Analysis of an Idealized System of Demand Adaptive Paired-Line Hybrid Transit

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Abstract

This paper proposes and analyzes a new transit system that integrates the traditional fixed-route service with a demand-adaptive service. The demand-adaptive service connects passengers from their origin/destination to the fixed-route service in order to improve accessibility. The proposed hybrid design is unique in that it operates the demand-adaptive service with a stable headway to cover all stops along a paired fixed-route line. Pairing demand-adaptive vehicles with a fixed-route line simplifies the complexity of on-demand routing, because the vehicles can follow a more predictable path and can be dispatched on intervals coordinated with the fixed-route line. The design of the two services are closely coupled to minimize the total system cost, which includes both the transit agency’s operating cost and the user cost. The optimal design model is formulated as a mixed integer program and solved using a commercially available metaheuristic. Numerical experiments are conducted to compare the demand adaptive paired-line hybrid transit (DAPL-HT) system with two related transit systems that may be considered its special cases: a fixed-route system and a flexible-route system. We show that the DAPL-HT system outperforms the other two systems under a wide range of demand levels and in various scenarios of input parameters. A discrete-event simulation model is also developed and applied to confirm the correctness of the analytical results.

Keywords: hybrid transit system; demand adaptive service; fixed-route service; discrete-event simulation

1 Introduction

The auto-dependent surface transportation system in the U.S. has not only created chronic traffic congestion that costs over $100 billions per year (Schrank et al., 2010), but also severely limited mobility options, especially for those who have no access to private autos. Such a system also exacerbates the environmental impacts of travel. In the U.S., transportation consumes 28.1% of all energy, of which 92.2% comes from petroleum use; it also contributes 34.7% of greenhouse gas (GHG) emissions (Davis et al., 2014). Promoting

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public transportation is widely regarded an important measure in the transition to more efficient, equitable and sustainable transportation (Vuchic, 2005).

Being a mode of collective transportation, transit tends to thrive in densely populated areas. In countries such as the U.S., unfortunately, suburb sprawl has made the condition progressively unfavorable to transit development since 1950s. In fact, the population density in the major urban areas has dropped more than 50% between 1950 and 1990 in the U.S. (Mieszkowski and Mills, 1993; Kahn, 2000). Not surprisingly, transit has been largely marginalized across the country. It is easy to see the dilemma faced by a traditional fixed-route transit service: on one hand, the potential ridership may not warrant the cost of providing high-quality coverage; on the other hand, the financially viable service is so inadequate that it attracts few non-captive riders. To reverse the trend of declining transit use thus requires a concerted effort of planning, police-making and engineering. Indispensable in this effort is to transform the way by which transit systems are designed and operated.

Demand-response transit (DRT) was an earlier response to the low-density conditions in the U.S.. Of various forms of DRT, dial-a-ride transit (DART) is perhaps the most widely studied and also the most relevant to this work. In a DART system, a passenger requests a service and is provided a pickup time window. Unlike taxi service, DART allows ride-sharing and even transferring (Wilson et al., 1976; Stein, 1978). The basic mathematical problem underlying DART is a many-to-many pickup and delivery problem with time windows, which may be further classified as single- and multi-vehicle problems depending on the number of vehicles employed, or static and dynamic problems depending on whether or not the passenger requests are processed in real time. The DART problem is a special class of vehicle routing problems (VRP) that are known to be NP-hard (Cordeau and Laporte, 2003a). Many attempts have been made to develop efficient DART solution methods in the past three decades (e.g. Psaraftis, 1980; Cordeau and Laporte, 2003b; Cordeau, 2006; Melachrinoudis et al., 2007; Ropke and Cordeau, 2009).

A major shortcoming of DRT services such as DART is the high per-capita operating cost resulted from low occupancy (Black, 1995). This observation has inspired proposals of hybrid systems that strive to marry the flexibility of DRT with the efficiency of traditional fixed-route services (FRS). Stein (1978) considers a system which divides the service area into zones connected by fixed-route bus services. Passengers use DART to get to designated “transfer points”, from where they travel to other zones on fixed-route buses. This system, which was implemented in Ann Arbor, MI (Potter, 1976), bears conceptual similarities with the proposed system in this paper in that each journey is divided into pick-up, line-haul, and drop-off phases. Malucelli et al. (1999) and Crainic et al. (2001) propose a demand adaptive system (DAS) that combines FRS with a restricted DART. The DART in DAS is restrictive in that vehicles have to follow a schedule at compulsory stops, which are designated transfer points. However, optional stops between consecutive compulsory stops are served only by request. The DART system in DAS has been studied independently as a Mobility Allowance Shuttle Transit (MAST) (Quadrifoglio et al., 2006, 2007, 2008) or flexible-route transit (Pratelli et al., 2001; Alshalalfah, 2009). High-coverage point-to-point transit system (HCPPT) proposed by Cortés and Jayakrishnan (2002) may be viewed as a variant of the zone-based system of Stein (1978). Instead of using line-haul buses to connect zones, HCPPT connects them with the same vehicles that make pickup and delivery trips. The motivation is that this design eliminates the out-bound transfer. However, using the same vehicle to collect demand and provide line-haul travel may also lead to the use of a large number of small vehicles and extended in-vehicle time at low speeds, especially for high demand levels.

In this paper, we propose to investigate a new hybrid transit system that integrates FRS with DAS. DAS in our system functions as a moving connector between FRS and passengers’ origins/destinations (O-D)
and can only be used by reservation (via, e.g., a smart phone application). It bears some similarities to the flexible line systems such as studied in Crainic et al. (2001); Quadrifoglio et al. (2006); Nourbakhsh and Ouyang (2012), but also has several distinctive features. First, instead of offering a door-to-door service, the proposed DAS is designed primarily to improve accessibility, and it only serves passengers whose access distance - i.e., the distance between their desired O-D and the closest transit stop - exceeds certain threshold (which itself is a design parameter that can vary with demand levels). Second, it is operated in parallel with paired fixed-route transit lines, potentially with a fleet of vehicles smaller than regular size buses. Last and most important, the design of DAS is tightly integrated with FRS to maximize system efficiency under given demand scenarios. Given the above features, the proposed system will be referred to as a demand adaptive paired-line hybrid transit system, or DAPL-HT.

Compared to the existing zone-based hybrid transit systems, DAPL-HT promises greater simplicity and flexibility. For one thing, pairing fixed-route and demand-adaptive lines simplifies the complexity of on-demand routing, mainly because demand-adaptive vehicles have a more predictable path and can be dispatched on intervals coordinated with the fixed-service lines with which they are paired. Moreover, transit operators can easily adjust the hybrid design to cope with varying demand conditions by reallocating the resources between fixed-route and demand-adaptive services. At one extreme, the operator may completely halt fixed-route services, and use demand-adaptive vehicles to provide the line-haul service when the demand level is very low. At the other extreme, when demand is very high, the operator could reduce the design accessibility - so that less passengers would be eligible for demand-adaptive services - in order to shift more resources to fixed-route services for greater efficiency.

The overwhelming majority of transit design studies consider the problem of choosing a system of fixed-routes and corresponding frequencies to serve a given demand pattern (Ceder and Wilson, 1986; Baaj and Mahmassani, 1991; Zhao and Zeng, 2008). These design models typically lead to nonlinear, non-convex and combinatorial optimization problems that are very difficult to solve (Newell, 1979). To bypass such intractability, another line of thinking focuses on strategic design issues using simpler analytical models (Holroyd, 1967; Byrne, 1975; Wirasinghe et al., 1977; Newell, 1979; Daganzo, 2010a). This approach is adopted here to study the design trade-off in DAPL-HT under idealized conditions. Specifically, the hybrid model will be built for a grid route structure and with similar simplifying assumptions used in Daganzo (2010b) and Nourbakhsh and Ouyang (2012). We note that similar design analysis of hybrid transit systems is relatively sparse in the literature. Two notable exceptions are Aldaihani et al. (2004) and Errico et al. (2011). The former considers the optimal design of a zone-based hybrid system in a grid network using a continuous approximation approach, whereas the latter proposes a unifying planning framework for DAS (Malucelli et al., 1999) that consists of strategic, tactical and operations decisions. As explained above, however, the proposed design has two unique and attractive features: (1) operating the demand-adaptive service in parallel with paired fixed-route transit lines (with a stable headway), which simplifies routing and enables the use of a fleet of smaller vehicles for demand-adaptive service (because passengers only use the service for access); and (2) introducing a “walking zone to allow the system to perform a delicate tradeoff between the access and other costs, which increases the design flexibility.

The rest of the paper is organized as follows. Section 2 presents the proposed DAPL-HT system and Section 3 gives the formulation of the optimal design problem. Section 4 briefly discusses two related transit systems that may be considered as special cases of DAPL-HT. Sections 5 and 5 presents results of numerical and simulation experiments, respectively. Section 7 concludes the paper with a summary of main findings and suggestion for future research.
2 Proposed hybrid transit system

Consider a square service area of side length $D$. The streets in the service area form a grid network with constant spacing $s$. The service area generates $\lambda$ passenger trips per hour per unit area. The trip origins and destinations are uniformly and independently distributed in the service area according to a homogeneous spatial Poisson process. For the ease of comparison, the notation in Nourbakhsh and Ouyang (2012) is adopted in the following analysis.

The proposed DAPL-HT system consists of both fixed-route and demand-adaptive lines. As depicted in Figure 1, fixed-route lines are operated in both North-South (N-S) direction and East-West (E-W) direction, whereas demand-adaptive lines are paired with the fixed-route lines in the N-S direction. Since the demand-adaptive lines in the N-S direction could cover the entire service area and all the transit stops, it is not necessary to operate the demand-adaptive lines in the E-W direction.

We define the \textit{access distance} as the distance between a passenger’s origin/destination to the nearest transit stop. The demand-adaptive lines are designed to serve, upon request, passengers whose access distance exceeds certain threshold. Let $N$ be the number of fixed-route lines running in either of the N-S or E-W direction. Accordingly, the line spacing can be determined as $S = D/N$. Since the demand-adaptive lines are paired with the fixed-route lines only in the N-S direction, there are $N$ demand-adaptive lines in total. The headway for the fixed-route and the flexible-route lines is denoted as $H_1$ and $H_2$, respectively. The service is provided by vehicles with the following characteristics: the cruising speed including stops due to traffic and pedestrian interference, $v$ (km/h); the time lost per stop due to the deceleration and the acceleration, $\tau_1$ (seconds/stop); the time added per boarding passenger, $\tau_1'$ (seconds/pax); and the additional pick-up and drop-off time required per passenger for the demand-adaptive service, $\tau_2$ (hour/pax). Passengers can walk at speed $v_w$ (km/h).

To simplify the analysis, the following assumptions are introduced to define how passengers plan their trips. Except those related to demand-adaptive services, most assumptions are adopted from Daganzo (2010b).

\textbf{Assumption 1.} \textit{Passengers always use the stops closest to their origin and destination. If the access distance is less than $\beta D/N$ (where $\beta \in (0, 1]$ is a design variable), passengers cannot use the demand-adaptive service and thus have to walk to the stop; otherwise, passengers always request the demand-adaptive service} \footnote{Note that if a traveler’s origin and destination are both outside the walking zone, then s/he will ride the demand-adaptive service twice in the journey.}

This assumption implies that by design the demand-adaptive service does not offer access trips (to/from a stop) deemed too short. Can passengers whose access distance is greater than the threshold find that walking is actually a cheaper alternative? Such a possibility cannot be ruled out. Yet, it is unlikely that many passengers would have this issue; if they do, the average system cost will be driven up because, as we shall see, the boundary of walking zone (defined by $\beta$) will be optimized to minimize that cost\footnote{For several reasons $\beta$ is treated as a decision variable, rather than a parameter determined from empirical data. First of all, data may not exist to determine a desired walking zone since currently few transit users have access to a hybrid system. Optimizing the boundary of the walking zone would allow the operator to estimate how the walking cost should be optimally accounted for in the overall system design. Second, the designation of a walking zone does not necessarily deny customers the demand-adaptive services and force them to walk. Rather, it merely means that those living inside the zone may have to pay a little extra to offset the system cost. In this sense, the walking zone may be viewed as a pricing tool for the demand-adaptive service. Third, the presented optimization model can be easily revised to allow $\beta$ to be treated as a parameter instead of a decision variable, when empirical data become available.}. In other words, if allowing those very passengers to walk can reduce the average system cost, $\beta$ should be...
such defined that the walking zone would contain them. Thus, the design logic here is to achieve a system optimum in the sense of average cost, rather than focusing on detailed individual choices, which may render the derivation of closed form cost functions intractable.

Assumption 2. Passengers send their request for the demand-adaptive service to a control center prior to the desired departure time from the origin. Their request will be processed in a first-come-first-serve basis.

Assumption 3. Passengers travel between fixed-route transit stops with the least possible number of transfers and as directly as possible, i.e., at most one transfer if the origin station and the destination station are not in the same line.

Assumption 4. When transfer is needed, passengers randomly choose the initial direction of travel.

The fixed-route vehicles travel along the designated lines and make pre-scheduled stops. The demand-adaptive vehicles, on the other hand, makes lateral movements to pick up and drop off passengers while sweeping longitudinally back and forth along the fixed-route line that they are paired with, as shown by the dashed trajectory in Figure 1. Operational characteristics of the demand-adaptive vehicles are summarized below.

- A demand-adaptive vehicle always follows the fixed route line when it is not diverted by pickup and/or drop-off requests.
- Passenger requests are assembled at a control center and dispatched to each vehicle in real-time. Prior to arriving at a stop $i$, each vehicle receives the requests for service for the area between stop $i$ and the next stop $i + 1$ (see the shaded area in Figure 1).
- The vehicle confirms the set of requests it can handle without spending more time to travel between $i$ and $i + 1$ than allowed by a pre-determined service protocol. Extra requests will be passed on to the next vehicle in the line.
- The vehicle makes and executes an optimal routing plan to answer all agreed requests, and it always returns to stop $i + 1$ to start a new service cycle afterwards. The exact trajectory of the vehicle depends on the requests and route planning.

3 Optimal design problem

The design problem aims to determine the optimal design parameters, namely, $N$, $H_1$, $H_2$ and $\beta$, such that the total cost of the proposed DAPL-HT system is minimized. The total system cost consists of two components: agency and user costs. A couple of analytical results must be established first before these costs can be estimated. For the reader’s convenience, Table 1 lists the important notations used in the paper.

We first examine the percentage of passengers who use the demand-adaptive service, $p_y$, which is given by

$$p_y = 1 - p_n,$$  \hspace{1cm} (1)

where $p_n$ denotes the percentage of passengers who choose walking to their preferred transit station. By Assumption 1, passengers will choose walking if the walking distance is less than $\beta D/N$ ($0 < \beta \leq 1$),
Figure 1: Overview of structured hybrid transit system
Table 1: Description of notations used in the paper

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(km)$</td>
<td>Side length of the grid network</td>
</tr>
<tr>
<td>$s(km)$</td>
<td>Distance between two adjacent streets (street spacing)</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of lines in N-S(E-W) direction</td>
</tr>
<tr>
<td>$\lambda(pax/h/km^2)$</td>
<td>Passenger trips generated per hour per unit area</td>
</tr>
<tr>
<td>$H_1(h)$</td>
<td>Headway of the fixed-route lines</td>
</tr>
<tr>
<td>$H_2(h)$</td>
<td>Headway of the demand adaptive lines</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Parameter that determines maximum walking distance</td>
</tr>
<tr>
<td>$\mu($)/h)$</td>
<td>Value of time</td>
</tr>
<tr>
<td>$\tau_1(s)$</td>
<td>Time lost per stop due to deceleration and acceleration</td>
</tr>
<tr>
<td>$\tau'_1(s)$</td>
<td>Time added per boarding passenger for fixed-route vehicles</td>
</tr>
<tr>
<td>$\tau_2(s)$</td>
<td>Additional pick-up and drop-off time required per passenger</td>
</tr>
<tr>
<td>$v(km/h)$</td>
<td>Vehicles’ cruising speed</td>
</tr>
<tr>
<td>$v_w(km/h)$</td>
<td>Walking speed</td>
</tr>
<tr>
<td>$\delta(km)$</td>
<td>Transfer penalty expressed in terms of the equivalent distance walked</td>
</tr>
<tr>
<td>$Q($)$/veh · km)</td>
<td>Operating cost per vehicle distance</td>
</tr>
<tr>
<td>$M($)$/veh · h)</td>
<td>Operating cost per vehicle hour</td>
</tr>
<tr>
<td>$Q(km/h)$</td>
<td>Expected total distance traveled per hour of operation</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of vehicles required</td>
</tr>
<tr>
<td>$A(h)$</td>
<td>Average total walking time per passenger</td>
</tr>
<tr>
<td>$W(h)$</td>
<td>Average total waiting time per passenger</td>
</tr>
<tr>
<td>$T(h)$</td>
<td>Average total in-vehicle travel time per passenger</td>
</tr>
<tr>
<td>$e_T$</td>
<td>Expected number of transfers required per passenger</td>
</tr>
</tbody>
</table>

![Diagram](a) $0 < \beta \leq 0.5$

![Diagram](b) $0.5 < \beta \leq 1$

Figure 2: Illustration of walking area around a transit station
where $\beta$ is a parameter and $D/N$ denotes the stop spacing. $p_n$ can be approximated by the portion of walking-feasible area around the transit station, see Figure 2 for illustration. In Figure 2, the transit station is centered at $O$, the area around the station is a square with length $D/N$. Since the proposed system operates in a grid network, the walking distance between a point $P(x,y)$ and the station is approximately the sum of the horizontal distance and the vertical distance, i.e., $|x| + |y|$. The shaded areas in Figure 2a and 2b represent the walking-feasible areas when $0 < \beta \leq 0.5$ and $0.5 < \beta \leq 1$, respectively. Thus, for $0 < \beta \leq 0.5$, we have

$$p_n = \frac{2\beta D^2}{N^2} = 2\beta^2,$$

(2)

and for $0.5 < \beta \leq 1$, we have

$$p_n = \frac{(D/N)^2 - 4 \times \frac{1}{2} \times \left(\frac{D}{N} - \beta \frac{D}{N}\right)^2}{(D/N)^2} = 1 - 2(1 - \beta)^2.$$

(3)

We now turn to another important measure: the expected number of transfers for an average user of the fixed-route service. It is worth emphasizing that here transfers needed to connect the demand-adaptive service and the fixed-route service are not considered. These can be separately estimated based on the percentage of passengers who choose walking.

The probability of requiring no transfer between the fixed-route lines is equivalent to the probability that the origin and the destination of the trip are within the corridor served by the same line. Once the origin is given, the horizontal and vertical line serving the origin are determined. Thus, the destination must be in one of the $2N - 1$ zones around the same horizontal and vertical line out of the total $N^2$ zones such that no transfer between the fixed-route lines is needed. The no-transfer probability can be calculated as

$$\Pr(Tr = 0) = \frac{2N - 1}{N^2}.$$  

(4)

Accordingly, the probability of requiring one transfer for a trip is

$$\Pr(Tr = 1) = 1 - \Pr(Tr = 0) = \frac{(N - 1)^2}{N^2}.$$  

(5)

Since at most one transfer is needed in a grid route system, the expected number of transfers can be computed as

$$e_T = 0 \times \Pr(Tr = 0) + 1 \times \Pr(Tr = 1) = \frac{(N - 1)^2}{N^2}.$$  

(6)

### 3.1 Agency costs

The agency costs are determined by the expected total vehicle distance traveled per hour of operation, $Q$, and the expected total fleet size in operation, $M$.

In the DAPL-HT system, $Q$ includes two parts: the expected total vehicle distance traveled per hour of operation by fixed-route vehicles, $Q_1$ and that by demand-adaptive vehicles $Q_2$.

The total vehicle distance is given by the product of the number of lines, the expected travel distance of one bus round trip, and the inverse of the bus headway (Nourbakhsh and Ouyang, 2012). For the fixed-route
buses, the round-trip distance per line is $2D$, thus the expected distance traveled per hour by the fixed-route buses is

$$Q_1 = \frac{4ND}{H_1}. \quad (7)$$

For the demand-adaptive vehicles, the total distance per round trip includes two components: (i) the necessary round-trip longitudinal distance to traverse from north to south and then back forth; (ii) the lateral distance to pick up and drop off passengers. For the first component, the distance is $2D$. For the second part, the expected lateral distance per passenger is $D_3$ and the passenger trips generated along the route per round trip is $\lambda H_2 D_2$. Thus, considering only the portion of passengers who use the demand-adaptive service, the total expected lateral distance per round trip can be expressed as $2p_y \lambda D_3^3$ (including both pickup and drop off, see e.g. Daganzo, 1984). Since the demand-adaptive vehicles will stop at each station, they will incur extra lateral distance. The average lateral distance between demand-adaptive vehicle and the station is simply $D_4$, thus the extra lateral distance per round trip is $D_2$. Therefore, the expected distance traveled per hour by the demand-adaptive vehicles is

$$Q_2 = N \times \left[ 2D + \frac{2p_y \lambda H_2 D_3}{3N^2} + \frac{D}{2} \right] \times \frac{1}{H_2} = \frac{5ND}{2H_2} + \frac{2p_y \lambda D_3}{3N}. \quad (8)$$

where $p_y$ is defined in Equation (3). The overall expected vehicle distance traveled per hour is then given by

$$Q = Q_1 + Q_2 = \frac{4ND}{H_1} + \frac{5ND}{2H_2} + \frac{2p_y \lambda D_3}{3N}. \quad (9)$$

For the fixed-route vehicles, the time required to travel distance $Q_1$ in one hour ($\frac{Q_1}{v_{c1}}$, where $v_{c1}$ represents the average speed of fixed-route vehicles) should include the time consumed while: (i) overcoming distance ($\frac{Q_1}{v}$); (ii) stopping ($\frac{2\tau_1 N^2}{H_1}$); and (iii) collecting passengers ($\tau'_1 (1 + e_T) \lambda D^2$) (Daganzo, 2010b). Hence, we have

$$\frac{Q_1}{v_{c1}} = \frac{Q_1}{v} + \frac{2\tau_1 N^2}{H_1} + \tau'_1 (1 + e_T) \lambda D^2. \quad (10)$$

For the demand-adaptive vehicles, the time required to traverse the distance $Q_2$ in one hour ($\frac{Q_2}{v_{c2}}$, where $v_{c2}$ is the average speed of demand-adaptive vehicles) should include the time consumed while: (i) overcoming distance ($\frac{Q_2}{v}$); and (ii) picking up and dropping off passengers ($2\tau_2 p_y \lambda D^2$). Hence, we have

$$\frac{Q_2}{v_{c2}} = \frac{Q_2}{v} + 2\tau_2 p_y \lambda D^2. \quad (11)$$

Since $Q_1(Q_2)$ is, by definition, the distance traveled by $\frac{Q_1}{v_{c1}} (\frac{Q_2}{v_{c2}})$ vehicles in one hour, the size of the fleet required to operate the fixed-route (flexible) service is simply

$$M_i = \frac{Q_i}{v_{ci}}, i = 1, 2.$$ 

The required fleet size is then given by $M = M_1 + M_2$. 

9
3.2 User costs

In the hybrid transit system, important metrics of user costs include the following components:

- **Walking time** \((A)\)

  As per Assumption 1, passengers may walk from their origins to the nearest transit stops or walk from transit stops to their destinations if the walking distance is short enough.

- **Waiting time** \((W)\)

  Passengers may need to wait for the demand-adaptive vehicle at their origin location or at the destination stop. Passengers may also need to wait for the fixed-route vehicles at either the starting stop or the transfer stop.

- **In-vehicle travel time** \((T)\)

  The in-vehicle travel time includes the travel time in fixed-route vehicles \((T_1)\) and in demand-adaptive vehicles \((T_2)\).

- **Transfer penalty** \((\frac{\delta}{\tau_v} \varepsilon_T)\)

  Besides waiting, passengers may incur an additional penalty cost for each transfer taking place between two fixed-route lines. Note that the transfer between demand-adaptive and fixed-route lines is not subject to this penalty.\(^3\)

Note that for a specific trip, the passenger’s total cost does not necessarily have all of the above components. For design purpose, we will calculate the average per passenger for each of the above components.

First, Equation (6) gives the the expected number of transfers, which can be used to estimate the average transfer penalty.

As to walking, note that some passengers are assumed to walk from their origins to the origin stations and/or from their destination stations to the destinations. At each end, the maximum walking distance is \(\beta \frac{D}{N}\). We need to calculate the average walking distance for two cases, see Figure 2a and 2b. Because of the symmetry, we only need to consider the average walking distance when the origins/destinations are in the first quadrant. When \(0 < \beta \leq 0.5\), denote the area \(OHG\) and \(HMDNG\) as \(A_1\) and \(A_2\), respectively, then the average walking distance can be calculated as

\[
I = \frac{\iint (x + y)dx\,dy}{A_1} = \frac{\iint xdx\,dy}{A_1} + \frac{\iint ydx\,dy}{A_1} = \frac{1}{3} \frac{\beta D}{N} + \frac{1}{3} \frac{\beta D}{N} = \frac{2}{3N} \beta D.
\]

(12)

Note that the first and second part in the second equality is just the \(x\) and \(y\) coordinate of the center of mass of the triangle, respectively. We will use this property and the symmetry to derive the results for \(0.5 < \beta \leq 1\). When \(0.5 < \beta \leq 1\), denote the area \(OMSTN\) and \(SDT\) as \(A_1\) and \(A_2\), respectively. Due to symmetry, the average walking distance can be expressed as

\[
I = 2 \frac{\iint xdx\,dy}{A_1} = 2x_1.
\]

\(^3\)To see why this makes sense, note that if a passenger walks from origin to the nearest stop to ride a fixed-route service, it is not considered a transfer, although s/he will no doubt be subject to a waiting cost at the stop. Since here the demand-adaptive service replaces walking as an access mode, penalizing it as if it is a regular transfer in the fixed route is inappropriate. Indeed, the fact that the service is demand-adaptive implies a very different user experience, hence much lower (potentially negligible) transfer penalty.
To derive \( \bar{x}_1 \), considering the center of mass of the square \( OMDN \), we have

\[
(A_1 + A_2) \times \frac{1}{4N} = A_1 \bar{x}_1 + A_2 \bar{x}_2,
\]

(13)

where

\[
A_1 = \left( \frac{1}{2} \frac{D}{N} \right)^2 - \frac{1}{2} (1 - \beta)^2 \left( \frac{D}{N} \right)^2 = \left[ \frac{1}{4} - \frac{1}{2} (1 - \beta)^2 \right] \left( \frac{D}{N} \right)^2,
\]

(14)

\[
A_2 = \frac{1}{2} \left( \frac{D}{N} - \beta \frac{D}{N} \right)^2 = \frac{1}{2} (1 - \beta)^2 \left( \frac{D}{N} \right)^2,
\]

(15)

\[
\bar{x}_2 = \frac{1}{2} \frac{D}{N} - \frac{1}{3} \left( \frac{D}{N} - \beta \frac{D}{N} \right) = \left( \frac{1}{6} + \frac{1}{3} \beta \right) \frac{D}{N}.
\]

(16)

Solving the above equation gives

\[
\bar{x}_1 = \frac{3 - 4(1 - \beta)^2 (1 + 2\beta) D}{12 - 24(1 - \beta)^2} \frac{D}{N}.
\]

(17)

Then we have

\[
l = 2\bar{x}_1 = \frac{3 - 4(1 - \beta)^2 (1 + 2\beta) D}{6 - 12(1 - \beta)^2} \frac{D}{N}.
\]

(18)

Considering the walking at both ends, the expected walking time per passenger is simply given by

\[
A = \frac{2l}{v_w},
\]

(19)

where

\[
l = \begin{cases} 
\frac{2\beta D}{3N}, & 0 < \beta \leq 0.5, \\
\frac{3 - 4(1 - \beta)^2 (1 + 2\beta) D}{6 - 12(1 - \beta)^2} \frac{D}{N}, & 0.5 < \beta \leq 1. 
\end{cases}
\]

(20)

The expected total waiting time is equal to the sum of the expected waiting time at the origin, at the origin station, at the transfer station, and at the destination station. Waiting time at each waiting location is approximately half of the headway of the corresponding transit service. Considering that not all passengers need demand-adaptive service and/or transfer, the expected waiting time per passenger is approximately calculated as

\[
W = p_y \frac{H_2}{2} + \frac{H_1}{2} + \frac{H_1}{2} e_T + p_y \frac{H_2}{2} = p_y H_2 + \frac{H_1}{2} \left[ 1 + \frac{(N - 1)^2}{N^2} \right].
\]

(21)

where \( p_y \) is defined in Equation (3) and \( e_T \) is defined in Equation 6.

We note that a passenger requesting demand-adaptive service may not be served by the first incoming vehicle if it is fully occupied by other passengers. Such a crowding effect, however, can occur in a fixed-route service too - in mega cities like Beijing, failing to board a crowded bus/train during rush hours is not an unusual incident. Because it is difficult to accommodate this effect in a sketch design model, the commonly accepted practice in the literature is to assume the average waiting time is approximately half of the headway (see e.g. Daganzo, 2010b; Nourbakhsh and Ouyang, 2012; Aldaihani et al., 2004), which is also adopted in the above analysis. In the simulation experiments presented later, however, the extra delays caused by crowding will be explicitly considered.
We finally analyze the in-vehicle travel distance, which consists of two parts: (i) the distance traveled in fixed-route vehicles \((E_1)\); and (ii) the distance traveled in demand-adaptive vehicles \((E_2)\). For the first part, let \(d\) denote the average distance traveled by a passenger on only one fixed-route vehicle, thus the average distance traveled by passengers with and without transfer in fixed-route vehicles are \(d\) and \(2d\), respectively. Aldaihani et al. (2004) showed that the average distance \(d\) converges to \(0.34D\) when \(N\) gets larger. As in Aldaihani et al. (2004), we set the average distance \(d\) to its converging value of \(0.34D\). Consequently, we have

\[
E_1 = d\Pr(\text{Transfer} = 0) + 2d\Pr(\text{Transfer} = 1) = \frac{0.34D(2N^2 - 2N + 1)}{N^2}. \tag{22}
\]

For the second part, passengers are assumed to board the closest stops from their origins and get off at the closest stops to their destinations. We first calculate the average longitudinal distance between the origin (destination) and the origin (destination) station, see Figure 2 for reference. When \(0 < \beta \leq 0.5\), the average longitudinal distance can be expressed as

\[
l_y = \int \int ydx \, dy = \frac{A_2}{A_1} = \bar{y}_2 \tag{23}
\]

Note that, again, \(A_1\) and \(A_2\) here represent the areas of the polygon OHG and HMDNG, respectively, as shown in Figure 2. Considering the center of mass of the square \(OMDN\), we have

\[
(A_1 + A_2) \times \frac{1}{4N} = A_1 \times \frac{1}{3N} + A_2\bar{y}_2, \tag{24}
\]

where

\[
A_1 = \frac{1}{2} \left( \frac{\beta D}{N} \right)^2, \quad A_2 = \frac{1}{4} \left( \frac{D}{N} \right)^2 - \frac{1}{2} \left( \frac{\beta D}{N} \right)^2. \tag{25}
\]

Solving the above equation gives

\[
l_y = \bar{y}_2 = \frac{3 - 8\beta^3}{12 - 24\beta^2} \frac{D}{N}. \tag{26}
\]

For \(0.5 < \beta \leq 1\), we leave it to the reader to verify that

\[
l_y = \bar{y}_2 = \frac{1}{2} \frac{D}{N} - \frac{1}{3} \left( \frac{D}{N} - \frac{\beta D}{N} \right) = \left( \frac{1}{6} + \frac{1}{3\beta} \right) \frac{D}{N}. \tag{27}
\]

It is difficult to estimate the average lateral distance travelled by the demand-adaptive vehicles, because it depends on the number of passenger requests processed. We bypass the complexity by assuming that the ratio between the longitudinal distance and the total distance travelled by demand-adaptive vehicles can be used as a surrogate for the ratio between the passenger’s expected and total in-vehicle travel distances. Let \(\rho\) be the ratio between the total and longitudinal distance traveled by demand-adaptive vehicles. For these vehicles, the longitudinal distance in one vehicle round trip is \(2D\) and the total distance is \(\frac{Q_2H_2}{N}\), where \(Q_2\) is given in Equation (8). This gives \(\rho\) as

\[
\rho = \frac{Q_2H_2}{2ND}. \tag{28}
\]
Thus, considering the distance at both ends, the total in-vehicle distance for passengers riding demand-adaptive vehicles, denoted as $E_2$, is estimated by scaling $2p_yl_y$ by $\rho$, i.e.,

$$E_2 = 2p_\rho l_y = \frac{p_y l_y Q_2 H_2}{ND}. \quad (29)$$

The expected total in-vehicle travel distance is

$$E = E_1 + E_2 = \frac{0.34D(2N^2 - 2N + 1)}{N^2} + \frac{p_y l_y Q_2 H_2}{ND}, \quad (30)$$

and the expected total in-vehicle travel time per passenger trip is

$$T = \frac{E_1}{v_{cl_1}} + \frac{E_2}{v_{cl_2}}. \quad (31)$$

### 3.3 Formulation

We are now ready to present the optimal design model. To define the objective function consistently, all cost components are first converted into travel time equivalents. Let $\pi_Q$, $\pi_M$ and $\mu$ be the agency operation cost per vehicle-distance, the agency cost per vehicle hour, and the average monetary value of one passenger-hour, respectively. Then, $\pi_Q = \frac{S_Q}{AD\mu}$ and $\pi_M = \frac{S_M}{AD^2\mu}$ convert the corresponding agency costs into the equivalent travel time per passenger (see Daganzo, 2010b). Let $\delta$ measure the transfer penalty expressed in terms of the equivalent distance walked. Then $\frac{\delta v_w}{e_T}$ gives the equivalent travel time per transfer per passenger.

The total generalized cost of the proposed DAPL-HT system can be summarized as

$$z(N, H_1, H_2, \beta) = \pi_Q Q + \pi_M M + W + A + T + \frac{\delta}{e_T} e_T. \quad (32)$$

The corresponding optimization problem follows:

$$\min z(N, H_1, H_2, \beta) \quad (33)$$

$$\text{s.t. } H_1 > 0, H_2 > 0, N \in \{1, 2, \ldots, \left\lfloor \frac{D}{s} \right\rfloor\}, 0 < \beta \leq 1. \quad (34)$$

### 4 Related transit systems

The proposed DAPL-HT system may be viewed as a combination of a conventional fixed-route transit system and a demand-adaptive transit system. For the purpose of comparison, we examine and implement two related systems from the literature, one in each category, to benchmark the performance of the new system. Specifically, the fixed-route system proposed by Daganzo (2010b) and the demand-adaptive system proposed by Nourbakhsh and Ouyang (2012) are adopted because they use similar simplifying assumptions and idealized conditions as considered herein. It is worth noting that both Daganzo (2010b) and Nourbakhsh and Ouyang (2012) divide the square service area into two subareas: a central square area with side length $d \leq D$ and its periphery. Accordingly, another decision variable $\alpha = d/D$ is introduced to represent the relative size of the inner square. This route structure, first proposed by Daganzo (2010b), combines the grid and hub-and-spoke networks to provide a double coverage in the inner square, where the demand for transit is likely greater. Because it offers another dimension of flexibility, this hybrid route structure is expected to be more efficient than the grid route structure used in DAPL-HT. The reader is referred to Appendixes A and B for details regarding the analysis of the two systems.
Conceptually, the DAPL-HT system offers a few attractive features compared to these benchmark systems. Firstly, by paring the demand-adaptive (flexible) service with fixed-route lines, a larger spacing between fixed-route lines may be used because of the improved accessability, which may help reduce the agency costs. Secondly, the demand-adaptive vehicles only need a capacity large enough to serve passengers between two neighboring stops, as they must return to the next stop to unload all passengers. This feature allows smaller vehicles to be used to operate the demand-adaptive service than buses of typical size. Consequently, the operation cost per unit distance (mostly fuel costs) may be lower for the demand-adaptive services in DAPL-HT. Finally, the paired-line design in the proposed system helps simplify planning on-demand routing, because for each planning cycle vehicles follow a more predictable path within a relatively small area. The next section will put the three systems through a numerical test in order to better understand their relative performance.

5 Numerical analysis

5.1 Solution method

The optimal design problem of the proposed DAPL-HT system is a mixed-integer optimization problem. However, because of the small number of variables, it can be solved easily by most existing commercial solvers. Thus, we only briefly comment on solution methods, for the convenience of the reader.

One may simply relax the integer constraint on $N$, and solve the relaxed problem first. Then, the optimal value of $N$ (as a real number) is rounded to the two closest integers, and, with $N$ being fixed to either integer value, the nonlinear program is solved again. The final optimal solution can be found by choosing the integer $N$ that leads to a smaller total system cost. This continuous relaxation (CR) method has been used in Nourbakhsh and Ouyang (2012) to solve the optimal design problem of the demand-adaptive transit system. We note that CR is a heuristic that does not guarantee a global optimal solution for non-convex problems.

The mixed-integer problem may also be solved directly using a global optimization method such as the genetic algorithm (GA). In our study, we test both the GA and CR methods, all implemented using Matlab’s built-in functions. Specifically, for CR method we use Matlab’s fmincon function with default interior-point algorithm as the nonlinear program solver. For the GA method, Matlab’s built-in function ga is employed. Figure 3 reports the optimal system cost obtained by the two methods with two city sizes and under different demand scenarios (x axis represents the value of $\lambda$, the demand density). Overall, the two methods provide similar results in almost all cases. The only exception is for the small city with a low demand, where GA clearly offers a lower system cost, indicating that the CR method might not locate the true global optima in this case. In addition, the results produced by the GA methods for the fixed-route and demand-adaptive systems are also comparable to and in many case slightly better than those reported in Daganzo (2010b) and Nourbakhsh and Ouyang (2012). Therefore, the GA method will be used to solve the proposed and the two benchmark systems in the following.

5.2 Cost comparison

In this section, the proposed DAPL-HT system is compared against the fixed-route system (Daganzo, 2010b) and the demand-adaptive system (Nourbakhsh and Ouyang, 2012) based on the system cost of the corresponding optimal design.

Wherever possible, default values of the input parameters used in the numerical analysis are taken from
Nourbakhsh and Ouyang (2012), as listed in Table 2. The operation cost per vehicle distance is related to fuel consumption and the operation cost per vehicle hour is related to labor cost. Note that the operation cost per distance \( Q \) for demand-adaptive vehicles in the proposed DAPL-HT system is set to be 30\% of the value in Table 2 because this service can be operated with smaller vehicles, as explained before. Specifically, we postulate that for the demand-adaptive service a 15-passenger van would be enough, which has an average fuel efficiency of about 10-15 mpg\(^4\). In comparison, a typical bus used by Chicago Transit Authority has a fuel efficiency of about 3.28 mpg\(^5\). Thus, the proposed 70\% discount for the demand-adaptive vehicle’s operation cost per unit distance seems to fall in a reasonable range. Finally, note that the walking speed is set to be smaller than the realistic value (around 5 km/h) to reflect the discomfort associated with walking.

Table 2: Values of input parameters in the numerical experiment

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) (km)</td>
<td>0.15</td>
<td>the distance between two adjacent streets (street spacing)</td>
</tr>
<tr>
<td>( \mu ) ($/h)</td>
<td>20</td>
<td>value of time</td>
</tr>
<tr>
<td>( \tau_1 ) (s)</td>
<td>12</td>
<td>time lost per stop due to deceleration and acceleration</td>
</tr>
<tr>
<td>( \tau'_1 ) (s)</td>
<td>1</td>
<td>time added per boarding passenger for fixed-route vehicles</td>
</tr>
<tr>
<td>( \tau_2 ) (s)</td>
<td>13</td>
<td>additional pick-up and drop-off time required per passenger</td>
</tr>
<tr>
<td>( v ) (km/h)</td>
<td>25</td>
<td>vehicles’ cruising speed</td>
</tr>
<tr>
<td>( v_w ) (km/h)</td>
<td>2</td>
<td>walking speed</td>
</tr>
<tr>
<td>( \delta ) (km)</td>
<td>0.03</td>
<td>transfer penalty expressed in terms of the equivalent distance walked</td>
</tr>
<tr>
<td>( Q ) ($/veh \cdot km)</td>
<td>2</td>
<td>operation cost per vehicle distance</td>
</tr>
<tr>
<td>( S_M ) ($/veh \cdot h)</td>
<td>40</td>
<td>operation cost per vehicle hour</td>
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</tbody>
</table>

Figure 4 plots the optimal cost (per passenger) of the three systems versus demand level \( \lambda \) for two different network sizes: a) \( D = 10\text{km} \) (a small city); b) \( D = 20\text{km} \) (a mid-size U.S. city)(Daganzo, 2010b). In both cases, DAPL-HT outperforms the other two transit systems under all demand levels. In the small city case \( D = 10\text{km} \), the demand-adaptive transit system is better than the fixed-route system when demand is

\(^4\)http://www.fueleconomy.gov/feg/byclass/Vans__Passenger_Type2015.shtml
\(^5\)http://www.chicagobus.org/buses/1000
Figure 4: Optimal system cost vs demand level with default input parameters

low (e.g., $\lambda < 50 \text{pax/h/km}^2$). For the other case ($D = 20\text{km}$), the demand-adaptive transit system performs better than the fixed-route system only under extremely low demand levels (e.g., $\lambda < 10 \text{pax/h/km}^2$). These observations suggest that (1) the demand-adaptive transit system may be desirable only for relatively small and low-density areas compared with the fixed-route system, and (2) DAPL-HT performs the best among the three systems under all demand levels. There are a couple of reasons for the dominating performance of DAPL-HT. First and foremost, it enables the system to determine the trade-off between the walking cost and the agency cost. In contrast, all passengers must walk in the fixed-route system whereas the flexible-route system must pick up and drop off everyone. This design flexibility is crucial to bringing down the total cost. Second, DAPL-HT allows to use a diverse fleet in the operation. Because the demand-adaptive service only need to take care of passengers between two stops at any given time (thanks to the “paired-line” design), a smaller vehicle can be used to operate the service, which reduces the agency’s distance-related cost.

Table 3a shows the optimal design parameters and cost components for the proposed DAPL-HT system for $D = 20\text{km}$. As expected, when $\lambda$ increases, the number of lines increases (i.e., the line spacing decreases) and the headway decreases considerably. Most per-capita agency and user cost components decrease monotonically, albeit the in-vehicle travel cost and the transfer cost remain almost intact.

For comparison, Tables 3b and 3c list similar optimal results for the fixed-route and the demand-adaptive transit systems, respectively. Generally, the fixed-route transit system requires more lines than DAPL-HT (as expected) but it has longer headways. The agency costs in DAPL-HT ($Q$ and $M$) are higher than their counterparts in the fixed-route transit system. Yet, the increases in the agency costs is more than offset by the reduction in the user costs.

For the demand-adaptive transit system, it uses fewer lines than the fixed-route transit system when the level of demand is low. Yet, when demand increases, the number of lines required rise much more sharply. The demand-adaptive transit system also has higher agency costs compared to the fixed-route transit system. While it eliminates walking, the demand-adaptive system incurs much larger in-vehicle travel cost, likely the result of long detours required by the door-to-door service.

We note that in DAPL-HT, the cost of transfers between demand-adaptive and fixed-route lines is ignored (see Footnote 3). One may argue that such an assumption may give DAPL-HT an undue advantage when compared to the flexible-route system. However, a close look at Tables 3a reveals that the transfer cost
(reported in the second last column in these tables) is rather small (around 1 to 2%) in all three systems. Even if we triple the transfer cost in the proposed system (which would sufficiently account for the two additional transfers, one at the origin stop, and the other at the destination stop), it would still outperform the flexible-route option by a significant margin.

Table 3: Optimal system design and the corresponding cost components

(a) DAPL-HT system

<table>
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<tr>
<th>λ</th>
<th>β</th>
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<th>H₂(h)</th>
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(b) Fixed-route transit system

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(c) Demand-adaptive transit system

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<td>0.04</td>
<td>0.77</td>
<td>0.02</td>
<td>1.15</td>
</tr>
</tbody>
</table>

5.3 Sensitivity analysis

In this section, we conduct a sensitivity analysis to examine how the performance of the three transit systems is affected by key input parameters. The network size is set to be $D = 20km$ for all tests. Figure 5a plots the optimal system cost versus the demand level for the three transit systems when the delay per stop for the
vehicles is increased from $12s$ to $30s$. We can see that all curves follow the same trend as those in Figure 4b, but the optimal costs for the three transit systems have increased, more significantly for the demand-adaptive system than the other two. Compared to Figure 4b, we can see that longer dwell time makes the DAPL-HT system more favorable: in fact it outperforms the other two transit systems with a large margin under all demand levels.

Figure 5b compares the optimal cost curves for the three systems with a much lower walking speed. Such a low walking speed may be used in cases where walking is highly undesirable, such as under extreme weather or in unsafe neighborhood. In this situation, the fixed-route transit system is penalized dramatically because it offers no alternative access mode other than walking. The demand-adaptive system has a slight cost increase because the walking speed may affect the transfer cost. The cost of the DAPL-HT system also rises, although it can mitigate the impact by expanding the range eligible for demand-adaptive service ($\beta$). The figure shows that the DAPL-HT system is the most favorable under all demand levels. Figure 5c reports the same results for a higher walking speed (50% more than the default value). As shown in the plot, although the performance of DAPL-HT is still the best among the three systems, the cost reduction compared with the fixed-route system narrows. This is expected because reducing the access cost is the core objective of DAPL-HT, and for higher walking speeds, there is less room for improvement in the access cost. For the same reason, higher walking speed also hurts the relative performance of the demand-adaptive system, probably more so than for the DAPL-HT system.

Finally, Figure 5d reports the results for the case where the waiting time weighs 80% more than the in-vehicle time. Because the estimation of waiting cost in all the three systems is quite similar, the relative performance does not change much from the base case shown in Figure 4b.

6 Simulation results of the DAPL-HT system

To validate the analytical results, a discrete-event simulation model of the DAPL-HT system is developed and tested in this section. A brief description of the simulation model will be presented first, followed by a comparison between simulation and analytical results.

6.1 Simulation model

The simulation model is implemented in NetLogo\textsuperscript{6}, a multi-agent programmable modeling environment. There are three types of agents in the simulated DAPL-HT system, namely, passengers, fixed-route vehicles, and demand-adaptive vehicles. The behavior of each agent is described below. Note that the stops closest to passengers’ actual origins (destinations) are called origin (destination) stops.

- **Passengers**: Passengers will walk to their origin stop if the stop is within the walking range; otherwise, they will request and use the demand-adaptive service to reach their origin stop. After arriving at the origin stop, they will take the fixed-route transit to their destination stop (transfer will be made if necessary). After arriving at their destination stop, they will directly walk to the destination if it is within the walking range; otherwise, they will wait for the demand-adaptive service.

- **Fixed-route vehicles**: Fixed vehicles are dispatched from the terminal stop according to the optimal headway. They will stop at each transit stop to pick up and drop off passengers.

\textsuperscript{6}https://ccl.northwestern.edu/netlogo/
Figure 5: Optimal system cost vs demand level with different input parameters

(a) $\tau_1 = 30s$, $\tau_2 = 31s$ ($\tau_2 = \tau_1 + 1$ in our experiments)

(b) $v_w = 0.1km/h$

(c) $v_w = 3km/h$

(d) 1 unit of waiting time = 1.8 unit of in-vehicle time
- **Demand-adaptive vehicles**: Demand-adaptive vehicles will pick up passengers who request the service from the origin and take them to their origin stop. These vehicles will also pick up passengers who need the service at each transit stop and drop them off at their destination.

A key issue in the simulation is representing the route planning of the demand-adaptive vehicles. Our implementation follows operational characteristics discussed in Section 2. Additional details are described below. When a demand-adaptive vehicle arrives at a stop, it will first drop off passengers who need to depart from the stop and collect the passengers who just arrive at the stop. It will then determine the list of passengers who need to be picked up on its way to next transit stop. When all pick-up and drop-off locations are determined, the visit order will be simply determined based on the longitudinal distance from the current stop. Once a vehicle leaves the stop, no further requests between that stop and the next stop will be processed, which rules out the possibility of “backward” movement.

### 6.2 Simulation results

We perform the simulation on a grid street network with $D = 20km$, a constant street spacing $s = 150m$ and a demand density $\lambda = 50\text{ pax/h/km}^2$. The simulation is conducted in the regular settings (base scenario) and four other different settings as in Section 5.3. The five scenarios simulated are (1) base scenario, (2) long dwell time, (3) inconvenient walking, (4) fast walking, and (5) high weight of waiting time. The system is configured based on the design parameters obtained from the optimization model, as shown in Table 4. Note that in Table 4e the number of lines in simulation does not exactly match the optimal value because of the street spacing constraint.

Table 4 shows that the simulation results generally match the analytical results well in all five scenarios. Notably, no significant differences are found between the analytical and simulated waiting times ($A$). This implies that the crowding effect (i.e., passengers are unable to get in a crowded bus) is not prominent and the “half-headway assumption holds reasonably well at the tested demand level. The in-vehicle travel time is higher in the simulation results, although the relatively difference is still within 10%. The reason for the large discrepancy may be due to the primitive nature of the route planning currently implemented in the simulation model. We expect that the simulated cost can be further reduced if a better routing algorithm can be implemented, which will be considered in a future study. Nevertheless, in general the simulation results confirm the correctness of the analysis.

A couple of other points are worth mentioning in Table 4. First, the optimal number of lines does not change much with different input parameters. This is desirable because the line spacing is difficult to change once implemented (due to the need to build bus stops), whereas the other design parameters are relatively easy to adjust in real time. Also, for the inconvenient walking scenario, the optimal value for the parameter $\beta$ is almost 0, which means almost all passengers will use the demand-adaptive service when walking is dramatically penalized. Note that the transfer cost in this scenario is extremely high, this is expected because the transfer cost is evaluated in terms of the equivalent distance walked.

### 7 Conclusions

In this paper, we proposed a new transit system - dubbed the demand-adaptive paired-line hybrid transit (DAPL-HT) system - that integrates the traditional fixed-route service with a demand-adaptive service.
Table 4: Simulation results vs analytical results

(a) Base scenario

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$N$</th>
<th>$H_1(h)$</th>
<th>$H_2(h)$</th>
<th>$S_Q $</th>
<th>$S_M$</th>
<th>$A(h)$</th>
<th>$W(h)$</th>
<th>$T(h)$</th>
<th>$\delta/v_w e^t(h)$</th>
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<tbody>
<tr>
<td>Analytical</td>
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<td>0.10</td>
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<td>0.11</td>
<td>0.19</td>
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<td>0.14</td>
<td>0.61</td>
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<td>1.07</td>
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<td>0.05</td>
<td>0.10</td>
<td>0.17</td>
<td>0.01</td>
<td>0.12</td>
<td>0.64</td>
<td>0.01</td>
<td>1.05</td>
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</table>

(b) Long dwell time

<table>
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<tr>
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<th>$H_1(h)$</th>
<th>$H_2(h)$</th>
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<th>$A(h)$</th>
<th>$W(h)$</th>
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<td>0.11</td>
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<tr>
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<td>0.10</td>
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<td>0.12</td>
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(c) Inconvenient walking

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<th>$H_2(h)$</th>
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<th>$S_M$</th>
<th>$A(h)$</th>
<th>$W(h)$</th>
<th>$T(h)$</th>
<th>$\delta/v_w e^t(h)$</th>
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</thead>
<tbody>
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<td>0.11</td>
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<td>0.00</td>
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<td>0.00</td>
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(d) Fast walking

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<th>$S_M$</th>
<th>$A(h)$</th>
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<td>1.07</td>
</tr>
<tr>
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<td>0.10</td>
<td>0.05</td>
<td>0.09</td>
<td>0.16</td>
<td>0.03</td>
<td>0.12</td>
<td>0.64</td>
<td>0.01</td>
<td>1.05</td>
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</table>

(e) High weight of waiting time

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<th>$\delta/v_w e^t(h)$</th>
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</thead>
<tbody>
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<td>0.03</td>
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<td>0.23</td>
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<td>0.67</td>
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Similar to existing hybrid transit systems, the demand-adaptive service is mainly used to improve accessibility, i.e. connecting passengers from their origin/destination to the fixed-route service. Yet, a unique feature of the proposed hybrid design is that it operates the demand-adaptive service with a stable headway to cover all stops along a fixed-route line with which it is paired. Pairing fixed-route and demand-adaptive lines simplifies the complexity of on-demand routing, because demand-adaptive vehicles can follow a more predictable path and can be dispatched on intervals coordinated with the fixed-service lines. The design of the two services are closely coupled to minimize the total system cost, which incudes both the transit agency’s operation cost and the user cost. Therefore, transit operators can easily adjust the hybrid design to cope with varying demand conditions by reallocating the resources between the two services.

We analyzed all the agency and user costs associated with the proposed system under idealized conditions. The problem of determining the optimal design parameters (number of transit lines, service headway and accessability) is then formulated as a mixed integer program and solved using a commercially available metaheuristic (Matlab’s built-in generic algorithm). Numerical experiments are conducted to compare the DAPL-HT system with two related transit systems that may be considered its special cases: a fixed-route system and a demand-adaptive system. Main findings from these experiments are summarized below.

• In term of the total system cost, the DAPL-HT system outperforms the other two systems under all demand levels we tested and in various scenarios of input parameters. Compared to the traditional fixed-route system, it leads to higher agency costs, but the additional agency costs are more than offset by the savings in the user costs.

• When walking is more preferable, the relative performance of the fixed-route transit system improves and that of the demand-adaptive transit system degrades.

• The demand-adaptive system is desirable only under low demand levels and in relatively small service areas compared with the fixed-route system, which is well known in the literature.

The correctness of the proposed analysis method is also confirmed with a discrete-event simulation developed in this study.

For future research, the DAPL-HT system should be compared with other hybrid designs, such as the widely studied zone-based systems. Another possible direction is to consider alternative route structures in the DAPL-HT system, such as ring-radial structure and the hybrid route structure proposed by Daganzo (2010b). A future study can also address the important issue of incorporating the crowding effect (i.e., the likelihood of failing to board a bus due to crowdedness) in the system design. Last but not least, the simulation model can be further developed to implement various transit systems and to incorporate more sophisticated routing algorithms, communication protocols, and realistic passenger behavior.

**Acknowledgment**

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7While the present paper was under review, a follow-up study along this direction has been published (Chen and Nie, 2017).
References


Daganzo, C. F., 2010a. Public transportation systems: Basic principles of system design, operations planning and real-time control. Tech. rep., Institute of Transportation Studies, University of California at Berkeley.


A Fixed-route transit system

In Daganzo’s fixed-route transit system (Daganzo, 2010b), the total system cost includes those related to total vehicle distance, vehicle fleet size, passenger walking time, passenger waiting time, and transfers. Using similar notations the optimal design problem can be written as follows:

\[ \min z = \pi_Q Q + \pi_M M + W + A + T + \frac{\delta}{v_w} e_T, \]  

s.t. \( \alpha \in \left[ \frac{1}{N}, 1 \right], H > 0, N \in \{1, 2, ..., \left\lfloor \frac{D}{s} \right\rfloor\}, \)  

where \( H \) denotes the headway of the fixed-route line, \( N \) denotes the number of fixed-route lines in the N-S direction (E-W direction has the same number of fixed-route lines), \( \pi_Q \) and \( \pi_M \) are defined as in (32). We specify the other terms of the objective function in what follows. The reader is referred to Daganzo (2010b) for details. The expected total vehicle distance traveled per hour

\[ Q = \frac{2D^2}{SH}(3\alpha - \alpha^2), \]  

where \( S \) is line spacing. The bus fleet size \( M \) is given by

\[ M = \frac{Q}{v_c}, \]  

where the average speed of the fixed-route bus

\[ \frac{1}{v_c} = \frac{1}{v} + \frac{\tau_1}{S} + \frac{(1 + e_T)\tau'_1 \lambda S H}{2(3\alpha - \alpha^2)D^2}. \]  

The average walking and waiting time per passenger are

\[ A = \frac{S}{H} \quad \text{and} \quad W = \left[ \frac{2 + \alpha^3}{3\alpha} + \frac{(1 - \alpha^2)^2}{4} \right] H, \]  

respectively. The average in-vehicle travel time, \( T \), is given by

\[ T \approx \frac{D}{12} \left( \frac{1}{v} + \frac{\tau_1}{S} \right)(12 - 7\alpha + 5\alpha^3 - 3\alpha^5 + \alpha^7), \]  

and the average number of transfers per passenger

\[ e_T = 1 + \frac{1}{2}(1 - \alpha^2)^2. \]
B Flexible-route transit system

In the flexible-route transit system, the total system cost includes all items in Eq. (35) except the walking time. Note that there is no passenger walking time because the system essentially offers door-to-door service. The formulation of the optimal design problem reads (Nourbakhsh and Ouyang, 2012).

\[
\min z = \pi Q + \pi M + W + T + \frac{\delta}{v_w}e_T,
\]

s.t. \( \alpha \in \left[ \frac{1}{N}, 1 \right] \), \( H > 0 \), \( N \in \{1, 2, \ldots, \left\lfloor \frac{D}{s} \right\rfloor \} \),

where \( H, N, \pi_Q \) and \( \pi_M \) are similarly defined as in Eq (35). The formulae for calculating other terms in the objective function follow (see (Nourbakhsh and Ouyang, 2012) for details).

The expected total vehicle distance traveled per hour

\[
Q = \frac{2N}{H} \left[ D \sum_{i=2}^{\infty} (i-1)[\alpha P_c(i) + (1-\alpha)P_p(i)] + 2D + \frac{2\lambda HD^3\alpha^3}{3N^2} + \frac{2\lambda HD^2(1-\alpha^2)l_p}{N} \right],
\]

where

\[
P_c(i) = \frac{\left( \frac{\alpha Ds i H}{N} \right)^i e^{-\frac{\alpha Ds i H}{N}}}{i!},
\]

\[
P_p(i) = \frac{\left( \frac{(1+\alpha)Ds i H}{N} \right)^i e^{-\frac{(1+\alpha)Ds i H}{N}}}{i!}, \text{ and}
\]

\[
l_p = \left\{ \begin{array}{ll}
\frac{(1+\alpha)D}{6N} + \frac{2\alpha^3}{(1+\alpha)^3D^3H^2} - \frac{4\alpha^5}{3D^3(1+\alpha^2)H^3}, & \text{if } D^2(1+\alpha)^2\lambda H \geq 2N^2, \\
\frac{D(1+\alpha)H}{\Omega(1+\alpha)H}, & \text{otherwise}.
\end{array} \right.
\]

The vehicle fleet size is

\[
M = \frac{Q}{v_c} \quad \text{and} \quad \frac{1}{v_c} = \frac{1}{v} + \frac{4\tau_2 D^2(1+\alpha^2)}{Q}.
\]

The average passenger waiting time, \( W \), is given by

\[
W = \left[ \frac{N-1}{N} \left( 1 - \frac{\alpha^4}{N} \right) + \frac{(1-\alpha^2)^2}{2} + 1 \right] \frac{H}{2}.
\]

The average in-vehicle travel time, \( T \), is given by

\[
T = \frac{E}{v_c}
\]

where

\[
E \approx [4\phi(\alpha) + 5\varphi(\alpha)]D + 2D \sum_{i=0}^{\infty} (i-1)[\phi(\alpha)P_c(i) + \varphi(\alpha)P_p(i)] +
\]

\[
\frac{2\lambda HD^2}{N} \left[ \frac{D\alpha^2}{3N} \phi(\alpha) + (1+\alpha)l_p \varphi(\alpha) \right],
\]

\[
\phi(\alpha) = \frac{1}{12} (11\alpha - \alpha^3 - \alpha^5),
\]

\[
\varphi(\alpha) = \frac{1}{18} (2 - 3\alpha + \alpha^3).
\]
Finally, the average number of transfers per passenger

\[ e_T = \frac{N - 1}{N} \left( 1 - \frac{\alpha^4}{N} \right) + \frac{(1 - \alpha^2)^2}{2}. \]  

(55)