Real-Time Omnichannel Fulfillment Optimization
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Abstract
Motivated by the rapid development of e-commerce business, we study an online optimization problem in the context of omnichannel order fulfillment. Suppose a retailer can sell a product through an online website as well as multiple brick-and-mortar stores, each of which has an initial inventory that is non-replenishable. There are two streams of demands: online and offline. An online customer arriving at the website can either be either rejected or fulfilled by an online store, whereas an offline customer arriving at a local store must be satisfied by that store if it has remaining inventory. We assume that the offline customers have a higher profit margin than online customers. Interestingly, when there is only one store, our model reduces to the classical one-warehouse multi-channel optimization problem.

DEFINITION 1. A protection level algorithm $ALG_{(a_0, a_2)}$ sets a threshold $a_i$ for each location $i \in \{1, N\}$.
- Upon seeing an offline order at store $i$, fulfill it as much as possible by the remaining available inventory at store $i$.
- Suppose $S \leq \{N\}$ of stores has available inventories and their so far used inventories for online orders have not exceeded $a_i$. Upon seeing an online order, the order is fulfilled by all stores in $S$, keeping that the amount fulfilled from store $i$ is proportional to $\frac{a_i}{\sum_{j=1}^{N} a_j}$. If at any point the total fulfilled online demand from store $j$ reaches $a_j$, then $S = S \setminus \{j\}$ and the fulfillment continues until either we have fulfilled all demands in this online order, or $S$ becomes an empty set.

Algorithm 1 (Protection level alg)

Algorithm 2 (An Adaptive Algorithm for two stores)

DEFINITION 2. The adaptive algorithm $ALG_{(a_0, a_2)}$ works as follows. For each $t \in \{1, t_i | i \geq 1\}$, suppose the number of online items seen so far is $t$ and the next arriving customer is online (making it a total of $t + 1$ online orders). This adaptive algorithm computes $x(t)$ and $y(t)$, the amount of inventory used to fulfill the first $t$ online orders from stores 1 and 2, respectively, by solving

$$
\begin{align*}
&\min_{x,y} \max_{t \in [1, (t+1)q_n]} \min_{t \in [1, (t+1)q_n]} \\
&\begin{aligned}
&\left\{ x + y + (x+y)r + n - x + (x+y)r + y + \frac{y}{r} (r-1) + (1+q)n \right\} \\
&n + \frac{(x+y)]}{r} \end{aligned}
\end{align*}
$$

and then fulfill $x(t) - x(t-1)$ amount of online demand from store 1 (if inventory is available) and fulfill $y(t) - y(t-1)$ amount of online demand from store 2 (if inventory is available).

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References