Abstract

I investigate the optimal design of interventions to stabilize financial institutions subject to rollover risk. A policy-maker facing the potential default of a bank discloses information about the long-term profitability of its assets and its liquidity position to multiple audiences: short-term creditors, external investors, taxpayers, and the bank itself. I characterize the optimal comprehensive disclosure policy and show that when the quality of the assets is above a threshold, the test assigns a coarse pass grade. In turn, when the quality of the assets is poor, the test assigns one of multiple failing grades and complements that grade with a follow up pass-fail test on the bank’s short-run liquidity position. Additionally, the policy-maker imposes contingent capital requirements. I find that without the latter, disclosure of information about the bank’s fundamentals may be ineffective. When the regulator lacks the technology to timely respond to liquidity shocks, she designs a liquidity-provision program whereby the government offers to buy assets from the bank in exchange for cash and a public disclosure of the bank’s liquidity position. Interventions display a non-monotone pecking order: the private sector funds banks with either high or poor-quality assets, while institutions with assets of intermediate performance participate in the government’s liquidity program. My results shed light on the optimal way to disclose information in environments with multiple audiences and multi-dimensional fundamentals.

JEL classification: D83, G28, G33.

Keywords: Stress Tests, Information Design, Capital Requirements, Security Design, Mechanism Design.

*Email: nicolasinostroza2018@u.northwestern.edu. I am deeply indebted to my advisors Alessandro Pavan, Mike Fishman, and Jeff Ely for their continuous support and encouragement. This paper has greatly benefitted from extended conversations with Nicolas Figueroa. I also thank Eddie Dekel, Piotr Dworczac, Konstantin Milbradt and Isaias Villamizar for their valuable feedback. The usual disclaimer applies.
1 Introduction

Information disclosure has become a prominent tool in banking supervision since the global financial crisis. In February 2009, the Federal Reserve introduced the Supervisory Capital Assessment Program (SCAP), commonly known as the Fed’s stress test. The objective of the program was to assess whether the capital buffers of the 19 largest bank holding companies were enough to sustain lending in the event of an unexpectedly severe recession, and to communicate these results to the public. The supervisors’ disclosure came at a time when informational asymmetries between inside and outside market participants regarding the soundness of the banking system had disrupted credit channels, leading to unprecedented interbank lending rates, abrupt haircuts in the repo market, and the freeze of capital markets for banks. Many scholars and policy-makers believe that the disclosure of stress tests results was a critical inflection point in the financial crisis because it provided market participants with credible information about potential losses at banks which helped restore market confidence (Bernanke [2013]).

Since their introduction, stress tests and asset quality reviews have been regularly conducted both in the US and in the Eurozone. Despite the consensus on the benefits associated with providing greater transparency on the otherwise opaque banking system, there still exists disagreement regarding the form that such disclosures should take, and the set of policies that should accompany them. While the stress tests conducted by the Fed, for example, have combined granular data with a pass/fail grade\textsuperscript{1}, the European Central Bank decided in 2016 to not assign grades to banks in order to avoid stigmatization. Moreover, while both regulatory authorities complement their disclosures exercises with capital requirements, American regulators have chosen to publicly announce their decisions while their European counterparts have opted for private recommendations\textsuperscript{2}. Perhaps the reason behind such disagreements is that disclosing information is a double edged sword and may be harmful when not done correctly (Goldstein and Sapra [2014] offer an excellent review of the costs associated to disclosure of information).

Part of the difficulty associated with the design of such targeted disclosures comes from the complexity of the interactions among the different market participants involved. To illustrate, observe that when a regulator discloses relevant information about a given bank, it speaks to multiple audiences. Namely, potential investors interested in the quality of the assets on the bank’s balance sheet; short-term creditors concerned by the bank’s liquidity position and its ability to repay its claims; speculators interested in the fate of the bank; counterparties exposed to a potential default of the bank; taxpayers concerned with the use of public funds if a bailout takes place; and the bank itself, which strategically chooses its funding strategy in response to the information disclosed by the

\textsuperscript{1}In 2018, the Fed introduced for the first time an intermediate third grade: conditional non-objection, assigned to Goldman Sachs and Morgan Stanley. Both bank holding companies had to cut by half the amount they intended to distribute among shareholders in order to avoid failing the test.

\textsuperscript{2}The privacy policy does not apply to those companies publicly listed for which capital requirements count as inside information and must be disclosed.
policy-maker.

In this paper I consider the case of a bank that has private information about (i) the quality of its assets and (ii) its liquidity position. Throughout the paper I refer to these two variables as the fundamentals. The bank is solvent at present but faces a maturity mismatch between short-term liabilities and long-term assets and, hence, may be subject to rollover risk in the case of a liquidity shock. To meet former liabilities and as a precautionary measure to minimize the risk of default, the bank may approach external investors and sell claims on its assets (securities) in order to increase the amount of available funds. For instance, the bank can place a bond or issue new shares to increase its buffers, or, alternatively, it can approach the repo market and offer some assets as collateral. The funding strategy chosen by the bank depends on the private information it possesses. Investors understand that there exists a probability that the bank defaults and, consequently, demand a premium that compensates for the risk they incur. Asymmetric information about the bank’s fundamentals, together with the possibility of default, may trigger a freeze in the asset market, leaving the bank vulnerable to default.

A benevolent policy-maker concerned with the potential failure of the bank may decide to intervene. I assume in the first part of the paper that the only tools available to the policy-maker are (i) a technology that allows her to commit to disclose information about the bank’s fundamentals to other market participants, and (ii) the authority to impose capital requirements that limit the bank’s ability to distribute dividends unless the bank succeeds at raising a minimal amount of capital. Importantly, the policy-maker cannot commit to using public funds to help the bank recover in case of distress. I relax this assumption in the second part of the paper, where I allow the designer to act as a lender of last resort and purchase remaining claims on the bank’s assets with taxpayers’ money, under the constraint that the price paid does not exceed the fair price of the securities.

The questions that motivate this work are as follows: (i) What type of information should be disclosed, if any, when taking into account the interaction among market participants? (ii) Assuming that the designer has the same objective as the bank, and that the latter may engage in costly signaling by strategically designing the securities it issues, are there any benefits to allowing the policy-maker to disclose information about the bank’s fundamentals? (iii) What is the role of companion regulatory policies such as the imposition of capital requirements? Are these policies complements to information disclosures or substitutes to them? Also, should capital requirements be publicly disclosed, or should they remain private? (iv) Acknowledging that different audiences worry about variables that are determined at different points in time, how do early disclosures affect later disclosures, and vice versa? What are the trade-offs faced by the regulator when choosing the optimal comprehensive disclosure policy? (v) If the designer lacks the ability to measure some of the variables and, hence, needs to rely on elicitation mechanisms that induce the bank to self-report part of its private information, how do information disclosure and capital requirements interact with the effectiveness of the policy-maker’s program?

These questions are not restricted to the design of intervention policies. Rather, they apply to a
broad set of environments. In fact, they are expected to arise in any context where a firm is subject to a maturity mismatch between short-term liabilities and long-term assets, a common theme in the corporate finance literature. Think, for example, of a firm that wants to undertake a socially desirable project. The project promises to pay off in the future but requires an initial investment and, most likely, some liquidity injections before it starts delivering dividends. If the firm is cash-constrained it may need to sell claims on the project’s cash-flows to fund its operations. In such circumstances, how do information disclosures affect the firm’s ability to raise funds?

To answer these questions I consider a model with a policy-maker and four audiences: short-term creditors, investors, taxpayers, and the bank. Uncertainty about the bank’s fundamentals is gradually resolved. While the quality of the bank’s assets is determined early, its liquidity position is determined at a later stage after a shock materializes. The timing is meant to reflect the idea that the quality of the bank’s assets depends on investment decisions made in the past, while the liquidity position of the bank is subject to shocks and may vary precipitously. The policy-maker’s technology allows her to learn the realization of these variables as soon as they are determined, and to make public announcements as a function of them. As is standard in the information design literature, I assume that the policy-maker has commitment power and chooses the information disclosure policy before observing the true realization of the bank’s fundamentals.

The rich environment proposed in this paper emphasizes the strategic interaction among the multiple audiences, who care about different aspects of the bank’s fundamentals. Consider first external investors and short-term creditors. While the external investors are interested in the long-term profitability of the bank’s assets (e.g., the amount of non-performing loans), short-term creditors are concerned about the liquidity position of the bank and its ability to repay their claims. Nevertheless, external investors care about the disclosure of information regarding the bank’s liquidity position, as such information affects creditors’ beliefs about the bank’s liquidity buffers and, hence, their decisions of whether to keep rolling over the bank’s debt. Given that short-term creditors’ claims are senior to those of potential investors, the latter understand that they may be wiped out if creditors decide to stop pledging to the bank and, hence, they are indirectly affected by disclosures about the bank’s liquidity. In turn, short-term creditors care about the level of funds the bank is able to raise, which in turn depends on the information about the bank’s assets disclosed by the policy-maker. Additionally, the information revealed about the profitability of the bank’s assets affects the price the policy-maker is able to pay for them if she decides to bail the bank out, without violating the interests of tax-payers. Finally, information revealed by the regulator affects the choice of securities the bank sells to external investors.

My first result characterizes the equilibrium of the fund-raising game played by the informed bank and external investors, in the absence of government intervention. The bank issues claims on its assets in exchange for funds, which helps it meet former obligations and creates a precautionary buffer against possible adversarial liquidity shocks. I adopt the framework of Nachman and Noe [1994], who consider the security design problem of a seller with private (but imperfect) information
about the profitability of her assets, and who issues claims on them in exchange for funds that help her meet a former liability. I modify their setting by introducing a probability of default, which is determined in equilibrium. In contrast to their celebrated result, which shows existence of a unique equilibrium where all types of sellers pool over the same debt security, I show that there exist multiple equilibria of the fund-raising stage, and that when investors’ prior beliefs about the subsequent liquidity shock are pessimistic, the presence of a bank type with poor-quality assets is enough to induce market freezing, regardless of the aggregate quality of the assets. When the aggregate quality of the assets falls below the minimal amount of capital required to dissuade creditors from running, market freezing is the unique equilibrium of the fund-raising game. When this outcome occurs, the bank is left unprotected against short-term liquidity shortages, which induce the bank to default in case they materialize.

To prevent the bank’s default, the policy-maker may decide to disclose information about the quality of the bank’s assets and its liquidity position. I fully characterize the optimal comprehensive disclosure policy. The policy-maker first examines the long-term profitability of the bank’s assets. When their quality is above a threshold, a coarse pass grade is given and no further disclosures about the bank’s liquidity are necessary. When the quality of the assets instead falls below a threshold, the policy-maker assigns one of multiple failing grades. The optimal disclosure policy has a monotone partitional structure in which adjacent quality levels are pooled together under the same grade. To improve the bank’s chances of survival, the policy-maker conducts a liquidity examination. When the liquidity position of the bank is sufficiently good, the bank is assigned a pass grade, which convinces creditors to keep rolling over the bank’s debt. In the opposite case, the bank is given a failing grade, which prompts short-term creditors to run.

Importantly, I find that imposing contingent capital requirements is instrumental to implementing the optimal policy. In fact, I show that without the imposition of minimal capital requirements, information disclosure about the bank’s liquidity position may be ineffective and the regulator may fail to help the bank raise funds. A disclosure rule that is not complemented with capital requirements may backfire and prove worse than a laissez faire policy.

In the absence of government intervention, the threat of a run of short-term creditors serves as a discipline device toward the possibility that types with different asset qualities separate during the fund-raising stage, and hence may promote risk-sharing. Government interventions soften short-term creditors’ response to liquidity shocks from an ex-ante perspective which makes separation among bank types more likely to occur. This may have a negative impact on risk-sharing. Capital requirements thus substitute for the disciplining role served by creditors’ run, by threatening the bank to reduce the dividends that can be distributed among shareholders in case it fails to raise the funds specified by the government. Crucially, for the introduction of capital requirements to work, the policy-maker needs to publicly communicate its policy to all market participants.

I follow a robust approach and assume that when multiple outcomes are consistent with a given policy, the one that minimizes the policy-maker’s payoff (state by state) is selected. The type of
applications I have in mind justifies this conservative strategy. The comprehensive policy proposed in the paper implements the optimal solution to a broader mechanism design problem in which the policy-maker possesses the authority to dictate the type of securities and prices the bank should choose when approaching external investors. I show that conferring this authority to the policy-maker is not necessary, since the same outcome can be implemented by combining appropriately designed information disclosures with capital requirements.

In certain environments, the policy-maker may not be able to conduct an examination of the bank’s buffers after a liquidity shock. In the second part of the paper I consider a setting where the regulator cannot conduct such a liquidity examination process in a timely manner and thus needs to implement a liquidity-provision program that prompts the bank to self-report the magnitude of the liquidity shortage. To induce truthful reporting from the bank, I allow the regulator to publicly communicate with market participants and to purchase claims on the bank’s assets under the constraint that the price paid by the policy-maker not exceed the fair price of the securities purchased. The policy-maker may act as a lender of last resort but with a natural constraint on its ability to use public funds.

The problem of designing a liquidity-provision program that elicits information about the bank’s buffers is similar to the problem considered in Philippon and Skreta [2012] and Tirole [2012], in that a bank’s outside options are endogenous to the choice of the government’s program. A bank that refuses to participate in the program faces short-term creditors whose beliefs depend on the government’s mechanism. The novelty with respect to those earlier models is that the policy-maker may provide privacy to the bank and may engage in strategic information disclosure about the information elicited from the bank. These additional properties drastically change the set of equilibrium outcomes.

The optimal liquidity-provision program asks the bank to (confidentially) report its liquidity position and promises in return to assign a pass-fail grade. Contingent on assigning the pass grade, the policy-maker purchases the remaining claims on the bank’s assets. When the regulator announces the bank has passed the test, short-term creditors find it in their best interest to keep rolling over the bank’s debt. In turn, when the policy-maker fails the bank, short-term creditors willingly stop pledging to it. To induce all liquidity types to truthfully report their liquidity positions, the policy-maker needs to compensate those types that are passed with lower probability. This compensation is done by offering them higher prices for their securities. The optimal liquidity-provision program offers a passing grade to most illiquid banks with low probability but compensates them with higher prices for their assets, while more liquid (but still vulnerable) banks are assigned a pass grade with higher probability and lower prices for their remaining claims on their assets. In this manner, the government improves the average liquidity position of banks receiving the passing grade, which persuades creditors to keep pledging to the bank. Perhaps surprisingly, the optimal policy also requires to fail safe banks (i.e., those immune to rollover risk) with large probability. Given that these banks survive the potential run of creditors regardless of whether they trade with the policy-maker, the policy-maker may afford to assign bad grades to these banks to minimize the incentive
of vulnerable types to mimic them, so that incentive compatibility constraints are respected.

I use the characterization of the optimal-liquidity-provision program to show that interventions that involve simultaneous pledging by both the private and the public sector are suboptimal. To prove this result, I show that imposing capital requirements undermines the effectiveness of the government’s liquidity programs. In fact, a bank that retains a smaller fraction of its assets can be promised fewer funds from the government under the fair price constraint. Given that the effectiveness of the liquidity-provision program relies on compensating extremely vulnerable banks, which receive a passing grade less often than more liquid banks, with higher prices for the remaining claims on their assets, requiring that the bank sells a fraction of such assets to external investors decreases the elicitation capacity of the policy-maker once the liquidity shock materializes. Additionally, having the bank raise funds from external investors intensifies incentive compatibility issues in the regulator’s elicitation program. To see this, note that a positive amount raised at the fund-raising game indicates the possibility that the bank is immune to rollover risk and, hence, may survive a potential run of creditors, regardless of whether it participates in the policy-maker’s program, which amplifies vulnerable bank’s incentives to misreport. As a result, if a liquidity-provision program is implemented, capital requirements are minimized.

The policy-maker is thus confronted with the dilemma of choosing between private-sector financing, which maximizes the price of the bank’s securities by selling them before the liquidity shock occurs, and the government’s liquidity-provision program, which asks the bank to (confidentially) report information about its liquidity buffers and then reveals information to its creditors. I show that optimal comprehensive interventions display a non-monotone pecking order. Institutions with high-quality assets are given a pass grade by the stress test that assesses the long-term profitability of the assets, and they are required to raise enough capital in the private markets to persuade short-term creditors to rollover the bank’s debt. Banks with intermediate-quality assets are assigned one of multiple failing grades and are funded with the government’s liquidity-provision program. Finally, institutions with extremely poor-quality assets are failed with multiple failing grades and are induced to seek private-sector financing.

The rest of the paper is organized as follows. Below, I wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 describe the equilibria in the absence of government intervention. Section 4 studies the optimal comprehensive disclosure policy. Section 5 studies the case where the policy-maker designs an elicitation mechanism to learn the liquidity position of the bank. Proofs omitted in the text are in the Appendix or in the Supplementary Material.

Related literature. The paper is related to several strands of the literature. The first strand is the literature on stress test design and regulatory disclosures in the financial system. Close in spirit to this paper is the work by Faria-e Castro et al. [2016], who consider a similar model of information disclosure by a policy-maker in an environment with runnable liabilities and asymmetric information. The paper focuses on the interaction between the government’s fiscal capacity and
the optimal degree of transparency of stress tests. Crucially, that paper assumes that there exists a one-to-one relationship between liquidity and asset quality. In contrast, in the present paper, I relax the assumption that liquidity and asset quality are perfectly correlated, which allows me to examine the role of disclosure of different information to different audiences. In the second part of the paper, where I allow the policy-maker to purchase claims on the bank’s assets, I find that the degree of transparency of the stress test affects the amount of funds the policy-maker can commit to use, generating a trade-off between coarser disclosure policies - which allow banks to raise, on average, more funds- and the effectiveness of the regulator’s program at eliciting information from the bank. In other words, I show that stronger financial capacity need not come with more information disclosure, contrary to what is established in Faria-e Castro et al. [2016]. Goldstein and Leitner [2018] consider the stress test design problem of a regulator who wishes to facilitate risk sharing among banks endowed with assets of heterogeneous qualities. My model complements theirs by analyzing an environment where (i) the bank endogenously chooses its funding strategy in response to the regulator’s disclosure, (ii) the amount of additional funds needed by the bank is endogenously determined by the disclosure policy selected by the policy-maker and the interaction among the different market participants, and (iii) the fundamentals are multidimensional and comprise the quality of the bank’s assets and its liquidity buffers. Orlov et al. [2017] consider the joint design of stress tests and capital requirements in a setting where multiple banks have correlated exposure to an exogenous shock. Inostroza and Pavan [2018] explore optimal disclosure policies when the policy-maker faces multiple receivers endowed with heterogeneous information, under an adversarial approach. They show that optimal stress tests need not generate conformism in beliefs among market participants, but generate perfect coordination among their actions. Alvarez and Barlevy [2015] study the incentives of banks to disclose balance sheet (hard) information in a setting where the market is not able to observe the exposure to counterparty risks. In my model, banks cannot disclose hard information but may try to signal information through their funding strategy. Bouvard et al. [2015] study a credit rollover setting where a policy maker must choose between transparency (full disclosure) and opacity (no disclosure) but cannot commit to a disclosure policy. In contrast, I assume the policy maker can fully commit to her disclosure policy and allow for fully flexible information structures.

Optimal government interventions in markets plagued by adverse selection have been studied in Philippon and Skreta [2012], Tirole [2012], and Fuchs and Skrzypacz [2015]. These papers share the common feature that government interventions affect post-intervention outcomes and vice versa. The first two papers consider a static setting, and show that the policy-maker optimally chooses to purchase low quality assets to jump-start a frozen market, permitting banks with better assets to receive funding from the private sector. The third paper considers a dynamic model in which low quality assets are sold first, which gradually improves the pool of legacy assets. The paper shows that the regulator should subsidize trade early in the model, and then impose prohibitively high taxes that essentially shut-down the asset market. In the second part of the paper, I propose a model
that shares the common feature of these papers. Namely, that the policy-maker’s liquidity-provision program generates endogenous participation constraints. In my model, however, the policy-maker may also engage in information design when trading with the bank, and some banks are funded directly by the government, instead of the private sector.

The present paper also contributes to the extensive literature on security design under adverse selection, as in Myers and Majluf [1984], Nachman and Noe [1994], DeMarzo and Duffie [1999], and DeMarzo and Fishman [2007], among others. This paper connects this literature with the literature on information design. Recent developments along these lines include Daley et al. [2018], who consider the effect of ratings on security issuance; Yang [2015], who studies security design when the buyer may acquire information about asset quality at a cost; and Szydlowski [2018], who considers the problem of a firm that seeks financing and chooses both its information disclosure policy and the type of security it offers to external investors.

Finally, this paper relates to the literature on information design. This literature can be traced back to Myerson [1986], who introduced the idea that, in a general class of dynamic games of incomplete information, the designer can restrict attention to private incentive-compatible action recommendations to agents. Recent developments include Kamenica and Gentzkow [2011], Kamenica and Gentzkow [2016], and Ely [2017]. These papers consider persuasion with a single receiver. Persuasion with multiple receivers is less studied. Calzolari and Pavan [2006a] consider an auction setting in which the sender is the initial owner of a good and where the different receivers are bidders in an upstream market who then resell in a downstream market. Related to this paper is Dworczak [2016], who offers an analysis of persuasion in other mechanism design environments with aftermarkets. Alonso and Camara [2016a] and Bardhi and Guo [2017] consider persuasion in a voting context, whereas Mathevet et al. [2016] and Taneva [2016] study persuasion in more general multi-receiver settings. Bergemann and Morris [2016a] and Bergemann and Morris [2016b] characterize the set of outcome distributions that can be sustained as Bayes-Nash equilibria under arbitrary information structures consistent with a given common prior. Alonso and Camara [2016b] study public persuasion in a setting with multiple receivers with heterogeneous priors. Kolotilin et al. [2017] consider a screening environment whereby the designer elicits the agents’ private information prior to disclosing further information. Basak and Zhou [2017] and Doval and Ely [2017] study dynamic games in which the designer can control both the agents’ information and the timing of their actions.
2 Model

Players and Actions. The economy is populated by a bank, short-term creditors, external investors, and a policy-maker. There are 3 periods, $T \equiv \{1, 2, 3\}$. The bank is cash-less, risk-neutral and has two legacy assets: a risky asset, and a zero-coupon bond. Both assets mature at period $t = 3$. The risky asset delivers an observable stochastic cash flow, $y \in \mathbb{R}_+$, while the bond has a face value of $R$, but can be (partially) liquidated early in period 2. During the first period, in order to increase the amount of liquid funds available at the second period, the bank may sell claims on its assets to a competitive, risk-neutral set of investors. At the beginning of period 2, the bank suffers a liquidity shock, described in detail below, that prevents the bank from selling a fraction or the totality of her bond $^3$. Finally, a continuum of short-term creditors of mass one, uniformly distributed over $[0, 1]$, has a claim of $\$1$ if redeemed early, during the second period, and equal to $R$ if rolled over until $t = 3$. Let $a_i \in \{0, 1\}$ denote the action chosen by creditor $i$, where $a_i = 0$ represents the action of rolling over the bank’s debt, and $a_i = 1$ the decision of withdrawing by the end of the second period. I denote by $A \in [0, 1]$ the fraction of creditors who chooses to stop pledging to the bank.

Fundamentals. The fundamentals of the bank’s balance sheet are captured by the vector, $(\omega, y)$. The variable $y$ represents the asset’s cash flows which are drawn from the full-support cdf $F_y$ over $\mathbb{R}_+$. The variable $\omega$ represents the bank’s short-term liquidity. More specifically, $\omega$ represents the fraction of the bond that the bank can sell during the second period in order to obtain additional funds to repay its obligations. A value of $\omega < 1$ can be interpreted as an unexpected liquidity shock which prevents the bank from selling the totality of the bond (e.g., off-balance sheet items or the imposition of haircuts). We will frequently refer to $\omega$ as the bank’s liquidity position. We assume that the fraction of the bond which is not liquidated becomes available at $t = 3$ and can be used to repay late creditors.

Default. If the fraction of creditors who decide not to roll over the bank’s debt is large enough with respect to the bank’s available cash, bankruptcy is triggered. In that case, the bank’s risky asset is confiscated along with any available cash the bank possesses at that moment. For simplicity I suppose that the recovery rate associated to bankruptcy is 0.

Precautionary Fund Raising. To reduce the probability of default, the bank may raise funds at $t = 2$ by selling claims on its risky asset to external investors. If the bank raises $P$ units of funds, the amount of cash available to repay early withdrawals is given by $\omega + P$. Let $r \in \{0, 1\}$ represent the event of whether the bank defaults, with $r = 1$ in case of default, and $r = 0$ otherwise. We assume that the fate of the bank is determined by the linear rule $r = 1 \{A \geq \omega + P\}$.

Exogenous Information. We assume that there is gradual resolution of uncertainty. At $t = 1$, the bank’s long-term cash flows, $y$, are drawn from $F^y$. The bank then learns a signal $\theta$ about

$^3$Alternatively, we may think that there exists a stochastic obligation that needs to be paid during the second period in addition to the fraction of early withdrawals. Importantly, the bank will suffer a liquidity shortage with positive probability.
y, and forms beliefs about the realization of y according to the conditional cdf \( F^\theta_y \) (resp., pdf \( f^\theta_y \)), where \( \theta \) belongs to the set \( \Theta = \{ \theta_L, \theta_H \} \), with \( \theta_H > \theta_L \). I will refer to \( \theta \) as the bank’s asset quality type. I assume that the conditional pdf \( f^\theta_y \) satisfies log-supermodularity in \((y, \theta)\) (or, equivalently, that the realization of cash-flows of different types \( \theta \) are ordered according to MLRP). The cash flow realization cannot be observed by any market participant until \( t = 3 \). The liquidity shock \( \omega \) is drawn from \( F^\omega \in \Delta \Omega \) at the beginning of the second period and is observed by the bank. The rest of market participants only learn whether the shock materialized or not (i.e., whether \( \omega = 1 \) or \( \omega \in [0, 1) \)). These assumptions are made to reflect the idea that the profitability of the bank’s asset depends on investment decision made in the past, while the bank’s liquidity position is subject to unexpected contingencies and may vary precipitously. All market participants anticipate at \( t = 1 \) the possibility that a liquidity shock takes place in period 2 but do not know its severity. All agents in the economy share the prior belief \( F^\omega \) about the bank’s liquidity position. The policy-maker, investors and short-term creditors share a common prior \( \mu^\theta \in \Delta \Theta \) about the bank’s asset type.

**Payoffs.** For simplicity, I assume no discounting. If the bank raises \( P \) units of money during the second period, draws a liquidity shock \( \omega \), and a fraction \( A \) of creditors withdraws early, it survives as long as the available funds are greater than its obligations: \( \omega + P \geq A \). In such a case, the bank may use the remaining cash to buy a bond and obtain a payoff of \( R \times (P + \omega - A) \) at \( t = 3 \). Thus, the bank’s payoff when it raises \( P \) units of cash in period 2, cash flows are \( \tilde{y} \) during the third period, the liquidity shock is \( \omega \), and faces a fraction \( A \) of early withdrawals, is given by:

\[
U(P, \tilde{y}, \omega, A) = \left( R(P + \omega - A) + \left( R - \frac{\omega R}{\text{liquidated early}} - \frac{(1 - A)R}{\text{late withdrawals}} \right) + \tilde{y} \right) \times 1\{P + \omega \geq A\} = (PR + \tilde{y}) \times 1\{P + \omega \geq A\}. \tag{1}
\]

The creditors’ payoffs depend on their actions. I normalize the utility from withdrawing early to 0 and let \( u_i(\tilde{\omega}, A) \) be the utility of a creditor who decides to pledge to the bank, when the total amount of available cash held by the bank at \( t = 2 \) is \( \tilde{\omega} = \omega + P \) and the fraction of early withdrawals is \( A \). Observe that when the bank survives, a creditor who withdraws early obtains $1 w.p. 1, while he would have received \( R \) had he chosen to roll over the bank’s debt. We denote by \( g \equiv R - 1 > 0 \) the positive utility differential from rolling over the bank’s debt in case the bank does not default. When the bank defaults, a creditor who chooses to pledge to the bank at \( t = 2 \) receives a payoff \( b(\tilde{\omega}, A) \) at \( t = 3 \). The function \( b(\tilde{\omega}, A) \) is negative, non-decreasing in \( \tilde{\omega} \) and non-increasing in \( A \). That is,

\[
u_i(\tilde{\omega}, A) = \begin{cases} 
g & \text{if } r = 0 \\ 
b(\tilde{\omega}, A) & \text{if } r = 1. \\
\end{cases}
\]

11
Finally, we assume that the policy-maker’s obtains a positive payoff $W_{0}(A)$ when default is successfully avoided, and a payoff of 0 when that is not the case, with $W_{0}(\cdot)$ non-increasing.

$$U^{P}(\hat{\omega}, A) = W_{0}(A) \times 1 \{\hat{\omega} > A\}$$

**Asset Market.** After observing its private asset type, $\theta \in \Theta$, the bank proposes to the investors a security $s_{\theta}$, by the end of the first period, which corresponds to claim on future cash-flow realizations of the risky asset and belongs to $S \equiv \{f : \mathbb{R}_{+} \to \mathbb{R}_{+} \text{s.t. } (\text{LL}),(M),(\text{MR})\}$ where:

- (LL) $0 \leq s(y) \leq y \ \forall y \geq 0$
- (M) $s(y)$ non-decreasing
- (MR) $y - s(y)$ non-decreasing.

These assumptions are standard in the literature of security design. The first constraint represents *limited liability* and states that a security $s \in S$ is in fact a sharing rule of the asset’s cash-flows. The second constraint, the *monotonicity* condition, requires that the security is non-decreasing in the the cash-flows, since otherwise the bank would have the option of asking for (risk free) credit to a third party in order to boost the cash-flow realization and thus decrease the amount owed to the initial investors. Finally, the last constraint imposes that the share of cash-flows kept by the bank is non-decreasing for, otherwise, the bank would have incentives to burn part of the cash-flows to improve her payoff. The market observes the security $s_{\theta}$ and prices it according to the available information.

**Intervention Policies.** The policy-maker concerned with the possibility that the bank defaults may choose to intervene. The policy maker possesses a technology which allows her to *commit* to disclose information to all market participants and to give recommendations to the bank about the amount of capital to raise from external investors. The assumption of *gradual resolution of uncertainty* implies that the designer may disclose information about the cash-flows at $t = 1$, after $y$ has been determined, but can disclose information about the liquidity shock $\omega$, only at $t = 2$, after $\omega$ has been drawn. I denote by $\Gamma^{y}$ the joint policy of disclosing information about the profitability of the bank’s assets, $y$, and the choice of contingent capital requirements, and by $\Gamma^{\omega}$ the liquidity examination conducted in the second period about the bank’s liquidity position. I will refer to $\Gamma^{y}$ as the bank’s stress test. I make the implicit assumption that the technology needed to conduct the stress test $\Gamma^{y}$ is time-demanding and cannot be postponed until the liquidity shock takes place, since then the policy-maker might not be able to disclose information on time, before short-term creditors make their decisions. Moreover, I assume that any information learned while conducting stress test $\Gamma^{y}$ becomes public. That is, the policy-maker cannot choose to learn information about $y$ and not share it with market participants\(^4\).

\(^4\)Any information produced by the regulator that is kept hidden from the rest of agents, always leaks and therefore if the policy-maker wants the rest of market participants not to learn some information she should not produce it in the first place. A similar assumption is made by Faria-e Castro et al. [2016].
Timing. The sequence of events is as follows:

**Period 0.** The policy maker chooses a comprehensive policy \( \Gamma = \{\Gamma^y, \Gamma^\omega\} \), and publicly announces it.

**Period 1.** (a) \( y \) is drawn from \( F^y \). (b) The bank observes a private signal \( \theta \) about \( y \). (c) The policy-maker discloses information \( m^y \), according to the stress test \( \Gamma^y \). (d) The bank sells a security \( s \) to external investors at price \( P \geq 0 \). I refer to (d) as the *fund-raising stage*.

**Period 2.** (a) \( \omega \) is drawn from \( F^\omega \). (b) The policy-maker conducts liquidity examination \( \Gamma^\omega \) and discloses information \( m^\omega \). (c) Short-term creditors observe \( P \) and all information available with respect to \( \omega \), and decide whether to keep pledging to the bank. (e) The bank liquidates a fraction of her bond and its fate is determined according to whether \( \omega + P \) is greater than the fraction of early withdrawals, \( A \). Any excess of liquid funds is reinvested.

**Period 3.** Conditional on the bank’s survival, (a) \( y \) is materialized and \( s(y) \) is paid to investors. (b) The fraction of bond not liquidated early, and any amount reinvested at period 2, is collected with interest and late creditors are paid back.

3 Laissez Faire

3.1 Raising Capital to Persuade Creditors

I first study the case where the policy maker does not intervene. In this case, the bank observing its asset quality type \( \theta \), enters the *fund-raising stage* by approaching investors to whom it offers claims on its asset in order to raise funds that allow it to pay its obligations, and hence avoid an eventual default triggered by a creditors’ run. I follow an adversarial approach, and assume that when multiple action profiles are rationalizable, creditors coordinate on the most aggressive outcome from the perspective of the bank.

Let \( \mathbb{E}(u(P,\omega,A)) \) be the expected utility of a creditor who chooses to pledge when the fraction of early withdrawals is given by \( A \), the seller has successfully raised \( P \) units of capital, and the liquidity shock is \( \omega \). The adversarial approach then implies that all creditors choose to attack the bank whenever withdrawing early is a best response to everyone withdrawing early. That is, each
creditor withdraws early when

$$\mathbb{E}(u(\omega, P, 1)) \equiv \int_0^1 (g \times 1 \{P + \omega > 1\} + b (P + \omega, 1) \times 1 \{P + \omega \leq 1\}) F^\omega(d\omega) \leq 0.$$  

Define, then, $A(P)$ as the most aggressive fraction of early withdrawals, for a given recapitalization level, $P$. In what follows I assume that $\lambda$, the probability with which no liquidity shock occurs, is small enough so that, if the bank does not raise additional funds, creditors withdraw early. Then, let $K \geq 0$ be the minimum amount of capital that the bank needs to raise in order to persuade short-term creditors to keep rolling over its debt. That is,

$$K \equiv \sup \{ P \geq 0 : \mathbb{E}(u(\omega, P, 1)) \leq 0 \}.$$  

From the definition of $K$ above, we have that $A(P) = 1 \{ P \leq K \}$. To make the problem interesting we assume that the low type bank has an asset with expected cash-flows below $K$, while the expected cash-flows of the asset of type H are above it.

**Assumption 1.** $\frac{1}{R} \mathbb{E}_L(y) < K < \frac{1}{R} \mathbb{E}_H(y)$.

The bank understands that the only way to convince short-term creditors that it is liquid is by raising $K$ units of capital in the asset market. By the end of period 2, short-term creditors observe the recapitalization secured by the bank and decide whether or not to rollover the bank’s debt. If the bank raises at least $K$ units of capital, then no short-term creditor withdraws early, allowing the bank to survive and to re-invest the funds buying a 1-period bond. On the other hand, if the amount raised is smaller than $K$, then all creditors withdraw early, in which the survival of the bank depends on the amount of capital raised and on the realization of the liquidity shock $\omega$. Given the above observation, the maximal price that external investors are willing to pay for any security $s$ and is given by:

$$P(s, \mu) \equiv \sup \left\{ p \geq 0 : \frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P}\{\omega + p \geq A(p)\} \geq p \right\} \tag{2}$$

where $\mathbb{E}_\mu(s)$ is the expected value of security $s$ when the market holds beliefs $\mu \in \Delta\Theta$ about the bank’s type. Note that the definition of $P(s, \mu)$ implies that, in case the equation

$$\frac{\mathbb{E}_\mu(s)}{R} \times \mathbb{P}\{\omega + p \geq A(p)\} = p, \tag{3}$$

admits multiple solutions, the selected one is the one associated with the largest price\(^5\). The next assumption will be used for certain results, for it favors tractability.

---
\(^5\)This selection has a game-theoretic foundation similar in spirit to the one encountered in Bertrand competition models. Namely, if the market reached a price $\hat{P} < P(s, \mu)$ satisfying 3, any buyer could deviate and offer a greater price $\tilde{P}$ for which the LHS of equation 3 is strictly greater than the RHS, and obtain a positive gain in the process. Such deviation would be willingly accepted by the bank. As a result, $\hat{P}$ would be inconsistent with equilibrium play. $P(s, \mu)$ is thus the unique price consistent with competitive markets and immune to such deviations.
Assumption 2. The prior distribution of the liquidity level $\omega$, $F^{\omega}$, is concave.

Assumption 2 reflects the idea that the liquidity problem is severe. Intuitively, when $F^{\omega}$ is concave (i.e., when the density $f^{\omega}$ is non-increasing), low liquidity levels are more likely to occur. When this is the case, and additionally $\lambda = 0$ (that is, there is no mass point at $\omega = 1$), investors will to fund any project with NPV below $K$. To see this, note that in this case the LHS of inequality in (2), $\frac{\mathbb{E}_\mu(s)}{R} \mathbb{P} \{ \omega + p \geq A(p) \}$, is smaller than the RHS, $p$, meaning that the expected payoff an investor obtains from purchasing security $s$ is no greater than what he pays. As a consequence, the market refuses to purchase security $s$ because it expects a high probability of default. The intuition behind this result is that, under an adversarial approach, investors believe that short-term creditors will overreact to the inability of the bank of raising enough capital. This generates a negative feedback cycle since it invites the market to offer a lower price for the security issued by the bank. The bank’s inability to raise funds then makes a massive early withdrawal more likely, which in turn implies a higher probability of default and thus a lower price. Hence, when $\lambda = 0$ and assumption 2 holds, the bank survives only if the price collected is at least $K$.

3.2 Solution Concept: PBE consistent with D1

The government most preferred outcome, although possibly unfeasible, has all bank types issuing securities that allow them to survive the liquidity shock, and hence avoid bankruptcy. As is usually the case with signaling games, the fund raising game may be plagued with multiple equilibria. In order to focus on equilibria which take into account the propensity of bank types to deviate, we restrict attention to PBE satisfying the D1 criterion, and we refer to them simply as equilibria.

Let $V(P, s, \theta)$ be the utility of a bank of type $\theta$, selling a security $s$ and receiving funds in the amount of $P$. Without government intervention, the bank’s payoff can be written as:

$$V(P, s, \theta) \equiv \mathbb{E} \left((PR + y - s) \times 1 \{ \omega + P \geq A(P) \} \right)$$

$$= (PR + \mathbb{E}_\theta (y - s)) \mathbb{P} \{ \omega \geq A(P) - P \}. \quad (4)$$

I will say that $\{s^*_\theta\}_{\theta \in \Theta}, \mu^*, P^*, A^*$ is an equilibrium of the fund-raising game if:

- [Sequential Rationality]: $s^*_\theta \in \arg \max V(P^*(s), \theta, s)$
- [Competitive Investors]: $P^*(s) = \sup \left\{ P : \frac{\mathbb{E}_{\mu^*}(s)}{R} \mathbb{P} \{ \omega + P \geq A^*(P) \} \geq P \right\}$
- [Adversarial Coordination]: $A^*(P) = 1 \{ P < K \}, \forall P \geq 0$
- [Belief Consistency]: $\mu^*(s)$ computed according to Bayes rule on-path

Additionally, I impose that off-path beliefs associated with securities not observed in equilibrium, assign all probability weight to the seller type with the greatest propensity to deviate to them. Rigorously, define first the set of best response to some security $s$, $BR(s)$, as the set of prices which
are consistent with rationality of the investors under some belief about the asset quality type of the bank\(^6\):

\[
BR(s) \equiv \left\{ P : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P} \{ \omega + P \geq A^* (P) \} \geq P \right\}.
\]

Define then,

\[
\mathcal{D}(\theta|s) \equiv \left\{ P \in BR(s) : V(P, s, \theta) > V(P^* (s^*_H), s^*_H, \theta) \right\}
\]

\[
\mathcal{D}^0(\theta|s) \equiv \left\{ P \in BR(s) : V(P, s, \theta) = V(P^* (s^*_H), s^*_H, \theta) \right\}.
\]

The profile \( \{ s^*_\theta \}_{\theta \in \Theta}, \mu^*, P^*, A^* \) satisfies the D1 criterion if for any security \( s \in S \) with \( s \neq s^* (\theta) \) \( \forall \theta \in \Theta \), \( \mu^*(s) \) is such that \( \forall \theta, \theta' \) \( (\mathcal{D}(\theta|s) \cup \mathcal{D}^0(\theta|s)) \subset \mathcal{D}(\theta'|s) \) \( \Rightarrow \mu^*(\theta|s) = 0 \).

### 3.3 Equilibrium Characterization.

In what follows I characterize the set of equilibria that arise in the fund-raising game. My first proposition shows that, in any pooling equilibrium, both bank types issue debt. When this is the case, the price obtained by the bank is no larger than \( K \). Then I show that separating equilibria may exist only if the expected cash-flows of type L are sufficiently large, in which case type H chooses to raise less funds than needed to avoid default with certainty, and hence remains exposed to rollover risk. I prove this proposition in the Appendix for general distribution of the fundamentals; This will permit me to invoke the result also in the next sections, when additional information about \( y \) may be revealed by the policy-maker. The result is an adaptation of the results in Nachman & Noe (94) to the setting under examination where I incorporate the probability of default to the pricing of securities.

**Proposition 1.** (i) Let \( \{ s^\text{pool}_\theta = s \}_{\theta \in \Theta}, \mu, P, A \) be a pooling equilibrium outcome of the fund-raising game. Then necessarily, \( s = \min \{ y, D \} \) for some \( D > 0 \). Moreover, \( P(s) \leq K \). (ii) Let \( \{ s^\text{sep}_\theta \}_{\theta \in \Theta}, \mu, P, A \) be a separating equilibrium of the fund-raising game. Then, \( \mathbb{E}_H (s^\text{sep}_H) < \mathbb{E}_L (y) \).

My second result characterizes the set of equilibria that arise when \( \lambda = 0 \) (i.e., when the liquidity shock occurs with probability one) and Assumption 2 holds. I first show that the only type of equilibria in the fund-raising game are pooling equilibria, where both bank types issue debt contracts. I then prove that if the expected profitability of the asset of the L-type bank is low enough, then there exists an equilibrium where the asset market freezes and no security is issued. Furthermore, I show that when, in addition, the average quality of the bank’s asset is low, then market freezing is the unique equilibrium outcome of the fund-raising game. Given that these results obtain under

---

\(^6\)First-order stochastic dominance (which is implied by MLRP) means that

\[
\left\{ P > 0 : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P} \{ \omega + P \geq A^* (P) \} \geq P \right\} = \bigcup_{\mu \in \Delta \Theta} \left\{ P > 0 : \frac{\mathbb{E}(s, \mu)}{R} \times \mathbb{P} \{ \omega + P \geq A^* (P) \} \geq P \right\}
\]
the assumption that a liquidity shock occurs with certainty \((\lambda = 0)\), under such conditions the bank defaults with probability 1. Finally, I show that when the expected profitability of a type L-bank is good enough, the unique equilibrium of the game has both types of bank placing a debt contract which collects enough funds to dissuade creditors from running. This last result is simply a manifestation of Nachman and Noe [1994]’s celebrated uniqueness result.

**Proposition 2.** Suppose Assumption (2) holds and \(\lambda = 0\). Then,

1. In any equilibrium of the fund-raising game, \(s^*_\theta = \min\{y, D\}\) for all \(\theta \in \Theta\), and for some \(D \geq 0\).

2. (Market freeze) If \(\frac{1}{R}E_L(y) < K\), there exists an equilibrium where \(s^*_\theta = 0\) for all \(\theta \in \Theta\). Moreover, if \(\frac{1}{R}E(y) < K\), this is the unique equilibrium.

3. (Optimal pooling) If \(\frac{1}{R}E(y) \geq K\), there exists an equilibrium where \(s^*_\theta = \min\{y, D^{pool}\}\) with \(D^{pool}\) defined as the unique solution to \(\frac{1}{R}E(\min\{y, D^{pool}\}) = K\). Moreover, if \(\frac{1}{R}E_L(y) \geq K\), this is the unique equilibrium.

An immediate implication of proposition 2 is that, when bank expects a severe liquidity shock, the presence of a bank type with sufficiently poor assets is enough to guarantee the existence of an equilibrium where the market for the bank’s assets freezes, preventing the bank from raising funds to avoid the imminent run of short-term creditors. The investors’ ability to foresee the possibility of a run, and to price assets accordingly, together with the incentives of the banks of type H to separate from L, induce a fire sale so severe the bank is unable to raise any funds. As a consequence of this property, any security which a type H-bank type may try to issue is also issued by the type L-bank, generating contagion among bank types and provoking the freeze of the asset market.
4 Disclosure Policies

The policy-maker, concerned with the possibility that the asset market freezes, may choose to intervene. I distinguish between regular and critical stress tests. A regular stress test, \( \Gamma^y = \{ \{ M^y, \pi^y \}, \{ R_\alpha[m^y] \}_{m^y \in M^y} \} \), comprises a disclosure policy, \( \pi^y : \mathbb{R}_+ \to \Delta M^y \), where \( M^y \) is an arbitrary set of messages, and a recapitalization rule \( R_\alpha (\cdot | m^y) : \mathbb{R}_+ \to \{0,1\} \), where for any \( P \in \mathbb{R}_+ \), \( R_\alpha(P|m^y) = 0 \) if, given \( P \), the bank is allowed to distribute dividends in the excess of a fraction \( \alpha \) of the total bank’s payoff, and \( R_\alpha(P|m^y) = 1 \) if it is not allowed. A regular stress test thus discloses a message \( m^y \) according to \( \pi^y(y) \in \Delta M^y \), and conditional on it, specifies a recapitalization rule \( R_\alpha[m^y] \), \( \alpha \). The recapitalization policy should be interpreted as setting capital requirements. The decision of allowing shareholders to distribute only a fraction \( \alpha \) of the total amount of profit if the bank does not comply with the recapitalization requirement specified by the rule serves the purpose of enforcing the policy-maker’s recommendation.\(^9\) Note that although we confer the designer the authority to impose capital requirements, we do not allow her to repudiate any contract the bank agrees upon with the investors. That is, investors preserve their claims on the future cash-flows of the asset even if the bank does not comply with the capital requirements. Importantly, the government commits to not inject any type of funds to insulate creditors from the liquidity shock, and hence tax-payers’ money is not at stake. I relax this assumption in the next section. As I show below, imposing capital requirements serves as a discipline device to control separation incentives among bank types during the fund-raising game.

A critical stress test, \( \Gamma^\omega = \{ M^\omega, \pi^\omega[P,m^y] \} \), is a disclosure policy \( \pi^\omega \), so that the policy-maker discloses information about the bank’s liquidity shock according to the rule\(^10\) \( \pi^\omega [P,m^y] : \Omega \to \Delta M^\omega \). Hereafter, I refer to a "comprehensive policy" \( \Gamma = \{ \Gamma^y, \Gamma^\omega \} \) as the combination of a regular stress test and a critical stress test.

4.1 Period 1

During the first period, \( y \) is determined. The policy-maker then discloses information \( m^y \) according to the policy \( \pi^y \). Given \( m^y \) the policy then specifies a recapitalization rule \( R_\alpha \). The bank then approaches external investors and offers a security \( s \). The latter, after observing the security issued

---

\(^7\)The assumption that \( R_\alpha(P|m^y) \) takes only deterministic values is without loss of optimality as it will become clear later on.

\(^8\)Assuming that \( R_\alpha \) does not depend on \( y \) directly is wlog. We make this assumption so that the induced beliefs about the quality of the bank’s asset depend only on \( m^y \), and not on \( R_\alpha \).

\(^9\)Imposing capital requirements can be interpreted in different ways in this one-shot framework (as opposed to a repeated game setup). The favored interpretation is that it represents a limit on the amount that can be distributed as dividends if the bank fails to raise the required level of capital. It could also represent the decision of selling the firm to another institution, and \( \alpha \) in that case represents the discount applied to the value of the bank.

\(^10\)Given that the ownership of asset’s claims, and the true realization of \( y \) are irrelevant for short-term creditors, who care about the liquidity shock and the amount of funds collected by the bank, \( P \), restricting attention to policies \( \pi^\omega \) that only depend on \( \omega \) and \( P \) is without loss.
by the bank, form beliefs \( \mu \in \Delta \Theta \) about its asset quality type. I denote by \( P_\mu(s;m^\omega) \) the competitive price offered to the bank. Suppose that investors, which hold beliefs \( F^\omega \) about the seller’s liquidity position, expect the designer to disclose information about \( \omega \) according to \( \Gamma^\omega(P) = \{ M^\omega, \pi^\omega[P]\} \). Then,

\[
P_\mu(s;m^\omega) \equiv \sup \left\{ P : \frac{\mathbb{E}_\mu(s;m^\omega)}{R} \times \int_{\Omega} \left( \sum_{m^\omega \in M^\omega} \mathbb{P} \{ \omega + P \geq A(P, m^\omega) | m^\omega \} \pi^\omega(m^\omega|\omega, P) \right) F^\omega(d\omega) \geq P \right\},
\]

where \( A(P, m^\omega) \) represents the most aggressive fraction of early withdrawals, when the seller is able to raise \( P \) units of additional capital, and the designer discloses information \( m^\omega \) about \( \omega \).

### 4.2 Period 2

After the liquidity shock \( \omega \) materializes, and the amount of capital raised by the bank, \( P \), has been observed by all market participants, the designer conducts the critical stress test, \( \Gamma^\omega \). Assume that message \( m^\omega \in M^\omega \) is publicly disclosed as a result of the exercise. Let \( F^\omega(\cdot|m^\omega) \) be the posterior measure characterizing the beliefs about the liquidity shock, \( \omega \), of an arbitrary creditor who observes the public message \( m^\omega \). That is,

\[
F^\omega(\Lambda|m^\omega) = \frac{\int_{\Lambda} \pi^\omega(m^\omega|\omega) F^\omega(d\omega)}{\int_{\Omega} \pi^\omega(m^\omega|\omega) F^\omega(d\omega)}, \quad \forall \Lambda \subseteq \Omega.
\]

The most aggressive fraction of early withdrawals faced by the bank is then given by

\[
A^{\Gamma^\omega}(P, m^\omega) = 1\{ P < K^{\Gamma^\omega}(m^\omega) \}.
\]

where \( K^{\Gamma^\omega}(m^\omega) \) is defined as the minimal amount of capital needed to persuade creditors to keep pledging, when receiving \( m^\omega \). That is,

\[
K^{\Gamma^\omega}(m^\omega) \equiv \sup \{ P \geq 0 : \mathbb{E}(u(\omega, P, 1)|m^\omega; \Gamma^\omega) \leq 0 \}.
\]

This implies that for every recapitalization amount, \( P \), there exists a critical liquidity level, \( \hat{\omega}^{\Gamma^\omega}(P, m^\omega) \), above which the bank survives the creditors run. That is,

\[
\{ \omega : \omega \geq \hat{\omega}^{\Gamma^\omega}(P, m^\omega) - P \} = \{ \omega : \omega \geq \hat{\omega}^{\Gamma^\omega}(P, m^\omega) \}.
\]

As a result, the payoff that a bank of type \( \theta \) obtains when it issues security \( s \) at price \( P \), information \( m^\theta \) is disclosed at \( t = 1 \), and capital requirements are specified by the policy \( R_\alpha \), is given by:

\[
V(s, P, \theta; m^\theta, R_\alpha) = \max \left\{ 1 - R_\alpha(P|m^\theta), \alpha \right\} \times (PR + \mathbb{E}_\theta(y - s|m^\theta)) \times \\
\times \left( \int_{\Omega} \left( \sum_{m^\omega} \mathbb{P} \{ \omega \geq \hat{\omega}^{\Gamma^\omega}(P, m^\omega) | m^\omega \} \pi^\omega(m^\omega|\omega, P) \right) F^\omega(d\omega) \right)
\]

19
4.3 Stress tests as convex functions

Assume that an amount \( P \) has been raised during the fund-raising game. The next lemma shows that the problem of maximizing the policy-maker’s payoff by means of a policy \( \Gamma_\omega \) is equivalent to maximizing the probability that creditors keep pledging to the bank.

**Lemma 1.** Fix the amount raised by the bank during the fund-raising game, \( P \geq 0 \). The problem of maximizing the designer’s payoff:

\[
\max_{\Gamma_\omega \in \{\pi_\omega, M^\omega\}} \mathbb{E} \left( W_0 (A) \times 1 \{ \omega + P \geq A (P, m_\omega) \} \right)
\]

s.t: \( A (P, m_\omega) = 1 \{ \mathbb{E} (u (\omega, P, 1) | m_\omega) \leq 0 \} \),

is equivalent to the problem of maximizing the probability that creditors keep pledging to the bank under the most aggressive equilibrium outcome:

\[
\max_{\Gamma_\omega \in \{\pi_\omega, M^\omega\}} \mathbb{P} \{ \mathbb{E} (u (\omega, P, 1) ; \Gamma_\omega) > 0 \} = \sum_{m_\omega \in M^\omega} 1 \{ \mathbb{E} (u (\omega, P, 1) | m_\omega) > 0 \} \times \int_{\Omega} \pi_\omega (m_\omega | \omega) F_\omega (d\omega).
\]

(6)

We will thus focus on maximizing the expression in (6). Consider then any critical stress test \( \Gamma_\omega = \{ M^\omega, \pi_\omega \} \). Each message \( m_\omega \) disclosed by stress test \( \Gamma_\omega \) induces a posterior distribution over \( \omega \), \( F_\omega (\cdot | m_\omega) \). Thus, every message \( m_\omega \) disclosed with positive probability generates a posterior expectation of \( u (\omega, P, 1) \), the utility a creditor who pledges to the bank obtains when the latter raises \( P \) units of capital and when all other creditors withdraw. That is, each message \( m_\omega \) induces a new assessment:

\[
\mathbb{E} (u (\omega, P, 1) | m_\omega) = \int_{\Omega} (g \times 1 \{ \omega \geq 1 - P \} + b (\omega + P, 1) \times 1 \{ \omega < 1 - P \}) F_\omega (d\omega | m_\omega).
\]

The optimal critical stress test \( \Gamma_\omega \) can thus be characterized by the distribution of posterior means of \( u (\omega, P, 1) \) it induces. Let \( G_\omega (\cdot ; P) \) be the distribution of posterior means of \( u (\omega, P, 1) \) induced by policy \( \Gamma_\omega \). The next lemma shows that any critical stress test, \( \Gamma_\omega \), corresponds to a mean-preserving contraction of the distribution associated to the full-disclosure policy \( \Gamma_\omega^{FD} \), \( G_\omega^{FD} \), and a mean-preserving spread of the no-disclosure policy, \( G_0^\omega \). That is, \( G_\omega^{FD} \geq_{MPS} G_\omega \geq_{MPS} G_0^\omega \), where the partial order \( \geq_{MPS} \) is defined as follows:

**Definition 1.** Let \( F \) and \( G \) be distribution functions with support in \( X \subseteq \mathbb{R} \). We say that \( F \) dominates \( H \) in the MPS order, \( F \geq_{MPS} H \), if \( \int_X \phi (x) F (dx) \geq \int_X \phi (x) G (dx) \) for any convex function \( \phi \) in \( X \).

**Lemma 2.** [Blackwell] Let \( \Gamma_1^\omega = (M_1^\omega, \pi_1^\omega) \) and \( \Gamma_2^\omega = (M_2^\omega, \pi_2^\omega) \) be two critical stress tests. Assume that there exists \( z : M_1^\omega \times M_2^\omega \to [0, 1] \) such that:
(i) \( \pi^{\varphi}_{2} (m_2 | \omega) = \sum_{m_1} z(m_1, m_2) \pi^\omega_1 (m_1 | \omega) , \forall \omega \in [0,1], \forall m_2 \in M^\omega_2 \)

(ii) \( \sum_{m_2} z(m_1, m_2) = 1, \forall m_1 \in M^\omega_1. \)

Then the distributions of posterior expected utility of creditors, \( \mathbb{E} (u (\omega, P, 1)) \), induced by \( \Gamma^\omega_1 \) and \( \Gamma^\omega_2 \) are such that \( G_{\Gamma^\varphi} \succeq_{MPS} G_{\Gamma^\omega_2}. \)

Lemma 2 shows that disclosure policies that are more informative, in the Blackwell sense, induce distributions of posterior expected utility of pledging creditors, \( \mathbb{E} (u (\omega, P, 1)) \), that dominate in the MPS order defined above. As a result, \( G^\omega_{\Gamma^\varphi} \succeq_{MPS} G^\omega_{\Gamma^\omega_2}. \)

Define next the integral function \( G^\varphi (t; P) \equiv \int_{\bar{u} = u(0, P, 1)}^{t} G^\varphi (\bar{u}; P) d\bar{u} \). The optimal critical stress test \( \Gamma^\omega (P) \) can be characterized using the integral of the distribution of posterior means that it induces. The approach I follow borrows from the analysis in Gentzkow and Kamenica [2016] and Dworczak and Martini [2018]. Let \( G^\varphi_{\Gamma^\varphi} \) and \( G^\varphi_0 \) be the integral functions associated to the full-disclosure policy, \( \Gamma^\varphi \), and no-disclosure policy, \( \Gamma^\omega_0 \), respectively. The set of feasible critical stress tests, \( \Gamma^\omega \), coincides with the set of convex functions between \( G^\varphi_{\Gamma^\varphi} \) and \( G^\varphi_0 \).

**Lemma 3.** Consider an arbitrary critical stress test \( \Gamma^\omega \). Then \( G^\varphi (t; P) \) is convex and satisfies \( G^\varphi_{\Gamma^\varphi}(t) \geq G^\varphi(0) \geq G^\varphi_{\Gamma^\omega_0}(t) \) for all \( t \in [u(0, P, 1), u(1, P, 1)] \). Conversely, any convex function \( h() \), satisfying \( G^\varphi_{\Gamma^\varphi}(t) \geq h(t) \geq G^\varphi_{\Gamma^\omega_0}(t) \) for all \( t \in [u(0, P, 1), u(1, P, 1)] \), corresponds to the integral distribution function of some disclosure policy \( \Gamma^\omega \).

Consider then the problem of maximizing the likelihood that creditors keep pledging to the bank. Using lemmas (1)-(3), the policy-maker’s problem can be reformulated as maximizing

\[ \mathbb{P} \{ \mathbb{E} (u (\omega, P, 1); \Gamma^\varphi) > 0 \} = 1 - G^\varphi (0; P) \]

among all possible disclosure policies over \( \omega \). That is,

\[
\max_{G^{\varphi} \geq_{MPS} G^{\Gamma^\varphi}} 1 - G^{\varphi} (0) \\
\text{s.t.: } G^\varphi_{\Gamma^\varphi} \succeq_{MPS} G^\varphi
\]

The designer’s problem is thus equivalent to finding the policy \( \Gamma^\varphi \) which generates the convex function \( G^\varphi \), between \( G^\varphi_0 \) and \( G^\varphi_{\Gamma^\varphi} \), with minimal slope at \( t = 0 \). As can be seen from Figure 2, the solution to the designer’s problem is thus given by the monotone-binary policy \( \Gamma^\varphi_* = (\{0,1\}, \pi^\varphi_*) \) so that:

\[ \pi^\varphi_* (0 | \omega) = 1 \{ u(\omega, P, 1) \geq \bar{u}(\tau) \equiv u (\bar{\omega} (P), P, 1) \} = 1 \{ \omega \geq \bar{\omega} (P) \}, \]

where \( \bar{u}(\tau) \) corresponds to the point at which \( G^\varphi_{\Gamma^\varphi} \) is tangent to the line with minimal slope to the left of 0, which respects the convexity of \( G^{\Gamma^\varphi_*} \). The value of \( \bar{u}(\tau) \) can also be characterized by the liquidity level that it induces, \( \bar{\omega}(\tau) \), which can alternatively be defined as the liquidity cutoff for which

\[ \mathbb{E} (u (\omega, P, 1) | \omega \geq \bar{\omega} (P)) = 0. \]
To see this last point, note that the policy $\Gamma^\omega$ induces a distribution of posterior means $G^{\Gamma^\omega}$ which assigns positive probability to only two points, which coincide with the points at which $G^{\Gamma^\omega}$ changes slope. Finally, to see that the first point at which $G^{\Gamma^\omega}$ changes slope coincides with

$$E(u(\omega, P,1)|\omega < \bar{\omega}(P)),$$

note that the tangency condition implies that $G^{\Gamma^\omega}(\bar{u}(P)) = G_{FD}^{\omega}(\bar{u}(P))$ where the RHS corresponds to $P\{u(\omega, P,1) \leq \bar{u}(P)\}$, or equivalently, $P\{\omega \leq \bar{\omega}(P)\}$.

The optimal policy can thus be interpreted as a pass-fail announcement, where given the level of recapitalization, $P$, the policy-maker assigns a pass grade when the liquidity of the bank is above the cutoff $\bar{\omega}(P)$. Proposition 3 summarizes the above findings.

**Proposition 3.** Fix the amount of capital $P \geq 0$ raised by the bank at $t = 1$. Then, irrespective of the details of the regular stress test $\Gamma^y$ conducted at $t = 1$, the optimal critical stress test $\Gamma^\omega$ consists of a monotone pass-fail test. That is, there exists $\bar{\omega}(p)$, such that $\Gamma^\omega(P) = (\{0,1\}, \pi^\omega_* (P))$, with $\pi^\omega_*(0|\omega; P) = 1 \{\omega \geq \bar{\omega}(P)\}$.

When the government announces that the bank passed the critical stress test (i.e., when $\Gamma^\omega(P)$ discloses $m^\omega = 0$), all creditors keep rolling over the bank’s debt, and hence survival occurs with certainty. When instead the bank fails the critical stress test $\Gamma^\omega$ (i.e., when $\Gamma^\omega(P)$ discloses $m^\omega = 1$), all creditors withdraw early from the bank. Whether the bank defaults then depend on whether $\omega + P$ is larger than 1, which under the optimal policy never occurs since $\bar{\omega}(P) < 1 - P$.

4.4 Regular Stress Test

We now proceed to characterize the optimal stress test $\Gamma^y$, taking into account the optimal critical stress test $\Gamma^\omega$. We will see that the optimal stress test $\Gamma^y$, takes a very simple form: it combines a
recommendation to the bank about the minimal amount of capital to raise during the first period, along with some disclosure about $y$. To make sure that the capital requirement is followed by the bank, the policy-maker imposes a constraint on the bank’s ability to distribute dividends if the amount of capital falls short of the minimal level required.

As the next result shows, the policy-maker asks the bank to raise an amount equivalent to the minimum between the (ex-ante) expected value of the asset and the capital cutoff which prevents posterior runs, $\min \left\{ K, \frac{1}{R} \mathbb{E}(y|m^y) \right\}$. Given the authority’s commitment to forbid the bank to distribute the totality of bank’s payoffs as dividends when the bank does not meet the capital cutoff, the game played by the bank and external investors becomes similar to the one in Proposition 2 and, therefore, in the unique equilibrium of the game, both types of bank pool and offer a debt security, $s_p^\text{pool}$, satisfying:

$$\frac{1}{R} \mathbb{E}(s_p^\text{pool}|m^y) = \min \left\{ K, \frac{1}{R} \mathbb{E}(y|m^y) \right\}$$

On-path, capital requirements are always obeyed.

Consider the design of the regular stress test $\Gamma^y$. At $t = 1$, the designer discloses information about the realization of future cash-flows $y$ according to the rule $\pi^y : \mathbb{R}_+ \to \Delta M^y$. The first result shows that capital requirements are necessary to minimize the probability of default. By introducing capital requirements the policy-maker mitigates separation incentives among bank types during the fund-raising game. In fact, high-asset quality types generally have an incentive to separate from low-asset quality types, and offer less expensive securities. This is because the price they receive for the security they issue is below the fair price they would receive in the absence of lower types mimicking their chosen security. The imposition of a minimal capital requirement makes this type of deviations unprofitable for high-asset quality types, and foster risk-sharing among bank types. The following proposition shows that, whenever possible, the optimal policy asks the bank to raise at least $K$ funds so as to persuade creditors to keep rolling over the bank’s debt. Whenever this is not possible (i.e., whenever the value of the assets falls below $K$) the regulator asks that the bank sells the whole asset.

**Proposition 4.** The optimal regular stress test $\Gamma^y = \{ \{M^y, \pi^y\}, \{R_\alpha[m^y]\}_{m^y \in M^y} \}$, imposes capital requirements according to the rule:

$$R_\alpha (P|m^y) = 1 \left\{ P < \min \left\{ K, \bar{P}(\mathbb{E}(y|m^y)) \right\} \right\} .$$

for some $\alpha \in (0, 1)$, where

$$\bar{P}(z) \equiv \sup \left\{ P \geq 0 : \frac{z}{R} \mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P \right\}$$

represents the maximal fair price consistent with selling a security with expected cash-flow $z \geq 0$.

Proposition 4 is subtler than it may appear at first glance. Contrary to what might be conjectured based on Proposition 2, a stress test which reveals a message $m^y$ such that $\frac{1}{R} \mathbb{E}(y|m^y) \geq K$ need
not prevent the freeze of the asset market under the best equilibrium of fund-raising game for arbitrary critical stress tests $\Gamma^\omega$. In fact, when the policy-maker reveals information about the bank’s liquidity shock $\omega$ in period 2, market freeze may occur even in situations where, without government intervention the bank would have been able to raise enough funds to avert a subsequent default. The reason is is that the possibility of a run of short-term funds imposes market discipline on the bank at the fund-raising game and mitigates, to some extent, separation incentives among bank types. In the absence of a critical stress test, high-quality types reduce the amount of funds raised from external investors to minimize the underpricing of their securities due to pooling with low-quality banks. As Proposition 2 shows, they raise exactly $K$, the minimal amount required to prevent a massive withdrawal of short-term funds. When the bank and external investors expect the policy-maker to disclose information about $\omega$ at $t = 2$, their assessment about creditors’ expected response become more optimistic. This, in turn, makes it easier for high-quality types to separate from low-quality types and, hence, risk-sharing incentives are dissipated. Imposing contingent capital requirements then substitute for the disciplining role of creditors’ run by limiting the dividends if the capital cut-off is not met. This implies that disclosing information, even if favorable, is not generally effective at preventing disruption of capital markets in the absence of a capital requirement policy\textsuperscript{11}. The next example shows that information disclosure about the liquidity shock $\omega$ may become ineffective without capital requirements.

**Assumption 3.** Creditors’ conditional payoffs in case they choose to pledge, $b$ and $g$, are constant.

**Example 1.** Suppose Assumption 3 holds and let $c \equiv \frac{g}{g+|b|}$. Assume that the policy-maker disclosed information $m^y$ in period 1 such that $E(y|m^y) \geq 1 - c$, and also that:

$$f^{\omega}(\omega) = \begin{cases} 2 \left(1 - \frac{\omega}{c}\right) & \omega \leq c \\ \omega - c & \omega > c. \end{cases}$$

Then, without capital requirements, at the best equilibrium of the fund-raising game default occurs without positive probability, regardless of the critical stress test $\Gamma^\omega$. Imposing capital requirements reduces the probability of default to 0.

We now proceed to the characterization of the optimal regular stress test $\Gamma^y$. Any information $m^y$ disclosed with positive probability induces a posterior probability distribution over $y$, for each bank type, and hence a posterior mean, $E_q(y|m^y)$, $\theta \in \Theta$.\textsuperscript{12} As a result of proposition 4, we know that the optimal policy implements capital requirements that depend on the posterior mean of cash-flows, $E(y|m^y)$. Let $G^y$ be the distribution of posterior means induced by policy $\Gamma^y$. The set of possible

\textsuperscript{11}In environments where the policy-maker lacks commitment power to not disclose information about the bank’s liquidity position if a shock materializes, the possibility of conducting a critical stress test may backfire if capital requirements are not imposed and equilibrium outcomes may prove worse than under a laissez-faire regime.

\textsuperscript{12}Note that regardless of the disclosure policy chosen by the policy-maker, $\pi^y$, there will never be a re-ordering of types. This a consequence of the fact that MLRP ordering is robust to bayesian updating.
distributions of posterior means that can be induced with a disclosure policy coincides with the set of distributions which are a mean-preserving contraction of the prior \( F^y \) (Blackwell [1953], Gentzkow and Kamenica [2016]).

Define then \( G^\Gamma^y(t) \equiv \int_t^1 G^\Gamma^y(z) \, dz \) as the integral of the distribution of posterior expected cash-flows induced by policy \( \Gamma^y \). Let \( G^\Gamma^y_{FD} \) and \( G^\Gamma^y_\emptyset \) be the functions associated to the full-disclosure policy and no-disclosure policies. Observe that under full-disclosure, each message generates a degenerate posterior distribution with all weight assigned to the true realization of \( y \), which also coincides with the posterior mean generated by the message. As a result, \( G^\Gamma^y_{FD}(t) = \int_t^1 F^y(z) \, dz \). Next, notice that under no-disclosure, the posterior mean remains unchanged and equal to \( \mathbb{E}(y) \). Thus, \( G^\Gamma^y_\emptyset(t) = \int_t^1 1 \{ z \geq \mathbb{E}(y) \} \, dz \). Finally, note that the distribution of posterior means \( G^\Gamma^y \) a garbling of the full-disclosure policy and a mean-preserving spread of the no-disclosure policy. That is, \( G^\Gamma^y_{FD} \succeq_{MPS} G^\Gamma^y \succeq_{MPS} G^\Gamma^y_\emptyset \). An argument analogous to the one provided for the critical stress test then implies that any convex function \( h \) satisfying \( G^\Gamma^y_{FD}(t) \geq h(t) \geq G^\Gamma^y_\emptyset(t) \) can be induced by some disclosure policy, and that the set of convex functions between \( G^\Gamma^y_\emptyset \) and \( G^\Gamma^y_{FD} \) are characterize the whole set of implementable disclosure policies.

Next, fix a message \( m^y \) and the induced expected value of the bank’s asset \( \mathbb{E}(y|m^y) \). Proposition 4 implies that the policy-maker may choose capital requirements so that the cutoff defining whether the bank survives or not can get arbitrarily close to \( \bar{\omega} \min \{ K, \bar{P}(\mathbb{E}(y|m^y)) \} \). Recall next that the function \( \bar{\omega} \) identifies the critical value of the liquidity shock below which the bank defaults when the capital raised at \( t = 1 \) is equal to \( P \). This value is equal to 0 for any \( P \geq K \), and that \( \bar{P}(K \times R) = K \).

Therefore we can write the relevant cutoff as \( \bar{\omega} \bar{P}(\mathbb{E}(y|m^y)) \). The designer’s objective function can thus be written as:

\[
\mathbb{E}(W_0 \left( 1 \{ \omega < \bar{\omega} \bar{P}(\mathbb{E}(y|m^y)) \} \right) \times 1 \{ \omega \geq \bar{\omega} \bar{P}(\mathbb{E}(y|m^y)) \} ),
\]

or equivalently,

\[
W_0(0) \times (1 - F^\omega(\bar{\omega} \bar{P}(\mathbb{E}(y|m^y)))).
\]

Thus, the policy-maker’s problem reduces to:

\[
\max_{G^\Gamma^y} \int_0^\infty (1 - F^\omega(\bar{\omega} \bar{P}(\tau))) G^\Gamma^y(d\tau)
\]

s.t: \( F^y \succeq_{MPS} G^\Gamma^y \)

Define \( Z^\Gamma^y \) as the auxiliary function that allows us to take mean-preserving contractions of \( F^y \). That is, for any mean-preserving contraction \( G^\Gamma^y \), we define \( Z^\Gamma^y \) such that \( G^\Gamma^y = F^y + Z^\Gamma^y \). Any such \( Z^\Gamma^y \) must respect the condition below:

**Condition 1.** \( Z^\Gamma^y \) is such that \( F^y + Z^\Gamma^y \) is (i) positive, (ii) non-decreasing, and (iii) right-continuous. Additionally, (iv) \( Z^\Gamma^y \) belongs to the set:
Z \equiv \left\{ Z : \mathbb{R}_+ \to \mathbb{R} : \int_0^{\bar{y}} Z(y)dy \leq 0 \ (\forall \bar{y} \geq 0), \int_0^{\infty} Z(y)dy = 0, \ Z(\infty) = 0 \right\}.

We can thus rewrite the designer’s period 1 problem in terms of $Z^{\Gamma^y}$ as follows:

$$\max_{Z^{\Gamma^y}} \int_0^{\infty} (1 - F^w(\tilde{\omega}(\tau))) Z^{\Gamma^w}(d\tau)$$

s.t: $Z^{\Gamma^w}$ satisfies condition (1)

As the next theorem shows, the optimal regular stress test consists of a monotone partition signal, where different values of $y$ are pooled (if at all) with adjacent realizations (i.e., within the same interval). Moreover, we show that under Assumptions 2 and 3, the optimal stress test $\Gamma^y$ takes a simple form. Namely, it fully discloses the realization of $y$ for any realization below a cutoff $y^+$, and pool all realizations above $y^+$ under a single message, say $m^y_+$. The posterior mean induced by message $m^y_+$, $E(y|y \geq y^+) \geq P^+$, where $P^+$ corresponds to the level of funds for which no further disclosure is required in the next period. That is, $P^+$ is the smallest amount for which $\tilde{\omega}(P^+) = 0$. Equivalently, it corresponds to the lowest amount of capital that persuades creditors to rollover the bank’s debt, under the prior beliefs characterized by $F^w$. In other words,

$$\tau^+ = K.$$

**Theorem 1.** The optimal regular stress test consists of a monotone partitional signal. That is, there exists a monotone partition $\mathcal{P} = \{(y_i, y_{i+1}]\}_{i \in I}$ of $\mathbb{R}_+$ such that the optimal regular stress test $\Gamma^y = \{m^y_i\}_{i \in I}, \pi^y_i$ satisfies $E(y|m^y_i) < E(y|m^y_j)$ for all $i < j$. Moreover, the highest cell in the partition always include $K$. Furthermore, under Assumptions 2 and 3, the optimal regular stress test is given by $\Gamma^y = \{[0, y^+] \cup m^y_{\text{pass}}, \pi^y\}$, with $\pi^y(\bar{y}|y) = 1_{\{\bar{y} = y\}}$ and $\pi^y(m^y_{\text{pass}}|y) = 1_{\{y \geq y^+\}}$ for all $\bar{y} \in [0, y^+]$, and all $y \geq 0$, where $y^+$ is defined by:

$$y^+ = \sup \left\{ y \geq 0 : \int_y^K (F^y(y) - F^y(\tau))d\tau + \int_0^\infty (1 - F^y(\tau))d\tau \geq 0 \right\}. \quad (10)$$

Theorem 1, along with the former results, imply that under assumptions 2 and 3, the optimal comprehensive disclosure policy $\Gamma = \{\Gamma^y, \Gamma^w\}$ has a simple structure. The policy $\Gamma$ assigns a single grade to all banks which meet a minimum standard in terms of profitability of their assets. This grade should be thought of as passing the policy-maker stress test on the quality of the bank’s asset. Any bank failing to meet this minimal standard receives a grade that fully reveals the quality of its assets. The policy $\Gamma$ also specifies a capital requirement rule that asks the bank to either raise enough funds to prevent a creditors’ run, or to sell the whole asset to external investors when its quality is low. Finally, the optimal policy entails a follow up stress test on the bank’s liquidity position which takes the form of a monotone pass-fail test that fails all banks with a liquidity position below an optimal cut-off, and passes the other.
Corollary 1. The optimal comprehensive disclosure policy $\Gamma = \{\Gamma^y, \Gamma^\omega\}$ can be sequentially implemented by:

(1) Conducting a regular stress test which (i) assigns a passing grade $m^y_{\text{pass}}$ to all banks with assets generating cash-flow above $y^+$, and assigns a failing grade $m^y_i$ to any assets delivering cash-flows $y \in (y_i, y_{i+1}]$, and (ii) imposing capital requirements which dictates that the bank raise $K$ when receiving the passing grade, and to sell the asset when falling below cut-off $y^+$.

(2) Conducting a critical stress test that informs creditors of wether the liquidity shock is above the cut-off $\bar{\omega} \left( \bar{P} \left( \mathbb{E}(y|m^y) \right) \right)$.
5 Screening Liquidity Position

In this section I consider interventions when the policy-maker does not have access to a disclosure technology that allows her to respond to liquidity shocks in a timely manner and, therefore, has to rely on information directly reported by the bank when such shocks occur. I also relax the assumption that the policy-maker cannot use public funds to help the bank survive an adversarial liquidity shock. I assume that the policy-maker may purchase securities from the bank using taxpayers’ money, but I impose the constraint that the price paid by the policy-maker not exceed the fair price of the securities, taking into consideration the probability of default.

The timing of the game remains identical to the one in Section 4, with the single modification that instead of allowing the policy-maker to conduct the critical stress test during at $t = 2$, the policy-maker runs a screening mechanism which asks the bank to self-report its private information $(\omega, \theta)$ and, conditional on the report, offers funding in exchange for a claim on the bank’s asset, and specifies a public disclosure about the bank’s information. I also assume that the policy maker cannot force the bank to accept the deals she offers. This assumption is made to rule out solutions that involve confiscation by the policy-maker.

5.1 Period 2: Screening

Assume that the policy-maker has disclosed information $m^y$ in period, according to the stress test $\Gamma^y$, and that the bank has successfully raised $P$ units of capital. Recall that if the recapitalization level, $P$, is such that

$$\int_0^{1-P} b(\omega + P, 1) f^w(\omega) d\omega + g \times (1 - F^w(\omega)) \leq 0,$$

(11)

creditors withdraw early in the absence of any disclosure by the policy-maker. In this case, the policy-maker offers the bank screening mechanism $\Upsilon^w = \{\{M^w, \pi^w\}, t, s\}$, which asks the bank to report its asset quality type $\theta$ and its liquidity position $\omega$ and, as a function of the report $(\hat{\theta}, \hat{\omega})$, offers to purchase a claim on the bank’s asset $s(\hat{\omega}, \hat{\theta})$, with $s\left(y|\hat{\omega}, \hat{\theta}\right) \in [0, y - s^*(y)]$, at a price $t\left(\hat{\omega}, \hat{\theta}\right) \geq 0$. In addition, $\Upsilon$ discloses a message $m^w$ to all market participants according to the disclosure policy $\pi^w[\hat{\omega}, \hat{\theta}] \in \Delta M^w$. The mechanism $\Upsilon$ must be (interim) (i) incentive compatible and (ii) individually rational. That is, (i) the bank must be at least as well-off by disclosing its private information than by reporting any other value of $\theta$ and $\omega$, and (ii) the bank must be at least as well-off by participating in the designer’s mechanism than by opting-out of it. Given that the designer can always induce the same conditions that the bank would face when opting-out of the program, it is wlog to assume that all bank types participate in the policy-maker’s program.\footnote{Philippon and Skreta [2012] and Tirole [2012] study a similar problem, but focus on indirect mechanisms where the bank has to decide whether to participate in the government program or not. This leads them to a mechanism design problem with endogenous participation constraints. In contrast, this paper follows a direct mechanism approach, where...}
An argument similar to the one establishing the Revelation Principle implies that it is without loss of optimality to restrict attention to mechanisms where the messages sent to the creditors take the form of action recommendations that creditors are willing to follow. This means that we can restrict the analysis to disclosure mechanisms with $M^\omega = \{0, 1\}$, where message $m^\omega = 0$ is interpreted as the recommendation to rollover the bank’s debt, and $m^\omega = 1$ as the recommendation to stop pledging funds. We will distinguish between the security and price offered by the designer when disclosing message $m^\omega = 0$, $(t_0 (\tilde{\omega}, \tilde{\theta}), s_0 (\tilde{\omega}, \tilde{\theta}))$, and the contract offered when recommending $m^\omega = 1$, $(t_1 (\tilde{\omega}, \tilde{\theta}), s_1 (\tilde{\omega}, \tilde{\theta}))$. Obedience requires that when the policy-maker a discloses recommendation message $m \in \{0, 1\}$, it must be the case that

$$
\mathbb{E} (u (P + t_0 (\omega, \theta), \omega, 1) | m = 0) = \frac{\sum_\theta \mu_\theta \times \int_{\Omega} u (\omega, P + t_0 (\omega, \theta), 1) \pi^\omega (0 | \omega, \theta) F^\omega (d \omega)}{\sum_\theta \mu_\theta \times \int_{\Omega} \pi^\omega (0 | \omega, \theta) F^\omega (d \omega)} > 0, \quad (12)
$$
and

$$
\mathbb{E} (u (P + t_1 (\omega, \theta), \omega, 1) | m = 1) = \frac{\sum_\theta \mu_\theta \times \int_{\Omega} u (\omega, P + t_1 (\omega, \theta), 1) \pi^\omega (1 | \omega, \theta) F^\omega (d \omega)}{\sum_\theta \mu_\theta \times \int_{\Omega} \pi^\omega (1 | \omega, \theta) F^\omega (d \omega)} \leq 0. \quad (13)
$$

Hereafter I refer to conditions (12) and (13) as obedience constraints. As shown in the former section, the policy-maker’s optimal disclosure policy, absent incentive compatibility constraints, consists of failing all banks with a liquidity position below the cutoff $\tilde{\omega} (P)$, so that banks with liquidity positions above $\tilde{\omega} (P)$ may survive. However, no bank vulnerable to runs (i.e., a bank with $\omega < 1 - P$) would ever choose to report its true type if this leads the designer to recommend short-term creditors to attack with certainty. In order to solve this conflict, the policy-maker may offer less liquid banks to purchase their assets at better terms in exchange of a lower passing probability. This implies that more liquid (but still vulnerable) banks have to receive lower prices for their remaining claims on the asset. The fact that these banks would default in the absence of a deal with the policy-maker then makes the mechanism incentive compatible. In what follows I provide a proof to the arguments explained above.

Let $U_P (\tilde{\omega}, \tilde{\theta}, \omega, \theta)$ be the utility of a bank with private information $(\omega, \theta)$ which has successfully raised $P$ units of capital in period 1, and chooses to report $(\tilde{\omega}, \tilde{\theta})$. Thus,

$$
U_P (\tilde{\omega}, \tilde{\theta}, \omega, \theta) = \sum_{m \in \{0, 1\}} \pi^\omega (m | \tilde{\omega}, \tilde{\theta}) \times 1 \{\omega + P + t_m (\tilde{\omega}, \tilde{\theta}) \geq A (P, m)\} \times \\
\times \left( (P + t_m (\tilde{\omega}, \tilde{\theta})) R + \mathbb{E}_\theta (y - s^* - s_m (\tilde{\omega}, \tilde{\theta})) \right)
$$

where $A (\tau, m)$ corresponds to the most aggressive fraction of early withdrawals consistent with observing the bank raising $P$ units of capital and the policy-maker disclosing message $m \in \{0, 1\}$. It is wlog to assume participation by all types. This distinction is obviously inconsequential for the allocations that are induced on-path. What makes my analysis fundamentally different from these works is that I enrich the designer’s problem by allowing her to disclose information in addition to purchasing assets.
Note then that the obedience constraints (12) and (13) imply that $A(\tau, m) = m$. That the mechanism satisfies incentive compatibility then translates to:

$$(\omega, \theta) \in \arg \max_{\tilde{\omega}, \tilde{\theta}} U_P \left( \tilde{\omega}, \tilde{\theta}; \omega, \theta \right).$$

Next, observe that offering to purchase claims on the bank’s asset when the policy-maker has assigned the failing grade only makes obedience constraint (13) and incentive compatibility constraints harder to satisfy and does not provide any benefit. Thus, it is without loss of optimality to set $t_1(\omega, \theta) = 0$ and $s_1[\omega, \theta] = 0$ for all $\omega$ and $\theta$. Moreover, given that any vulnerable bank will fail if not helped by the government, we can restrict attention to mechanisms which set $s_0[\omega, \theta_L] = y - s^*$ for all $\omega < 1 - P$, since this allows the policy-maker to offer higher prices for the bank’s securities. This property need not be satisfied for a type-H bank. To see this, note that it might be in the interest of the policy-maker to offer type-H banks to retain a fraction of their asset. This might be useful to alleviate incentive constraints. Also observe that the precise type of securities purchased by the policy maker is irrelevant. The only thing that matters is the fraction of the expected value of the security retained by the bank. Let $z_\theta = \mathbb{E}_H (y - s^*_\theta)$ be the value of the claims on asset of a type-$\theta$ bank net the cash flows promised to external investors under security $s^*_\theta$. Let $\phi(\omega, \theta_H)$ denote the fraction of $z_H$ the bank retains on its balance sheet. Next, note that incentive compatibility requires that banks do not have incentives to pretend to have neither a different liquidity position nor a different type. This means that the utility of vulnerable banks must be equalized across all $\omega < 1 - P$, for a given asset quality type, since otherwise the bank would report the message that yields best terms. That is,

$$\pi (0|\omega, \theta_L) \times ((P + t_0(\omega, \theta_L)) R) = V_L, \quad \forall \omega \leq 1 - P;$$

and

$$\pi (0|\omega, \theta_H) \times ((P + t_0(\omega, \theta_H)) R + \phi(\omega, \theta_H) z_H) = V_H, \quad \forall \omega \leq 1 - P.$$  

At the same time, banks must not have incentives to pretend to have a different asset quality type. This means that a vulnerable type L-bank must not want to pretend to be a type H bank:

$$V_L \geq \pi (0|\omega, \theta_H) \times ((P + t_0(\omega, \theta_H)) R + \phi(\omega, \theta_H) z_L), \quad \forall \omega \leq 1 - P.$$  

Similarly a vulnerable type H-bank must not have incentives to mimic a type L-bank:

$$V_H \geq V_L, \quad \forall \omega \leq 1 - P.$$  

We now characterize global incentive constraints. Namely, we make sure that safe banks do not want to mimic vulnerable ones and that vulnerable banks do not want to pretend to be safe ones. We start with the former case. A liquid bank with high quality assets (i.e., a bank with $\omega > 1 - P$ and $\theta = \theta_H$) would never accept any deal to sell any security $\hat{s}$ on its assets at a
price less than $\mathbb{E}_H (s|m^y)$. Any deal that pays a security $s$ at least $\mathbb{E}_H (s|m^y)$ would prompt safe banks with low-quality assets and vulnerable banks of both asset quality to pretend to be safe and having a high-quality asset, unless they are also offered an equally attractive deal. The fair-price constraint mentioned above (and made explicit below) however implies that the policy-maker cannot afford to pay type- $L$ as if it were type- $H$. The combination of the IC constraints with the fair-price constraint then imply that the policy-maker must not buy any security from safe banks. That is, $t_0(\omega, \theta) = t_1(\omega, \theta) = 0$ for any $\omega \geq 1 - P$, and any $\theta \in \Theta$.

Additionally, if the designer were to pass safe banks with probability one, then all vulnerable bank types would claim to be safe. In particular, vulnerable banks with high-quality assets would claim to be safe, thus avoiding default and being pooled with low-quality types. To overcome this problem the policy-maker must fail safe banks with positive probability. Let $\pi_s$ be the probability with which the policy-maker passes a safe bank. Incentive compatibility then requires that $V_L \geq \pi_s \times (P + z_L) R$, and that $V_H \geq \pi_s \times (P + z_H) R$, so that no vulnerable bank type has incentives to claim to be safe.

We can restate both inequalities as:

$$\pi_s \leq \min \left\{ \frac{V_L}{PR + z_L}, \frac{V_H}{PR + z_H} \right\}. \quad (18)$$

The fact that liquid banks cannot be offered the passing grade with high probability makes obedience constraint (13) hard to satisfy.

Next, consider the conditions guaranteeing that safe banks do not pretend to be vulnerable. The incentives problem is most severe for safe banks with low quality assets. By pretending to be vulnerable such banks would which receive the payment $t_0$ in case they receive a pass grade, and irrespective of the grade would never fail. For a safe type $L$-bank to not have incentives to claim to be vulnerable it must be that:

$$PR + z_L \geq \max_{\omega} \pi (0|\omega, \theta_L) \times (P + t_0 (\omega, \theta_L)) R + (1 - \pi (0|\omega, \theta_L)) (PR + z_L),$$

and

$$PR + z_L \geq \max_{\omega} \pi (0|\omega, \theta_H) \times ((P + t_0 (\omega, \theta_H)) R + \phi (\omega, \theta_H) z_L) + (1 - \pi (0|\omega, \theta_L)) (PR + z_L).$$

These constraints impose a bound on the amount the policy-maker can pay to vulnerable banks. In fact, the above constraints together, imply that

$$t_0 (\omega, \theta_L) \leq \frac{z_L}{R}, \quad \forall \omega < 1 - P, \quad (19)$$

and

$$t_0 (\omega, \theta_H) \leq (1 - \phi_H (\omega, \theta_H)) \frac{z_L}{R}, \quad \forall \omega < 1 - P. \quad (20)$$

Finally, consider the requirement that the price paid by the policy-maker not exceed the fair price of the security purchased. This means that:

$$t_0(\omega, \theta) \leq \frac{\mathbb{E}_H (s|\omega; m^y|m^y)}{R}, \quad (21)$$
Note that (21) uses the property that the mechanism is _obedient_, and hence the probability of default equals 0 when a pass grade \( m^\omega \) is given.

Summarizing, the policy-maker’s problem can be reduced to finding a passing probability \( \pi(0|\cdot) \) and transfer \( t_0(\cdot, \cdot) \), which maximize the probability of passing vulnerable banks, subject to the obedience constraints (12) and (13), incentive constraints among vulnerable banks (14), (15), incentive compatibility constraints guaranteeing that safe banks do not want to mimic vulnerable ones and vice versa (18), (19), and (20), and the constraint that imposes that the policy-maker does not pay more than the fair price (21) for the security she purchases from the bank:

\[
\max_{\{(0,1),\pi^\omega\}, t_0} \int_0^{1-P} \pi^\omega (0|\omega) F^\omega (d\omega) \\
\text{s.t: (12), ..., (21).} \tag{22}
\]

Let \( L(P) \equiv \int_0^{1-P} b(\omega + P, 1) f^\omega (\omega) d\omega + g \times (1 - F^\omega (1 - P)) \) be the creditors’ expected payoff under the _Laissez Faire_ regime, when they pledge to the bank and the latter successfully raises \( P \) units of capital during the fund raising game. That is, the government intervenes whenever the amount raised by the bank is such that \( L(P) \leq 0 \), or equivalently, \( P \leq K \). Our next result characterizes the optimal mechanism when the designer constraints herself to mechanisms that only elicit information about the bank’s liquidity, and disregard information about the quality of the bank’s asset. Under such type of mechanisms

\[
t_0 (\omega, \theta_L) = t_0 (\omega, \theta_H) = t_0(\omega), \quad \pi (0|\omega, \theta_L) = \pi (0|\omega, \theta_H) = \pi(0|\omega), \quad \& \phi(\cdot, \theta) = 0. \tag{23}
\]

I refer to this type of programs as _liquidity-eliciting mechanisms_. As I show below, the optimal _unconstrained_ elicitation program can be described relying on the characterization of the best liquidity-eliciting mechanism.

**Proposition 5.** Assume that \( P \leq K \). Then, the optimal liquidity-eliciting mechanism (i.e., satisfying (23)) is given by by:

\[
t_0(\omega) = \begin{cases} 
P + z_L/R & \omega < \hat{\omega} \\
1 - P - \omega & \omega \in [\hat{\omega}, \bar{\omega}] \\
1 - P - \bar{\omega} & \omega \in (\bar{\omega}, 1 - P) \\
0 & \omega \geq 1 - P 
\end{cases}, \quad \pi(0|\omega) = \begin{cases} 
\bar{V}/F + z_L & \omega < \hat{\omega} \\
\bar{V}/(1-\omega)R & \omega \in [\hat{\omega}, \bar{\omega}] \\
\bar{V}/(1-\omega)R & \omega \in (\bar{\omega}, 1 - P) \\
\bar{V}/F + z_H & \omega \geq 1 - P 
\end{cases}
\]

where \( \bar{\omega} \equiv 1 - P - z_L \), and \( \hat{\omega} \) and \( \bar{\omega} \) and \( \bar{V} \) are chosen so that:

\[
\int_{\bar{\omega}}^{\hat{\omega}} \frac{f^\omega (\omega)}{(1-\omega)R} d\omega + \frac{F^\omega (1 - P) - F^\omega (\bar{\omega})}{(1-\omega)R} = \frac{1}{g} \int_{0}^{\hat{\omega}} \frac{b(\omega + P, 1)}{PR + z_L} f^\omega (\omega) d\omega - (1 - F^\omega (1 - P)) \left( \frac{PR + z_H}{PR + z_H} \right).
\]

\[
\bar{V} \equiv \min \left\{ (1 - \hat{\omega}) R, \left\{ \frac{\left| L(P) \right|}{\int_0^{1-P} \frac{b(\omega + P, 1)}{(P + t_0(\omega))R} f^\omega (\omega) d\omega - g \times (1 - F^\omega (1 - P))}{PR + z_H} \right\} \right\}.
\]

32
The optimal liquidity-eliciting mechanism characterized in Proposition 5 is illustrated in Figure 4. The mechanism pays $z_L$, the maximal possible amount consistent with incentive compatibility constraints (19, 20), to all types below $\hat{\omega}$, defined as the most adversarial liquidity shock that has raised $P + \frac{z_L}{R}$ may survive when all creditors withdraw early. These banks are passed with the smallest probability consistent with incentive compatibility constraints. Banks observing a liquidity shock $\omega \in [\hat{\omega}, \bar{\omega})$ receive the precise amount of cash that allows them to survive an adversarial withdrawal and are passed with larger probability. Banks with a liquidity shock $\omega \in [\bar{\omega}, 1 - P)$ receive the smallest amount of funds provided within the policy-maker’s liquidity program and are given in return the largest passing probability. Finally, safe banks do not trade with policy-maker and are given a passing grade with small probability, which guarantees that vulnerable banks do not want to report to be safe.

To persuade creditors to follow the recommendation to rollover the bank’s debt, the policy-maker has to modify the likelihood of the bank’s survival. The bound on the price that can be pledged by policy-maker implies that banks with a buffer smaller than $\hat{\omega} + P + \frac{z_L}{R}$ default when all creditors withdraw early. The policy-maker then minimizes the passing probability assigned to these liquidity-types and compensates by paying the maximal price consistent with constraints (19, 20). All banks with liquidity positions above $\hat{\omega}$ receive enough funds to prevent default under an adversarial withdrawal. Incentive compatibility among vulnerable banks impose a negative relationship between the passing probability and the price paid by the policy-maker. Banks observing a liquidity shock $\omega \in [\hat{\omega}, \bar{\omega})$ receive the smallest price that allows them to survive a massive withdrawal in order to maximize the probability of assigning a passing grade. Any vulnerable bank with a liquidity shock above $\bar{\omega}$ is given the maximal passing probability and the minimal price within the liquidity program. The level $\bar{\omega}$ is chosen so that obedience constraint is satisfied. Intuitively, the smaller the value of $\bar{\omega}$, the more liquidity-types receive the maximal passing probability and, hence, the large
the aggregate survival probability. The optimal liquidity-eliciting mechanism chooses the minimal value of $\omega$ consistent with obedience constraint (12).

5.2 Period 1: Stress Test

In this section I study the design of information disclosure policies concerning the quality of the bank’s asset, along with contingent capital requirements, when any information about the bank’s liquidity position must be elicited. As I show below, the policy-maker faces an important trade-off when designing the capital requirements rule to impose in the first period: On the one hand, smaller capital requirements allow the bank to retain a greater fraction of the asset on its balance sheet. In turn, this increases the price the policy-maker can offer to banks, thus, enhancing the effectiveness of the liquidity program. On the other hand, more stringent capital requirements permit the bank to raise capital before the liquidity shock materializes. This helps decrease the premium bank has to pay to compensate for rollover risk.

In order to implement successful liquidity-provision programs (i.e., for the policy-maker to be able to assign informative grades about the bank’s liquidity position), the bank needs to own remaining claims on its asset with a value above a minimum threshold. When this cut-off is not met, the regulator can not induce the bank to self-report private information regarding its liquidity buffers. As discussed in the previous subsection, the key trade-off that allows the regulator to induce the bank to self report its liquidity position involves a negative relation between the amount of funds offered to the bank, and the probability of receiving a passing grade. When the value of the remaining claims on the bank’s asset is small, the bank for its assets is to low to discourage most vulnerable banks from mimicking more liquid ones and, hence, information elicitation about $\omega$ does not take place. Let $E$ be the minimal expected value of the bank’s remaining claims necessary for information elicitation. The next theorem shows that $E$ can be characterized by

$$E \equiv \inf_{E} \left\{ E \geq 0 : \int_{0}^{1-\frac{E}{R}} \frac{b}{E} \times \frac{f^{\omega}(\omega)}{E} d\omega + \int_{1-\frac{E}{R}}^{1} \frac{g \times f^{\omega}(\omega)}{(1 - \omega) R} d\omega \geq 0 \right\}. \quad (24)$$

Theorem 2 characterizes the optimal recapitalization policy and liquidity-provision program for any message disclosed by the disclosure rule $\{M^y, \pi^y\}$. I show that for intermediate ranges of asset quality $y$ policy-maker induces the bank to report its liquidity position and discloses information to the bank’s creditors according to a stochastic rule which assigns a pass grade in a monotone manner (that is, more liquid banks are passed with higher probability). The price the policy-maker pays for the bank’s assets is decreasing in the bank’s liquidity. Moreover, in this case the regulator does not impose capital requirements during the first period, and effectively asks the bank not to approach external investors. For low values of $y$ the policy-maker, instead, is unable to elicit information about the bank’s liquidity position. In that case, the policy-maker recommends the bank to raise capital from external investors before the liquidity shock materializes, which helps the bank maximize the amount of funds it gets in exchange for claims on its assets. Similarly, when the value of $y$ is large,
the policy-maker asks the bank to seek private sector financing (i.e., from external investors). In this
case the bank is asked to raise enough funds to persuade short-term creditors to rollover.

**Theorem 2.** Fix a message \( m^y \) disclosed with positive probability under \( \Gamma^y \). The optimal recapitalization policy and liquidity-provision program can be characterized as a function of the expected value of the asset’s cash-flows, \( y \equiv \mathbb{E}(y|m^y) \), as follows:

(i) If \( z \geq K \), the optimal recapitalization policy is given by \( R_\alpha(P) = 1 \{ P < K \} \) for some \( \alpha > 0 \), and no liquidity-provision program is required.

(ii) If \( E < z < K \), then either the bank is funded by external investors and no liquidity program is used, in which case \( R_\alpha(P) = 1 \{ P < \bar{P}(z) \} \), or the bank is asked to not raise capital from external investors, in which case the policy-maker uses the following liquidity-provision program to solicit information about \( \omega \):

\[
t_\theta^*(\omega; z) \equiv \begin{cases} \frac{\hat{z}}{R} & \omega < 1 - \frac{\hat{z}}{R}, \\ 1 - \omega & \omega \in \left[ 1 - \frac{\hat{z}}{R}, \tilde{\omega} \right], \\ 1 & \omega \in (\tilde{\omega}, 1] \end{cases} \quad \pi^*(0|\omega; z) \equiv \begin{cases} \frac{(1 - \omega)R}{z} & \omega < 1 - \frac{\hat{z}}{R}, \\ \frac{1 - \omega}{(1 - \omega)} & \omega \in \left[ 1 - \frac{\hat{z}}{R}, \tilde{\omega} \right], \\ 1 & \omega \in (\tilde{\omega}, 1] \end{cases}
\]

with \( \tilde{\omega} \) implicitly defined by:

\[
g \times \left( \int_{1 - \frac{\hat{z}}{R}}^{\tilde{\omega}} \frac{f^\omega(\omega)}{(1 - \omega)R} d\omega + \frac{1 - F^\omega(\tilde{\omega})}{(1 - \tilde{\omega})R} \right) = \int_0^{\frac{\hat{z}}{R}} b \left( \frac{z}{\hat{z}}, 1 \right) f^\omega d\omega.
\]

(iii) If \( z \leq E \), the bank is asked to seek funding from external investors and the capital requirement policy \( R_\bar{\alpha}(P) = 1 \{ P < \bar{P}(z) \} \) for some \( \bar{\alpha} > 0 \) is imposed.

Theorem 2 shows that interventions inducing simultaneous pledging by the market and the government are sub-optimal. The intuition behind this result is that, as explained above, inducing the bank to raise capital from external investors reduces the effectiveness of the policy-maker’s liquidity-provision program. Recall that a bank that retains a smaller fraction of its asset can be offered less funds by the government under the *fair price* constraint. Given that the effectiveness of the liquidity-provision program relies on compensating extremely vulnerable banks, which are passed less often than more liquid ones, with higher prices for the remaining claims on their assets, requiring that the bank sells a fraction of its assets to external investors decreases the elicitation capacity of the policy-maker once the liquidity shock materializes. Additionally, having the bank raising funds from external investors intensifies incentive compatibility issues in the regulator’s elicitation program. In fact, any amount of capital \( P > 0 \) raised during the fund-raising game makes a bank safe to a run for all \( \omega > 1 - P \) regardless of the policy-maker’s program. The larger \( P \) is, the larger the set of liquidity shocks under which the bank survives. Furthermore, the larger \( P \) is, the smaller the amount of cash the policy-maker can pay to to vulnerable banks and the smaller the probability a pass grade can be assigned to highly liquid safe banks. At the optimum, the policy-maker then either maximizes \( P \)
and then forgoes using a liquidity-provision program, or set $P = 0$ (thus asking the bank to refrain from raising funds) and then uses a liquidity-provision program.

The next theorem completes the analysis by characterizing the structure of the optimal comprehensive intervention as a function of the quality of the bank’s asset.

**Theorem 3.** The optimal comprehensive policy $\Gamma = (\Gamma^y, \Gamma^o)$ is characterized by a monotone partition $\mathcal{P} = \{(y_i, y_{i+1})\}_{i \in I}$ of $\mathbb{R}_+$ such that the optimal regular stress test $\Gamma^y = \{\{m_i^y\}_{i \in I}, \pi^y\}$ satisfies $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ for all $i < j$. Moreover, the highest interval always include $y^+$. Furthermore,

1. If $y \geq y^+$, the policy-maker passes the bank and sets capital requirements according to the policy $R_\alpha(P) = 1 \{P < K\}$ for some $\alpha > 0$.

2. If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \in (E, K)$, either the bank is funded only by the private sector, in which case $R_\alpha(P) = 1 \{P < \bar{P}(z)\}$, or the bank is funded only by the government only the government through the liquidity-provision program characterized by $t^*_0(\omega; \mathbb{E}(y|m_i^y)), \pi^*_0(\omega; \mathbb{E}(y|m_i^y))$, where $t^*_0$ and $\pi^*_0$ are as defined in (25).

3. If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \leq E$, the bank is asked to seek external funding, the government imposes capital requirements according to $R_\alpha(P) = 1 \{P < \bar{P}(z)\}$ for some $\alpha > 0$, and no liquidity program is used.

Theorem 3 shows that the optimal comprehensive policy features a non-monotone pecking order. Institutions with high-quality assets are given a pass grade by the stress test $\Gamma^y$ and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades and are funded with the government’s liquidity-provision program. Finally, institutions with extremely poor-quality assets, are failed with multiple failing grades and are induced to seek funding from the private sector.
6 Conclusions

In this paper, I study government interventions aimed at stabilizing financial institutions subject to rollover risk. I consider a rich environment which emphasizes the interaction among multiple audiences who care about different aspects of the bank’s multi-dimensional fundamentals. I show that complementing disclosure policies with minimal capital requirements is instrumental to maximizing the probability of bank’s survival. By combining appropriately designed information disclosures with capital requirements, the policy-maker is able to implement the optimal solution to a broader mechanism design problem where she has the authority to dictate the type of securities and the price the bank should choose when approaching external investors. Conferring such authority to the policy-maker is however not necessary. Perhaps surprisingly, the optimal stress test is opaque when the institution has high-quality assets and assigns a unique pass grade. In contrast, the optimal stress test is more transparent with banks with low-quality assets, in which case multiple failing grades are assigned to the bank as a function of the precise quality of the assets, which also triggers a follow-up stress test on the bank’s liquidity position.

When the policy-maker lacks the ability to examine the bank’s liquidity position and, hence, needs to elicit information from the bank, the initial stress test is followed by a liquidity-provision program, whereby the government offers to buy assets from the bank, in exchange of cash and a public disclosure of the bank’s liquidity position. I show that, in this case, imposing capital requirements undermines the effectiveness of the government’s liquidity program. I also show that simultaneous pledging by the government and the private sector is suboptimal. I find that optimal comprehensive policies feature a non-monotone pecking order: Institutions with high-quality assets are given a pass grade by the stress test that assess the long-term profitability of the bank’s assets and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades, and are funded with the government’s liquidity-provision program. Finally, institutions with extremely poor-quality assets are failed with multiple failing grades and are induced to seek private sector financing.

The above results are worth extending in several directions. The analysis in the present paper assumes the policy maker knows the distribution of future liquidity shocks when she designs the optimal comprehensive policy. Such knowledge may come from previous experience with banks of similar fundamentals. While this is a natural starting point, there are many environments in which it is more appropriate to assume that the designer lacks information about the joint distribution of the underlying fundamentals. In future work, it would be interesting to investigate the optimal disclosure policy in such situations. One idea is to apply a robust approach to the policy-maker’s problem, whereby the designer expects nature to select the information structure that minimizes her payoff. The characterization of the optimal policy in this environment is highly relevant both from a theoretical standpoint and for the associated policy implications.
The analysis in the present paper assumes that uncertainty regarding the bank’s liquidity is resolved after the bank approaches external investors. However, creditors’ runs are intrinsically dynamic phenomena. If the fundamentals are partially persistent over time, the optimal policy must specify the timing of information disclosures. In future work, it would be interesting to extend the analysis in this direction.
7 Appendix A: General Mechanisms

In this section we consider general mechanisms and show that the solutions found in sections (4) and (5) are optimal in a more general sense than the one adopted in the main text.

Consider a mechanism \( \Gamma = \{ \{ \pi^y, M^y \}, s, P, x, T \} \), where \( \{ \pi^y, M^y \} \) corresponds to a signal structure with disclosure rule \( \pi^y \), and message space \( M^y \):

\[
\pi^y : \Theta \times \mathbb{R}_+ \rightarrow \Delta M^y
\]

\[
(\theta, y) \in \pi^y[\theta, y],
\]

\( (s, P) \) corresponds to a public recommendation made to the bank and external investors, which are told to trade a security \( s \) at a price \( P \), under the constraint that investors willingly accept the contract. We assume that \( (s, P) \) is measurable only with respect to the public message disclosed by the designer, \( m^y \), which rules out the possibility that the choice of \( (s, P) \) convey information about \( y \), and therefore the only source of additional information regarding the quality of the asset will be given by message \( m^y \). Thus, the contract \( (s, P) \) satisfies:

\[
(s, P) : \Theta \times \mathbb{R}_+ \times M^y \rightarrow \Delta (S \times \mathbb{R})
\]

\[
(\theta, y, m^y) \rightarrow (s[m^y], P(m^y))
\]

s.t: \( \frac{1}{h} \mathbb{E} (x(\theta, m^y, s, P, \omega) \cdot s|m^y, P, s) \geq P. \)

Finally, \( (x, T) \) corresponds to the recommendation made to short-term creditors during the second period, after \( \omega \) has been realized, of whether to keep rolling over the bank’s debt, \( x \), and a transfer, \( T \), from the policy-maker to the bank, in exchange of a fraction of the remaining claims on the asset:

\[
(x, T) : \Theta \times M^y \times S \times \mathbb{R} \times \Omega \rightarrow \Delta \{ \{ 0, 1 \} \times \mathbb{R} \}
\]

\[
(\theta, m^y, s, P, \omega) \rightarrow (x(\theta, m^y, s, P, \omega), T(\theta, m^y, s, P, \omega))
\]

s.t: \( \mathbb{E} (u(\omega, P + T, 1)|m^y, s, P, x = 1) \leq 0, \)

\( \mathbb{E} (u(\omega, P + T, 1)|m^y, s, P, x = 0) > 0, \)

\( \mathbb{E} (T(\theta, m^y, s, P, \omega)|m^y, s, P) \leq \mathbb{E} (y - s|m^y) \)

A few comments are in order. First, we rule out the possibility that \( (x, T) \) directly depend on \( y \). This is without loss since at the moment of recommendation the true value of \( y \) is immaterial. All what short-term creditors care about is whether the (post-transfers) liquidity position \( \omega + P + T \) is above the mass of creditors refraining from rolling-over the bank’s debt. Secondly, that \( x \) takes the form of a recommendation \( (x \in \{ 0, 1 \}) \) which is obeyed by short-term creditors (conditions (b) and (c)) follows from the Revelation Principle. Finally, the third constraint imposes that the designer does not pay more than the expected value of the assets.

Let \( A(m^y, s, P, x) \) be the most aggressive size of attack that follows the public announcement of signal and recommendations \((m^y, s, P, x)\). That is,

\[
(\text{e}) \mathbb{E} (u(\omega, P + T, 1)|m^y, s, P, x) \leq 0 \Rightarrow A(m^y, s, P, x) = 1.
\]
The designer then maximizes:

$$\max_{\{\pi^y, M^y\}, s, P, x, T} \mathbb{E} (W_0 (A) \times 1 \{\omega + T + P \geq A\})$$

s.t.: $$(a) - (e).$$

We assume that the policy-maker has the authority to forbid the bank to net positive payoffs if she does not comply with the recommendation $(s, P)$. This is the reason why we do not specify incentive constraints for the bank. At the same time, this implies that it is without loss to set $s(y) \equiv y$, since this maximizes the amount of funds the bank may obtain from the market, which increases the likelihood of survival. Next, we note that even if $\theta$ were observable by the policy-maker, there would be no gain in allowing the set of recommendations to depend on it, since $\theta$ corresponds to a signal about $y$, and (i) the policy-maker perfectly observes the latter, and (ii) the ban’s incentive compatibility constraints will always be satisfied.

7.1 Cash-less Government

We consider a setting similar to the one introduced in section 4, where the government cannot use public funds to act as a lender of last resort. In the current setting, this means that the government is constrained to use policies for which $T = 0$. In order to solve the policy-maker’s problem we relax the assumption that the set of possible disclosures regarding the bank’s liquidity position corresponds to binary recommendations. Instead we assume that designer may disclose any message in the arbitrary set $M^\omega$ according to the disclosure rule:

$$\pi^\omega : M^y \times \mathbb{R} \times \Omega \to \Delta M^\omega$$

$$(m^y, P, \omega) \to \pi^\omega [m^y, P, \omega].$$

The policy-maker’s problem can then be written as:

$$\max_{\{\pi^y, M^y\}, P, \{\pi^\omega, M^\omega\}} \mathbb{E} (W_0 (A) \times 1 \{\omega + P \geq A\})$$

s.t.: $$(a') \quad \frac{1}{R} \mathbb{E} (1 \{\omega + P \geq A(m^y, P, m^\omega)\} \cdot y|m^y, P) \geq P,$$

$$(b') \quad \mathbb{E} (u(\omega, P, 1)|m^y, P, m^\omega) \leq 0 \Rightarrow A(m^y, P, m^\omega) = 1,$$

$$(c') \quad \mathbb{E} (u(\omega, P, 1)|m^y, P, m^\omega) > 0 \Rightarrow A(m^y, P, m^\omega) = 0.$$  

Fix some message $m^y \in M^y$ and a price $P > 0$. Lemma 1 and Proposition 3 imply that the optimal disclosure rule under constraints $(b')$ and $(c')$ is given by: $\{M^\omega = \{0, 1\}, \tilde{\pi}^\omega\}$ with $\tilde{\pi}^\omega (0|m^y, P, \omega) = 1 \{\omega \geq \tilde{\omega}(P)\}$. Given that this result follows for any non-increasing function $W_0(\cdot)$, it must hold for the case where it equals a constant. The result follows from observing that:

$$\frac{1}{R} \mathbb{E} (1 \{\omega + P \geq A(m^y, P, m^\omega)\} \cdot y|m^y, P) = \mathbb{E} (1 \{\omega + P \geq A(m^y, P, m^\omega)\} |m^y, P) \mathbb{E} (y|m^y),$$
which implies that $\pi^*$ maximizes the value that investors are willing to accept for the bank’s asset. As a result, the policy-maker’s problem simplifies to:

$$\max_{\{\pi^*, M^v\}, P} \mathbb{E} (W_0 (1 \{ \omega \geq \bar{\omega} (P) \}) \times 1 \{ \omega \geq \bar{\omega} (P) \})$$

s.t.: $(a'') \quad \mathbb{E} (y|m^y) \times P \{ \omega \geq \bar{\omega} (P) \} \geq P,$

or equivalently,

$$\max_{G^y} W_0(0) \times \int_0^\infty (1 - F^\omega (\bar{\omega} (\bar{P} (\tau)))) G^y (d \tau)$$

s.t.: $F^y \succeq_{MPS} G^y,$

which corresponds to the same problem considered in theorem 1, proving that the solution found in section 4 is, in fact, optimal. □

8 Appendix B: Omitted Proofs

Definition 2. We say a function $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies single crossing from above (SCFA), if the following holds true: if there exists some $y \in Y$ such that $g(y) < 0$, then $\forall \bar{y} > y$, $g(\bar{y}) \leq 0$. Similarly, we say that $h : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies single crossing from below (SCFB), if the following holds true: if there exists some $y \in Y$ such that $h(y) > 0$, then $\forall \bar{y} > y$, $h(\bar{y}) \geq 0$.

Lemma 4. Suppose that $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies SCFA and that $f(y, t)$ is log-supermodular for all $(y, t) \in Y \times T \subseteq \mathbb{R}^2$. Define $\phi(t) \equiv \int_Y g(y) f(y, t) dy$ and let $y_0 \equiv \inf \{ y \in Y : g(y) < 0 \}$. Then, $\forall \bar{t} > t \in T$:

$$\phi (\bar{t}) = 0 \Rightarrow \phi (t) > 0.$$  

Proof. That $f(y, t)$ is log-SM implies that $\frac{f(y, t)}{f(y, \bar{t})}$ is non-increasing. Then,

$$\phi (t) = \int_Y 1 \{ y \leq y_0 \} g(y) \frac{f(y, t)}{f(y, \bar{t})} f(y, \bar{t}) dy + \int_Y 1 \{ y > y_0 \} g(y) \frac{f(y, t)}{f(y, \bar{t})} f(y, \bar{t}) dy$$

$$\geq \left( \frac{f(y_0, t)}{f(y_0, \bar{t})} \right) \phi (\bar{t})$$

which implies the result. □

Lemma 5. Assume that $s_D (\cdot) = \min \{ \cdot, D \}$ and that $s \in S$ satisfies $\mathbb{E}_L (s_D) \leq \mathbb{E}_L (s)$. Then,

$$0 \geq \mathbb{E}_L (s_D - s) > \mathbb{E}_H (s_D - s)$$

Proof. See Nachman & Noe (94), Lemma A.3. □
Proof of Lemma 1.

Consider an arbitrary policy $\Gamma^\omega = \{\pi^\omega, M^\omega\}$. Assume that there exists some message $\bar{m}$ disclosed with positive probability under $\Gamma^\omega$ for which (i) $A(P, \bar{m}) = 1$, and (ii)

$$\mathbb{P}\{\{\omega : \omega + P \geq 1\} \cap \{\omega : \pi^\omega(\bar{m}|\omega) > 0\}\} > 0.$$That is, message $\bar{m}$ induces all creditors to stop pledging to the bank, and the set of realizations of $\omega$ in which the bank would have survived under the most aggressive size of attack, which are pooled under message $\bar{m}$, is of positive measure. Consider the alternative policy $\tilde{\Gamma}^\omega = \{\tilde{\pi}^\omega, \tilde{M}^\omega = M^\omega \cup \{\tilde{m}_0, \tilde{m}_1\}\}$ constructed as follows: for any $m \in M^\omega$ different from $\bar{m}$, $\pi^\omega(m|\cdot) = \pi^\omega(m|\cdot)$, for all $\omega \in \Omega$, $\tilde{\pi}^\omega(\tilde{m}_0|\omega) = \pi^\omega(\bar{m}|\omega) \times 1\{\omega + P \geq 1\}$, and $\tilde{\pi}^\omega(\tilde{m}_1|\omega) = \pi^\omega(\bar{m}|\omega) \times 1\{\omega + P < 1\}$. Policy $\tilde{\Gamma}^\omega$ preserves the probability that the bank survives and decreases the size of the attack, and hence weakly dominates $\Gamma^\omega$. As a result, the optimal policy maximizes the probability that creditors refrain from attacking. $\square$

Proof of Lemma 2.

Let $f^{m_i} \in \Delta[0, 1]$ be the posterior pdf after observing message $m_i \in M^\omega_i$, and $\pi_i^\omega(m_i) = \int \pi^\omega(m_i|\omega) f^\omega(\omega) d\omega$ the total probability of observing disclosure $m_i$, under policy $\Gamma^\omega_i$, $i \in \{1, 2\}$. Observe that bayesian updating together with property (i) imply that for any message $m_2 \in M^\omega_2$ with $\pi_2^\omega(m_2) > 0$ we have:

$$f^{m_2}(\omega) = \sum_{m_1 \in M^\omega_1} \frac{\pi_1^\omega(m_1) z(m_1, m_2)}{\pi_2^\omega(m_2)} f^{m_1}(\omega).$$

This implies that for any convex function $\phi$:

$$\sum_{m_2 \in M^\omega_2} \pi_2^\omega(m_2) \phi \left(\int_0^1 \omega f^{m_2}(\omega) d\omega\right) = \sum_{m_2 \in M^\omega_2} \pi_2^\omega(m_2) \phi \left(\sum_{m_1 \in M^\omega_1} \frac{\pi_1^\omega(m_1) z(m_1, m_2)}{\pi_2^\omega(m_2)} \int_0^1 \omega f^{m_1}(\omega) d\omega\right)$$

$$\leq \sum_{m_2 \in M^\omega_2} \sum_{m_1 \in M^\omega_1} \pi_1^\omega(m_1) z(m_1, m_2) \phi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega\right)$$

$$= \sum_{m_1 \in M^\omega_1} \pi_1^\omega(m_1) \phi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega\right),$$

where the second inequality arises from Jensen’s inequality and the last equality from using property (ii). As a result, $G^{\tilde{\Gamma}} \succeq_{MPS} G^{\Gamma}$ $\square$

Proof of Lemma 3.

Under full-disclosure, each message generates a degenerate posterior distribution with all weight assigned to $u(\omega, P, 1)$ when $\omega$ is realized, which also coincides with the posterior mean induced by
the message. As a result, \( G_{FD}^\omega(t; P) = \int_{u(0, P, A = 1)}^{t} G_{FD}^\omega(\tilde{u}; P) \, d\tilde{u} \), where

\[
G_{FD}^\omega(\tilde{u}; P) = \int_{u(0, P, A = 1)}^{\tilde{u}} \frac{f_\omega(u^{-1}(z; P, A = 1))}{\partial_u u(u^{-1}(z; P, A = 1), \tau, A = 1)} \, dz
\]

corresponds to the distribution of \( u(\omega, P, 1) \) under full-disclosure. Next, notice that under no-disclosure, the posterior mean remains unchanged and equal to \( \mathbb{E}(u(\omega, P, A = 1) | \emptyset) \). Thus, \( G_{\emptyset}^\omega(t; P) = \int_{u(0, P, A = 1)}^{t} \{ \tilde{u} \geq \mathbb{E}(u(\omega, P, A = 1) | \emptyset) \} \, d\tilde{u} \). To save on notation, hereafter we will omit the dependence on \( P \) of all disclosure policies and associated distributions.

Any disclosure policy \( \Gamma^\omega \), induces a function \( G^{\Gamma^\omega}(t) \equiv \int_{u(0, P, A = 1)}^{t} G^{\Gamma^\omega}(\tilde{u}) \, d\tilde{u} \). That \( G_{FD}^\omega \succeq_{MPS} G_{\emptyset}^\omega \) implies that \( G_{FD}^\omega(t) \geq G^{\Gamma^\omega}(t) \geq G_{\emptyset}^\omega(t) \) for all \( t \in [u(0, P, A = 1), u(1, P, A = 1)] \), which can be seen from applying the definition of \( \succeq_{MPS} \) to the convex function \( \max \{ \omega - t, 0 \} \). Moreover, \( G^{\Gamma^\omega} \) is convex since \( G^{\Gamma^\omega} \) is non-decreasing. Conversely, any non-decreasing, convex function \( h \) in \([u(0, P, A = 1), u(1, P, A = 1)]\), which satisfies that \( G_{FD}^\omega(t) \geq h(t) \geq G_{\emptyset}^\omega(t) \) can be induced by some policy \( \Gamma^\omega \). To see this note that \( h \) is differentiable almost everywhere and its right derivative is always well-defined since it is convex. Let \( G(\tilde{u}) \equiv h'(\tilde{u}^+) \) be the right-derivative of \( h \) at \( \tilde{u} \). Observe next that \( \lim_{\tilde{u} \to -\infty} G(\tilde{u}) = 1 \), and thus \( G \) is a distribution. Finally, note that \( G_{FD}^\omega \) is a mean-preserving spread of \( G \) and therefore there must exist a policy that induces it by Strassen’s Theorem (See Theorem 1.5.20 in Müller and Stoyan [2002]).

**Proof of Proposition 1.**

The proof is divided in two parts. First, I show that in any pooling equilibrium sellers place a debt security. The proof is general in that it applies regardless of whether the designer has disclosed information about the fundamentals \((y, \omega)\) by conducting stress tests or not. We assume that the probability that the bank survives can be written as \( \mathbb{P}\{\omega \geq \omega^z(\tau)\} \), where \( \omega^z(\cdot) \) represents a decreasing function of the capital raised by the bank, \( P \). In the context of section 3, \( \omega^z = A^*(P) - P \), while in the context of section 4, \( \omega^z = \bar{\omega} \). Define \( \Phi(\mathbb{E}_u(s)) \) as the set of prices which induce a non-negative profit to any investor when a security of value \( \mathbb{E}_u(s) \) is purchased. That is

\[
\Phi(\mathbb{E}_u(s)) \equiv \left\{ P \geq 0 : \frac{\mathbb{E}_u(s)}{R} \times \mathbb{P}\{\omega \geq \omega^z(P)\} \geq P \right\}.
\]

**Claim 1:** If \( \Phi(\mathbb{E}(y)) = \{0\} \), then the unique equilibrium of the game is \( s_{H}^* = s_{L}^* = 0 \).

\( \Phi(\mathbb{E}(y)) = \{0\} \) implies that \( \mathbb{E}(y) < K \). We prove first that \( s_{H}^* = s_{L}^* = 0 \) is, in fact, an equilibrium. Consider the deviation to any security \( \hat{s} \) satisfying \( \frac{1}{R} \mathbb{E}_H(\hat{s}) \geq K \) (which is the only relevant case since \( \Phi(\mathbb{E}(y)) = \emptyset \)). Observe that \( BR(\hat{s}) = \left[ K, \frac{1}{R} \mathbb{E}_H(\hat{s}) \right] \), since any price below \( K \) induces default with certainty when assumption (2) holds, and any \( P \geq K \) dissuades all creditors from running, and hence prevents default w.p. 1. As a consequence, a low-quality type can profitably deviate and place security \( \hat{s} \) for any price \( P \in BR(\hat{s}) : \)
Thus, \( D(\theta_L; \hat{s}) = BR(\hat{s}) \), implying that market beliefs that assign \( \mu(\theta_L, s) = 1 \) for any such \( s \in S \) are consistent with \( D_1 \). This amounts to say that any feasible deviation is always attributed to type \( L \), and therefore no bank type gets funded. Uniqueness follow from the fact that \( \mathbb{E}(y) < K \) and, hence, even if bank sell the whole asset funds are not enough to secure positive funds. Moreover, any security issued by type \( H \) that obtains a positive price may always be mimicked by type \( L \) and, therefore, cannot occur in equilibrium.

**Claim 2.** \( \Phi(\mathbb{E}(y)) \neq \{0\} \) implies that pooling may only occur over debt contracts.

Suppose that there exists an equilibrium of the fund-raising game, \( \left\{ \{\sigma_\theta\}_{\theta}, \mu, P, A \right\} \), and a security \( \hat{s} \in S \) with \( \sigma_\theta(\hat{s}) > 0 \), for all \( \theta \in \Theta \). We prove that any such security needs to be a debt contract. To see this, suppose that \( \hat{s} \) is not a debt contract. Define the debt security \( \hat{s}_D \equiv \min \{ y, D \} \) where \( D \) is such that \( \mathbb{E}_H(s_D - \hat{s}) = 0 \). Note that \( s_D - \hat{s} \) satisfies *single crossing from above* (SCFA) and hence lemma 4 implies that \( \mathbb{E}_L(s_D - \hat{s}) > 0 = \mathbb{E}_H(s_D - \hat{s}) \). Thus,

\[
\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D) > \mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s})
\]  

(26)

Next, define \( \Delta V_\theta(P) \) as the difference in payoffs, for seller \( \theta \), obtained by switching to security \( s_D \), and sell it at price \( P \), instead of issuing security \( \hat{s} \) and receiving the market price \( \hat{P}(\hat{s}) \). That is,

\[
\Delta V_\theta(P) = V(P, s_D, \theta) - V(\hat{P}(\hat{s}), \hat{s}, \theta)
\]

\[
= (PR + \mathbb{E}_\theta(y - s_D)) \times \mathbb{P}\left\{ \omega \geq \omega^*(P) \right\} - \left( \hat{P}(\hat{s}) R + \mathbb{E}_\theta(y - \hat{s}) \right) \times \mathbb{P}\left\{ \omega \geq \omega^*(\hat{P}(\hat{s})) \right\},
\]

Inequality (26) together with the fact that \( y - s_D \) and \( y - \hat{s} \) are monotone then imply that:

\[
\Delta V_H(\tau) - \Delta V_L(\tau) = (\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)) \times \mathbb{P}\left\{ \omega \geq \omega^*(P) \right\}

- (\mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s})) \times \mathbb{P}\left\{ \omega \geq \omega^*(\hat{P}(\hat{s})) \right\}

> 0, \ \forall P \geq \hat{P}(\hat{s}).
\]  

(27)

Note next that

\[ \Phi(\mathbb{E}(\hat{s})) \subset \Phi(\mathbb{E}_H(\hat{s})) = BR(s_D), \]

where the last equality arises from \( \mathbb{E}_H(s_D) = \mathbb{E}_H(\hat{s}) \). By definition, we have that

\[ P(\hat{s}) = \sup \Phi(\mathbb{E}(\hat{s})), \]

and hence, \( \hat{P}(\hat{s}) \in \text{int}(BR(s_D)) \).

Finally, notice that \( \mathbb{E}_L(y - \hat{s}) > \mathbb{E}_L(y - s_D) \), and therefore \( \Delta V_L(\hat{P}(\hat{s})) < 0 \). On the other hand, \( \Delta V_H(\hat{P}(\hat{s})) = 0 \), and thus \( \Delta V_H(\hat{P}(\hat{s}) + \epsilon) > 0 > V_L(\hat{P}(\hat{s}) + \epsilon) \) for \( \epsilon > 0 \) small enough so
that $\hat{P}(\hat{s}) + \epsilon \in BR(s_D)$. As a result, $\mathcal{D}(\theta_L|s_D) \cup \mathcal{D}^0(\theta_L|s_D) \subset \mathcal{D}(\theta_H|s_D)$, and consequently market beliefs consistent with $D_1$ must necessarily assign $\mu(\theta_H|s_D) = 1$. This implies that $P(s_D) > \hat{P}(\hat{s})$, since bank $H$ is not pooled with $L$ when placing $s_D$, and therefore by definition of $s_D$ we have that $\Delta V_H(P(s_D)) > 0$, which contradicts the assumption that $\{\sigma_{\theta}, \mu, P, A\}$ is an equilibrium.

Next, we prove that the price of any debt security, $s_d$, which is placed by both type of banks in equilibrium, cannot be larger than $K$. Assume by contradiction that $P(s_d) \equiv \min\{y, d\} > K$ (and hence $P(s_d) > K$). Consider the alternative debt contract $s_c = \min\{y, d - \epsilon\}$ with $\epsilon > 0$ small. We show that type $H$ can always profitably deviate and issue $s_c$ instead. Observe that $s_d - s_c$ is an increasing function. FOSD then implies that:

$$E_H(s_d - s_c) > E_L(s_d - s_c),$$

or equivalently,

$$E_H(y - s_c) - E_L(y - s_c) > E_H(y - s_d) - E_L(y - s_d).$$

(28)

Similar to what we did above, let $\Delta V_\theta(P) = V(P, s_\theta, \theta) - V\left(\hat{P}(\hat{s}), s_d, \theta\right)$. Inequality 28 implies that:

$$\Delta V_H(P) - \Delta V_L(P) = (E_H(y - s_c) - E_L(y - s_c)) \times \mathbb{P}\left\{\omega \geq \omega^*_H(P)\right\}$$

$$- (E_H(y - s_d) - E_L(y - s_d)) \times \mathbb{P}\left\{\omega \geq \omega^*_H(P(s_d))\right\}$$

$$> 0, \ \forall P \geq K.$$

(29)

For small values of $\epsilon$ we have:

$$\Phi\left(E(s_d)\right) \subset \Phi\left(E_H(s_c)\right) = BR(s_c),$$

and hence $P(s_d)$ which is the maximal element in $\Phi\left(E(s_d)\right)$ is contained in $BR(s_c)$. Moreover, given that $s_c$ is smaller than $s_d$, we must have that $\Delta V_\theta(P(s_d)) > 0$ for both $\theta \in \Theta$. Finally, by choosing $\epsilon$ small enough, and using inequality 29, we obtain that there must exists some $\hat{P} \in (K, P(s_d))$ for which $\Delta V_H(\hat{P}) > 0 > V_L(\hat{P})$. Thus, $\mathcal{D}(\theta_L|s_c) \cup \mathcal{D}^0(\theta_L|s_c) \subset \mathcal{D}(\theta_H|s_c)$, and consequently market beliefs consistent with $D_1$ must necessarily assign $\mu(\theta_H|s_D) = 1$, which implies that type $H$ can profitably deviate and separate from type $L$. This is a contradiction and therefore any debt contract under which both types must have a price no larger than $K$.

Claim 3. $\Phi\left(E(y)\right) \neq \{0\}$ implies that in any equilibrium in which there exists a security, $s_H$, only issued by type $H$ (i.e., $\sigma_H(s_H) > 0 = \sigma_L(s_H)$), we must have that $E_H(s_H) \leq \frac{E_L(y)}{R}$.

To see this, assume by contradiction that $P(s_H) > \frac{1}{R} E_L(y)$. Denote by $s_L$ any security issued with positive probability by type $L$. Observe that the separating nature of the equilibrium requires that:

$$P(s_L) = \sup \Phi(s_L) \leq \frac{E_L(s_L)}{R}.$$
Hence, the amount collected by type $H$ must be such that:

$$
P(s_H)R \geq P(s_L)R + \mathbb{E}_L (y - s_L). \quad (30)
$$

As a result, type $L$ has incentives to mimic type $H$. To see this last point, let $P(s_0)$ obtained when issuing and observe that:

$$
V(\tau(s_H), s_H, \theta_L) - V(\tau(s_L), s_L, \theta_L) = (\tau(s_H)R + \mathbb{E}_L (y - s_H)) \times \mathbb{P}\left\{ \omega \geq \omega^x(\tau(s_H)) \right\}
$$

$$
- (\tau(s_L)R + \mathbb{E}_L (y - s_L)) \times \mathbb{P}\left\{ \omega \geq \omega^x(\tau(s_L)) \right\}
$$

$$
> (\tau(s_H)R + \mathbb{E}_L (y - s_H)) - (\tau(s_L)R + \mathbb{E}_L (y - s_L)) \times \mathbb{P}\left\{ \omega \geq \omega^x(\tau(s_L)) \right\}
$$

$$
> 0,
$$

where the first inequality arises from the fact that $P(s_H) > P(s_L)$ and that $\omega^x$ is a decreasing function of the capital raised by the bank. The second inequality, in turn, is a consequence of equation (30). This is a contradiction and hence $P(s_H) \leq \frac{1}{R}\mathbb{E}_L (y|m^y)$. □

**Proof of Proposition 2.**

To prove (1) we show that under assumption 2 and $\lambda = 0$, there cannot be any separating, nor semi-separating equilibrium. Assume that type $H$ is the only type which chooses a particular security $s_H$ with positive probability. If $\frac{1}{R}\mathbb{E}_H(s_H) \geq K$, then $P(s_H) = \frac{1}{R}\mathbb{E}_H(s_H)$ since at this price default is avoided. Then either of the two following cases must be true: (i) both types place different securities and no pooling occurs, or (ii) there exists a different security $\tilde{s}$ which is placed by both sellers with positive probability. Let $\omega^x(P)$ be the cutoff liquidity level for which the bank defaults if it raises $P$ from external investors. In the first case, type $L$ has a strict incentive to deviate and pretend to be type $H$, since at any security placed by $L$ with positive probability we have that:

$$
V(P(s_L), s_L, \theta_L) - V(P(s_H), s_H, \theta_L) = (P(s_L)R + \mathbb{E}_L (y - s_L)) \times \mathbb{P}\left\{ \omega \geq \omega^x(P(s_L)) \right\}
$$

$$
- (P(s_H)R + \mathbb{E}_L (y - s_H)) \times \mathbb{P}\left\{ \omega \geq \omega^x(P(s_H)) \right\}
$$

$$
< (P(s_L)R + \mathbb{E}_L (y - s_L)) - (P(s_H)R + \mathbb{E}_L (y - s_H))
$$

$$
< (\mathbb{E}_H(s_H) - \mathbb{E}_L(s_H))
$$

$$
< 0,
$$

where the second inequality obtains from $P(s_L)R = \mathbb{E}_L (s_L) \times \mathbb{P}\left\{ \omega \geq \omega^x(P(s_L)) \right\}$, and the last one from FOSD and the fact that $s_H$ is non-decreasing. In the second case, in turn, type $H$ strictly
we apply the density and has at most one mass point at is continuous comes from the fact that (i) \( \bar{\omega} \) is continuous, (b) non-decreasing, and (c) satisfies

\[
L \quad \text{proves that the only type of equilibria that prevail in the fund-raising game are pooling equilibria. Proposition 1 then implies that both types must pool under debt contracts only, which completes the proof of (1).}

We next prove (2). Suppose first that \( \frac{1}{R} E_L(y) < K \leq E(y) \). Consider the deviation to any security \( \hat{s} \) satisfying \( \frac{1}{R} E_{\theta_H}(\hat{s}) \geq K \), which is the only relevant case since the market would never fund the low type. Observe that \( BR(s) = [K, \frac{1}{R} E_{\theta_H}(s)] \), since any price below \( K \) induces default with certainty when assumption (2) holds, and any \( P \geq K \) dissuades all creditors from running, and hence prevents default w.p. 1. As a consequence, bank \( L \) can profitably deviate and place security \( \hat{s} \) for any price \( \tau \in BR(s) \):

\[
V(\tau, \theta_L, \hat{s}) = (\tau R + E_{\theta_L}(y - s)) \times \mathbb{P} \{ \omega \geq \hat{\omega}(\tau) \} > 0.
\]

Thus, \( D(\theta_L; s) = BR(s) \), implying that market beliefs that assign \( \mu(\theta_L, s) = 1 \) for any such \( s \in S \) are consistent with D1. This amounts to say that any feasible deviation is always attributed to type \( L \), and therefore no seller type gets funded. If \( \frac{1}{R} E(y) < K \) instead, then claim 1 in the proof of Proposition 1 implies that \( s_\theta = 0 \) for all \( \theta \in \Theta \) is the unique equilibrium.

Finally, we prove (3). Assume then that \( \frac{1}{R} E(y) \geq K \). The result follows directly from Theorem 4 in [Nachman & Noe (94)].

**Proof of Theorem 1.**

Define \( \phi(\tau) \equiv 1 - F^\omega(\hat{\omega}(\hat{P}(\tau))) \). We first prove that \( \phi \) satisfies the following properties: \( \phi \) is (a) continuous, (b) non-decreasing, and (c) satisfies \( \phi(0) = 0 \), and \( \phi(\tau) = 1 \) for all \( \tau \geq K \). That \( \phi \) is continuous comes from the fact that (i) \( \hat{\omega} \cdot (\cdot) \) is continuously differentiable, (ii) \( F^\omega(\cdot) \) admits a density and has at most one mass point at \( \omega = 1 \), and (iii) \( \hat{P} \) is continuous. To see this last point, we apply the maximum theorem to the definition of \( \hat{P} \):

\[
\hat{P}(\tau) = \max P \quad \text{s.t.} \quad P \in \Gamma(\tau) \equiv \left\{ P \geq 0 : \frac{\tau}{R} \times \mathbb{P} \{ \omega \geq \hat{\omega}(P) \} \geq P \right\}
\]
where \( \Gamma(\cdot) \) is a compact valued and continuous correspondence. To see (b), we note that \( \bar{P} \) is non-decreasing and that \( \bar{\omega} \) is non-increasing which implies the result. Finally, (c) is by definition of functions \( \bar{P} \) and \( \bar{\omega} \). Conditions (a)-(c) guarantee that \( \phi \) satisfies the regularity assumption in Dworczak and Martini [2018]. That the optimal disclosure policy consists of monotone partitions thus follows proposition 2 in their paper. Next, to prove that the highest partition includes \( K \), we observe that (b) and (c) imply that \( \bar{\omega} \) is (weakly) concave for any \( y \) above some \( \bar{y} < K \), and therefore the second bullet point in proposition 2 in Dworczak and Martini [2018] applies.

We show the second part of the theorem with an independent proof. Under assumption 2 and 3, (d) \( \bar{\omega} \) is convex in some interval \([0, k] \subseteq [0, K]\), and (e) \( \phi'(0) < \lim_{\tau \to K^-} \phi'(\tau) \). To see (d) assume first that \( \bar{P}(\tau) > 0 \) for positive values of \( \tau \) (this is always the case if \( \lambda > 0 \)). We can then use equation 7, which implicitly defines \( \bar{\omega} \), and compute:

\[
\phi'(\tau) = c f^{\omega}(1 - \bar{P}(\tau)) \bar{P}'(\tau) = \frac{c f^{\omega}(1 - \bar{P}(\tau)) \phi(\tau)}{R - c f^{\omega}(1 - \bar{P}(\tau)) \tau} \geq 0, \quad \forall \tau \geq 0
\]  

(31)

where \( \bar{P}' \) can be obtained from its definition in equation (9) and equals:

\[
\bar{P}'(\tau) = \frac{1}{R} \times (\tau \phi'(\tau) + \phi(\tau)).
\]

Differentiating equation (31) once more, we obtain that the sign of \( \phi'' \) coincides with the sign of:

\[
1 - \frac{c}{R} f^{\omega}(1 - \bar{P}(\tau)) \tau
\]

which is positive for small values of \( \tau \), proving (d). Finally, to prove (e) we can use (31) to verify that:

\[
\phi(0) = \frac{c f^{\omega}(1) m}{R} < \frac{c f^{\omega}(1 - K)}{R - c f^{\omega}(1 - K) K} = \lim_{\tau \to K^-} \phi'(\tau),
\]

where the inequality follows from assumption (2) and (c).

Using integration by parts, we can rewrite the policy-maker’s objective function as:

\[
\int_0^\infty \phi(\tau) Z(d\tau) = (\phi(\tau) Z(\tau)) |_0^\infty - \int_0^\infty \phi'(\tau) Z(\tau) d\tau
\]

\[
= - \int_0^\infty \phi'(\tau) Z(\tau) d\tau.
\]

As a result, the designer’s problem is equivalent to:

\[
\min_{Z} \int_0^\infty \phi'(\tau) Z(\tau) d\tau \quad \text{s.t.} \quad Z \text{ satisfies condition (1)}.
\]

Finally, note that property (d) implies that \( \phi' \) is non-decreasing in \([0, k]\), and equal to 0 for any \( \tau \geq K \). Observe that the constraint \( F^y + Z \leq 1 \) (everywhere), together with the requirement that \( \int_0^\infty Z(\tau) d\tau = 0 \), impose a bound on the mass that \( Z \) can assign to values in \([0, K]\). In fact, we must
have that $\int_{0}^{\infty} Z(\tau) d\tau \leq \int_{K}^{\infty} (1 - F^{y}(\tau)) d\tau$, and hence there exists a lower bound on the value that $\int_{0}^{K} Z(\tau) d\tau$ may take. That $\phi'$ is non decreasing in $[0,k]$, and satisfies $\phi'(0) = \lim_{\tau \rightarrow K^-} \phi'(\tau)$, implies that the optimal choice of $Z$ is given by:

$$Z(\tau) = \begin{cases} 
0 & \tau \leq y^+ \\
F^{y}(y^+) - F^{y}(\tau) & \tau \in (y^+, K) \\
1 - F^{y}(\tau) & \tau \geq K
\end{cases}$$

where $y^+$ is chosen so that $\int_{y^+}^{\infty} Z(\tau) d\tau = 0$, or equal to 0 if $\int_{0}^{\infty} Z(\tau) d\tau \leq 0$. More precisely,

$$y^+ = \inf \left\{ y \geq 0 : \int_{y}^{K} (F^{y}(y) - F^{y}(\tau)) d\tau + \int_{K}^{\infty} (1 - F^{y}(\tau)) d\tau \leq 0 \right\}.$$ 

That $Z(\tau) = 0$ for all $\tau \leq y^+$ implies that $G(\tau) = F^{y}(\tau)$ for such $\tau$, or equivalently, that $G$ coincides with the full-disclosure policy for all $y \leq y^+$. On the other hand, that $G(\tau) = F^{y}(y^+)$ for all $\tau \in (y^+, K)$, and $G(\tau) = 1$ for all $\tau \geq K$, means that the optimal policy pools all the realizations of $y$ above $y^+$ under a single message, so that the induced posterior mean is given by $K$. □

**Proof of Proposition 5.**

Fix a message $m^y$ disclosed with positive probability under $\Gamma^y$, and let $z_{\theta} = E_{\theta}(y - s^*|m^y)$. We can write the designer’s problem in (22) as:

$$\max_{V,t_0(),\pi(0|\cdot)} \int_{0}^{1-P} \pi(0|\omega)f^{\omega}(\omega)d\omega$$

s.t: (i) $\int_{0}^{1-P} \left( (b - g) \cdot 1_{\{P+t_0 + \omega < 1\}} + g \right) \pi(0|\omega)F^{\omega}(d\omega) + \pi_s \cdot g \cdot (1 - F^{\omega}(1 - P)) \geq 0$

(ii) $V \times \int_{0}^{1-P} \pi(0|\omega)F^{\omega}(d\omega) \leq \frac{L(P)}{b} + \frac{|g|}{b} \times \pi_s \times (1 - F^{\omega}(1 - P))$

(iii) $\pi(0|\omega) \cdot ((P + t_0(\omega))R) = V, \forall \omega \leq 1 - P$

(iv) $\pi_s \leq \frac{V}{PR + z_H}$

(v) $t_0(\omega) \leq \min \left\{ \frac{z_L}{R}, B \right\}$

where the first two constraints are the obedience constraints associated to messages 0 and 1, respectively, and the last three correspond to incentive compatibility constraints: (iii) imposes that the payoff of any bank reporting a liquidity position below $1 - P$ must be the same, (iv) guarantees that vulnerable type-H banks do not have incentives to mimic safe banks, and (v) requires that safe type-L banks do not want to be thought as vulnerable banks, and at the same time imposes that the funds respect the regulator’s budget constraint

Let $\bar{\omega} \equiv 1 - P - z_L$ be the liquidity level threshold above which liquidity buffers are sufficiently high that the bank would survive even if all creditors stopped rolling over the bank’s debt. Define next $\rho$ as follows:
\[ \rho \equiv \left| \frac{b}{g} \right| \cdot \frac{F^\omega(\hat{\omega})}{PR + z_L} - \frac{1 - F^\omega(1 - P)}{PR + z_H}. \]

We will characterize the optimal elicitation mechanism as a function of the value of \( \rho \). Assume first that

\[ \rho \in \left( \frac{F^\omega(1 - P) - F^\omega(\hat{\omega})}{PR + z_L}, \int_0^{1-P} \frac{f^\omega(\omega)}{(1 - \omega)R} d\omega \right). \]

We note next that inequality (iv) must bind since this relaxes (i) and (ii), does not affect neither (iii) nor (v), and therefore allows to improve the policy-maker’s objective function. Next, constraint (iii) implies that we can write the policy-maker’s problem as a function of \( V \) and \( t_0 \). Thus, the set of relevant constraints is given by:

1. \( (i') \int_0^{1-P} ((b - g) \cdot 1_{\{P+t_0+\omega<1\}} + g) \left( \frac{F^\omega(d\omega)}{P + t_0(\omega)} \right) \geq 0 \)
2. \( (ii') V \times \left( \int_0^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))R} d\omega \right) \leq \frac{L(P)}{b} + \frac{g}{b} \times V \times \frac{(1 - F^\omega(1 - P))}{PR + z_H} \)
3. \( (v) t_0(\omega) \leq \min \left\{ \frac{z_L}{R}, B \right\} \)
4. \( (vi) \frac{V}{(P + t_0(\omega))R} \leq 1 \quad \forall \omega \leq 1 - P, \)

where the new constraint (vi) is added so that probabilities are well defined.

**Claim 1:** \( t_0(\omega) = z_L \) for all \( \omega < \hat{\omega} \equiv 1 - P - z_L \).

To see this, suppose by contradiction that this is not true. Let \( \mathcal{Y} = \{m, m, \pi(\cdot|m)\}_{m \in \{0,1\}} \) be the optimal mechanism. We will show that we can find another mechanism which strictly improves upon \( \mathcal{Y} \). Consider the alternative \( \mathcal{Y}^\epsilon \) which offers the alternative price \( t_0^\epsilon \) which modifies the value of \( t_0 \) for values of \( \omega \) for which the bank would not survive a run, in the following way:

\[ t_0^\epsilon(\omega) \equiv \begin{cases} 
\varepsilon z_L + (1 - \varepsilon)t_0(\omega) & \omega \leq \hat{\omega} \\
t_0(\omega) & \omega > \hat{\omega}.
\end{cases} \]

Let \( V^\epsilon \) be the value of \( V \) which preserves the LHS of (ii). That is,

\[ V^\epsilon \times \left( \int_0^{1-P} \frac{f^\omega(\omega)}{(P + t_0^\epsilon(\omega))R} d\omega \right) = V \times \left( \int_0^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))R} d\omega \right). \]

This modification relaxes (i), since \( b < 0 \), and increases the value of \( V \), which then relaxes (ii) since the RHS increases (recall that (iv) binds). As a result, the designer can increase \( \pi(0|\omega) \) without violating any constraint. This is a contradiction, and hence we must have that \( t_0(\omega) = z_L \) for all \( \omega < \hat{\omega} \equiv 1 - P - z_L \).

**Claim 2:** \( \exists \hat{\omega} \in [\hat{\omega}, 1 - P] \) so that \( t_0(\omega) = \max \{1 - \omega, 1 - \hat{\omega}\} \) for all \( \omega \in [\hat{\omega}, 1 - P] \).

Consider an arbitrary policy \( t_0 \) with \( t_0(\omega) \in [z_L, 1 - \omega] \) for all \( \omega \in [\hat{\omega}, 1 - P] \). Construct the alternative policy \( \hat{t}_0(\omega) \equiv \max \{1 - \omega, 1 - \hat{\omega}\} \) for all \( \omega \in [\hat{\omega}, 1 - P] \), where \( \hat{\omega} \) is chosen so that:

\[ \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))R} d\omega = \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + \max \{1 - \omega, 1 - \hat{\omega}\})R} d\omega. \]
To see this, we notice that that constraints (i’), (ii’), (v) remain unchanged with the alternative policy, but constraint (vi) relaxes, and hence it is with loss of optimality to assume that at the optimal policy $t_0(\omega) = \max \{ 1 - \omega, 1 - \tilde{\omega} \}$ for all $\omega \in [\tilde{\omega}, 1 - P]$...where we have assumed that the policy maker provides funds in excess of $1 - P - \omega$ for each $\omega > \tilde{\omega}$. We show at the end of the proof that this ambitious strategy is optimal for the policy-maker.

**Claim 3:** Constraint (i’) must bind.

This constraint corresponds to obedience constraint (12), and requires that creditors have an incentive to follow the recommendation to keep rolling over the bank’s debt. Again by contradiction, assume that this is not the case. Then,

$$ b \cdot \int_{0}^{\tilde{\omega}} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega + g \cdot \int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega + g \cdot \frac{(1 - F^\omega(1 - P))}{(PR + z_H)} > 0, \quad (32) $$

Assume that (ii’) is binding. Consider the following deviation from the optimal mechanism $Y$: We modify $t_0$ between $[\tilde{\omega}, 1 - P]$, so that the new price, $t'_0 = \max \{ 1 - \omega, 1 - \tilde{\omega} \}$, where $\tilde{\omega} < \tilde{\omega}$ satisfies that

$$ \int_{0}^{\tilde{\omega}} \frac{f^\omega(\omega)}{(P + t'_0(\omega))} R d\omega = \int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t'_0(\omega))} R d\omega - \epsilon, $$

for some $\epsilon > 0$ small enough so that the inequality above is respected. Next, let $\tilde{V}(\epsilon)$ be the maximal value that $V$ may take under the new policy so that (ii’) remains unchanged. That is,

$$ \tilde{V}(\epsilon) \times \left( |b| \times \left( \int_{0}^{\tilde{\omega}} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega + \int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t'_0(\omega))} R d\omega \right) - g \times \frac{(1 - F^\omega(1 - P))}{PR + z_H} \right) = C, $$

where $C \in (0, |L(P)|)$ is a constant. We assume that $C > 0$ since otherwise the constraint is trivially satisfied which, coupled with the assumption that (i) does not bind, delivers the contradiction, since then the designer can increase $\pi(0|\omega)$ without violating any constraint. Next, differentiating against $\epsilon$ and then taking the limit from the right as $\epsilon$ goes to 0, we get that:

$$ \lim_{\epsilon \to 0^+} \tilde{V}'(\epsilon) = \frac{\tilde{V}(0) \cdot |b|}{\left( |b| \times \int_{0}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega - g \times \frac{(1-F^\omega(1-P))}{PR + z_H} \right)}. $$

This allows us to compute the effect of such modification on the policy-maker’s payoff

$$ W = \tilde{V} \cdot \int_{0}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega $$

---

\(^{14}\)The existence of such $\epsilon$ comes from (32), since this inequality implies:

$$ \int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))} R d\omega > \rho > \frac{F^\omega(1 - P) - F^\omega(\tilde{\omega})}{P + z_L}. $$
for small values of $\epsilon$. In fact,
\[
\lim_{\epsilon \to 0^+} \frac{dW}{d\epsilon} = \left( \lim_{\epsilon \to 0^+} \tilde{V}'(\epsilon) \right) \cdot \frac{W(0)}{\tilde{V}(0)} - \tilde{V}(0)
\]
\[
= \frac{V^2}{C} \cdot (g - b) \cdot \frac{(1 - F^\omega(1 - P))}{PR + z_H} > 0.
\]

which contradicts the optimality of $\Upsilon$. Next, assume that (vi) is the binding constraint (defining $V$). Consider the alternative policy
\[
\tilde{\tau}_0(\omega) \equiv \begin{cases} 
    z_L & \omega \leq \tilde{\omega} \\
    \max \{1 - \omega, 1 - \tilde{\omega}^\epsilon\} & \omega \in (\tilde{\omega}, 1 - P], \\
    0 & \omega > 1 - P
  \end{cases}
\]

with $\epsilon$ small enough so that (ii') is still satisfied. Let $\tilde{V}(\epsilon)$ be the maximal value that $V$ may take under the new policy so that (vi) remains unchanged. That is,
\[
\frac{\tilde{V}(\epsilon)}{1 - \tilde{\omega}^\epsilon} = \frac{V}{1 - \tilde{\omega}}.
\]

This implies that $\tilde{\pi}(0|\omega) \equiv \frac{\tilde{V}(\epsilon)}{(P + t_0(\omega))R} > \pi(0|\omega)$ for all $\omega \leq \tilde{\omega}^\epsilon$ and $\tilde{\pi}(0|\omega) = \pi(0|\omega)$ for all $\omega > \tilde{\omega}^\epsilon$, and hence the policy-maker’s payoff must increase. This is a contradiction and hence (i') must be satisfied with equality. This means that
\[
\int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(P + t_0(\omega))R} d\omega = \rho \in \left( \frac{F^\omega(1 - P) - F^\omega(\tilde{\omega})}{PR + z_L}, \int_{\tilde{\omega}}^{1-P} \frac{f^\omega(\omega)}{(1 - \omega)R} d\omega \right),
\]

which is feasible.

Therefore, we choose $t_0(\omega)$ in $[\tilde{\omega}, 1 - P]$ among all the policies satisfying (i') so that $V$ is largest. Let $\tilde{\omega}$ be implicitly defined by:
\[
\int_{\tilde{\omega}}^{\omega} \frac{f^\omega(\omega)}{1 - \omega} d\omega + \frac{F^\omega(1 - P) - F^\omega(\tilde{\omega})}{1 - \tilde{\omega}} = \frac{|b|}{g} \times \frac{F^\omega(\tilde{\omega})}{P + z_L/R} - \frac{1 - F^\omega(1 - P)}{P + z_H/R}.
\]

That is, $\tilde{\omega}$ is the cutoff defining the price $t_0$ which maximizes the upper bound $\min_{\omega \leq 1-P} (P + t(\omega))R$, while respecting (i'). The optimal policy is thus given by:
\[
t_0(\omega) = \begin{cases} 
    P + z_L/R & \omega < \tilde{\omega} \\
    1 - P - \omega & \omega \in [\tilde{\omega}, \tilde{\omega}] \\
    1 - P - \tilde{\omega} & \omega \in (\tilde{\omega}, 1 - P) \\
    0 & \omega \geq 1 - P
  \end{cases}, \quad \pi(0|\omega) = \begin{cases} 
    \frac{\tilde{V}}{PR + z_L} & \omega < \tilde{\omega} \\
    \frac{\tilde{V}}{(1 - \omega)R} & \omega \in [\tilde{\omega}, \tilde{\omega}] \\
    \frac{\tilde{V}}{(1 - \omega)R} & \omega \in (\tilde{\omega}, 1 - P) \\
    \frac{\tilde{V}}{PR + z_H} & \omega \geq 1 - P
  \end{cases}
\]
where \( \bar{V} \) is chosen so that (ii) and (vi) hold:

\[
\bar{V} \equiv \min \left\{ \left(1 - \hat{\omega}\right) R, \frac{|L(P)|}{|b|} \times \int_0^{1-P} \frac{f^\omega(\omega)}{(P + \lambda|\omega|)R} d\omega - g \times \frac{(1 - F^\omega(1-P))}{(P + \mathbb{E}_H(y-s^\ast))R} \right\}.
\]

Finally, assume that

\[
\rho \geq \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{1 - \omega} d\omega.
\]

Then, the designer is unable to successfully dissuade creditors from running on the bank with positive probability. In other words, \( \pi(0|\cdot) = 0 \). To see this, it suffices to rewrite the inequality above as:

\[
|b| \cdot \frac{F^\omega(\hat{\omega})}{P + z_L/R} - g \cdot \frac{1 - F^\omega(1-P)}{P + z_H/R} \geq g \cdot \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{1 - \omega} d\omega,
\]

or equivalently,

\[
\mathbb{E}(u(\omega, P, 1)|0) = b \cdot \frac{F^\omega(\hat{\omega})}{P R + z_L/R} + g \cdot \int_{\hat{\omega}}^{1-P} \frac{f^\omega(\omega)}{(1 - \omega) R} d\omega + g \cdot \frac{1 - F^\omega(1-P)}{P R + z_H/R} \leq 0.
\]

That is, creditors obtain a negative payoff if they pledge to the bank (and the rest does not), even if the designer were to offer enough funds so that every bank with \( \omega > \hat{\omega} \) is able to survive the liquidity strain caused by all creditors refraining from rolling over the bank’s debt. As a result, under the most adversarial equilibrium all creditors run on the bank. The policy-maker thus cannot engage in disclosing informative messages about the bank’s liquidity buffer, and may only try to increase the likelihood of the bank’s survival by purchasing claims on its asset. The optimal strategy for the policy-maker consists of purchasing the totality of the remaining claims on the asset at the largest price allowed by the fair price constraint. Thus, the government purchases \( y - s^\ast \) at price \( t^\theta \) defined by:

\[
t^\theta \equiv \sup \left\{ \tau \leq B : \frac{\sum g \mu_\theta \mathbb{E}_\theta(y - s^\ast)}{R} \cdot \mathbb{P}\{\omega + P + \tau \geq 1\} \geq \tau \right\}.
\]

\( \Box \)

**Proof of Theorem 2**

Fix a message \( m^y \) disclosed with positive probability under \( \Gamma^y \). Assume that \( \mathbb{E}(y|m^y) \geq \mathbb{E} \). This means, as it will become clear below, that there exists a non-empty set of elicitation policies. We will characterize the optimal capital requirement and subsequent elicitation mechanism that follows the disclosure \( m^y \).

Consider the following alternative problem: assume that the bank does not receive any signal with respect to the quality of its asset. This means that the bank has a unique, average, asset quality type. The strategy of the proof consists in finding the optimal policy of the alternative problem, and then showing that the optimum under the new setting respects all the constraints of the original problem, and weakly dominates any feasible mechanism. Suppose that the bank successfully raises
that securities purchased by the government are not penalized follows from the fact that the policy-maker only purchases when assigning the passing grade, in which case the probability that the bank fails equals 0. The proof shows that even if we assume that external investors pay the default-free price of the claims (λ = 1), the designer still prefers to minimize the claims that are sold in the asset market. We will show that when \( E(y|m^y) \) is high enough so that elicitation is, in fact, possible, the policy-maker always prefers to minimize the amount raised by the bank during the fund-raising game in order to increase the value of \( z \) which will provide her with more elicitation power.

**Claim 1:** The best liquidity provision program under the alternative setting weakly dominates any feasible mechanism under the original problem (i.e., satisfying (12)-(21)).

Suppose the bank places an arbitrary security \( s \in S \) during the fund-raising game. Consider any feasible elicitation mechanism satisfying (12)-(21), \( \Upsilon[P] = \left\{ \tilde{V}_\theta, \tilde{t}_0(\cdot, \theta), \tilde{\pi}(0|\cdot, \theta), \tilde{\phi}(\cdot, \theta), \tilde{\pi}_s \right\}_{\theta \in \Theta} \). Constraints (16) and (17) imply that:

\[
t_0(\omega, \theta) \leq \frac{E_L(y - s|m^y)}{R} \leq \frac{E(y - s|m^y)}{R}, \ \forall \theta \in \Theta.
\]  
(33)

That is, the maximal amount of funds the policy-maker can commit to purchase securities from bank \( \theta \) under the original problem is smaller than under the alternative setting. Next, suppose that under the new setting the designer offers the average, stochastic mechanism, \( \Upsilon^{avg}[P] \), which randomizes between the following allocation rule \( \left\{ \tilde{V}_\theta, \tilde{t}_0(\cdot, \theta) + \tilde{\phi}(\cdot, \theta) \times \frac{g + \pi_s}{b}, \tilde{\pi}(0|\cdot, \theta), 0, \tilde{\pi}_s \right\} \), with probability \( \mu_\theta \), for each \( \theta \in \Theta \), without soliciting information about the asset quality type. The new mechanism

\(^{15}\text{Equality only holds in the absence of default risk, which can occur either when } E(y|m^y) \geq K, \text{ or when } \lambda = 1.\)
satisfies constraint (i)-(v) from *ex-ante* perspective, and delivers the same payoff than \( \Upsilon[P] \) to the policy-maker. That constraints (i)-(iii) are satisfied under \( \Upsilon_{\text{avg}}[P] \) is straight-forward. Constraint (iv) follows from (18), which implies that:

\[
\pi_s \times \left( PR + \mu_H z_H + \mu_L z_L \right) = \frac{E(y|m^y)}{R} \leq \mu_H \bar{V}_H + \mu_L \bar{V}_L \equiv \bar{V}.
\]

Finally, (v) follows from (33). Thus, the policy-maker can replicate the same payoff delivered by \( \Upsilon[P] \), with mechanism \( \Upsilon_{\text{avg}}[P] \), and hence the optimal solution of the alternative problem dominates any feasible mechanism under the original setting. \( \square \)

Note that although \( \Upsilon_{\text{avg}}[P] \) satisfies (i)-(v), it does not respect incentive compatibility. We will find the solution to the alternative problem, which therefore dominates \( \Upsilon_{\text{avg}} \), when \( P \) is optimally chosen by the policy-maker, and show that it respects incentive compatibility constraints.

**Claim 2:** \( \mathbb{E}(y|m^y) \geq \bar{E} \) implies that the set of potential policies satisfying (i)-(v) is non-empty.

By definition of \( \bar{E} \), when \( \mathbb{E}(y|m^y) \geq \bar{E} \) there exist transfers \( t_0(\cdot) \) and probability \( \pi_s \) so that (i) holds. Moreover, there always exist policies satisfying constraint (ii)-(iv), which can be seen by choosing \( V \) small enough, and then choosing \( \pi(0)\cdot \) consistently. \( \square \)

Using arguments analogous to those establishing proposition 5, we can solve the designer’s alternative problem. The fact that proposition 5 assumes no elicitation of the asset quality dimension makes the analysis similar to this case. The solution can be characterized as a function of \( P \) and \( z \) by:

\[
(t_0(\omega), \pi(0|\omega)) = \begin{cases} 
\frac{\mathbb{E}(y|m^y)}{R}, & \omega < \hat{\omega} \\
1 - P - \omega, & \frac{\bar{V}}{1-\omega}R \\
1 - P - \omega, & \left(1 - \frac{\bar{V}}{1-\omega}R\right) \omega \in (\hat{\omega}, 1 - P) \\
0, & \frac{\bar{V}}{PR+z} \omega \geq 1 - P 
\end{cases}
\]

with \( \hat{\omega} \) and \( \bar{V} \) are chosen so that:

\[
\int_{1 - \frac{\mathbb{E}(y|m^y)}{R}}^{\hat{\omega}} \frac{f^\omega(\omega)}{(1 - \omega) R} d\omega + \frac{F^\omega(1 - P) - F^\omega(\hat{\omega})}{(1 - \hat{\omega}) R} = |b| \times \frac{F^\omega \left( \frac{\mathbb{E}(y|m^y)}{R} \right) - (1 - F^\omega(1 - P))}{\mathbb{E}(y|m^y)}. \tag{34}
\]

\[
\bar{V} \equiv \min \left\{ (1 - \hat{\omega}) R, \frac{|L(P)|}{|b| \times \int_0^{1-P} \frac{f^\omega(\omega)}{(P+\omega)R} d\omega - \frac{1 - F^\omega(1 - P)}{PR+z}} \right\}. \tag{35}
\]

Next, we show that the designer can improve her payoff by decreasing \( P \) and increasing \( z \) accordingly so that \( PR + z \leq \mathbb{E}(y|m^y) \).

**Claim 2:** The optimal comprehensive intervention either sets \( P = 0 \), or \( P = \mathbb{E}(y|m^y) \) (i.e., optimal interventions either involves the government, or the private sector, but not both) for any \( \lambda \in [0, 1] \).
To see this, assume that \( \lambda = 1 \), so that the liquidity shock is a 0-probability event. This assumption exacerbates the incentives to let external investors (the private sector) purchase securities from the bank during the fund-raising game, since the bank avoids discounts (haircuts) on its asset to compensate for default risk.

Consider the following function:

\[
\varphi^+(P, \tilde{\omega}) \equiv \int_0^1 \frac{b \left( \frac{\mathbb{E}(y|m^y)}{R}, 1 \right) f^\omega(\omega)}{\mathbb{E}(y|m^y)} d\omega + \frac{1 - \mathbb{E}(y|m^y)}{R} \int_1^{\frac{1 - \mathbb{E}(y|m^y)}{R}} f^\omega(\omega) d\omega + \frac{\mathbb{E}(y|m^y)}{R} \int_1^{\frac{1 - \mathbb{E}(y|m^y)}{R}} f^\omega(\omega) d\omega + \left( 1 - \frac{\mathbb{E}(y|m^y)}{R} \right) \left( \int_1^{1 - \mathbb{E}(y|m^y)} f^\omega(\omega) d\omega + \frac{1 - \mathbb{E}(y|m^y)}{R} \right).
\]

It corresponds to the expected payoff of creditors, under the optimal elicitation mechanism, under message '0' (pass). Function \( \varphi^+ \) decreases with \( P \) (or equivalently, increases with \( z \)) if we keep the rest of variables (other than \( z \)) constant, since \( (1 - \tilde{\omega}) R < \mathbb{E}(y|m^y) \). The case in which \( (1 - \tilde{\omega}) R = \mathbb{E}(y|m^y) \) corresponds to the situation in which the policy maker can avoid default altogether (with certainty) and thus is not considered here. This implies that (i) is relaxed when we decrease the value of \( P \), or equivalently, when we increase the value \( z \). We only consider this obedience constraint since the other constraint never binds under \( P = 0 \).

Define \( \tilde{\omega}(P) \) as the optimal cutoff associated to any price \( P \in [0, \frac{E}{R}] \), as in (34). That is, \( \tilde{\omega}(P) \) is chosen so that \( \varphi^+(P, \tilde{\omega}(P)) = 0 \). Consider the case where \( P = 0 \). The optimal elicitation mechanism is then given by:

\[
(t_0^{P=0}(\omega), \pi^{P=0}(0|\omega)) = \begin{cases} 
\frac{\mathbb{E}(y|m^y)}{R}, & \omega < 1 - \frac{\mathbb{E}(y|m^y)}{R} \\
1 - \omega, & \omega \in \left[ 1 - \frac{\mathbb{E}(y|m^y)}{R}, \tilde{\omega}(0) \right] \\
1 - \tilde{\omega}(0), & \omega \in (\tilde{\omega}(0), 1].
\end{cases}
\]

Choose any alternative policy in which the bank raises a price \( \tilde{P} \in (0, 1 - \tilde{\omega}(0)) \) from external investors. That \( \varphi^+ \) decreases with \( P \) implies that \( \tilde{\omega}(\tilde{P}) > \tilde{\omega}(0) \), since \( \tilde{\omega}(\tilde{P}) \) satisfies \( \varphi^+(\tilde{P}, \tilde{\omega}(\tilde{P})) = 0 \). This means that \( \pi^{P=0}(0|\omega) > \pi^{\tilde{P}}(0|\omega) \) for all \( \omega \leq \tilde{\omega}(P) \), and \( \pi^{P=0}(0|\omega) = \tilde{\pi}(0|\omega) = 1 \) for all \( \omega > \tilde{\omega}(P) \). As a result, the policy-maker’s payoff is strictly greater at \( P = 0 \). Finally, consider the case where \( \tilde{P} \leq 1 - \tilde{\omega}(0) \). We note that:

\[
\int_0^{1 - \frac{\mathbb{E}(y|m^y)}{R}} b \left( \frac{\mathbb{E}(y|m^y)}{R}, 1 \right) f^\omega(\omega) d\omega + g \cdot \left( \int_1^{1 - \frac{\mathbb{E}(y|m^y)}{R}} f^\omega(\omega) d\omega + \frac{1 - \mathbb{E}(y|m^y)}{R} \right) \left( \int_1^{\frac{1 - \mathbb{E}(y|m^y)}{R}} f^\omega(\omega) d\omega + \frac{1 - \mathbb{E}(y|m^y)}{R} \right) < \varphi^+(0, \tilde{\omega}(0)) = 0,
\]

which means that the policy-maker is unable to convince creditors to keep pledging to the bank, regardless of her chosen elicitation mechanism. Clearly, if best elicitation mechanism does not require external investors under \( \lambda = 1 \), it won’t require them for \( \lambda < 1 \). Thus, the best liquidity provision

56
program sets $P = 0$, which confirms that the optimal intervention will never involve the government, and the private sector at the same time. □

**Claim 3:** There exists a lower bound $E_{\min} \geq E$, such that government does not intervene if $\mathbb{E}(y|m^y) < E_{\min}$.

Claim 2 shows that liquidity provision programs and private-sector funding never occur simultaneously. Let $W^{LPP}$ and $W^{PS}$ the probability that the bank survives under the best liquidity provision program, and under private sector financing, respectively. That is:

$$W^{LPP} = (1 - \tilde{\omega}(0)) R \times \frac{F^\omega \left(1 - \frac{\mathbb{E}(y|m^y)}{R}\right)}{\mathbb{E}(y|m^y)} + \int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^1 \min \left\{ \frac{1 - \tilde{\omega}(0)}{1 - \omega}, 1 \right\} f^\omega(\omega)d\omega,$$

and

$$W^{PS} = 1 - F \left(1 - P \left(\frac{\mathbb{E}(y|m^y)}{R}\right)\right).$$

$W^{LPP}$ and $W^{PS}$ are increasing with respect to $^{16} \mathbb{E}(y|m^y)$. Moreover, that $L(\mathbb{E}(y|m^y)) \leq 0$ implies that:

$$W^{LPP} (\mathbb{E}) = 0 \leq 1 - F \left(1 - P \left(\frac{\mathbb{E}}{R}\right)\right) = W^{PS} (\mathbb{E}),$$

and

$$\lim_{\mathbb{E}(y|m^y) \to \mathbb{E}^{-}} W^{LPP} = 1 > 1 - F \left(1 - \frac{\bar{E}}{R}\right) \geq \lim_{\mathbb{E}(y|m^y) \to \mathbb{E}^{-}} W^{PS},$$

which proves the claim. □

---

$^{16}$Note that $\tilde{\omega}(0)$ decreases with $\mathbb{E}(y|m^y)$. 

57
References


