Using Advanced Inspection Technologies to Support Investments in Maintenance and Repair of Transportation Infrastructure Facilities

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Abstract: We present a statistically rigorous and computationally efficient framework that can exploit the extensive capabilities of advanced inspection technologies to support investment decisions in maintenance and repair of transportation infrastructure facilities. The framework consists of two components: a state-estimation problem that involves processing arrays of condition data and using them to develop condition forecasts; and an optimization problem whose solution yields maintenance and repair policies. Through a computational study, we illustrate how the framework can be used to quantify the effect of uncertainties both in the deterioration process and in the data collection process on the optimal life-cycle costs of managing infrastructure facilities. We also show how the framework can be used to quantify the benefits associated with combining inspection technologies to monitor infrastructure facilities, and therefore can serve as a tool to develop deployment strategies for these technologies.

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Introduction

Recent developments in remote sensing and communications technologies allow agencies to install sensors within infrastructure facilities, such as pavements or bridge decks, in order to collect condition-related data. In addition, a plethora of automated, nondestructive inspection technologies, e.g., video, radar, and laser, have become commonplace in evaluating and measuring distresses on transportation infrastructure. In theory, such data should be processed and used as a key component to support maintenance and repair (M&R) decision-making. The reality facing public works agencies that have adopted these technologies is that vast amounts of data are accumulated but not used to address management needs. The goal of the research described herein, therefore, is to develop methodological tools to exploit the extensive capabilities of advanced monitoring technologies to support M&R investment decisions. In particular, the framework we propose allows agencies to both process condition data efficiently and use them to support M&R decision-making.

The framework can also be used to quantify the value of combining different monitoring technologies, and hence can serve as a tool to develop deployment strategies for advanced inspection technologies.

The emergence of advanced inspection technologies poses serious methodological and computational challenges to the development of optimization models to support M&R decisions. This is due to the potentially large quantities of data being generated, as well as the need to account for correlations, errors, and uncertainties in the data. In this paper, we present a framework that is both statistically rigorous and computationally efficient to address the problem of developing optimal M&R investment policies for transportation infrastructure facilities. The framework addresses two subproblems: a state-estimation problem that involves processing condition data and using them to develop condition forecasts; and an optimization problem whose solution yields M&R policies. The approach differs from the existing literature in that the optimization model accommodates sophisticated statistical tools, and therefore can fully exploit the capabilities of advanced technologies to support M&R investment decisions.

The remainder of the paper is organized as follows. “Literature Review” presents a review of the relevant literature and discusses the shortcomings that motivate the need for the framework that we propose. The framework and the assumptions that we use to solve the problem are discussed in “Model Formulation.” In “Computational Study,” we use an empirical study to illustrate how the framework can be used to quantify the effect of uncertainties in the deterioration and data-collection processes on optimal life-cycle costs. We also illustrate how the framework can be used to quantify the value of combining different technologies for condition assessment, and consequently can be used to evaluate strategies to adopt inspection technologies. A summary of the contributions of this research is presented in “Conclusions.”
In this section, we present an overview of optimization models used to support investment decisions for M&R of transportation infrastructure. We also discuss the limitations that motivate the need to develop a framework that can exploit the capabilities of advanced monitoring technologies.

Optimization models to support M&R decision-making constitute applications, perhaps the most successful, of the “Equipment Replacement Problem” introduced by Terborgh (1949) and formulated as a dynamic control problem by Bellman (1955) and Dreyfus (1960). Friesz and Fernandez (1979) and Golabi et al. (1982) extended the models to support M&R of transportation infrastructure. State-of-the-art models are formulated as Markov decision processes. Golabi et al. (1982), for example, present a mixed-criteria, constrained, Markov decision process (MDP) for pavement management in the state of Arizona (a network of 12,000 km of highways). Savings of $14 million were reported in the first year of implementation, and $101 million was forecast for the following 4 years. The same optimization model drives Pontis (Golabi and Shepard 1997), a bridge management system used in over 40 states. The success and impact of these models is related to the magnitude of investments in M&R of transportation infrastructure which in the United States is on the order of tens of billions of dollars per year. Recent reviews of optimization models for transportation infrastructure management are presented in Gendreau and Soriano (1998) and Durango (2002).

Optimization models to support M&R investments evaluate both the short- and long-term economic consequences associated with M&R decisions. This involves processing data related to the current infrastructure condition and using it to forecast the effect of actions on a future condition. The economic consequences associated with M&R decisions are then predicted by assuming a correspondence between infrastructure condition and costs. Information about current infrastructure condition is obtained through distress measurements. Distress measurements can be collected manually or automatically and are comprised of multiple measurements and/or (subjective) ratings that can be either discrete or continuous. Examples of distresses in pavement management include roughness, type and extent of cracking, rut depth and profile, extent of surface patching, and raveling. Information about future condition, i.e., condition forecasts, are generated with deterioration models. A deterioration model is a statistical expression that relates a condition to a set of explanatory variables and infrastructure condition. The deterioration model captures the inherent uncertainty in generating condition forecasts.

Empirical studies (Ben-Akiva and Ramaswamy 1993; Ben-Akiva and Gopinath 1995) have shown that latent performance models are appropriate to generate condition forecasts of transportation infrastructure, i.e., the goodness-of-fit measures are better than those reported using other statistical methods. This led Madanat and Ben-Akiva (1994) to include latent performance models into a framework to support M&R investments by formulating the underlying optimization problem as a latent MDP.

Unfortunately, the latent MDP suffers from computational limitations that make it impractical to support investments in M&R of transportation infrastructure when multiple technologies are used simultaneously to measure different distresses. This is because the process of finding an optimal action for a given period involves enumerating and assigning a probability to every possible outcome of the data-collection process [see, for example, Eq. (9) in Madanat and Ben-Akiva (1994)]. The number of outcomes, and hence the number of probabilities that need to be estimated and the computational effort to obtain M&R policies increases exponentially with the number of distresses being measured. For example, if technologies, each with a range discretized as \{1,2,...,10\}, require enumerating and estimating probabilities for 10^n possible outcomes. To a large extent, this difficulty explains why previous studies in the literature have only considered the case of inspections that yield a single distress measurement [cf. Madanat and Ben-Akiva (1994); Smilowitz and Madanat (2000); Guillaumot et al. (2003)]. This limitation serves as motivation for the framework proposed in this paper which constitutes an alternative to the latent MDP that is both statistically rigorous and computationally efficient.

Model Formulation

In this section, we describe the framework we propose to obtain optimal M&R policies. Prior to presenting the framework’s two components: optimization and state-estimation, we describe in detail the problem we consider. We then present a generic optimization model to address the problem, and discuss the assumptions that we make to solve it.
Problem Description and Mathematical Formulation

We consider an agency that manages a facility under a periodic review policy over \( T \) periods. At the start of every period, \( t=1,2,...,T \), the agency collects sets of distress measurements, represented with the vectors \( \widetilde{Z}_t \), and decides to take an action \( A_t \).

The condition data, \( \widetilde{Z}_n \), are related to a facility’s condition represented by the random variable \( X_t \). The choice of action depends on the set of information that an agency has at its disposal and results in a cost \( g(X_t,A_t) \in \mathbb{R} \). We use \( I_t \) to represent the set of information that an agency has at its disposal at the start of period \( t \). Using our notation

\[
I_t = \{ \widetilde{Z}_1, A_1, \widetilde{Z}_2, A_2, \ldots, \widetilde{Z}_{t-1}, A_{t-1}, \widetilde{Z}_t \} = \{ I_{t-1}, A_{t-1}, \widetilde{Z}_t \},
\]

\( t = 1, \ldots, T + 1 \) (1)

The cost structure depends both on the action and on the current facility condition, and therefore can be used to capture both agency and operating costs. In the management of transportation infrastructure, agency costs correspond to the costs of applying M&R actions and operating costs to both agency and operating costs. In the management of transportation systems, i.e., its physical deterioration process.

1. The variables in the model are assumed to be defined over continuous spaces, i.e., \( X_t, A_t \in \mathbb{R} \), where \( t=1,2,\ldots,T+1 \) and \( \tilde{Z}_t \in \mathbb{R}^n \), \( t=1,2,\ldots,T+1 \).

2. The period cost function can be represented (or approximated) by a second order polynomial, i.e.,
\[
g(X_t,A_t) = a_t X_t^2 + b_t X_t A_t + c_t A_t^2 + d_t X_t + e_t A_t + f_t
\]
\( (a_t, b_t, c_t, d_t, e_t, f_t) = (a, b, c, d, e, f) \), \( \forall t \).

3. The salvage value function can be represented (or approximated) by a second order polynomial, i.e.,
\[
s(X_{T+1}) = -p_{T+1} X_{T+1}^2 - q_{T+1} X_{T+1} + r_{T+1}
\]

4. The system equation can be represented (or approximated) by a linear model, i.e.,
\[
D(X_t, A_t) = g_t X_t + h_t A_t + \epsilon_t.
\]

The polynomial can be obtained by estimating an autoregressive moving average with exogenous input (ARMAX) model. We assume that \( \epsilon_t \) follows a normal distribution with mean \( \bar{\epsilon}_t \) and finite variance \( \sigma^2_{\epsilon_t} \) and

5. We assume that the measurement-error model can be represented (or approximated) by a linear model, i.e.,
\[
\mathcal{M}(X_t) = H_t X_t + \xi_t.
\]

The vector \( \xi_t \) is assumed to follow a Gaussian distribution with finite covariance matrix, \( R_t \).

Prior to discussing a solution procedure for the problem, we note that the above assumptions are not overly restrictive. Specifically,

- The framework does not require a discrete representation of condition and distress measurements, as is the case with MDP formulations. This is attractive because the process of discretizing these elements involves approximations that induce errors, as described by Madanat et al. (1995) and Mishalani and Madanat (2002);
- The decision variables in the framework, \( A_t \), \( t=1,2,\ldots,T \), can be interpreted as investment magnitudes or as maintenance rates;
- The assumptions about costs are not restrictive because, for example, it is possible to obtain optimal M&R investment policies for general cost structures by solving a finite sequence of problems. In each problem the cost structure is approximated by a second-order Taylor series expanded about a different point. This issue is discussed further in Dreyfus (1977);
- The linear structure assumed for \( D(\cdot) \) and \( \mathcal{M}(\cdot) \) is actually quite general as a number of transformations can be employed to capture complex patterns/structures in the data. ARMAX models represent a broad class of time series models; and
- The assumptions imposed on the random error terms \( \epsilon_t \) and \( \bar{\epsilon}_t \) are mild because they are consistent with obtaining adequate estimates of the models (unbiased parameters).

Optimization Problem

With the assumptions discussed in the preceding section, the optimization problem presented earlier can be formulated as a dynamic program. The optimal objective function, \( \mathcal{V}(I_t) \), is defined as the minimum expected discounted cost from the start of \( t \) until the end of the horizon given the information available at the start of \( t, I_t \). The recurrence relation is as follows:

\[
\mathcal{V}(I_t) = \min_{A_t} \left\{ E_{X_t} \left[ g(X_t, A_t) + \delta T \sum_{n=1}^{T} \mathcal{V}(I_{n+1}) - \delta T \mathcal{V}(I_{n+1}) \right] \right\}
\]

(6)

The boundary condition for the problem is as follows:

\[
\mathcal{V}(I_{T+1}) = E_{X_{T+1}} \left[ p_{T+1} X_{T+1}^2 + q_{T+1} X_{T+1} + r_{T+1} \right]
\]

(7)

Assumptions

We proceed to state and discuss the assumptions that we use to solve the mathematical model used to compute M&R policies. The assumptions are that:

- The linear structure assumed for \( D(\cdot) \) and \( \mathcal{M}(\cdot) \) is actually quite general as a number of transformations can be employed to capture complex patterns/structures in the data. ARMAX models represent a broad class of time series models; and
- The assumptions imposed on the random error terms \( \epsilon_t \) and \( \bar{\epsilon}_t \) are mild because they are consistent with obtaining adequate estimates of the models (unbiased parameters).
Under the assumptions discussed in the previous section, the dynamic programming formulation leads to a solution that can be obtained inductively and expressed in closed-form with parameters computed recursively as follows:

\[ A_t^o = \left[ \frac{b_t + 2p_{t+1}g_t h_t^2}{2c_t + 2p_{t+1}h_t^2} E[X_t|I_t] + \frac{2p_{t+1}h_t e_t}{2c_t + 2p_{t+1}h_t^2} \right] \]

\[ t = T, \ldots, 1 \]  

(8)

\[ p_t = a_t + p_{t+1}g_t \]

\[ (b_t + 2p_{t+1}g_t h_t^2)^2 \]

\[ 4[c_t + p_{t+1}h_t^2] \]

\[ t = T, \ldots, 1 \]  

(9)

\[ q_t = d_t + 2p_{t+1}e_t g_t + q_{t+1}g_t \]

\[ \frac{(b_t + 2p_{t+1}g_t h_t^2)[c_t + p_{t+1}h_t^2]}{2[c_t + p_{t+1}h_t^2]} \]

\[ t = T, \ldots, 1 \]  

(10)

\[ r_t = f_t + p_{t+1}(\bar{e}_t^2 + \sigma^2_t) + q_{t+1}e_t + r_{t+1} \]

\[ \frac{[c_t + p_{t+1}h_t^2]}{2[c_t + p_{t+1}h_t^2]} \]

\[ t = T, \ldots, 1 \]  

(11)

where \( A_t^o \) represents the optimal investment level for period \( t \), and is expressed as a function of the parameters \( p_t, q_t, r_t \).

As shown in Dreyfus (1977) and Bertsekas (2000), these equations are derived from the first-order/necessary conditions for the problem. The second-order/sufficiency conditions are satisfied because the objective function is convex for \( c_t \geq 0, \forall t \).

The equations are evaluated recursively noting that \( p_{T+1}, q_{T+1}, r_{T+1} \) are the parameters that define the salvage value function. Using the solution to the above system of equations allows us to write the optimal objective value function as

\[ v(I_t) = p_t E[X_t^2|I_t] + q_t E[X_t|I_t] + r_t, \quad t = 1, \ldots, T \]

(12)

The computational effort to obtain an optimal M&R policy, \( A_t^o, t = 1, 2, \ldots, T \), does not depend on the size of the vector \( \tilde{z}_t \). However, we note that to implement the optimal policy and to evaluate the optimal objective value function it is necessary to compute the conditional expected state given the set of information in each period. This step is referred to as the state-estimation problem and it is discussed further in the following section. An important feature of the dynamic programming formulation we propose is that the aforementioned conditional expectations are computed independently and separately from the optimal policy for the problem [Eqs. (8)–(11)]. The key to processing distress measurements generated simultaneously by multiple technologies is to compute these expectations efficiently.

State Estimation Problem

The state estimation problem consists of finding the conditional expected state given the set of information in each period, \( E[X_t|I_t], t = 1, 2, \ldots, T \). Under the assumptions discussed earlier, the expectation can be computed with a recursive algorithm known as the Kalman filter. The algorithm (for the special case where \( \tilde{z}_t \) follows a Gaussian distribution with zero mean) is presented below.

Kalman filter algorithm

Repeat at the start of each period:

- Given: \( E[X_t|I_{t-1}], \mathbb{V}(X_t|I_{t-1}), A_{t-1} \), and \( \tilde{z}_t = \tilde{z}_t \)
- Define: \( \hat{X}_{t-1} = E[X_{t-1}|I_{t-1}], P_{t-1} = \mathbb{V}(X_{t-1}|I_{t-1}) \), and \( I_t = (I_{t-1}, A_{t-1}, \tilde{z}_t) \)
- Time update:
  \( \hat{X}_t = g_{t-1}\hat{X}_{t-1} + h_{t-1}A_{t-1} \)
  \( P_t = g_{t-1}^2P_{t-1} + \sigma^2_{t-1} \)
- Measurement update:
  \( K_t = P_t H^T(P_t H H^T + R)^{-1} \)
  \( E[X_t|I_t] = \hat{X}_t + K_t(\tilde{z}_t - H\hat{X}_t) \)
  \( \mathbb{V}(X_t|I_t) = (1 - K_t H)P_t \)

The time update step uses the system equation to project the estimates of the conditional expectation and variance, i.e., the first two moments of the state distribution (which is normal under the assumptions presented earlier). The measurement update step revises (with Bayes’ law) the expectation and variance taking into account the new set of measurements obtained at the start of period \( t \), \( \tilde{z}_t \). The computational effort of the Kalman filter (and hence the effort of the proposed framework) increases polynomially with the size of the vectors \( \tilde{z}_t \) which means that the framework does not suffer from the shortcomings of the latent MDP approach. This is because the number of operations in the Kalman filter increases polynomially with the dimensions of the matrices \( H \) and \( R \), which increase linearly with the size of the vectors \( \tilde{z}_t \).

Computational Study

In this section, we present a computational study where we:

1. Provide numerical examples to illustrate the methodology presented above to address the state-estimation problem in the proposed framework. In particular, we use a set of simulated data to show how the Kalman filter processes distress measurements to update the state distribution;
2. Use the framework to study the effect of uncertainty both in the deterioration process and in the data-collection process on the optimal life-cycle cost of managing infrastructure facilities; and
3. Use the framework to quantify the value of combining different technologies for condition assessment.

Numerical Example: State-Estimation Problem

To illustrate how the Kalman filter addresses the state-estimation problem in the above framework, we consider the management of a pavement over a 40 years planning horizon. The initial condition of the pavement is 10 given in a scale with range [0, 100]. The deterioration and measurement-error models in the example are given by

\[ x_{t+1} = x_t + 8 - A_t \]  

(13)
Fig. 1. Updated state-distribution: First moments

Fig. 2. Updated state-distribution: Second moments
$$Z_t = X_t + \xi_t;$$

where $\xi_t$ is normally distributed with $\mu_{\xi_t} = 0$ and $\sigma_{\xi_t}^2$. (14)

That is, we assume that the pavement deteriorates deterministically at a rate of 8 units per year, and that the distress measurements correspond to the actual condition plus a random error term/white noise. The parameter $\sigma_{\xi_t}^2$ represents the precision of the technology used to collect the distress measurements.

We also assume that the pavement is restored to its initial condition every 10 years, i.e.,

$$A_t = \begin{cases} 
80; & t = 11, 21, 31, 41 \\
0; & \text{otherwise} 
\end{cases} \quad (15)$$

Finally, we assume that the initial, estimated state-distribution is normal with $E[X_1|I_1]=25$ and $\text{Var}(X_1|I_1)=20$.

To illustrate how the Kalman filter uses the sequence of distress measurements to update the state-distribution we simulated an instance of the above process. The solid line in Fig. 1 represents the true condition of the pavement over time. The triangles represent a set of randomly generated distress measurements that are consistent with the measurement-error model in Eq. (14). We use $\sigma_{\xi_t}^2=10$. The dashed line corresponds to the first moment of the estimated state-distribution. The figure shows how the condition estimate converges to the true condition of the pavement (over time, the dashed line traces the solid line).

Fig. 2 shows how the Kalman filter updates the second moment of the estimated state-distribution. In this part of the study we considered the effect of technologies of different precisions to collect distress measurements. Specifically, we consider cases where $\sigma_{\xi_t}^2=0, 2, 10$, i.e., “perfect,” “fine,” and “coarse” technologies used to collect measurements. We also consider the case where two coarse technologies with $\sigma_{\xi_t}^2=10$ were used to collect distress measurements simultaneously (the technologies were assumed to be independent of each other). We observe that the variance in the estimated state-distribution becomes very small very quickly. The asymptote and the convergence rate are properties of the technologies. The key observation is that the variance in the estimated state distribution is well within the precision of each technology, i.e., the procedure filters out the random error/noise in the measurements. For example, the variance in the state distribution when measurements are collected with the coarse technology ($\sigma_{\xi_t}^2=10$) converges to approximately 1 after 10 years.

Table 1. Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.024</td>
</tr>
<tr>
<td>b</td>
<td>−0.006</td>
</tr>
<tr>
<td>c</td>
<td>0.003</td>
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<td>d</td>
<td>−1.262</td>
</tr>
<tr>
<td>e</td>
<td>0.795</td>
</tr>
<tr>
<td>f</td>
<td>14.650</td>
</tr>
</tbody>
</table>

![Fig. 3. Life-cycle costs versus deterioration process variance](image-url)
Empirical Study of the Effect of Uncertainty on Life-Cycle Costs

We present an empirical study to investigate the effect of uncertainties, both in the deterioration process and in the data-collection process, on the minimum expected discounted costs of managing pavements. This type of study allows agencies to quantify the value of using different technologies, and therefore provides critical decision-support in the process of adopting advanced inspection technologies for condition assessment.

In this part of the study we consider instances of managing the pavement described in the previous section. In order to use the framework that we proposed, it is necessary to specify a second-order polynomial that represents the per-period cost function $g(x,t)$. To this end, we adapted the cost functions presented in Madanat and Ben-Akiva (1994) (the details are presented in Appendix A). The parameters to specify the cost function are presented in Table 1. We set the residual value of a facility to zero $s(x_r)=0$, and used a discount rate of 5% $(\delta=0.9524)$. 

In this part of the study we considered the same technology choices that we used in the previous section. However, instead of considering a deterministic deterioration process we let $\sigma_d^2$ be 0.1, 1, 2, 4, and 8. For each combination of technology and deterioration process variance we calculated the average expected cost of managing 100 pavement sections. The optimal policies were obtained by solving the optimization model presented earlier. The results appear in Fig. 3.

From the figure we observe that, as expected, the costs to manage the pavements increase as the uncertainty in the deterioration process grows. Also as expected, coarse data collection technologies result in higher costs incurred. Interestingly, we notice that the cost of using the combination of coarse technologies falls roughly halfway in between the costs of using the coarse or the fine technologies independently. This type of analysis can be used together with the costs of adopting technologies to obtain an effective data collection strategy. For example, it is conceivable that the cost of adopting the fine technology is not justified by the benefits that will accrue from using it in the management process. In the next section, we further explore the issue of quantifying the value of combining different technologies for condition assessment.

Combining Multiple Technologies for Condition Assessment

As discussed earlier, agencies often use multiple technologies to collect distress measurements simultaneously. From the previous section, technology choice would seem to be dictated by precision. If this were true, then what would be the value of collecting additional data (with coarse technologies)? Here, we show that combining various technologies can result in benefits even when compared to a single technology with high precision. The analysis also provides insights that can be used when adopting inspection technologies.

To obtain a measurement-error model for the analysis, we build on the statistical work in Ben-Akiva and Gopinath (1995). They consider the case of collecting five distress measurements: roughness (RQI), cracking (CRX), rutting (RDMN), surface patching (SPAT), and raveling (RAV). The model is presented in Appendix B.

Numerical Results

In our study we considered the same setup as in “Empirical Study of the Effect of Uncertainty on Life-Cycle Costs” (deterioration model, cost functions, planning horizon, and discount rate). We assumed that the deterioration model described the progression of roughness. In order to highlight the effect of technology precision we set the variance in the deterioration process, $\sigma_d^2$, to be 0.1.

Table 2 presents the average (over 100 instances) of the minimum expected costs for all possible combinations of the different technologies. The third column in the table corresponds to the percentage of the difference between the costs of using

<table>
<thead>
<tr>
<th>Technology</th>
<th>Average minimum cost (dollars)</th>
<th>Percentage of largest difference</th>
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</thead>
<tbody>
<tr>
<td>RQI CRX RDMN SPAT RAV</td>
<td>338.5218</td>
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</tr>
<tr>
<td>SPAT</td>
<td>356.5109</td>
<td>89.46</td>
</tr>
<tr>
<td>RAV</td>
<td>358.7532</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Table 3. Discretization and Transformation of Pavement Condition Index (PCI) Scale

<table>
<thead>
<tr>
<th>State</th>
<th>PCI range</th>
<th>Modified roughness scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0–20</td>
<td>80–100</td>
</tr>
<tr>
<td>1</td>
<td>20–40</td>
<td>60–80</td>
</tr>
<tr>
<td>2</td>
<td>40–50</td>
<td>50–60</td>
</tr>
<tr>
<td>3</td>
<td>50–60</td>
<td>40–50</td>
</tr>
<tr>
<td>4</td>
<td>60–70</td>
<td>30–40</td>
</tr>
<tr>
<td>5</td>
<td>70–80</td>
<td>20–30</td>
</tr>
<tr>
<td>6</td>
<td>80–90</td>
<td>10–20</td>
</tr>
<tr>
<td>7</td>
<td>90–100</td>
<td>0–10</td>
</tr>
</tbody>
</table>
a particular combination of technologies and a case of perfect inspections yielding the true condition of the pavement (average minimum expected costs $337.4802 per m²). This difference is taken relative to the difference in costs that results when the system is managed while collecting only raveling measurements. The main observations from the simulation are as follows:

- We can see from Table 2 that the least minimum expected cost occurs when all five technologies are combined together. This shows that we do obtain a better performance by combining different technologies;
- We also notice that the costs are not only dependent on technology precision, \( \sigma_{T}^2 \), but also on how the distress measurements relate to each other. For example, the technology used to collect measurements of surface patching (SPAT) is highly precise when compared to other technologies. However, the value of \( \lambda_2 \) is 0.167. Cracking (CRX) is the least accurate of all the technologies but \( \lambda_2=1.503 \) which means that the measurements are closely related to the latent variable that we are trying to measure. We notice that collecting measurements of cracking is more cost-effective than collecting measurements of raveling.

Conclusions

We have developed an optimization framework to provide support for investments in preservation and improvement of transportation infrastructure facilities that are inspected periodically with sensors or other advanced technologies. This work was motivated by recent developments in remote sensing and communications technologies that have increased the availability and cost-effectiveness of using advanced technologies; and by statistical and computational limitations associated with existing optimization models to support investment decisions. These limitations are related to their inability to process condition data collected simultaneously using multiple technologies.

The framework we presented involves formulating the underlying decision problem as a discrete-time, stochastic optimal control problem and consists of two components: a state-estimation problem that involves processing arrays of condition data and using them to develop condition forecasts; and an optimization problem whose solution yields M&R investment policies. Our approach differs from the literature in that both elements are fully integrated. This, in turn, leads to a framework that is both statistically rigorous and computationally efficient, i.e., capable of providing effective decision-support.

Through empirical studies, we provide numerical examples to illustrate the methodology presented above to address the state-estimation problem in the above framework. In particular, we show how the Kalman filter processes distress measurements to update the state distribution. We then showed how the proposed framework can be used to study the effect of uncertainties in the deterioration process and in the process of collecting distress measurements on the optimal life-cycle cost of managing infrastructure facilities. We also illustrated how the framework can be used to quantify the value of combining different technologies for condition assessment. This is the first study in the area of transportation infrastructure management to quantify the costs incurred when inspection technologies are combined for condition assessment.

Acknowledgment

The writers gratefully acknowledge the financial support of the Midwest Regional University Transportation Center.

Appendix A. Cost Parameter Generation

To estimate the parameters needed to specify the period cost function \( g(X_t,A_t) \), we used data from the empirical study presented in Madanat and Ben-Akiva (1994) with minor modifications. In that study, the states used to represent pavement condition are obtained by discretizing the PCI scale into 8 states. From this discretization we constructed a modified roughness scale to be consistent with the assumption that as the variable used to represent condition, \( X_t \), increases, the condition worsens. The two scales are shown in Table 3.

The agency and user costs used in the aforementioned study are presented in Table 4 and are a function of the discrete states shown above and four M&R actions. Each entry in the table is labeled with a corresponding state-action pair in the domain of the period cost function. The “do nothing” action was assumed to have no effect on facility condition, i.e., \( A_t=0 \). “Routine maintenance” was assumed to prevent the facility from deteriorating, i.e., \( A_t=8 \). The effects of the other two actions on improvements (measured as reductions on the modified roughness scale) were

Table 4. Agency and User Costs [Adapted from Madanat and Ben-Akiva (1994)] ($/m²)

<table>
<thead>
<tr>
<th>State</th>
<th>Do nothing</th>
<th>Routine maintenance</th>
<th>Overlay (2 in.)</th>
<th>Reconstruction</th>
<th>User costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(90,0) → 0</td>
<td>(90,8) → 6.9</td>
<td>(90,51.5) → 21.81</td>
<td>(90,91.5) → 25.97</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>(70,0) → 0</td>
<td>(70,8) → 2</td>
<td>(70,41.5) → 12.31</td>
<td>(70,71.5) → 25.97</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>(55,0) → 0</td>
<td>(55,8) → 1.4</td>
<td>(55,36.5) → 10.69</td>
<td>(55,56.5) → 25.97</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>(45,0) → 0</td>
<td>(45,8) → 0.83</td>
<td>(45,36.5) → 9.06</td>
<td>(45,46.5) → 25.97</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>(35,0) → 0</td>
<td>(35,8) → 0.65</td>
<td>(35,36.5) → 6.64</td>
<td>(35,36.5) → 25.97</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>(25,0) → 0</td>
<td>(25,8) → 0.31</td>
<td>(25,26.5) → 4.11</td>
<td>(25,26.5) → 25.97</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>(15,0) → 0</td>
<td>(15,8) → 0.15</td>
<td>(15,16.5) → 3.91</td>
<td>(15,16.5) → 25.97</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>(5,0) → 0</td>
<td>(5,8) → 0.04</td>
<td>(5,6.5) → 3.81</td>
<td>(5,6.5) → 25.97</td>
<td>0</td>
</tr>
</tbody>
</table>
the data used to estimate the model are presented in Table 5. The statistics that describe different combinations of technologies in Cuba, Brazil, and Mexico are also necessary to estimate the parameters using linear regression. The data come from Table 4.

### Appendix B. Latent Performance and Measurement-Error Models

The performance and measurement-error models from Ben-Akiva and Gopinath (1995) are presented below:

\[
X = \alpha_1 \text{AGER} + \alpha_2 \text{ESAX} + \alpha_3 \text{CP} + \epsilon \quad (16)
\]

where \(X\) is latent variable representing condition. The condition is specified to be influenced by the following factors:

- AGER = time since the last rehabilitated pavement structural number;
- ESAX = cumulative equivalent standard axle load since last rehabilitation;
- CP = cumulative precipitation since last rehabilitation.

The error variance is obtained by calculating the expected improvement using the transition probabilities in Madanat and Ben-Akiva (1994). To obtain the parameters presented in Table 1 we assumed that \(g(X, \lambda)\) could be represented as a second-order polynomial and estimated the parameters using linear regression. The data come from Table 4.

### Table 6. Parameter Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>3.562</td>
<td></td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>3.897</td>
<td></td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>1.654</td>
<td></td>
</tr>
<tr>
<td>(\lambda_1)</td>
<td>1.000</td>
<td>0.37</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>1.503</td>
<td>0.35</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>0.167</td>
<td>0.45</td>
</tr>
<tr>
<td>(\lambda_4)</td>
<td>0.256</td>
<td>0.09</td>
</tr>
<tr>
<td>(\lambda_5)</td>
<td>0.531</td>
<td>0.04</td>
</tr>
</tbody>
</table>

obtained by calculating the expected improvement using the transition probabilities in Madanat and Ben-Akiva (1994).

The transition probabilities in Madanat and Ben-Akiva (1994) were inferred from the square multiple correlations (SMCs) reported in the study. The SMC measures the fraction of the variance explained by the model. The error variances for the five distress measurements are presented in Table 7.

### References


### Table 7. Precision Estimates

<table>
<thead>
<tr>
<th>Technology</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roughness</td>
<td>148.6356</td>
</tr>
<tr>
<td>Cracking</td>
<td>429.3185</td>
</tr>
<tr>
<td>Rut depth</td>
<td>7.570225</td>
</tr>
<tr>
<td>Surface patching</td>
<td>68.088</td>
</tr>
<tr>
<td>Raveling</td>
<td>372.5664</td>
</tr>
</tbody>
</table>