A Proof of Theorem 1: Existence of Equilibrium

First, we establish the existence of equilibrium in an auxiliary game where the buyer’s initial offer belongs to $P_b$ and the seller’s initial offer belongs to $P_s$. Then we show that deviations to offers outside of these sets are unprofitable for the players, which establish the existence of equilibrium where players can offer any price. We consider the case in which $\Theta = \{\theta_1, \ldots, \theta_n\}$ with $n \geq 2$, which will allow us to establish equilibrium existence for the more general model studied in Section 5.

Let $\sigma_b \in \Delta(P_b)$ and $\sigma_s = (\sigma_{\theta})_{\theta \in \Theta}: P_b \rightarrow [\Delta(P_s)]^n$ be a strategy for the buyer and the seller, respectively. We equip the probability spaces $\Delta(P_b)$ and $\Delta(P_s)$ with the topology of weak convergence, which is induced by the Prokhorov metric (Billingsley, 2013). Let $\pi: P_b \times P_s \rightarrow \Delta(\Theta)$ and $\varepsilon_s: P_b \times P_s \rightarrow [0, 1]$ be a system of beliefs for the buyer, assigning, respectively, a probability distribution over the rational-type seller’s production cost and a probability that the seller is a commitment type, after every pair of offers $(p_b, p_s) \in P_b \times P_s$. In order to avoid confusion, throughout this section, we use $\pi_j = \pi(\theta_j)$ to denote the (exogenous) prior probability that the seller is rational and has production cost $\theta_j$. Likewise, let $\varepsilon_b: P_b \rightarrow [0, 1]$ denote the seller’s posterior belief that the buyer is a commitment type after observing offer $p_b \in P_b$.

Fix a continuation equilibrium in the war-of-attrition game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ that follows bargaining postures $(p_b, p_s)$ with $p_b < p_s$ and beliefs given by $(\hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$. Such an equilibrium always exists given the results in Abreu and Gul (2000). Its construction for the case in which $p_b > \theta_1$ and $p_s < 1$ was provided in Lemmas 4.2 and 4.3. In the remaining cases, if $p_b > \theta_1$ and $p_s = 1$, we set players’ strategies to be equal to the limit of the strategies described in Lemmas 4.2 and 4.3 as $p_s \rightarrow 1$. Likewise, if $p_b \leq \theta_1$ and $p_s < 1$, we extend players’ strategies in Lemmas 4.2 and
4.3 by taking the limit as \( p_b \to \theta_1 \). Otherwise, if \( p_b \leq \theta_1 \) and \( p_s = 1 \), there exists a continuation equilibrium in the war-of-attrition in which neither player concedes. Let \( V_b(p_b, p_s, \hat{\theta}_b, \hat{\theta}_s, \hat{\pi}) \) and \( V_\theta(p_b, p_s, \hat{\theta}_b, \hat{\theta}_s, \hat{\pi}) \) be the buyer’s and type-\( \theta \) seller’s equilibrium continuation values in the war-of-attrition game \( \Gamma(p_b, p_s, \hat{\theta}_b, \hat{\theta}_s, \hat{\pi}) \). Notice that our equilibrium construction implies that \( V_b \) and \( V_\theta \) are continuous in each of their arguments.

Suppose the seller’s strategy is \( \sigma_s \) and players’ system of beliefs is induced by \((\pi, \varepsilon_s, \varepsilon_b)\), then the buyer’s expected payoff from demanding \( p_b \) is

\[
U_b(p_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi) \equiv \sum_{j=1}^{n} \left( \sum_{p_s \leq p_b} \pi_j \sigma_{\theta_j}(p_s | p_b) \left( 1 - \frac{p_s + p_b}{2} \right) \right)
+ \sum_{p_s > p_b} \pi_j \sigma_{\theta_j}(p_s | p_b) V_b(p_b, p_s, \varepsilon_b(p_b), \varepsilon_s(p_b, p_s), (1 - \varepsilon_s(p_b, p_s)) \pi(p_b, p_s)) \right).
\]

Likewise, we can write the seller’s expected payoff from offering \( p_s \) after the buyer offers \( p_b \) when his type is \( \theta \in \Theta \) as:

\[
U_\theta(p_s, p_b, \varepsilon_b, \varepsilon_s, \pi) \equiv \begin{cases} 
\frac{p_s + p_b}{2} - \theta, & \text{if } p_s \leq p_b \\
V_\theta(p_b, p_s, \varepsilon_b(p_b), \varepsilon_s(p_b, p_s), (1 - \varepsilon_s(p_b, p_s)) \pi(p_b, p_s)), & \text{if } p_s > p_b
\end{cases}.
\]

We define an equilibrium of the bargaining game to be an assessment \((\varepsilon_b, \varepsilon_s, \pi, \sigma_b, \sigma_s)\) such that strategies are sequentially rational and players’ beliefs \((\varepsilon_b, \varepsilon_s, \pi)\) are consistent with Bayes’ rule at every information set \((p_b, p_s) \in P_b \times P_s\). Since \( \mu_b \) and \( \mu_s \) have full support, all pairs of offers arise with positive probability and beliefs are thus always pinned down by Bayes’ rule.

The proof follows from a fixed point argument, with the only caveat that we have to deal with the fact that final payoffs depend non-trivially on beliefs. We circumvent this by constructing a correspondence whose fixed point is an equilibrium assessment, specifying both strategies and beliefs, and then we show that such a fixed point exists.

Let \( \mathcal{K} \equiv \Delta(P_b) \times [\Delta(P_s)]^{|P_b|} \times [0, 1]^{|P_b|} \times [0, 1]^{|P_s|} \times \Delta(\Theta)^{|P_b|} \times [P_s] \) be the set of possible strategy profiles. Define \( \Sigma : \mathcal{K} \ni [0, 1]^{|P_b|} \times [0, 1]^{|P_s|} \times \Delta(\Theta)^{|P_b|} \times [P_s] \rightarrow \mathcal{K} \) to be the correspondence that maps every strategy profile \((\sigma_b, \sigma_s) \in \mathcal{K}\) to a system of beliefs \((\varepsilon_b, \varepsilon_s, \pi)\) obtained at every information set using Bayes’ rule applied to the strategies \((\sigma_b, \sigma_s)\). Since there is no off-path event when \( \varepsilon > 0 \), we know that \( \Sigma(\sigma_b, \sigma_s) \) is non-empty, single-valued (and hence compact and convex), and upper hemi-continuous.

Next, we define the best response correspondence at every information set for a given strategy
According to the Kakutani-Fan-Glicksberg fixed point theorem (Corollary 17.55 in Aliprantis and Border, 2006), the seller’s strategy to specify a counteroffer for every player’s offer is an equilibrium of the game where the buyer’s and the seller’s offers are restricted. This assessment is an equilibrium of the game where the buyer’s and the seller’s offers are restricted to lie, respectively, in \( P_b \) and \( P_s \). Let \( BR_\theta(p_b, \varepsilon_b, \varepsilon_s, \pi) \) be the correspondence

\[
BR_\theta(p_b, \varepsilon_b, \varepsilon_s, \pi) = \arg \max_{\sigma_\theta \in \Delta(P_s)} \sum_{p_s \in P_s} \sigma_\theta(p_s)U_\theta(p_s, p_b, \varepsilon_b, \varepsilon_s, \pi)
\]

every \( \theta \in \Theta, p_b \in P_b \).

And let \( BR_\theta(\varepsilon_b, \varepsilon_s, \pi) = \prod_{p_b \in P_b} BR_\theta(p_b, \varepsilon_b, \varepsilon_s, \pi) \). Our equilibrium characterization implies that payoffs in the war-of-attrition game are continuous in beliefs \( (\varepsilon_b, \varepsilon_s, \pi) \). Players’ payoffs are also continuous in \( (p_b, p_s) \), which ensures that they are continuous in \( (\sigma_b, \sigma_s) \) by the Portmanteau Theorem (Theorem 2.1 in Billingsley, 2013). Moreover, compactness of \( P_b \) and \( P_s \) ensures that the spaces \( \Delta(P_b) \) and \( \Delta(P_s) \) are compact in the weak topology (Theorem 15.11 in Aliprantis and Border, 2006), and therefore so is the product space \( \mathcal{K} \times [0, 1]^{P_b} \times [0, 1]^{P_s} \times \Delta(\Theta)^{P_b} \times \Delta(\Theta)^{P_s} \). It then follows from the Maximum Theorem (Theorem 17.31 in Aliprantis and Border, 2006) that best-response correspondences are upper-hemi-continuous with nonempty and compact values. Linearity of payoffs in \( \sigma_b \) and \( \sigma_s \) ensures that they are also convex-valued.

Let \( M \equiv [0, 1]^{P_b} \times [0, 1]^{P_s} \times \Delta(\Theta)^{P_b} \times \Delta(\Theta)^{P_s} \times \mathcal{K} \), which is a locally convex Hausdorff space. Then, we can define the correspondence \( E : M \rightrightarrows M \) to be:

\[
E(\varepsilon_b, \varepsilon_s, \pi, \sigma_b, \sigma_s) = \Sigma(\sigma_b, \sigma_s) \times BR_\theta(\sigma_s, \varepsilon_b, \varepsilon_s, \pi) \times BR_{\theta_1}(\sigma_b, \varepsilon_b, \varepsilon_s, \pi) \times \ldots \times BR_{\theta_n}(\sigma_b, \varepsilon_b, \varepsilon_s, \pi).
\]

According to the Kakutani-Fan-Glicksberg fixed point theorem (Corollary 17.55 in Aliprantis and Border, 2006), \( E \) has a fixed point. Let \( (\varepsilon_b^*, \varepsilon_s^*, \pi^*, \sigma_b^*, \sigma_s^*) \) be a fixed point of \( E \). By construction, this assessment is an equilibrium of the game where the buyer’s and the seller’s offers are restricted to lie, respectively, in \( P_b \) and \( P_s \).

We conclude by using \( (\varepsilon_b^*, \varepsilon_s^*, \pi^*, \sigma_b^*, \sigma_s^*) \) to construct an equilibrium of the original game where players can choose any offer. In this construction, the buyer’s strategy is \( \sigma_b^* \in \Delta[0, 1] \). We extend the seller’s strategy to specify a counteroffer for every \( p_b \in [0, 1] \) as follows:

\[
\overline{\sigma}_\theta(\cdot | p_b) = \begin{cases} 
\sigma_b^*(\cdot | p_b), & \text{if } p_b \in P_b \\
\delta_{\{1\}}, & \text{if } p_b \in [0, 1] \setminus P_b,
\end{cases}
\]

where \( \delta_{\{1\}} \) is the Dirac measure on 1. Since all pairs of offers in \([0, 1]^2 \setminus P_b \times P_s \) are off-path, we
extend players’ systems of beliefs to be:

\[
\begin{align*}
\bar{\varepsilon}_b(p_b) &= \begin{cases} 
\varepsilon^*_b(p_b), & \text{if } p_b \in P_b \\
0, & \text{if } p_b \in [0, 1] \setminus P_b
\end{cases} \\
\bar{\varepsilon}_s(p_b, p_s) &= \begin{cases} 
\varepsilon^*_s(p_b, p_s), & \text{if } (p_b, p_s) \in P_b \times P_s \\
0, & \text{if } p_s \in [0, 1] \setminus P_s \\
\varepsilon\mu_*(p_s) + (1-\varepsilon) \sum_{j=1}^{n} \pi_j \sigma_{\theta_j}(p_s|p_b), & \text{otherwise}
\end{cases}
\end{align*}
\]

We extend \(\pi^*(p_b, p_s)\) to assign probability one to type \(\theta_1\) for all \((p_b, p_s) \in [0, 1]^2 \setminus (P_b \times P_s)\). Let \(\pi : [0, 1]^2 \rightarrow \Delta(\Theta)\) be such extension. We argue that the assessment \((\bar{\varepsilon}_b, \bar{\varepsilon}_s, \pi, \sigma^*_b, \sigma_s)\) is an equilibrium. To do so, we first check that the buyer doesn’t benefit from deviating to an offer \(p_b \in [0, 1] \setminus P_b\). If she does so, the seller demands the entire surplus, and the resulting payoff is 0 which is worse than her equilibrium payoff guarantee of \(1 - \theta_n\).

Finally, we verify sequential rationality for the seller. Following \(p_b \in P_b\), it suffices to check that offers \(p_s \in [0, 1] \setminus P_s\) do not dominate \(\sigma^*_b(\cdot|p_b)\). This is straightforward when \(p_b > \theta_n\), since demanding \(p_s \in [0, 1] \setminus P_s\) implies revealing rationality and is therefore equivalent to conceding immediately. When \(p_b \in P_b \cap [0, \theta_n]\), deviating to \(p_s \in [0, 1] \setminus P_s\) also involves immediate concession for types \(\theta\) such that \(\theta < p_b\) due to our construction of off-path beliefs. Further, because the buyer believes that she’s facing the (rational) type \(\theta_1\) with probability one, we can set the buyer’s strategy to be such that she delays concession indefinitely after the seller demands \(p_s \in [0, 1] \setminus P_s\), which makes this offer dominated for types \(\theta > p_b\) as well. After the buyer offers \(p_b \in [0, 1] \setminus P_b\), she is perceived to be rational with probability one. Hence, there is a continuation equilibrium where the seller demands the entire surplus (as stipulated by \(\pi_s\)) and the buyer concedes immediately. Therefore, players’ strategies are sequentially rational. This concludes the proof that \((\bar{\varepsilon}_b, \bar{\varepsilon}_s, \pi, \sigma^*_b, \sigma_s)\) is an equilibrium of the bargaining game.

### B Proof of Theorem 2: Existence of Equilibrium

We modify the existence proof of Theorem 1 in order to allow for an additional stage of the game in which the seller decides which production technology to adopt. In that stage, the seller’s action space is \(\Theta\), which determines the distribution of the seller’s production cost in the bargaining game. In order to establish existence for the more general case studied in Section 5, we focus on the case where \(\Theta = \{\theta_1, \ldots, \theta_n\}\). Let \(c_j\) denote the cost of choosing technology \(j\) for every \(j \in \{1, \ldots, n\}\).

Similar to the existence proof of Theorem 1 in Online Appendix A, we start by showing existence
in the modified game where players’ bargaining postures are restricted to belong to the set $P_b \times P_s$.

Throughout, we use the notation introduced in that proof. Let the payoff of the seller from choosing technology $\theta \in \Theta$, given the strategies at the bargaining stage $(\sigma_b, \sigma_s) \in K$ and players’ system of beliefs $(\epsilon_b, \epsilon_s, \pi) \in [0, 1]|P_b| \times [0, 1]|P_s| \times \Delta(\Theta)|P_b| \times |P_s|$ be given by

$$U_s(\theta_j, \sigma_b, \sigma_s, \epsilon_b, \epsilon_s, \pi) = U_{\theta_j}(\sigma_s, \sigma_b, \epsilon_b, \epsilon_s, \pi) - c_j.$$ 

We construct a correspondence whose fixed point is an equilibrium of the game with endogenous technology adoption. Let $\tilde{\pi} \in \Delta(\Theta)$ denote a strategy of the seller at the investment stage of the game. Let $J \equiv \Delta(\Theta) \times K$ be the set of possible strategy profiles in the bargaining game with investment. Define $\Sigma^I : J \Rightarrow [0, 1]|P_b| \times [0, 1]|P_s| \times \Delta(\Theta)|P_b| \times |P_s|$ to be the correspondence mapping every action profile $(\tilde{\pi}, \sigma_b, \sigma_s) \in J$ to a system of beliefs $(\epsilon_b, \epsilon_s, \pi)$ obtained at every information set using Bayes’ rule applied to the strategies $(\tilde{\pi}, \sigma_b, \sigma_s)$. This is always well defined under our assumption that all irrational types $(p_b, p_s) \in P_b \times P_s$ occur with strictly positive probability. Thus, $\Sigma^I(\sigma_b, \sigma_s)$ is non-empty, single-valued, and upper-hemi-continuous.

Let the seller’s best response correspondence at the investment stage $BR_0 : K \times [0, 1]|P_b| \times [0, 1]|P_s| \times \Delta(\Theta)|P_b| \times |P_s| \Rightarrow \Delta(\Theta)$ be given by

$$BR_0(\sigma_b, \sigma_s, \epsilon_b, \epsilon_s, \pi) = \arg \max_{\tilde{\pi} \in \Delta(\Theta)} U_s(\tilde{\pi}, \sigma_b, \sigma_s, \epsilon_b, \epsilon_s, \pi).$$ 

Since $U_\theta$ is continuous (established in Online Appendix [A]), $BR_0$ is non-empty, compact, convex, and upper hemi-continuous. Therefore, the equilibrium correspondence $E^I : [0, 1]|P_b| \times [0, 1]|P_s| \times \Delta(\Theta)|P_b| \times |P_s| \times J \Rightarrow [0, 1]|P_b| \times [0, 1]|P_s| \times \Delta(\Theta)|P_b| \times |P_s| \times J$ defined as

$$E^I(\epsilon_b, \epsilon_s, \pi, \tilde{\pi}, \sigma_b, \sigma_s) =$$

$$\Sigma^I(\tilde{\pi}, \sigma_b, \sigma_s) \times BR_0(\sigma_b, \sigma_s, \epsilon_b, \epsilon_s, \pi) \times BR_b(\sigma_b, \epsilon_b, \epsilon_s, \pi) \times BR_\theta(\sigma_b, \epsilon_b, \epsilon_s, \pi) \times \ldots \times BR_\theta_n(\sigma_b, \epsilon_b, \epsilon_s, \pi),$$

admits a fixed point according to the Kakutani-Fan-Glicksberg fixed point theorem. Let $(\epsilon^*_b, \epsilon^*_s, \pi^*, \tilde{\pi}^*, \sigma^*_b, \sigma^*_s)$ be a fixed point of $E^I$. This fixed point is an equilibrium of the game with price grids $P_b \times P_s$. Using the extension provided in Online Appendix [A], one can apply the same argument to verify that the assessment $(\overline{\pi}_b, \overline{\pi}_s, \overline{\pi}, \tilde{\pi}^*, \sigma^*_b, \sigma^*_s)$ is an equilibrium of the game where players’ can choose any bargaining posture from the unit interval.
We generalize Lemma 4.1 in the main text to environments with more than two possible production costs. Let \( \Theta \equiv \{ \theta_1, ..., \theta_n \} \) with \( 0 < \theta_1 < \theta_2 < ... < \theta_n < 1 \). Fix any equilibrium, let \( \hat{\epsilon}_b(p_b) \) be the probability that the buyer is the commitment type after she offers \( p_b \). We state the generalization as Lemma C.1 which will be used in the subsequent proofs for Theorems 3 and 4.

**Lemma C.1.** In any equilibrium, after the buyer offers \( p_b \in P_b \) with \( \hat{\epsilon}_b(p_b) < 1 \), every type of the seller with production cost no less than \( p_b \) will demand 1.

**Proof.** Suppose by way of contradiction that after the buyer offers \( p_b \), there exists a type \( \theta_k \geq p_b \) that demands some \( p'_s \) with positive probability. Our richness assumption implies that \( (p_s, 1) \cap P_s \) is non-empty. For every \( p'_s \in (p_s, 1) \cap P_s \), there exists \( \theta_j \) with \( \theta_j < p_b \) such that type \( \theta_j \) offers \( p'_s \) with positive probability. This is because otherwise, the buyer will rule out types lower than \( p_b \) after observing \( p'_s \) and will therefore concede immediately. If this is the case, then type \( \theta_k \) has a strict incentive to deviate by offering \( p'_s \) instead of \( p_s \). Without loss of generality, let \( \theta_j \) be the highest type that (i) has cost strictly lower than \( p_b \) and (ii) offers \( p'_s \) with positive probability. It is optimal for type \( \theta_j \) to offer \( p'_s \) and then concede after the rational-type buyer finishes conceding.

After the seller offers \( p_s \) and \( p'_s \), respectively, let \( T \) and \( T' \) be the times at which the rational-type buyer finishes conceding, let \( c_b \) and \( c'_b \) be the buyer’s concession probabilities at time 0, and let \( A \) and \( A' \) be the discounted probability of trade when the buyer is rational and the seller never concedes. On the one hand, the high-cost type \( \theta_k \) weakly prefers \( p_s \) to \( p'_s \), which implies that

\[
A(p_s - \theta_k) \geq A'(p'_s - \theta_k)).
\]  

(C.1)

On the other hand, it is optimal for the low-cost type \( \theta_j \) to offer \( p'_s \) and concede at time \( T' \), so he prefers this strategy to offering \( p_s \) and conceding at time \( T \). This incentive constraint implies that:

\[
e^{-rT'}\hat{\epsilon}_b(p_b)(p_b - \theta_j) + (1 - \hat{\epsilon}_b(p_b))A'(p'_s - \theta_j) \geq e^{-rT}\hat{\epsilon}_b(p_b)(p_b - \theta_j) + (1 - \hat{\epsilon}_b(p_b))A(p_s - \theta_j),
\]

which is equivalently to

\[
(e^{-rT'} - e^{-rT})\frac{\hat{\epsilon}_b(p_b)}{1 - \hat{\epsilon}_b(p_b)}(p_b - \theta_j) \geq A(p_s - \theta_j) - A'(p'_s - \theta_j).
\]  

(C.2)
Inequality (C.1) implies that
\[ A(p_s - \theta_j) - A'(p_s' - \theta_j) = A(p_s - \theta_k) - A'(p_s' - \theta_k) + (A - A')(\theta_k - \theta_j) \geq (A - A')(\theta_k - \theta_j), \]
and therefore,
\[ (e^{-rT'} - e^{-rT}) \frac{\hat{\epsilon}_b(p_b)}{1 - \hat{\epsilon}_b(p_b)} (p_b - \theta_j) \geq (A - A')(\theta_k - \theta_j). \] (C.3)

Since \( p_s < p_s' \), inequality (C.1) implies that \( A > A' \). Inequality (C.3) together with \( A > A' \) implies that \( T' < T \), which further implies that \( T > 0 \). As a result, there exists a type with production cost is strictly lower than \( p_b \) who will offer \( p_s \) with positive probability in equilibrium. This is because otherwise, the buyer will concede immediately following \( p_s \), which contradicts our earlier conclusion that \( T > 0 \).

Let \( \theta \) be the lowest type that offers \( p_s \) in equilibrium and let \( \theta' \) be the lowest type that offers \( p_s' \) in equilibrium. Our earlier conclusion implies that \( \theta, \theta' < p_b \). Consider the payoffs of type \( \theta \) and type \( \theta' \) under the following two strategies (i) offering \( p_s \) and conceding right after time 0, and (ii) offering \( p_s' \) and conceding right after time 0. Type \( \theta \) weakly prefers the first strategy, which gives:
\[ c_b(p_s - p_b) + p_b - \theta \geq c_b'(p_s' - p_b) + p_b - \theta. \]
Type \( \theta' \) weakly prefers the second strategy, which gives:
\[ c_b(p_s - p_b) + p_b - \theta' \leq c_b'(p_s' - p_b) + p_b - \theta'. \]
The two inequalities together imply that
\[ c_b(p_s - p_b) = c_b'(p_s' - p_b). \] (C.4)
Since \( p_s' > p_s \), the above inequality implies that \( c_b \geq c_b' \), where equality holds if and only if \( c_b = c_b' = 0 \). Recall our formulas for players’ concession rates. After the seller offers \( p_s \), the buyer will concede at rate
\[ \lambda_b(\theta) = \frac{r(p_b - \theta)}{p_s - p_b} \] (C.5)
when type $\theta$ seller is conceding. After the seller offers $p'_s$, the buyer will concede at rate

$$\lambda_b'(\theta) = \frac{r(p_b - \theta)}{p'_b - p_b}$$  \hspace{1cm} (C.6)$$

These expressions imply that (i) $\lambda_b(\theta) > \lambda'_b(\theta)$ for every $\theta < p_b$, and (ii) $\lambda_b(\theta') > \lambda_b(\theta)$ and $\lambda'_b(\theta') > \lambda'_b(\theta)$ for every $\theta' < \theta < p_b$.

Let $\bar{\Lambda}(t)$ be the buyer’s concession rate at time $t$ when the seller offers $p_s$ and let $\bar{\Lambda}'(t)$ be the buyer’s concession rate at time $t$ when the seller offers $p'_s$, which are well-defined except for a finite number of points. Let $\Lambda(t) \equiv \lim_{t \to \cdot} \bar{\Lambda}(t^n)$ and let $\Lambda'(t) \equiv \lim_{t \to \cdot} \bar{\Lambda}'(t^n)$.

First, we show that $\Lambda(\eta) > \Lambda'(\eta)$ for $\eta = 0$ as well as every $\eta$ that is sufficiently close to 0. This is because otherwise, the lowest type that offers $p_s$, denoted by $\theta$, is strictly greater than the lowest type that offers $p'_s$, denoted by $\theta'$. If this is the case, then type $\theta$ has a strict incentive to deviate by offering $p'_s$ and conceding at time $\varepsilon \approx 0$. This is because (C.4) implies that type $\theta$ is indifferent between offering $p_s$ and conceding at time 0 and offering $p'_s$ and conceding at time 0, from which he obtains his equilibrium payoff since he offers $p_s$ and concedes at time 0 with positive probability. By definition, type $\theta'$ is indifferent between offering $p'_s$ and conceding at time 0 and offering $p'_s$ and conceding at time $\varepsilon > 0$, for $\varepsilon$ small enough. Therefore, type $\theta$ strictly prefers offering $p'_s$ and conceding at time $\varepsilon$ to his equilibrium strategy, which leads to a contradiction.

Let $t^*$ be the smallest $t \in \mathbb{R}_+$ such that $\Lambda(t) < \Lambda'(t)$. Such $t^*$ exists since otherwise, $\Lambda(t) \geq \Lambda'(t)$ for every $t$ and the previous step implies that $\Lambda(t) > \Lambda'(t)$ for some $t$. Since $c_b \geq c'_b$, the rational-type of the buyer will finish conceding sooner when the seller offers $p_s$ compared to the case in which the seller offers $p'_s$. This contradicts our earlier conclusion that $T > T'$.

Given the existence of such $t^*$, we know that the type of seller who starts to concede at time $t^*$ is strictly greater under offer $p_s$, which we abuse notation and denote it by $\theta$, compared to that under offer $p'_s$, which we abuse notation and denote it by $\theta'$. This is because $\lambda_b(\theta) > \lambda'_b(\theta)$ for every $\theta < p_b$ and both $\lambda_b(\cdot)$ and $\lambda'_b(\cdot)$ are strictly decreasing functions of $\theta$. Let $A(t^*)$ be the discounted probability that the buyer concedes to the seller before time $t^*$ when the seller offers $p_s$. Let $B(t^*)$ be the probability that the buyer has not conceded by time $t^*$ when the seller offers $p_s$. Let $A'(t^*)$ and $B'(t^*)$ denote the same variables when the seller offers $p'_s$. Type $\theta'$ weakly prefers offering $p'_s$ and conceding at $t^*$ to offering $p_s$ and conceding at $t^*$, which gives:

$$A'(t^*)(p'_s - \theta') + B'(t^*)e^{-rt^*}(p_b - \theta') \geq A(t^*)(p_s - \theta') + B(t^*)e^{-rt^*}(p_b - \theta').$$  \hspace{1cm} (C.7)
Notice that $A(t^*) + e^{-rt^*} B(t^*)$ depends only on the expected delay in trade when the seller offers $p_s$ and concedes at time $t^*$, and $A'(t^*) + e^{-rt^*} B'(t^*)$ depends only on the expected delay in trade when the seller offers $p'_s$ and concedes at time $t^*$. The definition of $t^*$ implies that $\Lambda(t) \geq \Lambda'(t)$ for every $t < t^*$ with strict inequality holds for every $t$ that is close enough to 0. Therefore, less delay is incurred when the seller offers $p_s$, which implies that

$$A(t^*) + e^{-rt^*} B(t^*) > A'(t^*) + e^{-rt^*} B'(t^*) \tag{C.8}$$

Since $\theta > \theta'$, inequalities (C.7) and (C.8) together imply that

$$A'(t^*)(p'_s - \theta) + B'(t^*) e^{-rt^*}(p_b - \theta) > A(t^*)(p_s - \theta) + B(t^*) e^{-rt^*}(p_b - \theta). \tag{C.9}$$

Inequality (C.9) suggests that type $\theta$ strictly prefers offering $p'_s$ and conceeding at time $t^*$, to offering $p_s$ and conceeding at time $t^*$. However, type $\theta$ is supposed to play the latter strategy with positive probability in equilibrium. This implies that type $\theta$ has a strictly profitable deviation, which leads to a contradiction.

The above contradiction implies that when the buyer offers $p_b \in P_b$ that is offered with positive probability by the rational-type buyer, every type of the seller with cost weakly greater than $p_b$ will demand 1 with probability 1.

\[\Box\]

## D Proof of Theorem 3

We follow the same steps as in the Proof of Theorem 1. Fix $\pi \in \Delta(\Theta)$, and let $(\sigma_b, \sigma_s, \tau_b, \tau_s)$ be an equilibrium strategy profile of the bargaining game. Let

$$\hat{\varepsilon}_b(p_b) = \frac{\varepsilon \mu_b(p_b)}{\varepsilon \mu_b(p_b) + (1 - \varepsilon) \sigma_b(p_b)}, \tag{D.1}$$

$$\hat{\varepsilon}_s(p_b, p_s) = \frac{\varepsilon \mu_s(p_s)}{\varepsilon \mu_s(p_s) + (1 - \varepsilon) \sum_{j=1}^{n} \pi(\theta_j) \sigma_s(p_s | \theta_j, p_b)}, \tag{D.2}$$

$$\hat{\pi}_j(p_b, p_s) = \frac{(1 - \varepsilon) \pi(\theta_j) \sigma_s(p_s | \theta_j, p_b)}{\varepsilon \mu_s(p_s) + (1 - \varepsilon) \sum_{j=1}^{n} \pi(\theta_j) \sigma_s(p_s | \theta_j, p_b)}, \text{ for every } j \in \{1, \ldots, n\}. \tag{D.3}$$

Fix $(p_b, p_s)$ and the resulting $(\hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi}_1, \ldots, \hat{\pi}_n)$. We characterize equilibrium strategies in the
continuation game $\Gamma(p_b, p_s, \hat{\epsilon}_b, \hat{\epsilon}_s, \hat{\pi})$, when $\theta_1 < p_b < p_s < 1$. To do so, as in Section 4.1, let

$$m \equiv \max\{j \in \{1, \ldots, n\} : \theta_j < p_b\},$$

And for every $j \in \{1, \ldots, m\}$, define $\lambda_b^j \equiv \frac{r(p_b - \theta_j)}{p_s - p_b}$, and

$$T_s^j \equiv \frac{\log(\beta_s + \sum_{i > j} \hat{\pi}_i)}{\lambda_s}, \quad T_b \equiv \frac{-\log(\beta_b) - \sum_{j=1}^{m-1} (\lambda_b^j - \lambda_b^{j+1}) T_b^j}{\lambda_b^m},$$

$$L \equiv \frac{-\lambda_s \log \beta_b}{-\sum_{j=1}^{m} (\lambda_b^j - \lambda_b^{j+1}) \log(\beta_s + \hat{\pi}_{j+1})},$$

$$\hat{c}_s^j \equiv 1 - \left(\frac{\beta_s - \lambda_b^j}{\beta_s + \hat{\pi}_{j+1}}\right)^{\lambda_b^j - \lambda_b^{j+1}} \frac{1}{\lambda_b^j}, \quad \hat{c}_b = 1 - \hat{c}_b \exp\left\{\sum_{i=1}^{m} \lambda_b^j (T_s^j - T_s^{j-1})\right\}.$$ 

The next series of Lemmas extends the equilibrium characterization of the game $\Gamma(p_b, p_s, \hat{\epsilon}_b, \hat{\epsilon}_s, \hat{\pi})$ to the case in which $\Theta$ has more than two elements. The arguments are exactly the same as in the two-type setting.

Let $j^* \equiv \min\{j \in \{1, \ldots, m\} : \hat{c}_s^j < \sum_{i < j} \hat{\pi}_i\}$.

**Lemma D.1.** Fix offers $(p_b, p_s)$ with $1 > p_s > p_b > \theta_1$. In any equilibrium of $\Gamma(p_b, p_s, \hat{\epsilon}_b, \hat{\epsilon}_s, \hat{\pi})$, the buyer concedes with positive probability at time zero if and only if $L > 1$ and the seller concedes with positive probability at time 0 if and only if $L < 1$. Players’ concession probabilities at time 0 are $c_b \equiv \max\{0, \hat{c}_b\}$ and $c_s \equiv \max\{0, \hat{c}_s^j\}$, respectively.

Let $T_j = T_s^j + \frac{\log(1-c_s)}{\lambda_s}$ for all $j \in \{j^*, \ldots, m-1\}$ and $T_m \equiv \min\left\{\frac{-\log(\beta_b) - \sum_{j=1}^{m-1} (\lambda_b^j - \lambda_b^{j+1}) T_b^j}{\lambda_b^m}, T_s^m\right\}$.

**Lemma D.2.** In every equilibrium of the war-of-attrition game $\Gamma(p_b, p_s, \hat{\epsilon}_b, \hat{\epsilon}_s, \hat{\pi})$ in which $p_b > \theta_1$ and $p_s < 1$, players’ equilibrium concession times $\tau_b$ and $\tau_s(\theta)$ must satisfy:

1. For every $j \in \{j^*, \ldots, m\}$, the buyer concedes at rate $\lambda_b^j$ when $t \in (T_j-1, T_j)$ with $T_j^*-1 = 0$.

2. The seller with production cost $\theta \in \{\theta^*_1, \ldots, \theta_m\}$ concedes at rate $\lambda_s$ when $t \in (T_j-1, T_j)$ with $T_j^*-1 = 0$.

3. The seller never concedes if his production cost is strictly greater than $\theta_m$.

Next, we characterize players’ concession probabilities at time 0 in the limit where $\varepsilon \to 0$. Consider an infinite sequence $\{\varepsilon^k\}_{k=0}^{\infty}$ satisfying $\varepsilon^k \to 0$ as $k \to \infty$. Let $(\sigma_b^k, \sigma_s^k)$ be players’ equilibrium
bargaining strategies when the ex ante probability of commitment types is $\varepsilon^k$, and $(\sigma^\infty_b, \sigma^\infty_s)$ be a subsequential limit. Let $(\hat{\varepsilon}_b^k, \hat{\varepsilon}_s^k, \hat{\tau}^k)$ be given by [D.1], [D.2] and [D.3] using $(\varepsilon^k, \sigma^k_b, \sigma^k_s)$, and let $\lim_{k \to \infty} \hat{\tau}^k_j = \hat{\tau}^\infty_j$ for every $j \in \{1, ..., n\}$ and $\hat{\varepsilon}^\infty_i = \lim_{k \to \infty} \hat{\varepsilon}^k_i$ for every $i \in \{b, s\}$.

Lemma D.3. Suppose $\{\varepsilon^k\}_{k=1}^\infty$ is such that $\varepsilon^k \to 0$ as $k \to \infty$. Let $(c^\infty_b, c^\infty_s)_{k=1}^\infty$ be given according to Lemma [D.1] in the game $\Gamma(p_b, p_s, \hat{\varepsilon}^k_b, \hat{\varepsilon}^k_s, \hat{\tau}^k)$ with $\theta_1 < p_b < p_s < 1$, and let $(c^\infty_b, c^\infty_s)$ be the limit as $k \to \infty$. Then

1. If $\sum_{j=1}^n \sigma^\infty_s(p_s|\theta_j, p_b) > 0$ and $\lambda^b > \lambda_s$, then $c^\infty_s(p_b, p_s) = 1$.

2. If $\sigma^\infty_b(p_b) > 0$, $\hat{\tau}^\infty_j(p_s, p_b) > 0$, and $\lambda_s > \lambda^b_j$ or $p_b \leq \theta_j$ for some $j \in \{1, ..., n\}$, then $c^\infty_b(p_b, p_s) = 1$.

3. If $\sigma^\infty_b(p_b) > 0$, and $\hat{\varepsilon}^\infty_s(p_b, p_s) > 0$ or $\lambda_s > \lambda^1_b$, then $c^\infty_b(p_b, p_s) = 1$.

We now derive the seller’s equilibrium strategy at the bargaining stage. Let $p_b \in P_b \cap (\theta_1, p_{\theta_n}]$ be an offer in the support of the buyer’s strategy. We show that all types $\theta \leq \theta_m$ (with $m$ defined as above) offer the same price that is approximately max$\{p_b, p_m(p_b)\}$ with $p_m(p_b) = 1 + \theta_m - p_b$, and all types $\theta > \theta_m$ demand 1. According to Lemma [C.1] for any $p_b$ that belongs to the support of the buyer’s strategy, all types with production cost $\theta > \theta_m$ will demand the entire surplus. Next, consider types $\theta \leq \theta_m$. Suppose first that $p_b < p_m(p_b)$. If type $\theta_m$ offers $p_s < p_m(p_b)$, then his payoff is approximately $p_s - \theta_m$, by part (2) of Lemma D.3. In order to prevent this type from deviating to $p_m(p_b)$, it must be that some type $\theta' < \theta_m$ offers $p_m(p_b)$ in equilibrium. Letting $\bar{\theta}$ be the highest type below $\theta_m$ that offers $p_m(p_b)$ in equilibrium, type $\theta_m$’s payoff from deviating to $p_m(p_b)$ is bounded below by

$$\left( c_b + (1 - c_b) \frac{p_b - \bar{\theta}}{p_m(p_b) - \bar{\theta}} \right) (p_m(p_b) - \theta_m) \geq \frac{p_s - \bar{\theta}}{p_m(p_b) - \bar{\theta}} (p_m(p_b) - \theta_m) > p_s - \theta_m,$$

Which gives rise to a contradiction. If otherwise type $\theta_m$ demands $p_s > p_m(p_b)$, then the fact that all types higher than $\theta_m$ demand 1 almost surely implies that type $\theta_m$ has to concede immediately after he demands $p_s$, which is again dominated by demanding $p_m(p_b)$ instead. Since type $\theta_m$ makes an offer close to $p_m(p_b)$ with probability close to one, it immediately follows that type $\theta < \theta_m$ finds it optimal to do so as well. Otherwise, if $p_b > p_m(p_b)$, all types $\theta \leq \theta_m$ have to concede immediately when they offer anything higher than $p_b$, and thus any such strategy is equivalent to offering $p_b$.

Given the above, any offer $p_b \in P_b \cap (\theta_1, p_{\theta_n}]$ in the support of the buyer’s equilibrium strategy yields a payoff of approximately $\pi[\theta_1, \theta_m](1 - \max\{p_b, p_m(p_b)\})$, which is maximized at
$p_b \approx \min\{p_{\theta_i^*}, \theta_{i+1}^*\}$. Therefore, the buyer offers $\min\{p_{\theta_i^*}, \theta_{i+1}^*\}$ with probability close to one when $\varepsilon$ is small enough.

Given the above equilibrium strategies, the outcome is approximately efficient conditional on the seller’s type being $\theta \leq \theta_{i^*}$. Conditional on type $\theta > \theta_{i^*}$, we use the seller’s incentive compatibility constraint to derive bounds on expected delay. First, in order to ensure that type $\theta_{i^*}$ doesn’t benefit from deviating to demanding 1 after the buyer offers $\min\{p_{\theta_i^*}, \theta_{i+1}^*\}$ it must be that

$$\max\{p_{\theta_i^*}, 1 + \theta_{i^*} - \theta_{i+1}^*\} - \theta_{i^*} \geq (1 - \theta_{i^*})E[e^{-r\tau_b}|\theta > \theta_{i^*}],$$

Which is also sufficient to ensure that types $\theta < \theta_{i^*}$ don’t benefit from deviating. On the other hand, when $\varepsilon$ and $\nu$ are small, the condition preventing type $\theta > \theta_{i^*}$ from deviating to $\bar{p}_s$ after the buyer offers $\min\{p_{\theta_i^*}, \theta_{i+1}^*\}$ and waiting for the buyer to concede is

$$(1 - \theta)E[e^{-r\tau_b}|\theta > \theta_{i^*}] \geq \max\{p_{\theta_i^*}, 1 + \theta_{i^*} - \theta_{i+1}^*\} - \theta_{i^*} \frac{1}{1 - \theta_{i^*}}(1 - \theta).$$

The two conditions combined give rise to the expected welfare in (5.2). The proof of Theorem 3 is completed by showing equilibrium existence, which has been established in Online Appendix A.

### E Proof of Theorem 4

Let $V_\theta$ denote type $\theta$’s equilibrium payoff net of the cost of adopting technologies. Let $\pi \in \Delta(\Theta)$ be the seller’s equilibrium adoption decision.

We begin by showing the second part of Theorem 4. Suppose that condition (5.3) is satisfied and moreover $C$ satisfies

$$c_1 - c_j < \frac{c_1(1 - \theta_1) - (\theta_n - \theta_1)(1 - \min\{p_{\theta_1}, \theta_n\})}{(\theta_n - \theta_1)(\min\{p_{\theta_1}, \theta_n\} - \theta_1)}(\theta_j - \theta_1), \quad \forall j = 2, \ldots, n - 1, \quad (E.1)$$

$$c_1 \in \left(\max\left\{\frac{1}{2}, \frac{1 - \theta_n}{1 - \theta_1}\right\}(\theta_n - \theta_1), \theta_n - \theta_1\right). \quad (E.2)$$

The set of production costs satisfying (E.1), and (E.2) is open (in $\mathbb{R}^{n-1}$, given that we are fixing $c_n = 0$). It is also non-empty, given that the set of conditions are satisfied by $C$ such that $c_1$ satisfies (E.2), and ($c_2, \ldots, c_{n-1}$) are sufficiently close to $c_1$.

Conditions (E.1) and (E.2) combined imply that $j^\circ = 1$. We construct an equilibrium where the seller adopts $(\theta_{j^\circ}, c_{j^\circ})$ with probability strictly less than one. In particular, consider the adoption
strategy for the seller where
\[
\pi(\theta^{*}) \approx \frac{p_{\theta_{n}} - \theta_{n}}{\min\{p_{\theta^{*}}, \theta_{n}\} - \theta^{*}}, \quad \pi(\theta_{n}) = 1 - \pi(\theta^{*}).
\]

Note that (5.3) implies that \(\pi(\theta^{*}) \leq 1 - \theta_{n}^{2}(\theta_{n} - \theta^{*}) < 1\).

Consider the buyer’s strategy at the bargaining stage that assigns probability arbitrarily close to one to:

- offer \(p_{\theta_{n}}\) with probability \(\rho \in (0, 1)\), and offer \(\min\{p_{\theta^{*}}, \theta_{n}\}\) with probability \(1 - \rho\).

The exact value of \(\pi(\theta^{*})\) will depend on \(\varepsilon\), and is chosen so as to ensure that the buyer is indifferent between offering \(p_{\theta_{n}}\) and making the screening offer \(\min\{p_{\theta^{*}}, \theta_{n}\}\). Moreover, as argued in the proof of Theorem 3, the buyer assigns vanishing probability to any other offer. We also showed there that, after the buyer offers \(p_{\theta_{n}}\), an agreement is reached with negligible delay; and after the buyer offers \(\min\{p_{\theta^{*}}, \theta_{n}\}\), type \(\theta^{*}\) trades almost immediately at a price close to \(\hat{p} \equiv 1 + \theta^{*} - \min\{p_{\theta^{*}}, \theta_{n}\}\), and type \(\theta_{n}\) demands 1 and trades with expected delay \(\hat{p} - \theta^{*}\). Therefore, in order to ensure that the seller is indifferent between adopting \(\theta^{*}\) and \(\theta_{n}\), we choose \(\rho\) so as to satisfy

\[
\rho(p_{\theta_{n}} - \theta_{n}) + (1 - \rho)(\hat{p} - \theta^{*})(1 - \theta_{n}) \approx \rho(p_{\theta_{n}} - \theta^{*}) + (1 - \rho)(\hat{p} - \theta^{*}) - c^{*}
\]

\[\iff \rho \approx \frac{c^{*}(1 - \theta^{*}) - (\theta_{n} - \theta^{*})(\hat{p} - \theta^{*})}{(\theta_{n} - \theta^{*})(1 - \hat{p})}.
\]

Where again we are writing the limiting value of \(\rho\) that achieves the seller’s indifference, while keeping in mind that its exact value will depend on \(\varepsilon\). \(\rho \in (0, 1)\) is ensured by (E.2).

It remains to verify that the seller doesn’t benefit from deviating to an alternative technology \((\theta_{j}, c_{j})\), with \(j \notin \{1, n\}\). To do so, we use to following auxiliary result. Let \(V_{\theta}(p_{b})\) be type \(\theta\)’s continuation payoff after the buyer offers \(p_{b} \in [0, 1]\).

**Lemma E.1.** *In any equilibrium, \(V_{\theta}(p_{b})\) is weakly decreasing in \(\theta\).*

**Proof.** Take \(\theta, \theta' \in \Theta\) and suppose that \(\theta' > \theta\). Let \(p_{b} \in [0, 1]\) be any offer by the buyer, and let \(p \in [0, 1]\) be an offer in the support of type \(\theta'\)’s equilibrium strategy in the bargaining game after the buyer offers \(p_{b}\). Let \(T \in \mathbb{R}_{+} \cup \{+\infty\}\) be any concession time in the support of \(\tau_{s}(\theta', p_{b}, p)\). His
continuation payoff following the buyer’s offer $p_b$ can be written as

$$
V_{\theta'}(p_b) = \int_0^T e^{-rt} d\tau_b(p_b, p)(p - \theta') + \tau_b(p_b, p)[T, +\infty]e^{-rT}(p_b - \theta') \leq \int_0^T e^{-rt} d\tau_b(p_b, p)(p - \theta) + \tau_b(p_b, p)[T, +\infty]e^{-rT}(p_b - \theta).
$$

Because type $\theta$ can always mimic type $\theta'$’s strategy, it must be that the right-hand-side of the inequality is weakly less than $V_{\theta}(p_b)$. Therefore, type $\theta'$’s equilibrium payoff is higher following every offer by the buyer.

Using Lemma [E.1] we have that the payoff from deviating to technology $j$ with $j \notin \{1, n\}$ is at most

$$
\rho(p_{\theta_n} - \theta_1) + (1 - \rho)(\hat{p} - \theta_{j^o}) - c_{j^o} = V_{\theta_{j^o}} - c_{j^o},
$$

This verifies that the seller’s adoption decision is sequentially rational.

In order to show the third part of Theorem 4, suppose that $p_{\theta_{j^o}} < \theta_n$ and that

$$
c_{j^o} \in \left(\frac{\theta_n - \theta_{j^o}}{2}, \theta_n - \theta_{j^o}\right). \quad (E.3)
$$

We argue that $\pi(\theta_{j^o})$ is bounded away from 1 in any equilibrium. Suppose by contradiction that $\pi(\theta_{j^o}) > 1 - \eta$ for all $\eta > 0$. By Lemma [C.1] all sellers with type $\theta > p_{\theta_{j^o}}$ will demand 1 after the buyer offers $p_{\theta_{j^o}}$. As in the proof of Theorem 2, it then follows that the buyer’s optimal offer when $\pi(\theta_{j^o})$ is high enough is $p_{\theta_{j^o}}$, and thus the seller’s equilibrium payoff is approximately $p_{\theta_{j^o}} - \theta_{j^o} - c_{j^o}$. On the other hand, by deviating to $\theta_n$ (the fact that $c_{j^o} + \theta_{j^o} < \theta_n$ implies that $\theta_{j^o} \neq \theta_n$), demanding approximately 1 and waiting for concession after the buyer offers $p_{\theta_{j^o}}$, the seller can secure a payoff of $1 - \frac{\theta_n}{2}$. This is strictly better than his equilibrium payoff $p_{\theta_{j^o}} - \theta_{j^o} - c_{j^o}$, under condition [E.3].

This establishes the third part of Theorem 4. The argument for the first part is given in the main text (Section 5). The existence of equilibrium has been established in Online Appendix B.

References
