tau_1}{1 - 2\tau_1} \geq 1 \),

\[ \hat{\tau} - \tau_1 = \frac{1 - \lambda}{1 - 2\lambda} - \tau_1 = \frac{(1 - \tau_1) - \lambda (1 - 2\tau_1)}{1 - 2\lambda} > 0 \]

Note that \((R - 1)C \leq G \leq (R - 1)C + \hat{\tau} (1 - \hat{\tau}) w^2\), instead of the previous \((R - 1)C \leq G \leq (R - 1)C + \frac{1}{4} w^2\), leads to \( G < (R - 1)C + \hat{\tau} (1 - \hat{\tau}) w^2 \Rightarrow \hat{\theta} < \frac{R - 1}{R} \)

The interpretation here is that for a sufficiently large weight, \( \lambda \), the government sets \( \hat{\theta} < \theta^*, \hat{\tau} > \hat{\tau}^* \), i.e., the tax burden is switched from the concessionaire to the service providers. When \( 1 \leq \lambda \leq \frac{1 - \tau}{1 - 2\tau} \), the policy presented in Lemma 3.1 still applies.

References


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\(^{10}\) A simple derivation can show that when \( \hat{\tau} = \frac{1 - \lambda}{1 - 2\lambda} \), then \( \hat{\tau} (1 - \hat{\tau}) < \frac{1}{4} \).