Incentive-Compatible Information Design*

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Abstract

We study the design of mechanisms by an intermediary that generates information for a sender to persuade a receiver about an unknown attribute of the sender. The sender is initially privately, but imperfectly, informed about her attribute, and the receiver takes an action based on posterior beliefs about the sender's attribute and the sender's belief about the attribute. The design of the mechanism, therefore, confronts both incentive-compatibility constraints (for the sender) and obedience constraints (for the receiver). We characterize profit-maximizing mechanisms when the intermediary contracts with the sender, and we specialize to two applications: the design of college-admissions tests and the optimal use of consumer data on a digital market platform.

1 Introduction

Consider a firm that designs a college admissions test and sets a price to students for taking the test. The design of the test can reveal more or less information about the student's ability. Variations in the informativeness of the test affect how the college uses the test in admissions decisions and this affects the students willingness to pay for the test. In addition to these tradeoffs, the student's have prior private (but imperfect) information about their ability and this translates into private information about how they will perform on the test and thus how much

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they are willing to pay for the test. The design of the test must therefore grapple with screening frictions. These screening frictions are novel owing to the particular features of a test as a screening instrument. The design of a test is inherently a multi-dimensional screening problem (for example how finely to discriminate students among students in the upper tail versus students in the lower tail) but with some structure that is familiar from the information design literature.

What is the profit maximizing design of a college admissions test? What are the implications for patterns of college admissions, student welfare, and equity?

Consider next the design of a data policy by a digital market platform. The platform can use the data it collects about users to estimate consumers' values for new products. This information is useful both to the consumers on the platform (to improve their purchase decisions) and to sellers on the platform (to target buyers and set prices). The platform can, for example, offer data management policies to consumers which help consumers make informed purchases and which also incentivize firms to set prices which maintain high consumer surplus. Consumers are willing to pay for such policies and this can be a source of revenue for the platform. At the same time the consumers have their own private information about willingness to pay. The value of any data-management policy will be different for consumers with different private beliefs and so the platform faces screening frictions similar to that of the test-design firm. Data management policies are effective information design policies and these screening frictions are also disciplined by similar constraints.

These are two examples of a general class of problems we study in which an intermediary sells an information policy to a privately but imperfectly informed sender who will interact *ex post* with a receiver. The intermediary can design a policy which enhances the information of the sender while at the same time selectively disclosing information to the receiver to influence her ex post action. We combine tools of mechanism design and information design to characterize incentive-compatible information policies coupled with obedient decision rules for the receiver. We use these tools to solve for optimal mechanisms in the two examples described above.

2 Model

Setup A sender has attribute $\theta \in \{0, 1\}$, unknown to all. The sender is privately but imperfectly informed of θ as represented by her prior belief μ. Only the sender knows her prior μ, and we will refer to μ as the type of the sender.

There is a receiver who is initially uninformed about the sender's attribute θ and her type μ, knowing only that the latter is distributed according to a prior distribution function F which is continuous and has density f.

An intermediary will design an information policy which will generate new information about θ. The intermediary can reveal some of this information to the sender, allowing the sender to update her belief to a posterior ν. The intermediary can also reveal information to the receiver. This will allow the receiver to form a *second-order* belief, i.e. an element of $\Delta(\Theta \times \Delta\Theta)$, about both the attribute θ and the posterior belief ν of the sender.

After observing the information provided by the intermediary, the receiver will take an action $a \in A$ and obtain a payoff $\pi_R(a, \beta)$. The sender will earn payoff $\pi_S(a, \nu)$.

Examples In our first application the sender is a student and the receiver is a college. The intermediary sells a test to the student which will reveal information to the college to be used in admissions decision. Here θ is the student's ability, high ($\theta = 1$) or low ($\theta = 0$). The college chooses from the pair of actions $A = \{\texttt{admit}, \texttt{reject}\}$ and wants to admit students of high ability. Specifically the *ex-post* payoff from admitting the student depends only on θ and is given by the following matrix where $p \in (0, 1)$.

	$\theta = 0$	$\theta = 1$
admit	-1	$\frac{1-p}{p}$
reject	0	0

Given a belief β the college makes its admissions decision to maximize its expected payoff. In particular the student is admitted if and only if the college beleives with at least probability p that the student has high ability. The student earns a payoff of 1 from being admitted and 0 from being rejected, independent of θ.

To avoid trivialities we assume $\mathbf{E}_F\theta < p$ so that some information is necessary to induce the college to admit the student.

Our second application is to digital market platforms. Here the sender is a buyer whose willingness to pay for a product is θ. The receiver is the seller of the product; he offers a price p which the buyer will either accept or reject. Rejection leads to a payoff of zero for both parties and acceptance leads to payoff $\theta - p$ for the buyer and p for the seller. Thus, when the buyer has posterior ν, she will accept any price $p \leq \nu$ and her total payoff will be $\max\{\nu - p, 0\}$. Given belief β, the seller's payoff from offering price p is given by

$$p \cdot \text{Prob}(\nu \geq p).$$

Note that the seller's payoff depends on his second-order belief: his belief about the buyer's belief.

Information Policies A *test* is defined by two non-empty sets of messages ("test results") $\mathcal{M}_S$ and $\mathcal{M}_R$ and for each θ a distribution $\rho_\theta \in \Delta(\mathcal{M}_S \times \mathcal{M}_R)$. When the sender submits to a test, the results are drawn from ρ_θ conditional on the sender's true attribute. The result in $\mathcal{M}_S$ is privately disclosed to the sender and the result in $\mathcal{M}_R$ is privately disclosed to the receiver. An information policy, or a *mechanism* is defined by a non-empty set of messages $\mathcal{M}_\Sigma$ and a rule that specifies as a function of $\mathcal{M}_\Sigma$ both a payment made by the receiver and a test. The receiver selects a message from $\mathcal{M}_\Sigma$ and the resulting test is carried out. Importantly, we restrict payments to be *ex ante*, i.e. before the realization of the test.

In a *direct* mechanism $\mathcal{M}_\Sigma = \Theta$, $\mathcal{M}_R = A$ and $\mathcal{M}_S = \Delta\theta$. The sender reports her type $\mu \in \Theta$ and in response the intermediary charges the sender the fee $\phi(\mu)$, uses $\rho_\theta(\mu)$ to recommend an action $a \in A$ to the receiver and privately informs the sender of her posterior $\nu \in \Delta\theta$. Thus, $\rho_\theta(\mu) \in \Delta(A \times \Delta\theta)$. When the intermediary uses a direct mechanism and the realized recommendation is a, the receiver formulates the conditional belief $\beta(\cdot \mid a) \in \Delta(\theta \times \Delta\theta)$.

A direct mechanism is *obedient* if the recommended action is always the one that maximizes the receiver's payoff conditional on hearing the recommendation, i.e. with probability 1

$$a \in \text{argmax}_A \pi_R(\cdot, \beta(\cdot \mid a)).$$

Let $\tilde{\nu}(\mu) = \mu \, \text{marg}_{\Delta\theta} \, \rho_1(\mu) + (1 - \mu) \, \text{marg}_{\Delta\theta} \, \rho_0(\mu)$ be the total probability distribution of the posterior induced for type μ. A direct mechanism is *bayes-plausible* if for every μ,

$$\mathbf{E}\tilde{\nu}(\mu) = \mu.$$

Recall that Kamenica and Gentzkow (2011) bayes-plausibility implies that for a sender with prior μ, the conditional probability of $\theta = 1$ conditional on every suggested posterior ν is ν itself. On the other hand, should μ report some other type $\mu' \neq \mu$ to the mechanism and receive the posterior ν suggested for μ', she would use that suggestion to update her true prior μ and thereby obtain a possibly different posterior, call it $\delta_{\mu,\mu'}(\nu)$. By continuity of Bayesian updating, $\delta_{\mu,\mu'}(\nu)$ is continuous in μ for any given μ' and ν.

A direct mechanism yields *gross utility* for type μ equal to $v(\mu) = \mathbf{E}_{\rho(\mu)}\pi_S(a,\nu)$. We define the *indirect utility* function of a direct mechanism by

$$U(\mu) = v(\mu) - \phi(\mu)$$

When μ misreports $\mu' \neq \mu$ she instead earns gross utility

$$v(\mu' \mid \mu) = \mu \mathbf{E}_{\rho_1(\mu')}\pi_S(a, \delta_{\mu,\mu'}(\nu)) + (1-\mu)\mathbf{E}_{\rho_0(\mu')}\pi_S(a, \delta_{\mu,\mu'}(\nu))$$

A direct mechanism is *incentive compatible* if for every μ, μ'

$$U(\mu) \geq v(\mu' \mid \mu) - \phi(\mu').$$

A direct mechanism is *individually rational* if $U(\mu) \geq 0$ for all μ.

The goal of the intermediary is to maximize expected revenue. By the revelation principle we can restrict attention to obedient, bayes-plausible and incentive-compatible direct mechanisms (ρ, ϕ) designed to maximize

$$\Pi = \int_0^1 \phi(\mu) dF(\mu)$$

3 Characterization of Incentive Compatibility

The gross deviation utility $v(\mu' \mid \mu)$ is non-linear which complicates the usual envelope representation of incentive compatible mechanisms. However we can make use of the following observation to obtain a condition that is necessary in general and sufficient in some special cases.

Let (ρ, ϕ) be an incentive-compatible mechanism. Suppose for every μ' there exist numbers $q_0(\mu')$ and $q_1(\mu')$ such that for every μ

$$v(\mu' \mid \mu) \geq \mu q_1(\mu') + (1-\mu) q_0(\mu') \tag{1}$$

with equality when $\mu = \mu'$. Then the linear function on the right-hand size is a support function for the indirect utility function:

$$\begin{aligned} U(\mu) &\geq v(\mu' \mid \mu) - \phi(\mu') \\ &\geq \mu q_1(\mu') + (1-\mu) q_0(\mu') - \phi(\mu') \\ &= (\mu - \mu') q_1(\mu') + \left[(1-\mu) - (1-\mu')\right] q_0(\mu') + \mu' q_1(\mu') + (1-\mu') q_0(\mu') - \phi(\mu') \\ &= U(\mu') + (\mu - \mu') \left[q_1(\mu') - q_0(\mu')\right] \end{aligned}$$

This implies that $U(\cdot)$ is convex and hence absolutely continuous. It is differentiable at almost every μ with slope $q_1(\mu) - q_0(\mu)$. The indirect utility of type μ can be expressed as

$$U(\mu) = U(0) + \int_0^{\mu} \left[q_1(\mu') - q_0(\mu')\right] d\mu'$$

Turning things around, suppose that a mechanism (ρ, ϕ) yields a convex indirect utility function U and the inequality in Equation 1 holds with equality for all μ. If the integral representation above holds then the mechanism is incentive compatible because by convexity.

$$U(\mu) \geq U(\mu') + (\mu - \mu') \left[q_1(\mu') - q_0(\mu')\right] = v(\mu' \mid \mu) - \phi(\mu').$$

4 College Admissions

In the collge admissions application a direct mechanism recommends `admit` or `reject` to the college and the obedience constraint is

$$\text{Prob}(\theta = 1 \mid \alpha = \texttt{admit}) \geq p$$

Let $q_\theta(\mu) = \text{marg}_A\, \rho_\theta(\mu)\, [\texttt{admit}]$ be the probability that a student with type μ and ability θ is recommended for admission. Then

$$v(\mu \mid \mu') = \mu q_1(\mu') + (1-\mu) q_0(\mu')$$

for all μ and μ', and therefore the monotonicity condition, namely for all $\mu \geq \mu'$

$$q_1(\mu) - q_0(\mu) \geq q_1(\mu') - q_0(\mu')$$

together with the envelope formula are necessary and sufficient condition for incentive compatibility.

For any incentive compatible mechanism there must exist a type μ^0 whose participation constraint binds, i.e. $U(\mu^0) = 0$. If not, fees could be increased by a constant for all types, raising revenue without altering incentive constraints.

Then we may write

$$U(\mu) = \begin{cases} -\int_{\mu}^{\mu^0} [q_1(\mu') - q_0(\mu')]\, d\mu', & \text{if } \mu \leq \mu^0 \\ \int_{\mu}^{\mu^0} [q_1(\mu') - q_0(\mu')]\, d\mu', & \text{if } \mu \geq \mu^0 \end{cases}$$

Since U is convex and weakly positive, the slope must be non-positive below μ^0 and non-negative above. In particular $q_1(\mu) \leq q_0(\mu)$ for all $\mu \leq \mu^0$.

We may express the intermediary's expected revenue as expected gross utility minus expected indirect utility.

$$\Pi = \int_0^{\mu^0} \left\{ v(\mu) + \int_{\mu}^{\mu^0} \left[q_1(\mu') - q_0(\mu') \right] d\mu' \right\} f(\mu) d\mu$$
$$+ \int_{\mu^0}^{1} \left\{ v(\mu) - \int_{\mu}^{\mu^0} \left[q_1(\mu') - q_0(\mu') \right] d\mu' \right\} f(\mu) d\mu$$

We can see immediately that profit is increasing in $q_1(\mu)$ for all $\mu \leq \mu^0$. Moreover, increasing $q_1(\mu)$ adds slack to the obedience constraint and therefore the constraint $q_1(\mu) \leq q_0(\mu)$ must bind and we have $q_1(\mu) = q_0(\mu)$ for all $\mu \leq \mu^0$.

It follows that $q_1(\mu) \geq q_0(\mu)$ for all $\mu \in [0, 1]$ and in fact we may take $\mu^0 = 0$, that is $U(0) = 0$.

When we change the order of integration and simplify we obtain the following expression for expected *virtual surplus*

$$\Pi = \mathbf{E}\left[q_1(\mu) \left(\mu - \frac{1 - F(\mu)}{f(\mu)} \right) \right] + \mathbf{E}\left[q_0(\mu) \left(1 - \mu + \frac{1 - F(\mu)}{f(\mu)} \right) \right] \qquad (2)$$

The problem reduces to maximizing expected virtual surplus subject to monotonicity, obedience, and $q_1(\mu) \geq q_0(\mu)$.

Notice that the sum of the coefficients on $q_1(\mu)$ and $q_0(\mu)$ equals 1. If we ignore the obedience constraint, it would be optimal therefore to set $q_1(\mu) = q_0(\mu) = 1$ for all

μ. This is the mechanism that admits the student with probability 1 (and extracts all of the surplus), but this mechanism violates obedience because $\mathbf{E}\mu < p$.

It follows that the obedience constraint is binding at the optimum and the problem becomes one of restoring obedience at minimum cost in terms of foregone virtual surplus, and subject to monotonicity and $q_1(\mu) \geq q_0(\mu)$. One candidate mechanism is a threshold mechanism in which $q_1(\mu) = q_0(\mu)$ for all μ and these admission probabilities jump from 0 to 1 at the interior point $\tilde{\mu}$ defined by

$$\mathbf{E}(\mu \mid \mu \geq \tilde{\mu}) = p$$

This mechanism is feasible because it is monotonic, and satisfies both obedience and $q_1(\mu) \geq q_0(\mu)$. Moreover its indirect utility has a constant slope of zero so the mechanism extracts all of the surplus it generates.

Nevertheless this mechanism is typically not optimal. Instead a mechanism which selectively rejects some low-ability students (by setting $q_1(\mu) > q_0(\mu)$ for some types), can generate larger revenues despite yielding positive rents to the student.

Proposition 1. *The optimal revenue is achieved by allocation of the following form, for some values* $0 \leq \mu_0 \leq \overline{\mu} \leq 1$.

1. *If* $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ *is non-increasing then optimal allocation* (q_1, q_0) *is of the form*

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases}$$

   Where $\int_{\mu_0}^{1} \mu dF(\mu) = \frac{p}{1-p} \int_{\overline{\mu}}^{1} (1-\mu) dF(\mu)$.

2. *If* $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ *is non-decreasing then optimal allocation* (q_1, q_0) *is of the form*

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1-\alpha, 0) & \mu \in [\mu_0, \overline{\mu}) \\ (1,\alpha) & \mu \in [\overline{\mu}, 1] \end{cases}$$

   Where $(1-\alpha) \int_{\mu_0}^{\overline{\mu}} \mu dF(\mu) + \int_{\overline{\mu}}^{1} \mu dF(\mu) = \alpha \frac{p}{1-p} \int_{\overline{\mu}}^{1} (1-\mu) dF(\mu)$.

The optimal mechanism obtained in item 1 allocates informative tests to the higher types, and lower types are offered uninformative tests. Students with lower prior expected ability take easier tests (less informative) and contribute to a greater share of the intermediate's revenue. Students with higher prior expected ability take difficult tests (more informative) and earn greater information rent. In contrast, the optimal mechanism obtained in item 2 allocates informative tests more evenly among different applicant types. In fact, there is a reversal in the allocation of perfectly revealing tests. For item 2, types in the middle are admitted only if the student is high-ability, and higher types are admitted even if they are low-ability. Whereas in item 1, types in the middle are always admitted and higher types are admitted only if they are high-ability. For a welfare perspective, all types that are not excluded by the intermediary earn information rents in item 2, whereas the intermediary extracts all surplus from types that are allocated uninformative tests in item 1.

5 Digital Platform

Our next application is to digital marketplace platforms. A platform has access to detailed data which enables it to precisely estimate a buyer's willingness to pay for products. The platform can provide this information to the buyer but moreover to the seller. The former guides the buyer's purchases while the latter disciplines the prices set by sellers. The platform sells access to the buyer and by controlling the information available to sellers the platform also controls the consumer surplus provided to buyers and thus increases the value of membership on the platform.

Formally the buyer plays the role of the sender, with willingness to pay θ, and private information μ. The seller is the receiver, setting a price $p \in A = \mathbf{R}$. A sale at price p yields $\theta - p$ for the buyer and p for the seller. The payoff to both from no sale (including when the buyer is excluded from the platform) is zero.

A mechanism for the platform specifies whether the buyer will be admitted to the platform and if so provides information to the buyer leading to the posterior ν and recommends the price p to the seller. The seller infers the distribution of posteriors ν for buyers on the platform and adopts the recommended price p if and only if

$$p \cdot \text{Prob}(\nu \geq p \mid p) \geq p' \cdot \text{Prob}(\nu \geq p' \mid p) \tag{3}$$

The above system of inequalities constitutes the obedience constraint.

Participation Let $q_\theta(\mu)$ denote the probability that type μ joins the platform when having willingness to pay θ, and let

$$\bar{q}(\mu) = \mu q_1(\mu) + (1-\mu) q_0(\mu)$$

be the total probability that μ joins the platform. The buyer earns non-negative payoff only when joining the platform. The participation constraint therefore requires that the expected payoff for every type μ conditional on joining the platform and paying price p is non-negative:

$$\mu \cdot q_1(\mu) \geq p\bar{q}(\mu) \tag{4}$$

Incentive Compatibility The following observation enables us to use our general representation of incentive compatibility. Since the buyer's payoff is zero when excluded from the platform this payoff is equivalent to the payoff from purchasing the good at price equal to her willingness to pay θ. Thus we may without loss represent any mechanism as one in which sale occurs with probability 1 at a random price from the set $\{0, p, 1\}$.

In particular the expected price paid by type μ having willingness to pay θ is

$$\bar{p}_1(\mu) = p \cdot q_1(\mu) + 1 \cdot (1 - q_1(\mu)) \tag{5}$$

for $\theta = 1$ and

$$\bar{p}_0(\mu) = p \cdot q_0(\mu) + 0 \cdot (1 - q_0(\mu)) \tag{6}$$

for $\theta = 0$. Since the total (both on and off the platform) expected value of the buyer's posterior is equal to her type μ, this yields gross payoff

$$v(\mu) = \mathbf{E}\,(\nu - p) = \mu - (1-\mu)\bar{p}_0(\mu) - \mu\bar{p}_1(\mu)$$

where $\bar{p}_\theta(\mu)$ is the expected value of the random price offered to type μ, conditional on having true willingness to pay θ. The deviation payoff is

$$v(\mu' \mid \mu) = \mu' - \mu'\bar{p}_1(\mu) - (1-\mu')\bar{p}_0(\mu) = \mu'(1 - \bar{p}_1(\mu)) - (1-\mu')\bar{p}_0(\mu)$$

and therefore the monotonicity condition requires that $1 - \bar{p}_1(\mu) + \bar{p}_0(\mu)$ is weakly increasing. Or using the expressions in Equation 5 and Equation 6,

$$K(\mu) = (1-p)q_1(\mu) + pq_0(\mu) \quad \text{is weakly increasing in } \mu. \tag{7}$$

This condition together with the envelope formula

$$U(\mu) = \int_0^{\mu} \left[(1-p)q_1(\mu') + pq_0(\mu')\right] d\mu' + U(0) \tag{8}$$

are necessary and sufficient for incentive compatibility.[1]

Relaxed Obedience Constraint In the previous examples, the receiver's conditional beliefs about θ were sufficient to characterize the obedience constraint. Here by contrast, obedience for the seller depends on the full conditional distribution of the buyer's interim beliefs. To facilitate the analysis of obedience we will make use of results in a companion paper Chopra and Ely (2025) which in turn builds on the results of Roesler and Szentes (2017) to characterize obedient value distributions when the buyer has some initial private information.

Consider the following family of cumulative distribution functions parameterized by a target price p and a mass x.

$$H_x^p(s) = \begin{cases} 0 & s < p \\ x & s = p \\ 1 - \frac{p}{s} & 1 > s \geq \frac{p}{1-x} \\ 1 & s = 1 \end{cases}$$

Note that when $x = 0$ the distribution H_x^p is the unit-elastic demand studied by Roesler and Szentes (2017) and makes the seller indifferent between all prices in the interval $[p, 1]$. In other words H_0^p satisfies the obedience constraint Equation 3 with equality for all $p' \geq p$. As shown by Roesler and Szentes (2017) this is the way to maximize consumer surplus of a buyer with no initial private information.

In our setting the platform is a profit-maximizer screening a buyer with private information. The seller is therefore is willing to tradeoff consumer surplus (i.e. efficiency) in exchange for rent extraction. Moreover given the buyer's initial private information there typically does not exist an information policy that can generate

[1] With this formulation we are empowering the platform to enforce sale when the seller accepts the recommended price p. Thus, the mechanism needn't guard against "double deviations" in which a buyer of type μ misreports as type μ' and then selectively rejects p for some posteriors. Nevertheless, we show below that the optimal mechanism the platform never enforces undesirable purchases on path. Moreover, in Section B.6 we discuss the issue of double deviations further and show that allowing randomized prices deters all double deviations in any incentive-compatible and individually-rational mechanism.

an iso-elastic demand. Instead we will show that obedience can more generally be satisfied by targeting information to the buyer in such a way as to generate a distribution of posteriors of the form H^p_x for some $x \geq 0$. The resulting value distribution makes the seller indifferent between all prices in $\{p\} \cup [p/(1-x), 1]$.[2]

A distribution of buyer values can be transformed into one that satisfies obedience if and only if it is a mean-preserving contraction of some H^p_x. We can therefore replace the obedience constraint with the condition that $\operatorname{marg}_{\Delta\theta} \beta(\cdot \mid p)$ to be a mean-preserving contraction of H^p_x for some x. We will instead impose a relaxed constraint consisting of two necessary conditions. The first condition is that the mean valuation in $\operatorname{marg}_{\Delta\theta} \beta(\cdot \mid p)$ equals the mean of H^p_x.

$$\mathbf{E} \operatorname{marg}_{\Delta\theta} \beta(\cdot \mid p) = \mathbf{E} H^p_x. \tag{9}$$

When this condition holds and additionally the CDF of $\operatorname{marg}_{\Delta\theta} \beta(\cdot \mid p)$ crosses H^p_x once and from below then the former is a mean-preserving contraction of the latter. A necessary condition for this single crossing is

$$\operatorname{marg}_{\Delta\theta} \beta(p \mid p) \leq x, \tag{10}$$

that is the CDF of $\operatorname{marg}_{\Delta\theta} \beta(\cdot \mid p)$ is below H^p_x at the point p.

Note for future reference that $\mathbf{E} H^p_x$ is strictly decreasing in x.

Relaxed Problem When we express the platform's profit as the difference between expected gross payoff and expected buyer utility, represent expected utility by the envelope formula in with $U(0) = 0$[3] and then proceed with the standard manipulations we obtain the following objective function for the platform.

$$\Pi = \int_0^1 (1-p) \left(\mu - \frac{1-F(\mu)}{f(\mu)} \right) q_1(\mu) dF(\mu) - \int_0^1 p \left(1 - \mu + \frac{1-F(\mu)}{f(\mu)} \right) q_0(\mu) dF(\mu) \tag{11}$$

Our relaxed problem is to choose $q_\theta(\cdot)$ to maximize this profit function subject to monotonicity (Equation 7), participation (Equation 4), and relaxed obedience (Equation 9 and Equation 10).

[2] As it turns out however we identify a condition on the ex ante distribution F under which the optimal mechanism is indeed a unit-elastic demand with mass $x = 0$.

[3] From Equation 8 we see that $U(\mu) \geq U(0)$ for all μ and therefore if $U(0) > 0$ we could increase payments by the constant $U(0)$ without violating any constraints.

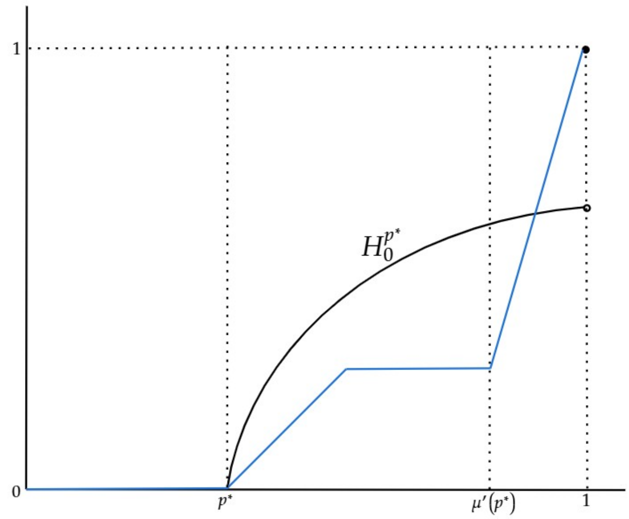

Figure 1: For uniform distribution, the optimal price $p^*(\approx 0.417)$ solves the relaxed problem and generates the distribution of buyer valuation, conditional on being recommended price p^*, represented above in blue. This distribution crosses $H_0^{p^*}$ once and from below and hence is a mean preserving contraction of $H_0^{p^*}$.

Proposition 2. *If $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non-increasing, then for any p one of the following $q(\mu) = (q_1(\mu), q_0(\mu))$ is a solution to the relaxed problem*

$$q(\mu) = \begin{cases} (0,0) & \text{if} \quad \mu \leq \mu_0 \\ \left(\frac{p}{1-p}\frac{1-\mu_0}{\mu_0}, 1\right) & \text{if} \quad \mu \in [\mu_0, \mu') \\ (1,1) & \quad \text{otherwise} \end{cases} \quad \text{or} \quad q(\mu) = \begin{cases} (0,0) & \text{if} \quad \mu \leq \mu_0 \\ (1,0) & \text{if} \quad \mu \in [\mu_0, \mu') \\ (1,1) & \quad \text{otherwise} \end{cases}$$

for some thresholds $0 \leq \mu_0 \leq \mu' \leq 1$

Notice that the likelihood ratio $q_1(\mu)/q_0(\mu)$ is greater than or equal to 1 for all types μ except those belonging to the second interval $[\mu_0, \mu')$. In that second interval the likelihood ratio is weakly increasing. It follows that there is at most one type such that Equation 4 holds with equality. More precisely, the distribution of values induced by the relaxed solution has no mass at p, i.e. $x = 0$. The induced value distribution therefore has the same mean as H_0^p, the iso-elastic demand implementing price p.

When we then ask whether the relaxed solution can be transformed into a fully obedient mechanism and therefore a solution to the original problem, we need only check whether this value distribution crosses H_0^p once and from below. We show in the appendix (Section B.5) that this is true in particular for the uniform distribution on $[0, 1]$. The structure of the optimal mechanism for this case is represented in Figure 1.

6 Related Literature

Corrao (2023) and Lizzeri (1999) study variations of an information design problem with incentive constraints but in their model the sender is already perfectly informed. The role of the intermediary is to turn the sender's soft information into information that is verifiable to the receiver. In Ali et al. (2020) the intermediary is generating new information but the sender has no private information *ex ante* and so there are no screening frictions. Bergemann et al. (2018) study a problem like ours where the information designer produces new information for an *ex ante* privately informed agent. However in Bergemann et al. (2018) the sender herself takes the action *ex post*, there is no third party/receiver.

Three concurrent and independent papers are closest to ours. Celik and Strausz (2025) and Weksler and Zik (2025) both study a buyer-seller interaction like our digital platform and Mäkimattila et al. (2024) study a version of our college admissions example. All three papers impose a restriction on the intermediary that the test chosen by the sender must be revealed to the receiver. By contrast we give the intermediary full flexibility in designing the information structure for the sender and the receiver. This ability to pool is valuable to the intermediary and shapes the optimal mechanism. As a benchmark Mäkimattila et al. (2024) do also examine the fully flexible case and derive the optimal mechanism in a case similar to our item 1.

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A College Admissions

A.1 Geometry of Monotone Tests

The slope of the indirect utility is given by $K(\mu) = q_1(\mu) - q_0(\mu)$. Substituting this into Equation 2 gives us

$$\Pi = \mathbf{E}\left[q_1(\mu)\right] - \mathbf{E}\left[K(\mu)\left(1 - \mu + \frac{1 - F(\mu)}{f(\mu)}\right)\right]$$

The obedience constraint is given by

$$\mathbf{E}\left[\mu q_1(\mu)\right] \geq \frac{p}{1-p}\mathbf{E}\left[(1-\mu)q_0(\mu)\right]$$

As we argued in the text the obedience constraint must bind at an optimum. We can express the binding constraint in terms of the slope function $K(\mu)$ as follows.

$$\mathbf{E}\left[(1-\mu)K(\mu)\right] = \mathbf{E}\left[\left(1 - \frac{\mu}{p}\right)q_1(\mu)\right]$$

Putting together,

$$\Pi = \mathbf{E}\left[\frac{\mu}{p}q_1(\mu)\right] - \mathbf{E}\left[K(\mu)\frac{1 - F(\mu)}{f(\mu)}\right] \tag{12}$$

If K is such that there exists some incentive compatible and obedient test (q_1, q_0) for which $K(\mu) = q_1(\mu) - q_0(\mu)$, the following holds

$$0 \leq K(\mu) \leq q_1(\mu) \leq 1 \tag{13}$$

and

$$\mathbf{E}\left[(1-\mu)K(\mu)\right] \geq \int_0^p \left(1 - \frac{\mu}{p}\right)K(\mu)dF(\mu) + \int_p^1 \left(1 - \frac{\mu}{p}\right)dF(\mu) \tag{14}$$

We call a slope K feasible if it satisfies Equation 13 and Equation 14 for some test allocation q_1. Define the following class of threshold allocations

$$(q_1^K(\mu),\, q_0^K(\mu)) := \begin{cases} (K(\mu),\, 0) & \mu < \mu^K \\ (1,\, 1 - K(\mu)) & \mu \geq \mu^K \end{cases}$$

For some $\mu^K \in [0, 1]$.

Lemma 1. *For given p and feasible $K(\mu)$, the optimal allocation is $q_1 = q_1^K$ for some μ^K.*

Proof. It is clear from Equation 12 and Equation 14 that the optimum requires $q_1(\mu) = 1$ for all $\mu \geq p$.

Consider for contradiction that there is some interval of types $[\mu_0, \overline{\mu})$ such that $K(\mu) < q_1(\mu) < 1$. It's sufficient to consider $\overline{\mu} \leq p$. Pick $\delta > 0$ such that $\overline{\mu} - \mu_0 \geq 2\delta$. We can improve the revenue by increasing q_1 on $[\overline{\mu} - \delta, \overline{\mu})$ by some $\varepsilon_1 > 0$ and by reducing q_1 on $[\mu_0, \mu_0 + \delta)$ by $\varepsilon_0 > 0$ where

$$\varepsilon_1 = \varepsilon_0 \frac{\int_{\mu_0}^{\mu_0+\delta} \left(1 - \frac{\mu}{p}\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \left(1 - \frac{\mu}{p}\right) dF(\mu)}$$

ensuring that Equation 14 is maintained.

The revenue from the new allocation is greater than the old allocation if the following holds:

$$\frac{\int_{\mu_0}^{\mu_0+\delta} \left(1 - \frac{\mu}{p}\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \left(1 - \frac{\mu}{p}\right) dF(\mu)} \geq \frac{\int_{\mu_0}^{\mu_0+\delta} \frac{\mu}{p} dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \frac{\mu}{p} dF(\mu)}$$

$$\iff \frac{\int_{\mu_0}^{\mu_0+\delta} dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} dF(\mu)} \geq \frac{\int_{\mu_0}^{\mu_0+\delta} \mu dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \mu dF(\mu)}$$

The last inequality follows from our choice of δ.

Now consider q_1 such that there are intervals $I_1 < I_2$ where $q_1(\mu) = K(\mu)$ for $\mu \in I_2$ and $q_1(\mu) = 1$ for $\mu \in I_1$. It suffices to consider $\sup(I_2) \leq p$ by the previous arguments. We construct an improvement similar to the above by slightly increasing q_1 on I_2 and reducing q_1 on I_1 to preserve the inequality in Equation 14. The revenue of this improvement is greater than the old allocation if the following holds:

$$\frac{\int_{I_1} dF(\mu)}{\int_{I_2} dF(\mu)} \geq \frac{\int_{I_1} \mu dF(\mu)}{\int_{I_2} \mu dF(\mu)}$$

This is implied by $\frac{\inf(I_2)}{\sup(I_1)} \geq 1$. □

By Lemma 1 we can, without loss, focus on allocations that have non-decreasing admission probability q_1. Let Λ be the set of all such allocations which are also incentive-compatible and have indirect utility with $U(0) = 0$. When viewed as a subset of the topological vector space (TVS) $\mathbb{L}_\infty([0,1] \to \mathbb{R}^2)$, the set Λ is algebraically compact and convex.[4] Moreover, its extreme points are such that for some $0 \leq \mu_0 \leq \overline{\mu} \leq 1$

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases}$$

The set of such allocations is $\mathrm{ex}(\Lambda)$.

Another consequence of Lemma 1 is that, fixing μ^K, the designer's objective can be represented as maximizing a linear functional, where the choice variable is the slope of the indirect utility, a monotone function! Thus, the slope K for an extreme test allocation is a non-decreasing single-step function. By virtue of a linear objective and a single linear obedience constraint, an optimal allocation can be represented as a convex combination of at most two elements of $\mathrm{ex}(\Lambda)$ (seeDubins (1960)). Thus, the optimal revenue is achieved by a mechanism where the slope of the indirect utility is a non-decreasing two-step function. In particular, the optimal test can be found among the ones with the following structure

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \mu_1) \\ (1,\alpha) & \mu \in [\mu_1, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases} \quad \text{or} \quad (q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1-\alpha,0) & \mu \in [\mu_0, \mu_1) \\ (1,\alpha) & \mu \in [\mu_1, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases}$$

The proposition then establishes sufficient conditions on the prior F under which 1) the optimal allocation is described by a single extreme point, 2) the optimal allocation is described by convex combination of two extreme points.

A.2 Proof of Proposition 1

Proof of Proposition 1. First, we note that a test such that $q_0 = 0$ is never optimal; this can be seen as the obedience constraint is always slack for such tests.
To prove item 1, we show by contradiction that a convex combination of two distinct tests can be improved. By Lemma 1 we can restrict attention to tests where

[4] A set Λ in a TVS is algebraically compact if the intersection of the set Λ with a line is always algebraically closed and bounded, see Barvinok (2012).

$q_1(\mu) = q_0(\mu)$ implies that either $q_1(\mu) = 0$ or $q_1(\mu) = 1$. First consider as a test (q_1, q_0) such that there exists $0 \le \mu_0 \le \mu_1 < \overline{\mu} \le 1$ and

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \mu_1) \\ (1,\alpha) & \mu \in [\mu_1, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases}$$

We will construct a more profitable test $q' = (q'_1, q'_0)$ such that $q'_1 = q_1$ for all μ and $q_0 = q'_0$ on the complement of $[\mu_1, \overline{\mu})$. On $[\mu_1, \mu_1 + \varepsilon)$ set $q'_0 = 1$ for some small $\varepsilon > 0$ and on $[\mu_1 + \varepsilon, \overline{\mu})$ set $q'_0 = (1-\delta)\alpha$ for small enough $\delta > 0$. Now we will argue that whenever $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is decreasing there exist $\varepsilon > 0$ and $\delta > 0$ such that q' is feasible and earns higher revenue. Revenue increases by

$$(1-\alpha)\int_{\mu_1}^{\mu_1+\varepsilon}\left[1-\mu+\frac{1-F(\mu)}{f(\mu)}\right]dF(\mu) - \delta\alpha\int_{\mu_1+\varepsilon}^{\overline{\mu}}\left[1-\mu+\frac{1-F(\mu)}{f(\mu)}\right]dF(\mu)$$

and the slack in the obedience constraint increases by

$$-(1-\alpha)\int_{\mu_1}^{\mu_1+\varepsilon}\frac{(1-\mu)p}{1-p}dF(\mu) + \delta\alpha\int_{\mu_1+\varepsilon}^{\overline{\mu}}\frac{(1-\mu)p}{1-p}dF(\mu)$$

For a fixed $\varepsilon > 0$ there exist a $\delta > 0$ such that both are positive if and only if

$$\frac{\int_{\mu_1}^{\mu_1+\varepsilon}[1-\mu+\frac{1-F(\mu)}{f(\mu)}]dF(\mu)}{\int_{\mu_1+\varepsilon}^{\overline{\mu}}[1-\mu+\frac{1-F(\mu)}{f(\mu)}]dF(\mu)} > \frac{\int_{\mu_1}^{\mu_1+\varepsilon}\frac{(1-\mu)p}{1-p}dF(\mu)}{\int_{\mu_1+\varepsilon}^{\overline{\mu}}\frac{(1-\mu)p}{1-p}dF(\mu)} = \frac{\int_{\mu_1}^{\mu_1+\varepsilon}(1-\mu)dF(\mu)}{\int_{\mu_1+\varepsilon}^{\overline{\mu}}(1-\mu)dF(\mu)}$$

and this inequality holds if and only if

$$\frac{\int_{\mu_1}^{\mu_1+\varepsilon}\frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\mu_1+\varepsilon}^{\overline{\mu}}\frac{1-F(\mu)}{f(\mu)}dF(\mu)} > \frac{\int_{\mu_1}^{\mu_1+\varepsilon}(1-\mu)dF(\mu)}{\int_{\mu_1+\varepsilon}^{\overline{\mu}}(1-\mu)dF(\mu)} \tag{15}$$

For $\varepsilon = 0$ both sides are zero. Now suppose $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is decreasing in μ. Then

$$\frac{1-F(\mu_1)}{(1-\mu_1)f(\mu_1)} \ge \int_{\mu_1}^{\overline{\mu}}\frac{1-F(\mu)}{(1-\mu)f(\mu)}dF(\mu) \ge \frac{\int_{\mu_1}^{\overline{\mu}}\frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\mu_1}^{\overline{\mu}}(1-\mu)dF(\mu)}$$

or

$$\frac{\frac{1-F(\mu_1)}{f(\mu_1)}}{\int_{\mu_1}^{\overline{\mu}} \frac{1-F(\mu)}{f(\mu)} dF(\mu)} \geq \frac{1-\mu_1}{\int_{\mu_1}^{\overline{\mu}} (1-\mu) dF(\mu)}$$

and the left-hand side is the derivative of the left-hand side in Equation 15 while the right-hand side is the derivative of the right-hand side in Equation 15, both at $\varepsilon = 0$. This guarantees that Equation 15 holds on a neighborhood of $\varepsilon = 0$.

Now consider some test (q_1, q_0) which has the following form

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1-\alpha, 0) & \mu \in [\mu_0, \mu_1) \\ (1, \alpha) & \mu \in [\mu_1, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1) \end{cases}$$

If $\mu_0 = \mu_1$ and $\mu_1 < \overline{\mu}$ then previous argument shows that (q_1, q_0) is not optimal. When $\mu_0 < \mu_1$ and $\mu_1 < \overline{\mu}$, then we can use the same ideas to construct a profitable deviation, let (q_1', q_0') such that $q_1' = q_1$ for all μ and $q_0 = q_0'$ on $([\mu_0, \overline{\mu})^C$. On $[\mu_0, \mu_0 + \varepsilon)$ set $q_0' = 1 - \alpha$ for some small $\varepsilon > 0$ and on $[\mu_1, \overline{\mu})$ set $q_0' = (1-\delta)\alpha$ for small enough $\delta > 0$. Now we will argue that whenever $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is decreasing there exist $\varepsilon > 0$ and $\delta > 0$ such that the proposed deviation is feasible and leads to a higher revenue. The increase in revenue and obedience slack are

$$(1-\alpha) \int_{\mu_0}^{\mu_0+\varepsilon} [1 - \mu + \frac{1-F(\mu)}{f(\mu)}] dF(\mu) - \delta\alpha \int_{\mu_1}^{\overline{\mu}} [1 - \mu + \frac{1-F(\mu)}{f(\mu)}] dF(\mu)$$

and

$$-(1-\alpha) \int_{\mu_0}^{\mu_0+\varepsilon} \frac{(1-\mu)p}{1-p} dF(\mu) + \delta\alpha \int_{\mu_1}^{\overline{\mu}} \frac{(1-\mu)p}{1-p} dF(\mu),$$

respectively. For a fixed $\varepsilon > 0$ there exist a $\delta > 0$ such that both are positive if and only if

$$\frac{\int_{\mu_0}^{\mu_0+\varepsilon} \frac{1-F(\mu)}{f(\mu)} dF(\mu)}{\int_{\mu_1}^{\overline{\mu}} \frac{1-F(\mu)}{f(\mu)} dF(\mu)} > \frac{\int_{\mu_0}^{\mu_0+\varepsilon} 1 - \mu \, dF(\mu)}{\int_{\mu_1}^{\overline{\mu}} 1 - \mu \, dF(\mu)}$$

which can be shown by a similar derivation as above to be true for small enough $\varepsilon > 0$. We have established item 1 that $(q_1, q_0) \in \text{ex}(\Lambda)$.

To prove item 2, consider some $(q_1, q_0) \in \text{ex}(\Lambda)$ and $0 \leq \mu_0 < \overline{\mu} < 1$ such that

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \overline{\mu}) \\ (1,0) & \mu \in [\overline{\mu}, 1] \end{cases}$$

We will show that there exist tests (q_1', q_0') and (q_1'', q_0'') such that a mixture of these is feasible and more profitable than $(q_1(\mu), q_0(\mu))$. To this end for some $\varepsilon > 0$ define

$$(q_1'(\mu), q_0'(\mu)) := \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \overline{\mu} - \varepsilon) \\ (1,0) & \mu \in [\overline{\mu} - \varepsilon, 1] \end{cases}$$

$$(q_1''(\mu), q_0''(\mu)) := \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1,1) & \mu \in [\mu_0, \overline{\mu} + \varepsilon) \\ (1,0) & \mu \in [\overline{\mu} + \varepsilon, 1] \end{cases}$$

Now define a "Revenue" and an "Obedience" function on $[\mu_0, 1]$;

$$R(\mu) := \int_{\mu_0}^{\mu} dF(\widetilde{\mu}) + \int_{\mu}^{1} [\widetilde{\mu} - \frac{1 - F(\widetilde{\mu})}{f(\widetilde{\mu})}] \, dF(\widetilde{\mu})$$

$$Q(\mu) := \int_{\mu_0}^{\mu} \frac{\widetilde{\mu} - p}{1 - p} dF(\widetilde{\mu}) + \int_{\mu}^{1} \widetilde{\mu} \, dF(\widetilde{\mu})$$

Note that R and $-Q$ are differentiable and increasing. Let

$$\alpha' := \inf\{\alpha \in [0,1] | \alpha R(\overline{\mu} - \varepsilon) + (1 - \alpha) R(\overline{\mu} + \varepsilon) = R(\overline{\mu})\}$$

$$\alpha'' := \inf\{\alpha \in [0,1] | \alpha Q(\overline{\mu} - \varepsilon) + (1 - \alpha) Q(\overline{\mu} + \varepsilon) = Q(\overline{\mu})\}$$

Also define the corresponding types $\mu' := \overline{\mu} + (1 - 2\alpha')\varepsilon$ and $\mu'' := \overline{\mu} + (1 - 2\alpha'')\varepsilon$. In particular, we get that

$$\alpha' = \frac{R(\overline{\mu} + \varepsilon) - R(\overline{\mu})}{R(\overline{\mu} + \varepsilon) - R(\overline{\mu} - \varepsilon)} \quad \text{and} \quad \alpha'' = \frac{Q(\overline{\mu}) - Q(\overline{\mu} + \varepsilon)}{Q(\overline{\mu} - \varepsilon) - Q(\overline{\mu} + \varepsilon)}$$

The proof follows by showing that $\mu' < \mu''$, for which it is sufficient to show $\alpha' > \alpha''$.

$$\frac{R(\overline{\mu} + \varepsilon) - R(\overline{\mu})}{R(\overline{\mu} + \varepsilon) - R(\overline{\mu} - \varepsilon)} > \frac{Q(\overline{\mu}) - Q(\overline{\mu} + \varepsilon)}{Q(\overline{\mu} - \varepsilon) - Q(\overline{\mu} + \varepsilon)}$$

$$\iff \frac{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}[1-\mu+\frac{1-F(\mu)}{f(\mu)}]dF(\mu)}{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}+\varepsilon}[1-\mu+\frac{1-F(\mu)}{f(\mu)}]dF(\mu)} > \frac{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)}{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)}$$

$$\iff \frac{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}\frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}+\varepsilon}\frac{1-F(\mu)}{f(\mu)}dF(\mu)} > \frac{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)}{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)}$$

$$\iff \frac{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}}\frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}\frac{1-F(\mu)}{f(\mu)}dF(\mu)} < \frac{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}}(1-\mu)dF(\mu)}{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)}$$

Whenever $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non-decreasing the required inequality holds as

$$\frac{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}\frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon}(1-\mu)dF(\mu)} \geq \frac{1-F(\overline{\mu})}{(1-\overline{\mu})f(\overline{\mu})} \geq \frac{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}}(\frac{1-F(\mu)}{f(\mu)})dF(\mu)}{\int_{\overline{\mu}-\varepsilon}^{\overline{\mu}}(1-\mu)dF(\mu)}$$

Recall that any test can be written as a convex combination of at most two extreme points. The above argument shows that if $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non-decreasing, then the optimal solution is represented by a convex combination of extreme points for which either $\overline{\mu} = 1$ or $\overline{\mu} = \mu_0$. Using Lemma 1, the optimal test can be expressed in the following form:

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu \in [0, \mu_0) \\ (1-\alpha, 0) & \mu \in [\mu_0, \overline{\mu}) \\ (1, \alpha) & \mu \in [\overline{\mu}, 1] \end{cases}$$

□

B Digital Platform

B.1 Relaxed Problem

Recall that $K(\mu) = (1-p)q_1(\mu) + pq_0(\mu)$ is the slope of the indirect utility. We may substitute into the objective in Equation 11 to obtain

$$\Pi = \int_0^1 \mu(1-p)q_1(\mu)dF(\mu) - \int_0^1 p\,(1-\mu)\,q_0(\mu)dF(\mu) \\ - \int_0^1 K(\mu)\left(\frac{1-F(\mu)}{f(\mu)}\right)dF(\mu)$$

For any incentive-compatible and obedient mechanism q there is a corresponding monotone slope function $K(\mu)$ taking values in $[0,1]$ and a target *platform mean* m defined by

$$\int_0^1 \mu q_1(\mu)dF(\mu) = \frac{m}{1-m}\int_0^1 (1-\mu)q_0(\mu)dF(\mu)$$

or

$$\int_0^1 \left[1-p-\mu\left(1-\frac{p}{m}\right)\right]q_1(\mu)dF(\mu) = \int_0^1 (1-\mu)K(\mu)dF(\mu) \tag{16}$$

Substituting again into the objective and re-arranging we arrive at:

$$\Pi = \int_0^1 \mu\left(1-\frac{p}{m}\right)q_1(\mu)dF(\mu) - \int_0^1 \frac{1-F(\mu)}{f(\mu)}K(\mu)dF(\mu) \tag{17}$$

We have organized the obective function in a way that isolates the gains from increasing $q_1(\mu)$ from the costs of increasing the slope $K(\mu)$. In the background we can adjust $q_0(\mu)$ to maintain a given slope $K(\mu)$ and target mean m provided $q_1(\mu) \in [0,1]$ satisfies

$$\frac{K(\mu)-p}{1-p} \leq q_1(\mu) \leq \frac{K(\mu)}{1-p}. \tag{18}$$

The first inequality ensures $q_0(\mu) \geq 0$ and the second ensures $q_0(\mu) \leq 1$. We can also express the participation constraint Equation 4 in terms of $K(\mu)$ as follows

$$q_1(\mu) \geq \frac{(1-\mu)K(\mu)}{1-p}. \tag{19}$$

We will say that a slope function $K(\mu)$ is *feasible* with respect to a target mean m if there exists an allocation $q_1(\mu)$ which satisfies Equation 16, Equation 18, and Equation 19.

B.2 Optimal Allocation for Fixed K

We can approach the problem by first finding the optimal q_1 for a given target mean m and feasible $K(\mu)$, and then optimizing the latter.

Define the following class of bang-bang allocations.

$$q_1^K(\mu) := \begin{cases} \max\left\{\frac{(1-\mu)K(\mu)}{1-p}, \frac{K(\mu)-p}{1-p}\right\} & \mu < \mu^K \\ \min\left\{1, \frac{K(\mu)}{1-p}\right\} & \mu \geq \mu^K \end{cases}$$

For some $\mu^K \in [0,1]$.

Lemma 2. *For given m and feasible $K(\mu)$, the optimal allocation is $q_1 = q_1^K$ for some μ^K.*

Proof. Consider any q_1 for which there is an interval of types, $[\mu_0, \overline{\mu}]$ such that $\max\left\{\frac{(1-\mu)K(\mu)}{1-p}, \frac{K(\mu)-p}{1-p}\right\} < q_1(\mu) < \min\left\{1, \frac{K(\mu)}{1-p}\right\}$. Pick $\delta > 0$ such that $\overline{\mu} - \mu_0 \geq 2\delta$. We can improve the revenue by increasing q_1 on $[\overline{\mu} - \delta, \overline{\mu})$ by some $\varepsilon_1 > 0$ and by reducing q_1 on $[\mu_0, \mu_0 + \delta)$ by $\varepsilon_0 > 0$ where

$$\varepsilon_1 = \varepsilon_0 \frac{\int_{\mu_0}^{\mu_0+\delta} \left(1 - p - \mu\left(1 - \frac{p}{m}\right)\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \left(1 - p - \mu\left(1 - \frac{p}{m}\right)\right) dF(\mu)}$$

ensuring that the target mean, i.e. Equation 16 is maintained. This adjustment doesn't violate the constraint in Equation 10 as it weakly decreases the size of any point mass at p without changing the target mean.

The revenue from the new allocation is greater than the old allocation if the following holds:

$$\frac{\int_{\mu_0}^{\mu_0+\delta} \left(1 - p - \mu\left(1 - \frac{p}{m}\right)\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \left(1 - p - \mu\left(1 - \frac{p}{m}\right)\right) dF(\mu)} \geq \frac{\int_{\mu_0}^{\mu_0+\delta} \mu\left(1 - \frac{p}{m}\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \mu\left(1 - \frac{p}{m}\right) dF(\mu)}$$

$$\iff \frac{\int_{\mu_0}^{\mu_0+\delta} (1-p)\, dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} (1-p)\, dF(\mu)} \geq \frac{\int_{\mu_0}^{\mu_0+\delta} \mu\left(1 - \frac{p}{m}\right) dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \mu\left(1 - \frac{p}{m}\right) dF(\mu)}$$

$$\iff \frac{\int_{\mu_0}^{\mu_0+\delta} dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} dF(\mu)} \geq \frac{\int_{\mu_0}^{\mu_0+\delta} \mu dF(\mu)}{\int_{\overline{\mu}-\delta}^{\overline{\mu}} \mu dF(\mu)}$$

The last inequality follows from our choice of δ.

Now consider q_1 such that there are intervals $I_1 < I_2$ where $q_1(\mu) = \max\left\{\frac{(1-\mu)K(\mu)}{1-p}, \frac{K(\mu)-p}{1-p}\right\}$ for $\mu \in I_2$ and $q_1(\mu) = \min\left\{1, \frac{K(\mu)}{1-p}\right\}$ for $\mu \in I_1$. We construct an improvement similar to the above by slightly increasing q_1 on I_2 while reducing q_1 on I_1 to keep the mean constraint binding. Finally, note that this improvement introduces more slack to the balanced constraint. The revenue of this improvement is greater than the old allocation if the following holds:

$$\frac{\int_{I_1} dF(\mu)}{\int_{I_2} dF(\mu)} \geq \frac{\int_{I_1} \mu dF(\mu)}{\int_{I_2} \mu dF(\mu)}$$

This is implied by $\frac{\inf(I_2)}{\sup(I_1)} \geq 1$. □

B.3 Optimal Choice of K

Notice that the monotonicity of $K(\mu)$ implies that there exists μ_0 such that $\frac{(1-\mu)K(\mu)}{1-p} \geq \frac{K(\mu)-p}{1-p}$ for all $\mu \leq \mu_0$ and $\frac{(1-\mu)K(\mu)}{1-p} < \frac{K(\mu)-p}{1-p}$ for all $\mu > \mu_0$. In particular the participation constraint in Equation 19 binds only for types $\mu \leq \mu_0$ and all mass x at the point p comes from these types. The next lemma states that $x = 0$ for a solution to the relaxed problem.

Lemma 3. *The solution to the relaxed problem has $K(\mu) = 0$ for all $\mu < \mu_0$.*

Proof. Consider a feasible allocation with target mean m and mass size x. Suppose there is a non-empty interval $[\underline{\mu}, \mu_0)$ consiting of types μ for which $q_1(\mu) = \frac{(1-\mu)K(\mu)}{1-p} > 0$. Then $x > 0$. Consider a new allocation which reduces $K(\mu)$ slightly at all points in $[\underline{\mu}, \mu_0)$ while keeping q_1 unchanged. The new allocation increases the mean slightly (by Equation 9) to say $m' > m$ and eliminates any mass point (Equation 4 will be slack). For m' close enough to m the new allocation is feasible for the relaxed problem with target mean m' and mass $x = 0$. That is, Equation 9 and Equation 10 are satisfied for m' and $x = 0$ because $\mathbf{E}H_0^p > \mathbf{E}H_x^p$.

By inspection of the objective in Equation 17 this is an improvement. Therefore a solution to the relaxed problem must have $K(\mu) = 0$ for all $\mu < \mu_0$. □

Lemma 2 and Lemma 3 imply that a solution to the relaxed problem can be described by μ_0 and two further thresholds $\mu_0 \leq \mu_1 \leq \overline{\mu}$ such that

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < \mu_0 \\ \left(\frac{K(\mu)-p}{1-p}, 1\right) & \mu \in [\mu_0, \mu_1) \\ \left(\frac{K(\mu)}{1-p}, 0\right) & \mu \in [\mu_1, \overline{\mu}) \\ \left(1, \frac{1}{p}\left(K(\mu) - (1-p)\right)\right) & \mu > \overline{\mu} \end{cases} \tag{20}$$

Lemma 4. *A solution to the relaxed problem has a threshold $\mu' \geq \overline{\mu}$ such that*

1. $K(\mu) = 1$ *for all* $\mu \geq \mu'$.
2. $q_0(\mu)$ *is constant on* $[\mu_0, \mu')$ *and equal to* 0 *or* 1.

Proof. We first show that any candidate solution can be weakly improved by one that has $q_0(\mu) \in \{0,1\}$ for all $\mu \in [\overline{\mu}, 1]$. If the candidate allocation does not already have that property then we construct a new allocation q' such that on $q'(\mu) = q(\mu)$ for $\mu \in [\overline{\mu}, 1]^C$. On the interval $[\overline{\mu}, \overline{\mu} + \varepsilon_0)$ let $q_0'(\mu) = q_0(\overline{\mu})$, on the interval $[1-\varepsilon_1, 1]$ let $q_0'(\mu) = 1$, and $q_0'(\mu) = q_0(\mu)$ for $\mu \in [\overline{\mu} + \varepsilon_0, 1 - \varepsilon_1)$. Where

$$\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon_0} (1-\mu)(q_0(\mu) - q_0(\overline{\mu}))dF(\mu) - \int_{1-\varepsilon_1}^{1} (1-\mu)(1-q_0(\mu))dF(\mu) = 0$$

As long as $\overline{\mu} + \varepsilon_0 \leq 1 - \varepsilon_1$, the allocation q' is well-defined and has the same target mean as q. Consider the difference in revenue between q and q':

$$\int_{\overline{\mu}}^{\overline{\mu}+\varepsilon_0} \left(1 - \mu + \frac{1-F(\mu)}{f(\mu)}\right)(q_0(\mu) - q_0(\overline{\mu}))dF(\mu) - \int_{1-\varepsilon_1}^{1} \left(1 - \mu + \frac{1-F(\mu)}{f(\mu)}\right)(1-q_0(\mu))dF(\mu)$$

$$= \int_{\overline{\mu}}^{\overline{\mu}+\varepsilon_0} \frac{1-F(\mu)}{f(\mu)}(q_0(\mu) - q_0(\overline{\mu}))dF(\mu) - \int_{1-\varepsilon_1}^{1} \frac{1-F(\mu)}{f(\mu)}(1-q_0(\mu))dF(\mu)$$

When $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non-increasing the change in revenue is positive as

$$\int_{1-\varepsilon_1}^{1} \frac{1-F(\mu)}{f(\mu)}(1-q_0(\mu))dF(\mu) \leq \frac{1-F(1-\varepsilon_1)}{\varepsilon_1 f(1-\varepsilon_1)} \int_{1-\varepsilon_1}^{1} (1-\mu)(1-q_0(\mu))dF(\mu)$$

$$\implies \int_{1-\varepsilon_1}^{1} \frac{1-F(\mu)}{f(\mu)}(1-q_0(\mu))dF(\mu) \leq \frac{1-F(1-\varepsilon_1)}{\varepsilon_1 f(1-\varepsilon_1)} \int_{\overline{\mu}}^{\overline{\mu}+\varepsilon_0} (1-\mu)(q_0(\mu) - q_0(\overline{\mu}))dF(\mu)$$

$$\implies \int_{1-\varepsilon_1}^{1} \frac{1-F(\mu)}{f(\mu)}(1-q_0(\mu))dF(\mu) \leq \int_{\overline{\mu}}^{\overline{\mu}+\varepsilon_0} \frac{1-F(\mu)}{f(\mu)}(q_0(\mu)-q_0(\overline{\mu}))dF(\mu)$$

The first implication follows from the choice of $\varepsilon_0, \varepsilon_1$. Thus any candidate allocation can be weakly improved by one for which there exists $\mu' \in [\overline{\mu}, 1]$ such that $q_0(\mu) = 0$ on $[\overline{\mu}, \mu')$ and $q_0(\mu) = 1$ on $[\mu', 1]$. Note that the latter implies item 1 in the statement of the Lemma in view of Equation 20.

In particular, by Equation 20, the improved allocation has $q_0(\mu) = 1$ for $\mu \in [\mu_0, \mu_1)$ and $q_0(\mu) = 0$ for $\mu \in [\mu_1, \mu')$. Next we claim that any such allocation for which μ_1 is strictly between μ_0 and μ' can be improved by one which satisfies item 2 in the statement of the Lemma. In other words, one for which μ_1 equals either μ_0 or μ'.

Assume for contradiction that the q is such that $\mu_0 < \mu_1 < \mu'$. We construct a new allocation q' such that on $q'(\mu) = q(\mu)$ for $\mu \in [\mu_0, \mu')^C$. On the interval $[\mu_0, \mu_1)$ we require $q_0'(\mu) = q_0(\mu) - \varepsilon_0$, on the interval $[\mu_1, \mu')$ we require $q_0'(\mu) = q_0(\mu) + \varepsilon_1$. Where

$$\varepsilon_1 = \varepsilon_0 \frac{\int_{\mu_0}^{\mu_1}(1-\mu)dF(\mu)}{\int_{\mu_1}^{\mu'}(1-\mu)dF(\mu)}$$

For small enough $\varepsilon_0 > 0$, the allocation q' is feasible with the same target mean as q. We argue that under the condition of the proposition, this q' achieves a weakly higher revenue. To see this, consider the difference in revenue between q and q':

$$\varepsilon_0 \int_{\mu_0}^{\mu_1} \left[1-\mu+\frac{1-F(\mu)}{f(\mu)}\right] dF(\mu) - \varepsilon_1 \int_{\mu_1}^{\mu'} \left[1-\mu+\frac{1-F(\mu)}{f(\mu)}\right] dF(\mu)$$

Similar to Proposition Proposition 1, when $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non-increasing the change in revenue is positive as

$$\frac{\int_{\mu_0}^{\mu_1} \frac{1-F(\mu)}{f(\mu)}dF(\mu)}{\int_{\mu_1}^{\mu'} \frac{1-F(\mu)}{f(\mu)}dF(\mu)} > \frac{\int_{\mu_0}^{\mu_1}(1-\mu)dF(\mu)}{\int_{\mu_1}^{\mu'}(1-\mu)dF(\mu)}.$$

□

B.4 Proof of Proposition 2

Proof of Proposition 2. It follows from Lemma 3, Lemma 4 and Equation 20 that a solution to the relaxed problem takes one of the following two forms.

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < \mu_0 \\ \left(\frac{K(\mu)-p}{1-p}, 1\right) & \mu \in [\mu_0, \mu') \\ (1,1) & \mu > \mu' \end{cases} \tag{21}$$

or

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < \mu_0 \\ \left(\frac{K(\mu)}{1-p}, 0\right) & \mu \in [\mu_0, \overline{\mu}) \\ (1,1) & \mu > \overline{\mu} \end{cases} \tag{22}$$

We complete the proof of Proposition 2 by identifying the slope in the middle regions. Consider the first case and let $K(\mu)$ be a feasible slope. By definition of μ_0 we have $K(\mu_0) - p \geq (1-\mu_0)K(\mu_0)$ implying $K(\mu_0) \geq \frac{p}{\mu_0}$. Therefore by monotonicity of K, for all $\mu \geq \mu_0$ we have $1 \geq K(\mu) \geq \frac{p}{\mu_0}$. We will show in fact that a solution to the relaxed problem in this case has $K(\mu) \in \left\{\frac{p}{\mu_0}, 1\right\}$ for all $\mu \in [\mu_0, \mu')$.

If $K(\mu)$ fails this property then we construct a new allocation that is determined by Equation 21 for slope function $\hat{K}$, where $\hat{K}(\mu) = K(\mu)$ for all $\mu \notin [\mu_0, \mu')$. Let $\hat{K}(\mu) = \frac{p}{\mu_0}$ for $\mu \in [\mu_0, \mu_0 + \varepsilon_0)$ and $\hat{K}(\mu) = 1$ for $\mu \in [\mu' - \varepsilon_1, \mu')$ and $\hat{K} = K$ otherwise. Choose $\varepsilon_0, \varepsilon_1 > 0$ such that $\mu_0 + \varepsilon_0 < \mu' - \varepsilon_1$ and the following holds

$$\int_{\mu'-\varepsilon_1}^{\mu'} \mu\,(1 - K(\mu))\, dF(\mu) - \int_{\mu_0}^{\mu_0+\varepsilon_0} \mu \left(K(\mu) - \frac{p}{\mu_0}\right) dF(\mu) = 0 \tag{23}$$

By construction the allocation corresponding to $\hat{K}$ is feasible as it has the same mean as the one corresponding to K, this follows from Equation 23 and Equation 16 and the fact that $q_1(\mu) = \frac{K(\mu)}{1-p} + \textit{constant}$ over $[\mu_0, \mu')$ (by Equation 21). By assumption and monotonicity, there must also be values of ε_0, ε_1 such that $\hat{K} \neq K$.

The change in revenue is given by

$$\int_{\mu'-\varepsilon_1}^{\mu'} \left[\frac{\mu(m-p)}{(1-p)m} - \frac{1-F(\mu)}{f(\mu)}\right](1-K(\mu))\,dF(\mu) - \int_{\mu_0}^{\mu_0+\varepsilon_0} \left[\frac{\mu(m-p)}{(1-p)m} - \frac{1-F(\mu)}{f(\mu)}\right]\left(K(\mu)-\frac{p}{\mu_0}\right)dF(\mu)$$

which by Equation 23 equals

$$= -\int_{\mu'-\varepsilon_1}^{\mu'} \frac{1-F(\mu)}{f(\mu)}(1-K(\mu))\,dF(\mu) + \int_{\mu_0}^{\mu_0+\varepsilon_0} \frac{1-F(\mu)}{f(\mu)}\left(K(\mu)-\frac{p}{\mu_0}\right)dF(\mu)$$

Note that $\frac{1-F(\mu)}{(1-\mu)f(\mu)}$ is non increasing, implies that $\frac{1-F(\mu)}{f(\mu)}$ is non increasing, thus the change in revenue is positive if

$$\frac{1-F(\mu_0+\varepsilon_0)}{\mu_0+\varepsilon_0}\left[\int_{\mu_0}^{\mu_0+\varepsilon_0}\left(K(\mu)-\frac{p}{\mu_0}\right)dF(\mu) - \int_{\mu'-\varepsilon_1}^{\mu'}(1-K(\mu))dF(\mu)\right] \geq 0$$

The inequality follows from the fact that $\frac{1-F(\mu_0+\varepsilon_0)}{\mu_0+\varepsilon_0}$ is positive and by Equation 23 which implies

$$\int_{\mu'-\varepsilon_1}^{\mu'}(1-K(\mu))dF(\mu) \leq \frac{\mu_0+\varepsilon_0}{\mu'-\varepsilon_1}\int_{\mu_0}^{\mu_0+\varepsilon_0}\left(K(\mu)-\frac{p}{\mu_0}\right)dF(\mu).$$

We conclude that a solution to the relaxed problem in the first case has $K(\mu) \in \left\{\frac{p}{\mu_0}, 1\right\}$ for all $\mu \in [\mu_0, \mu')$. We can now conclude the proof of the first case in the Proposition. It suffices to note that $K(\mu) = 0$ implies $q_1(\mu) = q_0(\mu) = 0$ and $K(\mu) = 1$ implies $q_1(\mu) = q_0(\mu) = 1$. Then for the interval $\mu \in [\mu_0, \mu')$ since $K(\mu) = \frac{p}{\mu_0}$ and $q(\mu) = \left(\frac{K(\mu)-p}{1-p}, 1\right)$ (by Equation 21) we obtain $q_1(\mu) = \frac{p}{1-p}\frac{1-\mu_0}{\mu_0}$.

The second case, Equation 22, is treated following similar lines. By definition of $\overline{\mu}$, we have $0 \leq K(\mu) \leq 1-p$ for $\mu \in [\mu_0, \overline{\mu})$. By arguments analogous to the first case we can show that a solution to the relaxed problem has $K(\mu) \in \{0, 1-p\}$ for $\mu \in [\mu_0, \overline{\mu})$. To conclude the proof note that for the interval $\mu \in [\mu_0, \overline{\mu})$ since $K(\mu) = 1-p$ and $q(\mu) = \left(\frac{K(\mu)}{1-p}, 0\right)$ (by Equation 22) we obtain $q_1(\mu) = 1$. □

B.5 Uniform Prior

So far, we have characterized the qualitative structure of the optimal relaxed mechanism under some distributional constraints. In this section, we demonstrate the power of our characterization by deriving the optimal primal mechanism for a uniform prior. Consider $F \sim \text{Unif}[0,1]$, this satisfies all conditions of Proposition 2, thus the optimal solution to the relaxed problem can be expressed as one of the mechanisms identified in the proposition. First, we look at the mechanism of the form

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < \mu_0 \\ (1,0) & \mu \in [\mu_0, \overline{\mu}) \\ (1,1) & \mu > \overline{\mu} \end{cases}$$

The mean constraint binding with no point mass at p implies that the relaxed problem can be written as

$$\max_{\mu_0, \overline{\mu}} \int_{\mu_0}^{1} (1-p)(2\mu - 1) d\mu - 2p \int_{\overline{\mu}}^{1} (1-\mu) d\mu$$

s.t.

$$0 \leq \mu_0 \leq \overline{\mu} \leq 1$$

$$\int_{\mu_0}^{1} \mu d\mu = \frac{\mathbf{E}H_0^p}{1 - \mathbf{E}H_0^p} \int_{\overline{\mu}}^{1} (1-\mu) d\mu$$

The objective can be rewritten as

$$\int_{\mu_0}^{1} (1-p)(2\mu - 1) d\mu - 2p \frac{1 - \mathbf{E}H_0^p}{\mathbf{E}H_0^p} \int_{\mu_0}^{1} \mu d\mu$$

Note that $\mu_0 = \mu^*(p) := \frac{1-p}{2\left(1 - \frac{p}{\mathbf{E}H_0^p}\right)}$ is the pointwise maximize of the objective. Let

$p_1 := \text{argmax}\left\{ p \in [0,1] \,\middle|\, \mu^*(p) \geq t(\mu^*(p)) \right\}$, where $t(\mu^*) = 1 - \sqrt{\frac{\left(1 - \mathbf{E}H_0^p\right)}{\mathbf{E}H_0^p} (1 - (\mu^*)^2)}$.

For $p \geq p_1$, the point-wise optimal mechanism is feasible for the relaxed problem. Moreover, the revenue is given by

$$\int_{\mu^*(p)}^{1} \left[2\mu \left(1 - \frac{p}{\mathbf{E}H_0^p} \right) - (1-p) \right] d\mu$$

The above is decreasing in p, the derivative of the revenue is

$$\int_{\mu^*(p)}^{1} 1 - \frac{2\mu}{p}\frac{1}{(1-\ln(p))^2}d\mu$$

This is negative as $\frac{1-3\ln(p)-p(1-\ln(p))}{-2p\ln(p)(1-\ln(p))^2} > 1$ for all $p \in [0,1]$.

For $p < p_1$, the point-wise optimal mechanism doesn't satisfy the interim participation constraint. The interim participation fails for $p < p_1$ as $\mu_1 < \mu^*(p)$. Note that the revenue decreases if $q_1(\mu)$ increases for $\mu < \mu^*(p)$. Thus, if $p < p_1$ then the optimal solution to the relaxed problem, restricted to mechanisms above, involves thresholds $\mu_0 = \overline{\mu}$. By the mean constraint, the threshold value μ_0 is a root of the following quadratic equation

$$(1-\overline{\mu})^2 = \frac{1-\mathbf{E}H_0^p}{\mathbf{E}H_0^p}(1-\overline{\mu}^2)$$

This has a single interior solution,$\mu_0 = \overline{\mu} = 2\mathbf{E}H_0^p - 1$, the revenue is thus given by

$$\int_{2\mathbf{E}H_0^p-1}^{1}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right)-(1-p)\right]d\mu$$

The above is negative if $p_1 > p \geq p_0 := \operatorname{argmax}\left\{p \in [0,1] \;\middle|\; \frac{1+p}{2} \geq p(1-\ln p)\right\}$, thus the optimal thresholds are such that is if $p \in [p_0, p_1]$ then $\mu_0 = \overline{\mu} = 2\mathbf{E}H_0^p - 1$ and if $p < p_0$ then $\mu_0 = 1$. Thus, to find the optimal mechanism in the first class of mechanisms from the Proposition 2, we will maximize the following objective with respect to $p \in [p_0, p_1]$.

$$R(p) := \int_{2\mathbf{E}H_0^p-1}^{1}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right)-(1-p)\right]d\mu = 2\left[1-\mathbf{E}H_0^p\right]\left[2\mathbf{E}H_0^p-(1+p)\right]$$

This function is concave in the domain and achieves an interior maximum. Unfortunately, the first order condition is a transcendental equation, so we rely on numerical methods to calculate the optimum. The optimal revenue for the relaxed problem among this class of mechanisms is $\simeq 0.0646$, which is achieved by $p \simeq 0.4364$, and $\mu_0 = \overline{\mu} \simeq 0.596$.

Now we will describe the optimal mechanism for the second class of mechanisms identified in Proposition 2;

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < \mu_0 \\ \left(\frac{p(1-\mu_0)}{(1-p)\mu_0}, 1\right) & \mu \in [\mu_0, \overline{\mu}) \\ (1,1) & \mu > \overline{\mu} \end{cases}$$

We only need to optimize over these mechanisms for price $p < p_1$, as for other prices the mechanism discussed above maximizes the revenue point-wise. We solve the following problem

$$\max_{\mu_0, \overline{\mu}} \int_{\mu_0}^{\overline{\mu}} (1-p)\frac{p(1-\mu_0)}{(1-p)\mu_0}(2\mu - 1)d\mu + \int_{\overline{\mu}}^{1}(1-p)(2\mu-1)d\mu - 2p\int_{\mu_0}^{1}(1-\mu)d\mu$$

s.t.

$$p \leq \mu_0 \leq \overline{\mu} \leq 1$$

$$\int_{\mu_0}^{\overline{\mu}} \mu \frac{p(1-\mu_0)}{(1-p)\mu_0} d\mu + \int_{\mu_0}^{1} \mu d\mu = \frac{\mathbf{E}H_0^p}{1-\mathbf{E}H_0^p}\int_{\mu_0}^{1}(1-\mu)d\mu$$

The objective can be rewritten as

$$\int_{\mu_0}^{\overline{\mu}} \frac{p(1-\mu_0)}{(1-p)\mu_0}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right) - (1-p)\right]d\mu + \int_{\overline{\mu}}^{1}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right) - (1-p)\right]d\mu$$

From the obedience we can express $\overline{\mu} = h(\mu_0) := \sqrt{\mu_0^2 + \frac{(1-p)\mu_0(1-\mu_0)}{(\mu_0-p)(1-\mathbf{E}H_0^p)}(1+\mu_0 - 2\mathbf{E}H_0^p)}$

For feasibility we require $p \leq \mu_0 \leq \overline{\mu}$, thus feasibility can be restated as $\max\{p, 2\mathbf{E}H_0^p - 1\} \leq \mu_0$. The optimization problem can be restated

$$\int_{\mu_0}^{h(\mu_0)} \frac{p(1-\mu_0)}{(1-p)\mu_0}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right) - (1-p)\right]d\mu + \int_{h(\mu_0)}^{1}\left[2\mu\left(1-\frac{p}{\mathbf{E}H_0^p}\right) - (1-p)\right]d\mu$$

s.t.

$$\max\{p, 2\mathbf{E}H_0^p - 1\} \leq \mu_0 \leq 1$$

This is a well-defined two-variable unconstrained optimization on a compact set. In particular, we can numerically derive the relevant features of the mechanism at the optimum. The optimal revenue for the relaxed problem among this class of mechanisms is $\simeq 0.0651$, which is achieved by $p \simeq 0.417$, $\mu_0 \simeq 0.578$, and $\overline{\mu} \simeq 0.629$. Thus optimal relaxed mechanism is given by

$$(q_1(\mu), q_0(\mu)) = \begin{cases} (0,0) & \mu < 0.578 \\ (0.522, 1) & \mu \in [0.578, 0.629) \\ (1,1) & \mu > 0.629 \end{cases}$$

The CDF for the receiver's second-order beliefs for the above mechanism is given by

$$\operatorname{marg}_{\Delta(\theta)} \beta(\mu \mid p = 0.417) = \begin{cases} 0 & \mu < 0.417 \\ w^{-1} \int_{0.578}^{\frac{\mu}{\mu + 0.522(1-\mu)}} (0.522s + (1-s))ds & \mu \in [0.417, 0.47) \\ w^{-1} \int_{0.578}^{0.629} (0.522s + (1-s))ds & \mu \in [0.47, 0.629) \\ \frac{\left[\int_{0.578}^{0.629} (0.522s + (1-s))ds + \int_{0.629}^{\mu} (s + (1-s))ds\right]}{w} & \mu \geq 0.629 \end{cases}$$

Where $w = \int_{0.578}^{0.629} 0.522s ds + \int_{0.629}^{1} ds + \int_{0.578}^{0.629} (1-s)\, ds$. As the mechanism solves the relaxed problem, $\operatorname{marg}_{\Delta(\theta)} \beta(\cdot \mid p = 0.417)$ has the same expectation as $H_0^{0.417}$. We claim that $H_0^{0.417} \succeq_{\text{mps}} \operatorname{marg}_{\Delta(\theta)} \beta(\cdot \mid p = 0.417)$, this follows from the fact that the distributions have the same support and that $\operatorname{marg}_{\Delta(\theta)} \beta(\cdot \mid p = 0.417)$ crosses $H_0^{0.417}$ exactly once from below.

B.6 Deterring Double Deviations

If the platform can leverage the risk-neutrality of the seller and enforce random prices with expected value p, then a scheme to deter double deviations is feasible. It works as follows. Consider any type μ which joins the platform with positive probability in the optimal mechanism. Let ν be the expected posterior obtained by μ conditional on joining the platform. We have that $\nu \geq p$. Now when any other type $\underline{\mu}$ misreports as μ, they will obtain a random posterior $\tilde{\nu}(\underline{\mu})$ with some expectation $\mathbf{E}\tilde{v}(\underline{\mu})$. Let $\underline{\mu} \leq \mu$ be the type such that the expected posterior that $\underline{\mu}$ would obtain from mis-reporting as μ equals p, i.e. $\mathbf{E}\tilde{v}(\underline{\mu}) = p$. The continuity and monotonicity of Bayesian updating guarantees that $\underline{\mu}$ exists and is unique.

When μ joins the platform require the firm to charge price equal to the realization of $\tilde{v}(\underline{\mu})$. Note that this price is in a one-to-one correspondence with the realized posterior of type μ as well as the posterior that would be obtained if any other type were to misreport μ. Note that this random price has expected value p so it leaves all payoffs, revenue, and incentive constraints unchanged.

Now consider double deviations. Consider any type μ' which misreports as μ and must then decide whether to accept the offered price. By the monotonicity of Bayesian updating if $\mu' \geq \mu$ the realized price is below $\tilde{v}(\mu')$ with probability 1. Therefore, types higher than μ find no double deviation profitable. On the other hand if $\mu' < \mu$ then after misreporting as μ, the type μ' finds every price realization strictly below $\tilde{v}(\mu')$, again by the strict monotonicity of Bayesian updating. Such a type therefore rejects every price offered and obtains a payoff from zero from the mis-report. Since the participation constraint already requires that the payoff from truth-telling is weakly greater than zero, these double deviations are never profitable.