Determinacy without the Taylor Principle*

George-Marios Angeletos† **Chen Lian**‡

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Abstract

Our understanding of monetary policy is complicated by an equilibrium-selection conundrum: because the same path for the nominal interest rate can be associated with multiple equilibrium paths for inflation and output, there is a long-standing debate about what the right equilibrium selection is. We offer a potential resolution by showing that small frictions in social memory and intertemporal coordination can remove the indeterminacy. Under our perturbations, the unique surviving equilibrium is the same as that selected by the Taylor principle, but it no more relies on it; monetary policy is left to play only a stabilization role; and fiscal policy needs to be Ricardian, even when monetary policy is passive.

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†MIT and NBER; angelet@mit.edu.

‡UC Berkeley and NBER; chen_lian@berkeley.edu.

1 Introduction

Can monetary policy regulate inflation and aggregate demand? Does the ZLB trigger a deflationary spiral? Does Ricardian equivalence hold when taxation is non-distortionary, markets are complete, and consumers have rational expectations and infinite horizons? One may be inclined to respond "yes" to all these questions. But the correct answer, at least within the dominant policy paradigm (the New Keynesian model), crucially depends on how equilibrium is selected.

The basic problem goes back to Sargent and Wallace (1975): the same path for the nominal interest rate is consistent with multiple equilibrium paths for inflation and output.[1] The standard approach selects a specific equilibrium by assuming that monetary policy satisfies the Taylor principle (Taylor, 1993), or equivalently that it is "active" in the sense of Leeper (1991). It is this selection that drives the model's customary predictions, including the "yes" to the aforementioned questions. But as stressed by Cochrane (2017, 2018), an alternative selection, based on the Fiscal Theory of the Price Level (FTPL), can lead to sharply different predictions. This approach elevates government debt and deficits to key drivers of inflation and output, even when these variables do not enter the model's three "famous" equations.[2]

Both approaches are equally coherent, at least in the sense of being consistent with rational expectations and the same micro-foundations. They are also hard to test, because they translate to different assumptions about off-equilibrium strategies of the monetary and fiscal authorities. As a result, the debate about which approach is "right" has never been settled.[3]

We shed new light on this conundrum, and offer a possible way out of it, by demonstrating how the indeterminacy problem of the New Keynesian model hinges on delicate assumptions about social memory and dynamic coordination. Once we perturb these assumptions, appropriately but tinily, the model's conventional solution emerges as the unique rational expectations equilibrium regardless of monetary policy. This reinforces the logic for answering "yes" to the questions raised in the beginning. And it allows one to think about *both* the Taylor principle and the FTPL in new ways, liberated from the equilibrium-selection conundrum.

Preview of results. A crucial stepping stone of our analysis is the translation of a New Keynesian economy into a dynamic game among the consumers. The details are spelled out in Section 2 but the basic idea is that an individual's optimal spending depends on her expectations of future ag-

[1] There is, however, an important difference between the New Keynesian framework and flexible-price models, such as that of Sargent and Wallace (1975), which we explain in due course.

[2] See Leeper (1991), Sims (1994) and Woodford (1995) for the genesis of the FTPL, Bassetto (2002) for a game-theoretic perspective, and Cochrane (2005, 2017, 2018) for extensions and reinterpretations.

[3] See Bassetto (2008) for a concise and balanced review of the debate; Canzoneri, Cumby, and Diba (2010) for how it fits in the broader context of the fiscal-monetary interaction; and Kocherlakota and Phelan (1999), King (2000), Bassetto (2002), Cochrane (2007), and Atkeson et al. (2010) for more on the role of off-equilibrium strategies.

gregate spending via GE feedbacks such as the intertemporal Keynesian cross. This explains both why our game-theoretic prism extends to a wide class of Keynesian economies (think HANK) and why the key issue is the ability of current and future consumers to coordinate on multiple self-fulfilling paths for aggregate spending. Our contribution is to expose the fragility of such coordination to a small friction in social memory.

Our main result, developed in Section 4, models the relevant friction as follows. There are overlapping generations of finitely-lived consumers. Consumers learn all the shocks (payoff-relevant or not) realized during their lifetime, but the intergenerational transmission of such knowledge need not be perfect: for any t, the fraction of the population who know or can otherwise condition their actions on shocks realized at any $\tau \leq t$ is $(1-\lambda)^{t-\tau}$, where $\lambda \in [0,1)$ parameterizes the erosion of social memory over time.

The standard paradigm is nested with $\lambda = 0$; it translates to assuming that any shock remains common knowledge in perpetuity; and it admits a continuum of sunspot and "backward-looking" equilibria whenever the Taylor principle is violated and fiscal policy is Ricardian. We instead show that all these equilibria unravel as soon as $\lambda > 0$. The only surviving equilibrium is the conventional one, known as the fundamental or minimum-state variable (MSV) solution.

This result allows knowledge of long histories of the exogenous shocks but abstracts from direct observation of past endogenous outcomes such as the past levels of aggregate spending and inflation. This abstraction is at odds with both the recursive equilibrium representation common in macroeconomics and a literature that has shown how endogenous outcomes can themselves serve as coordination devices (Angeletos and Werning, 2006). We address this concern in Section 5 with two additional results, both of which allow for direct observation of past output and past inflation. This requires an adjustment in the relevant perturbation—in particular, Proposition 4 requires immediate forgetting of a small component of the fundamentals—but the take-home lesson remains the same.[4]

Interpreting our contribution. The key logic behind our results echoes the literature on global games (Morris and Shin, 2002, 2003) and is subject to a similar qualification: indeterminacy may strike back if markets or other mechanisms facilitate enough coordination (Atkeson, 2000; Angeletos and Werning, 2006). Note, however, that our context's multiplicity is sustained by a self-fulfilling infinite chain over different generations of players: today's consumers are responding to a payoff-irrelevant variable because and only because they expect tomorrow's consumers to do the same on the basis of a similar expectation about behavior further into the future, and so on.

[4]Section 5 focuses on the observability of past output and inflation, because, as explained these variables emerge as the "right" endogenous sunspots in our baseline model. It is an open question whether *other* endogenous variables, such as public debt in the FTPL context, can serve as more potent coordination devices.

This suggests that the requisite coordination might be harder to attain in our context than in, say, a self-fulfilling bank run. But a formalization of this broader idea is elusive at this point.

All in all, we therefore view our contribution not as a definite resolution of the New Keynesian model's indeterminacy problem but rather in the following terms: (i) as a new lens for understanding this problem; (ii) as a formal justification for selecting the fundamental solution; and (iii) as an invitation to reconsider the applied meaning of *both* the Taylor principle and the FTPL. The first two points should be self-evident by now, so let us expand on the last.

Consider first the Taylor principle. Our result removes the need for equilibrium selection but leaves room for sunspot-like fluctuations along the MSV equilibrium path in at least the following two forms: overreaction to noisy public news (Morris and Shin, 2002); and shocks to higher-order beliefs (Angeletos and La'O, 2013; Benhabib et al., 2015). This in turn lets the slope of the Taylor rule play a new function: to regulate the macroeconomic complementarity and thereby the aforementioned kind of sunspot-like volatility. Our contribution is therefore not to rule out "animal spirits" altogether but rather to recast the Taylor principle as a form of on-equilibrium stabilization instead of an off-equilibrium threat.[5]

Consider next the FTPL. By guaranteeing that the MSV solution is the only possible solution regardless of monetary policy, our perturbations do not allow fiscal policy to be used for equilibrium selection. Our work thus redirects attention to how one can capture the following questions *outside* the equilibrium selection conundrum: what is the relation between persistent fiscal deficits and inflation (Sargent and Wallace, 1981); which authority is "dominant;" and whether a deficit can be self-financed by an automatic adjustment in the price level and/or the tax base along the MSV solution (Angeletos et al., 2023).

Local vs global determinacy. Like most of the literature, we work with the linearized New Keynesian model and require equilibria to be bounded. This focus has two rationales within the context of interest. First, unbounded equilibria (namely self-fulfilling hyper-inflations and self-fulfilling liquidity traps) are customarily ruled out by means other than the Taylor principle[6] and are therefore outside our scope. And second, we, as analysts, have more trust in the New Keynesian model's local properties than its global properties. Moving beyond the context of interest, the following seems a safe conjecture: our methods and results guarantee local determinacy around any given steady state, but do not necessarily speak to the question of global determinacy.

[5]While this is the exact opposite of the prevailing theoretical approach, it is arguably closer to how Taylor (1993) himself had originally thought about the issue.

[6]These are the type of escape clauses considered in, inter alia, Wallace (1981), Obstfeld and Rogoff (1983, 2021), Taylor (1993), Christiano and Rostagno (2001), Benhabib et al. (2001), and Atkeson et al. (2010).

Sticky vs flexible prices. Our results are not sensitive to the degree of nominal rigidity, as long as there is some of it. If instead prices are perfectly flexible, output and inflation are no longer demand determined, the economy can no longer be understood as a coordination game among the consumers, and our methods do not apply (at least not in their current form). This touches on a larger methodological question, whether flexible-price models are proper limits of sticky-price models (Kocherlakota, 2020). And it suggests that, contrary to conventional wisdom, the New Keynesian model's indeterminacy problem might *not* be a mere translation of the flexible-price counterpart (Sargent and Wallace, 1975).

Related literature. Kocherlakota and Phelan (1999), Buiter (2002), Niepelt (2004) and others have interpreted the non-Ricardian assumption as an off-equilibrium threat to blow up the government budget. Cochrane (2005, 2011) has fired back by arguing not only that this interpretation is misguided but also that the Taylor principle itself amounts to a threat to blow up inflation and interest rates. While these arguments emphasize the subtlety of both approaches, they do not help resolve the conundrum: Bassetto (2002, 2005) and Atkeson, Chari, and Kehoe (2010) have shown that *both* approaches can be supported with more sophisticated policies, which avoid such controversial threats and guarantee a proper continuation equilibrium always. By contrast, our paper seeks to remove the need for equilibrium selection of either kind.

Our main result, Proposition 2, recalls Rubinstein (1989) and the global-games literature (Morris and Shin, 1998, 2003): certain equilibria unravel because of a series of contagion effects related to higher-order beliefs. Our second result, Proposition 3, has the flavor of rational inattention: agents observe an endogenous coordination device with idiosyncratic noise. Our third result, Proposition 4, connects to Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012): it combines a purification in payoffs with finite social memory.[7] The common thread is the relaxation of common knowledge and the resulting coordination friction. But the precise connections between our results and the related literature deserve further exploration.

A large literature has already incorporated information/coordination frictions in the New Keynesian model (Mankiw and Reis, 2002; Woodford, 2003; Maćkowiak and Wiederholt, 2009; Angeletos and Lian, 2018). But it has *not* addressed the determinacy issue: it has focused exclusively on how such frictions improve the empirical properties of the model's MSV solution, while assuming away all other solutions (by invoking, implicitly or explicitly, the Taylor principle). We complement this literature by showing, in effect, that one can hit two birds with one stone.

[7]Bounded recall is well documented is psychology (Kahana, 2012) and has found important applications in economics (e.g., Gennaioli and Shleifer, 2010; da Silveira et al., 2020). While we welcome this interpretation, for our purposes, just as for the aforementioned papers, it suffices to have bounded *social* memory: the key is to introduce a friction in intertemporal coordination.

A different literature has studied which of the model's solutions are "learnable" in the sense of E-stability (McCallum, 2007; Christiano et al., 2018). Although this literature has produced mixed results,[8] it offers complementary light on the question of which solution is most sensible.

The determinacy problem we are after extends from Rational Expectations Equilibrium (REE) to a larger class of solution concepts that relax the perfect coincidence between subjective beliefs and objective outcomes but preserve a fixed-point relation between them. This class includes cognitive discounting (Gabaix, 2020) and diagnostic expectations (Bordalo et al., 2018), but not Level-K Thinking (García-Schmidt and Woodford, 2019; Farhi and Werning, 2019). The latter produces a unique solution because it rules out *entirely* any feedback from objective reality to subjective beliefs. This may be reasonable in the context of unprecedented experiences but seems less appropriate in the context of stationary environments, which is our focus here.

2 A Simplified New Keynesian Model

Here we introduce our baseline model. Time is discrete and is indexed by $t \in \mathbb{N}$. There are overlapping generations of consumers, each living two periods. Each generation has mass one half; the young and the old receive the same income; information is the only possible source of heterogeneity; and there is no fiscal authority and no public debt. These and a few other simplifications add transparency to the analysis but do not drive the results; Section 6 will discuss how our arguments extend to a large class of New Keynesian economies.[9]

The basics

Consider a consumer i born at t and let $C^1_{i,t}$ and $C^2_{i,t+1}$ denote her consumption when young and old, respectively. Her preferences are given by

$$u(C^1_{i,t}) + \beta u(C^2_{i,t+1}) e^{-\varrho_t}, \tag{1}$$

where $u(C) \equiv \frac{1}{1-1/\sigma} C^{1-1/\sigma}$, $\beta \in (0,1)$, and ϱ_t is an intertemporal preference shock (the usual proxy for aggregate demand shocks). Her budget constraints in the first and second period of life are

[8] For example, sunspot equilibria can be E-stable if the interest rate rule is written as a function of expected inflation (Honkapohja and Mitra, 2004). And there is a debate on how the E-stability of backward-looking solutions depends on the observability of shocks (Cochrane, 2011; Evans and McGough, 2018).

[9] In particular, it is worth emphasizing that our upcoming uniqueness result does not depend on the finite-horizon specification. Also, although finite horizons (or incomplete markets more generally) can break Ricardian equivalence, this is unrelated to the non-Ricardian assumption in FTPL; the latter regards exclusively equilibrium selection and operates with finite and infinite horizons alike.

given by, respectively,

$$P_t C^1_{i,t} + B_{i,t} = P_t Y_t \qquad \text{and} \qquad P_{t+1} C^2_{i,t+1} = P_{t+1}(Y_{t+1} - T_{t+1}) + I_t B_{i,t},$$

where P_t is the nominal price level at t, Y_t is the real aggregate (also average) income at t, $B_{i,t}$ is nominal saving/borrowing when young, T_{t+1} is a real lump-sum tax paid (or transfer received) when old, and I_t is the gross nominal interest rate between t and $t+1$.[10]

Private saving/borrowing is done in nominal claims against the central bank ("reserves"). The central bank sets the interest rate on these claims and clears their aggregate value with the aforementioned lump-sum taxes; that is, $P_t T_t = I_{t-1} B_{t-1}$, where $B_{t-1} \equiv \int B_{i,t-1} di$.[11] As it will become clear momentarily, the aggregate claims are necessarily zero in equilibrium ($B_t = 0$). Hence, it would have been without serious loss to abstract from the taxes and directly impose that nominal bonds are in zero net supply, as it is typically done in textbook treatments. The reason we spell out the additional details here is purely auxiliary: to let consumers have fully specified beliefs both on and off equilibrium.

Old consumers are "robots" in the sense that they have no optimization margin: their consumption mechanically adjusts to meet the second-period budget. By contrast, young consumers are "strategic" in the sense that they optimally choose between consumption and saving, on the basis of rational beliefs about the state of the economy.

For the time being, we take no stand on what information these beliefs are based on. We only require that individual consumption is optimal, given possibly arbitrary information. After the usual log-linearization,[12] this translates to the following optimal consumption function:

$$c^1_{i,t} = E_{i,t}\left[\tfrac{1}{1+\beta} y_t + \tfrac{\beta}{1+\beta}\left(y_{t+1} - \tau_{t+1}\right) - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right], \tag{2}$$

where $E_{i,t}$ denotes the rational expectation conditional on i's information. The familiar, full-information benchmark is nested by restricting $E_{i,t}[\cdot] = \mathbb{E}_t[\cdot]$ for all i and t, where $\mathbb{E}_t[\cdot]$ is the rational expectation conditional on perfect knowledge of all current and past shocks; our own contribution will rest on relaxing this restriction.

[10] To ease the exposition, we side-step labor supply. The missing details are filled in Appendix B.2 but the basic point is this: because output is demand-determined, the specification of labor supply is inconsequential. Also note that the young and the old earn the same income, which guarantees that the steady state interest rate is $\beta^{-1} > 1$ and there is no room for bubbly money a la Samuelson (1958).

[11] Throughout, $\int x_i di$ means the cross-sectional average of variable x.

[12] Throughout, we log-linearize around the steady state in which $\varrho_t = 0$, $\Pi_t \equiv \frac{P_t}{P_{t-1}} = 1$, and $I_t = \beta^{-1}$; and for any variable X_t with steady-state value X^{ss}, we define the corresponding lower-case variable as $x_t \equiv \log X_t - \log X^{ss}$ if $X^{ss} \neq 0$ and $x_t \equiv X_t / Y^{ss}$ if $X^{ss} = 0$. For example, $y_t = \log Y_t - \log Y^{ss}$ but $\tau_t \equiv T_t / Y^{ss}$. This is standard practice.

An intertemporal Keynesian cross (aka a Dynamic IS equation)

Pick any t. By aggregating the budgets of the old and using the fact that $P_t T_t = I_{t-1} \int B_{i,t-1} di$, we get $\int C^2_{i,t} di = Y_t$. By market clearing in the goods market, $C_t \equiv \frac{1}{2}\int C^1_{i,t} di + \frac{1}{2}\int C^2_{i,t} di = Y_t$. Combining, we infer that $\int C^1_{i,t} di = Y_t = C_t$. By aggregating the budgets of the young, we then verify that $\int B_{i,t} di = 0$. (As anticipated, it is as if the net supply of bonds were zero.)

More importantly, from the fact that $\int C^1_{i,t} di = Y_t = C_t$, we see that aggregate consumption coincides with the average consumption of the young. Translating this in log deviations, aggregating (2), and replacing $y_t = c_t$, we conclude that, for any process of the interest rate and inflation, the process for aggregate spending must satisfy the following equation:

$$c_t = \bar{E}_t \left[\tfrac{1}{1+\beta} c_t + \tfrac{\beta}{1+\beta} c_{t+1} - \tfrac{\beta}{1+\beta} \sigma (i_t - \pi_{t+1} - \varrho_t) \right], \tag{3}$$

where $\bar{E}_t [\cdot] = \int E_{i,t} [\cdot] \, di$ is the average expectation of the young.

As evident from its derivation, equation (3) makes no assumption about how interest rates and inflation are determined. It only combines consumer optimality with market clearing and, in so doing, it embeds the GE feedback between income and spending. This equation can thus be read interchangeably as a special case of the "intertemporal Keynesian cross" (Auclert et al., 2018) and as a Dynamic IS equation.

Connection to the standard New Keynesian model

Although our version of the Dynamic IS equation looks different from its textbook counterpart, it actually nests it when there is full information. In this benchmark, $\bar{E}_t$ can be replaced by $\mathbb{E}_t$, which henceforth denotes the rational expectation conditional on full information about the economy's history up to, and inclusive of, period t. Along with the fact that c_t and i_t must themselves be measurable in such information, this means that in this case equation (3) reduces to

$$c_t = \tfrac{1}{1+\beta} c_t + \tfrac{\beta}{1+\beta} \mathbb{E}_t [c_{t+1}] - \tfrac{\beta}{1+\beta} \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - \varrho_t),$$

or equivalently

$$c_t = \mathbb{E}_t [c_{t+1}] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - \varrho_t),$$

which is evidently the same as the Euler condition of a representative, infinitely-lived consumer.

This clarifies the dual role of the adopted OLG structure. With full information, it lets our model translate to the standard, representative-agent, New Keynesian model. And away from this benchmark, it eases the exposition by letting the intertemporal Keynesian cross take a particularly simple form and by equating the players in our upcoming game representation to the young consumers. These simplifications are relaxed in Section 6, without changing the essence.

A Phillips curve and a Taylor rule

For the main analysis, we abstract from optimal price-setting behavior (firms are "robots") and impose the following, ad hoc Phillips curve:

$$\pi_t = \kappa(y_t + \xi_t), \tag{4}$$

where $\kappa \geq 0$ is a fixed scalar and ξ_t is a "supply" or "cost-push" shock. The absence of a forward-looking term in (4) simplifies the exposition significantly, but does not drive the results: as shown in Section 6, our arguments directly extend to the fully micro-founded, forward-looking, New Keynesian Phillips curve. With either version of the Phillips curve, the essence (for our purposes) is that there is a positive GE feedback from aggregate output to inflation. Equation (4) merely stylizes this feedback in a convenient form.

We finally assume that monetary policy follows a Taylor rule of the following type:

$$i_t = z_t + \phi\pi_t, \tag{5}$$

where z_t is a random variable and $\phi \geq 0$ is a fixed scalar. This readily nests $i_t = i_t^* + \phi(\pi_t - \pi_t^*) + \zeta_t$, where i_t^* and π_t^* are state-contingent "targets" and ζ_t is a pure monetary shock. Also, no restriction is imposed on how z_t covaries with ϱ_t and ξ_t; for instance, z_t may track the natural rate of interest or lean against cost-push shocks. In the standard paradigm, this helps disentangle the stabilization and equilibrium selection functions of monetary policy: the former is served by the design of z_t, the latter by the restriction $\phi > 1$.[13] Our perturbations will dispense with the latter function and guarantee determinacy even under interest-rate pegs (herein nested by $\phi = 0$).

The model in one equation—and the economy as a game

From (4) and (5), we can readily solve for π_t and i_t as simple functions of y_t, which itself equals c_t. Replacing into (3), we conclude that the model reduces to the following single equation:

$$c_t = \bar{E}_t\left[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}\right] \tag{6}$$

where δ_0, δ_1 are fixed scalars and θ_t is a random variable.[14] These are given by

$$\delta_0 \equiv \frac{1-\beta\sigma\phi\kappa}{1+\beta} < 1, \quad \delta_1 \equiv \frac{\beta+\beta\sigma\kappa}{1+\beta} > 0, \quad \theta_t \equiv -\frac{1}{1+\phi\kappa\sigma}\left(\sigma z_t - \sigma\varrho_t + \sigma\phi\kappa\xi_t - \sigma\kappa\mathbb{E}_t[\xi_{t+1}]\right). \tag{7}$$

By construction, equation (6) summarizes private sector behavior and market clearing, for

[13] See King (2000) and Atkeson et al. (2010) for sharp articulations of this point. Also note that we are restricting $\phi \geq 0$. Letting $\phi < 0$ qualifies the Taylor principle (see footnote 17) but does not upset our own result. Finally, note that (5) has the monetary authority respond to current inflation. But as explained in Appendix B.5, our insights go through if the monetary authority responds to past inflation and/or expected future inflation.

[14] For the time being, we take no stand how much is known about θ_t or its components, which is why θ_t appears inside the expectation operator in (6). Also, the fact that θ_t is multiplied by $1-\delta_0$ is just a normalization.

any information structure and any monetary policy. Different information structures change the properties of $\bar{E}_t$ but do not change the equation itself. Similarly, different monetary policies map to different values for δ_0 or different stochastic processes for θ_t, via the choice of, respectively, a value for ϕ or a stochastic process for z_t. But for any given monetary policy, we can understand equilibrium in the private sector by studying equation (6) alone.

Equation (6) and the micro-foundations behind it also facilitate the interpretation of the economy as a game among an infinite chain of different generations of players. In this game, the players acting at t are the young consumers of that period (old consumers, firms, and the monetary authority are "robots," in the sense already explained), their actions are their consumption levels, and their payoffs are obtained as follows. Take the primitive preferences (1); use the budgets to express $C^2_{i,t+1}$ as functions of $C^1_{i,t}$ and of $(Y_t, Y_{t+1}, I_t, \Pi_{t+1})$; drop the superscript 1 from $C^1_{i,t}$ to ease the notation; and finally use the consumer's first-order knowledge of market clearing, the Phillips curve, and the Taylor rule to substitute out $(Y_t, Y_{t+1}, I_t, \Pi_{t+1})$ and express the consumer's realized utility as $U(C_{i,t}; C_t, C_{t+1}, \varrho_t, z_t, \xi_t)$, for some U.

Maximizing this payoff over a player's own action (and for arbitrary beliefs about the actions of other players) results in the following log-linearized best response, which is the individual-level counterpart of (6):

$$c^1_{i,t} = E_{i,t}\left[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}\right]. \tag{8}$$

Under this prism, δ_0 and δ_1 parameterize the intra- and inter-temporal degrees of strategic complementarity, respectively, while θ_t identifies the game's fundamental (i.e., the only payoff-relevant exogenous random variable). Finally, by regulating the strength of the underlying GE feedbacks, different values for β, κ, and ϕ map to different degrees of strategic complementarity.

This game-theoretic prism is not strictly needed: our formal arguments work directly with equation (6), which itself can be read as a "consolidated" equilibrium condition. Still, this prism helps translate the determinacy question from one about eigenvalues (Blanchard and Kahn, 1980) to one about intertemporal coordination, and in so doing it also allows us to import useful insights from the literature on global games and higher-order uncertainty.

Fundamentals, sunspots, and the equilibrium concept

Aggregate uncertainty is of two sources: fundamentals and sunspots. The former are herein conveniently summarized in θ_t. The latter are represented by a random variable η_t that is independent of the current, past, and future values of θ_t. As explained in Section 5, our arguments extend to essentially arbitrary specifications of these variables. To ease the exposition, the main analysis makes the following simplification:

Assumption 1 (**Simplification**)**.** *Both the fundamental* θ_t *and the sunspot* η_t *are i.i.d. over time, with means normalized to zero.*

Let h^t capture the history of all fundamentals and sunspots up to and including period t. To simplify the exposition, we assume that histories are infinite and, accordingly, focus on stationary equilibria. More precisely, we let $h^t \equiv \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$ and we define an equilibrium as follows:

Definition 1 (**Equilibrium**)**.** *An equilibrium is any solution to equation (6) along which: expectations are rational, although potentially based on imperfect information about* h^t*; and the outcome is given by*

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k} \tag{9}$$

where $\{a_k, \gamma_k\}$ *are known and uniformly bounded coefficients.*

Recall that consumer optimality, firm behavior, market clearing, and the policy rule have already been embedded in equation (6). It follows that the above is the standard definition of a Rational Expectations Equilibrium (REE), except for the addition of three "auxiliary" restrictions embedded in (9): linearity, boundedness, and stationarity. Linearity is needed for tractability. Boundedness amounts to saying that we are only concerned with local determinacy.[15] Finally, the stationarity restriction is without serious loss of generality; it only makes sure that all non-fundamental equilibria are treated as sunspot equilibria.[16]

Finally, and circling back to our game-theoretic prism, note that the following is true: because every consumer is infinitesimal, there is no need to specify off-equilibrium beliefs, and the economy's REE coincides with the corresponding game's Perfect Bayesian Equilibria (PBE).

3 The Standard Paradigm

In this section, we consider the full-information version of our model (which is, in essence, the standard New Keynesian model); we review its determinacy problem; and we finally contextualize our departures from this benchmark.

[15] As mentioned in the Introduction, there are two rationales for this focus: that the Taylor principle itself is exclusively about local determinacy; and that we think that the New Keynesian model is a priori designed to speak primarily to "local" phenomena as opposed to, say, hyper-inflations. Also note that the form of boundedness imposed in Definition 1 is implied by boundedness in the agents' strategies (i.e., by requiring that $c_{i,t}$ is a linear function of $\mathbb{I}_{i,t}$ with uniformly bounded coefficients, where $\mathbb{I}_{i,t}$ is a subset of h^t and denotes the agent's information set).

[16] We explain this in detail in Appendix B.3, but the basic idea is simple. Relaxing the stationarity restriction does not change the essence; it only lets some sunspot equilibria disguise as deterministic paths. But saying that something is "deterministic" amounts to saying that it is common knowledge in perpetuity, which would be in direct contradiction to the spirit of our upcoming perturbation (Assumption 2).

Full information, the MSV solution, and the Taylor principle

Suppose that all consumers know the entire h^t, at all t. As shown earlier, it is then *as if* there is a representative, fully informed and infinitely lived, consumer—just as in the textbook case. Accordingly, equation (6), which summarizes equilibrium, reduces to the following:

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}], \tag{10}$$

where $\mathbb{E}_t[\cdot] \equiv \mathbb{E}[\cdot|h^t]$ is the rational expectation conditional on full information and

$$\delta \equiv \frac{\delta_1}{1-\delta_0} = \frac{1+\kappa\sigma}{1+\phi\kappa\sigma} > 0.$$

Note that δ is necessarily positive but can be on either side of 1, depending on ϕ.

Because equation (10) is purely forward looking and θ_t is i.i.d., $c_t = c_t^F \equiv \theta_t$ is necessarily an equilibrium. This is known as the model's "fundamental" or "minimum state variable (MSV)" solution (McCallum, 1983), and is the basis of the conventional understanding of how monetary policy works. For instance, if the central bank can adjust z_t in response to the underlying demand and supply shocks, she can guarantee $\theta_t = 0$. This directly translates to $c_t = 0$ ("closing the output gap") under the MSV solution—but not under other solutions.

To rule out other solutions and justify conventional policy predictions, the standard approach imposes the Taylor principle. In our context, just as in the textbook treatment, this principle is defined by the restriction $\phi > 1$. This in turn translates to $\delta_0 + \delta_1 < 1$ and, equivalently, $\delta < 1$. The former can be read as "the overall degree of strategic complementarity is small to guarantee a unique equilibrium," the latter as "the dynamics are forward-stable." And conversely, $\phi < 1$ translates to "the complementarity is large enough to support multiple equilibria" ($\delta_0 + \delta_1 > 1$) and the "dynamics are backward-stable" ($\delta > 1$).

This discussion underscores the tight connection between our way of thinking about determinacy (the size of the strategic complementarity) and the standard way (the size of the eigenvalue). The next proposition verifies this point and also characterizes the type of equilibria that emerge in addition to the MSV solution once the Taylor principle is violated.[17]

Proposition 1 (**Full-information benchmark**)**.** *Suppose that h^t is known to every i for all t, which means in effect that there is a representative, fully informed, agent. Then:*

(i) There always exists an equilibrium, given by the fundamental/MSV solution c_t^F.

(ii) When the Taylor principle is satisfied ($\phi > 1$), the above equilibrium is the unique one.

[17] By restricting $\phi \geq 0$, we have restricted $\delta > 0$. If we allow $\delta < 0$, which is possible for ϕ sufficiently negative, Proposition 1 and the discussion after it continue to hold, provided that we recast the Taylor principle as $\delta \in (-1, 1)$, or equivalently as $\phi \in \left(-\infty, -\frac{2}{\kappa\sigma} - 1\right) \cup (1, +\infty)$. This echoes Kerr and King (1996). More importantly, our own uniqueness result does not hinge on $\delta > 0$.

(iii) When this principle is violated ($\phi < 1$)*, there exist a continuum of equilibria, given by*

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta, \tag{11}$$

where $a, b \in \mathbb{R}$ *are arbitrary scalars and* c_t^B, c_t^η *are given by*

$$\underbrace{c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k}}_{\text{backward-looking, pseudo-fundamental component}} \qquad \text{and} \qquad \underbrace{c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}}_{\text{pure sunspot component}}. \tag{12}$$

To understand the type of non-fundamental equilibria documented in part (iii) above, take equation (10), backshift it by one period, and rewrite it as follows:

$$\mathbb{E}_{t-1}[c_t] = \delta^{-1}(c_{t-1} - \theta_{t-1}). \tag{13}$$

Since η_t is unpredictable at $t-1$, the above is clearly satisfied with

$$c_t = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t, \tag{14}$$

for any $a \in \mathbb{R}$. As long as $\delta > 1$, we can iterate backward to obtain

$$c_t = -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k} + a\sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k} = c_t^B + ac_t^\eta. \tag{15}$$

This is both bounded, thanks to $\delta > 1$, and a rational-expectations solution to (13), by construction, which verifies that $c_t^B + ac_t^\eta$ constitutes an equilibrium, for any $a \in \mathbb{R}$. Part (iii) of the Proposition adds that the same is true if we replace c_t^B with any mixture of it and the MSV solution.

To illustrate what all these equilibria are, switch off momentarily the fundamental shocks. Then, $c_t^F = c_t^B = 0$ and (11) reduces to $c_t = ac_t^\eta$, which is a pure sunspot equilibrium of arbitrary amplitude. In this equilibrium, consumers respond to the current sunspot because and only because they expect future agents to keep reacting to it, in perpetuity.

Now let us switch off the sunspots and switch on the fundamentals. Multiplicity then takes the following form: the same path for interest rates or other fundamentals maps to a continuum of different paths for aggregate spending and inflation. Consider, for example, the solution given by $c_t = c_t^B$. Along it, aggregate spending is invariant to the current interest rate and *increases* with past interest rates. This may sound paradoxical but is sustained by basically the same self-fulling infinite chain as that described above: consumers spend more in response to higher interest rates because and only because they expect future consumers to do the same in perpetuity. The same is true for any equilibrium of the form (11) for $b \neq 0$, and explains why all such equilibria can be thought of as both non-fundamental and backward-looking.

All in all, the Taylor principle is therefore used not only to rule out sunspots but also to secure the logical foundations of the modern policy paradigm. The rest of our paper attempts to liberate these foundations from their strict reliance on the Taylor principle, or any substitute thereof.

Beyond the full-information benchmark: a challenge and the way forward

Consider conditions (14) and (15). Clearly, these are equivalent representations of the same equilibrium: the first is recursive, and the second is sequential. This equivalence means that all the equilibria that can be supported by perfect knowledge of $h^t = \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$ coincide with those that can be supported by perfect knowledge of $(\theta_t, \eta_t; \theta_{t-1}, c_{t-1})$. But what if agents lack such perfect knowledge?

Regardless of what agents know or don't know, one can *always* represent any equilibrium in a sequential form, or as in equation (9). This is simply because c_t *has* to be measurable in the history of exogenous aggregate shocks, fundamental or otherwise. But it is far from clear if and when there is an equivalent recursive representation. In fact, a finite-state recursive representation is generally impossible when agents observe noisy signals of endogenous outcomes, due to the infinite-regress problem first highlighted by Townsend (1983).

This poses a challenge for what we want to do in this paper. On the one hand, we seek to highlight how fragile all non-fundamental solutions can be to perturbations of the aforementioned kinds of common knowledge, or to small frictions in coordination. On the other hand, we need to make sure that these perturbations do not render the analysis intractable.

To accomplish this dual goal, in the rest of the paper we follow two strategies. Our main one, in Section 4, takes off from (15), or the sequential representation. An alternative, in Section 5, circles back to (14), the recursive representation. Both strategies illustrate the fragility of non-fundamental equilibria, each one from a different angle.

4 Uniqueness with Fading Social Memory

This section contains our main result. We introduce a friction in social memory and show how it yields a unique equilibrium regardless of monetary policy.

Main assumption

For the purposes of this and the next section, we replace the assumption of a representative, fully-informed agent with the following, incomplete-information variant:

Assumption 2 (**Social memory**). *In every period t, a consumer's information set is given by*

$$\mathbb{I}_{i,t} = \{(\theta_t, \eta_t), \cdots, (\theta_{t-s_{i,t}}, \eta_{t-s_{i,t}})\},$$

where $s_{i,t} \in \{0, 1, \cdots\}$ is an idiosyncratic random variable, drawn from a geometric distribution with parameter λ, for some $\lambda \in (0, 1]$.

To understand this assumption, note that herein $s_{i,t}$ indexes the random length of the history of shocks that a period-t agent knows. Next, recall that the geometric distribution means that $s_{i,t} = 0$ with probability λ, $s_{i,t} = 1$ with probability $(1-\lambda)\lambda$, and more generally $s_{i,t} = k$ with probability $(1-\lambda)^k\lambda$, for any $k \geq 0$. By the same token, the fraction of agents who know *at least* the past k realizations of shocks is given by $\mu_k \equiv (1-\lambda)^k$.

One can visualize this as follows. At every t, the typical player (young consumer) learns the concurrent shocks for sure; with probability λ, she learns nothing more; and with the remaining probability, she inherits the information of another, randomly selected player from the previous period (a currently old consumer). In this sense, λ parameterizes the speed at which social memory (or common-p belief of past shocks) fades over time.

Note that Assumption 2 does not influence the MSV solution itself, because $\mathbb{I}_{i,t}$ always contains the current fundamental. As mentioned in the Introduction, this helps isolate our contribution from the existing literature on informational frictions, which allows imperfect information about the current θ_t but does not address the determinacy issue.

Finally, note that Assumption 2 rules out direct observation of endogenous outcomes, including current income and current interest rates. This is consistent with our characterization of optimal consumption in (2) and by extension with our game representation in (6), because both of them are valid for arbitrary information. But it also means that we must envision consumers choosing their spending under uncertainty about current income and current interest rates. Such uncertainty can be motivated in its own right as the product of inattention, but is not strictly needed for our results.[18]

Main result

The full-information benchmark is nested with $\lambda = 0$; this indeed translates to $\mathbb{I}_{i,t} = h^t$ for all i, t, and h^t (i.e., perfect and common knowledge of the infinite history at all times). The question of interest is what happens for $\lambda > 0$, and in particular as $\lambda \to 0^+$. In this limit, the friction vanishes in the following sense: almost every agent knows the history of shocks up to an arbitrarily distant point in the past. But the following is also true: as long as λ is not *exactly* zero, we have that $\lim_{k\to\infty} \mu_k = 0$, which means that shocks are expected to be "forgotten" in the very distant future. As shown next, this causes all non-fundamental equilibria to unravel.

[18] First, this uncertainty vanishes as $\lambda \to 0^+$, in a sense we qualify in Appendix B.4. Second, our analysis goes through if consumers observe perfectly their own income and own interest rate, provided that we abstract from signal-extraction about payoff-irrelevant histories; see Appendix B.1. Finally, we can accommodate such signal-extraction if we adopt the variant perturbation of Section 5. We thus invite the reader to take Assumption 2 with an open mind: even though it may not be the most realistic specification of information one can think of, it allows us to introduce a plausible *perturbation* away from common knowledge.

Proposition 2 (**Determinacy without the Taylor principle**). *Suppose that social memory is imperfect in the sense of Assumption 2, for any $\lambda > 0$. Regardless of ϕ, or of δ_0 and δ_1, the equilibrium is unique and is given by the fundamental/MSV solution.*

The result is proven in Appendix A for arbitrary δ_0 and δ_1. To illustrate the main argument as transparently as possible, here we set $\delta_0 = 0$ and $\delta_1 = \delta$, for arbitrary $\delta > 0$ (including $\delta > 1$). This zeroes in on the role of coordination across time. We also abstract from fundamentals and focus on ruling out pure sunspot equilibria. That is, we specialize equation (6) to

$$c_t = \delta \bar{E}_t[c_{t+1}]; \tag{16}$$

we search for solutions of the form $c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k}$; and we verify that $a_k = 0$ for all k.

By Assumption 2, we have that, for all $k \geq 0$,

$$\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$$

where $\mu_k \equiv (1-\lambda)^k$ measures the fraction of the population at any given date that know, or remember, a sunspot realized k periods earlier. Future sunspots, on the other hand, are known to nobody. It follows that, in any candidate solution, average expectations satisfy

$$\bar{E}_t[c_{t+1}] = \bar{E}_t\left[a_0 \eta_{t+1} + \sum_{k=1}^{\infty} a_k \eta_{t+1-k}\right] = 0 + \sum_{k=0}^{+\infty} a_{k+1} \mu_k \eta_{t-k}.$$

By the same token, condition (16) rewrites as

$$\underbrace{\sum_{k=0}^{+\infty} a_k \eta_{t-k}}_{c_t} = \delta \underbrace{\sum_{k=0}^{+\infty} a_{k+1} \mu_k \eta_{t-k}}_{\bar{E}_t[c_{t+1}]}.$$

For this to be true for all sunspot realizations, it is necessary and sufficient that, for all $k \geq 0$,

$$a_k = \delta \mu_k a_{k+1}, \tag{17}$$

or equivalently

$$a_{k+1} = \frac{a_k}{\delta \mu_k}. \tag{18}$$

Because $\mu_k \to 0$ as $k \to \infty$, $|a_k|$ explodes to infinity, and hence a bounded solution does not exist, unless $a_0 = 0$. But $a_0 = 0$ implies $a_k = 0\ \forall k$, which proves that all sunspot equilibria are ruled out and only the MSV solution survives.[19]

[19] Note that this argument does not hinge on the sign of δ.

Comparison to full information and the importance of $\lim_{k\to\infty}\mu_k = 0$

We will explain the essence of our result momentarily. But first, it is useful to repeat the above argument for the knife-edge case with $\lambda = 0$. In this case, $\mu_k = 1\ \forall k$, and condition (18) becomes

$$a_{k+1} = \delta^{-1} a_k.$$

When $\delta < 1$ (equivalently $\phi > 1$), this still explodes as $k \to \infty$ unless $a_0 = 0$ and hence also $a_k = 0$ $\forall k$. But when $\delta > 1$, the above remains bounded, and indeed converges to zero as $k \to \infty$, for arbitrary $a_0 = a \in \mathbb{R}$. This explains how $\lambda = 0$ recovers the sunspot equilibria of Proposition 1.

Note next that the result does not depend on the assumption that memory decays at an exponential rate, but it depends on it vanishing asymptotically, i.e., on $\mu_k \to 0$ as $k \to \infty$. If instead $\mu_k \to \mu$ for some $\mu \in (0,1)$, multiplicity would have remained for $\delta > 1/\mu$; that is, the Taylor principle would have been relaxed but would not have been completely dispensed with. This is because, in this case, agents can count on a fraction μ of all future generations to be able to respond to the sunspot in perpetuity. Notwithstanding this point, let us emphasize that the key is not whether memory *actually* vanishes over time but rather how agents *reason* about the future. We expand on this below.

Finally, note how *both* the standard argument with $\lambda = 0$ and our variant with $\lambda > 0$ use the boundedness assumption, namely that a_k does not explode. But whereas in the full-information case the boundedness assumption—equivalently, the escape clauses articulated in Wallace (1981), Benhabib et al. (2001), Atkeson et al. (2010), etc.—must be complemented with the Taylor principle in order to rule out sunspot equilibria, the Taylor principle has become redundant under our perturbation. To put it differently, the monetary authority still has to commit to "do whatever it takes" to keep inflation or the output gap within some bounds, but it no more needs to commit to "blow up interest rates" in response to deviations from the MSV solution.

Intuition and the role of higher-order beliefs

Focus on the effects of the period-0 sunspot and let $\{\frac{\partial c_t}{\partial \eta_0}\}_{t=0}^{\infty}$ stand for the corresponding impulse response function (IRF). We can then rewrite condition (17) as

$$\frac{\partial c_t}{\partial \eta_0} = \delta \mu_t \frac{\partial c_{t+1}}{\partial \eta_0}.$$

This is the same condition as that characterizing the IRF of c_t to η_0 in a "twin" representative-agent, full-information economy, in which condition (6) is modified as follows:

$$c_t = \tilde{\delta}_t \mathbb{E}_t[c_{t+1}], \quad \text{with} \quad \tilde{\delta}_t \equiv \delta \mu_t.$$

Under this prism, it is *as if* we are back to the standard New Keynesian model but the relevant eigenvalue, or the dynamic macroeconomic complementarity, has become time-varying and has been reduced from δ to $\tilde{\delta}_t$. Furthermore, because $\mu_t \to 0$ as $t \to \infty$, we have that there is T large enough but finite so that $0 < \tilde{\delta}_t < 1$ for all $t \geq T$, regardless of δ. In other words, the twin economy's dynamic feedback becomes weak enough that c_t cannot depend on η_0 after T. By backward induction, then, c_t cannot depend on η_0 before T either.[20]

This interpretation of our result must be clarified as follows. Here we focused on the response of c_t to η_0. This means that our "twin" economy is defined from the perspective of period 0, and that $\tilde{\delta}_t = \mu_t \delta$ measures the feedback from $t+1$ to t in a very specific sense: as perceived by agents in period 0, when they contemplate whether to react to η_0. To put it differently, in this argument, t indexes not the calendar time but rather the belief order, or how far into the future agents reason about the effects of an innovation today.

Let us further explain. Because η_0 is payoff irrelevant in every t, period-0 agents have an incentive to respond to it only if they are confident that period-1 agents will also respond to it, which in turn can be true only if they are also confident that period-1 will themselves be confident that period-2 agents will do the same, and so on, ad infinitum. It is this kind of "infinite chain" that supports sunspot equilibria when $\lambda = 0$. And conversely, the friction we have introduced here amounts to the typical period-0 agent reasoning as follows:

> "I can see η_0. And I understand that it would make sense to react to it if I were confident that all future agents will keep conditioning their behavior on it *in perpetuity*. But I worry that future agents will fail to do so, either because they will be unaware of η_0, or because they may themselves worry, like me, that agents further into the future will not react to it. This makes it iteratively optimal not to react to η_0."

Three remarks help complete the picture. First, the reasoning articulated above, and the proof given earlier, can be understood as a chain of contagion effects from "remote types" (uninformed agents in the far future) to "nearby types" (informed agents in the near future) and thereby to present behavior. This underscores the high-level connection between our approach and the global games literature (Morris and Shin, 1998, 2003).

Second, the aforementioned worries don't have to be "real" (objectively true). That is, we can reinterpret Assumption 2 as follows: agents don't forget themselves but worry that others will forget. Strictly speaking, this requires a modification of the solution concept: from REE to PBE with misspecified priors about one another's knowledge, along the lines of Angeletos and Sastry

[20] Although this argument assumed $\delta_0 = 0$, it readily extends to $\delta_0 \neq 0$. In this case, the twin economy has both δ_0 and δ_1 replaced by, respectively, $\mu_t \delta_0$ and $\mu_t \delta_1$. That is, both types of strategic complementarity are attenuated.

(2021). But the essence is the same.

Last but not least, our argument, like the related arguments in the global games literature, relies on rational expectations (or more precisely on common knowledge of rationality, which itself is implied by REE). This cuts both ways. On the one hand, it lets our paper speak directly and precisely to the question of interest, namely the determinacy of rational expectations equilibria. On the other hand, it begs the question of how monetary policy should be designed if bounded rationality is itself the source of non-fundamental volatility. While this question is outside the scope of our paper, we touch again on it in Section 6.

5 Robustness and Complementary Perturbations

In this section, we explain how our uniqueness result generalizes to more flexible specifications of the fundamentals and the sunspots, provided that Assumption 2 is maintained. We next replace this assumption with two variants, which accommodate direct observation of past outcomes and, thereby, endogenous coordination devices. We finally comment on two other subtleties: the distinction between local and global determinacy; and the role of nominal rigidity. Readers interested in our paper's take-home lessons may skip this section and jump to Section 6.

Persistent fundamentals

In the main analysis, we assumed that the fundamental θ_t is uncorrelated over time. Relaxing this assumption changes the MSV solution but does not affect our determinacy result.

To illustrate, suppose that θ_t follows an AR(1) process: $\theta_t = \rho\theta_{t-1} + \varepsilon_t$, where $\rho \in (-1, 1)$ is a fixed scalar and $\varepsilon_t \sim \mathcal{N}(0, 1)$ is a serially uncorrelated innovation. As long as $\rho \neq 0$, an innovation affects payoffs not only today but also in the future. This naturally modifies the MSV solution. Indeed, if we guess that $c_t = \gamma\theta_t$ for some $\gamma \in \mathbb{R}$ and substitute this into (10), we infer that the guess is correct if and only if $\gamma = 1 + \delta\rho\gamma$. For this to admit a solution, it is necessary and sufficient that $\rho \neq \delta^{-1}$. Provided that this is the case, the MSV solution exists and is now given by $c_t^F = \frac{1}{1-\delta\rho}\theta_t$. Modulo this minor adjustment, Proposition 2 directly extends. This claim is verified in Appendix C, indeed for a more general specification of the fundamental uncertainty: such generality naturally modifies the MSV solution but does not interfere with our uniqueness argument.

Let us now zero in on the role of $\rho \neq \delta^{-1}$ in the above example. This restriction is used to guarantee the existence of the MSV solution. But it is *not* used in our argument for ruling out any other solution (Proposition 2); for that purpose, it suffices to assume that social memory fades over time (Assumption 2). Finally, note that the comparative statics of the MSV solution with respect to θ_t switch sign depending on whether ρ is lower or higher than δ^{-1}. In particular, when $\rho > \delta^{-1}$, the MSV solution exhibits the so-called neo-Fisherian property: a sufficiently persistent increase in the nominal interest rate triggers an *increase* in inflation and output. This raises a number of delicate questions, such as whether the MSV solution can be obtained by forward induction, whether the neo-Fisherian property is robust to bounded rationality (García-Schmidt and Woodford, 2019) or imperfect information about monetary policy (Angeletos and Lian, 2018), or even whether the New Keynesian model is mis-specified in a such way that makes it relatively less reliable for studying persistent changes in interest rates or other fundamentals ($\rho > \delta^{-1}$) as opposed to studying short-run fluctuations ($\rho < \delta^{-1}$). But these questions are clearly beyond the scope of our paper.

Persistent sunspots

Let us now revisit the assumption that the sunspot is serially uncorrelated. As in the case of fundamentals, this assumption can readily be relaxed (see Appendix C.2 for details), except for one special case: when η_t follows an AR(1) process with autocorrelation *exactly* equal to δ^{-1}. In this case, $c_t = c_t^F + a\eta_t$ is an equilibrium for any a and is supported by knowledge of the concurrent θ_t and η_t alone. Social memory of the distant past is no longer needed, because the exogenous sunspot happens to coincide with the *right* sufficient statistic of the economy's infinite history.

This situation seems unlikely insofar as the sunspot is an exogenous random variable: formally, the requisite sunspot is degenerate in the space of ARMA processes. But what if agents can devise an *endogenous* sunspot? For instance, could it be that agents coordinate on an equilibrium that lets an endogenous outcome, such as c_t itself, replicate the requisite sunspot variable? We already hinted that such coordination, too, can be fragile: in the limit as $\lambda \to 0^+$, agents were arbitrarily well informed about exogenous shocks and endogenous outcomes alike, and yet uniqueness was obtained. We now reinforce this message by showing how determinacy may remain with two variant information structures, which, unlike Assumption 2, allow for *direct* signals of endogenous outcomes.

Recursive sunspot equilibria: another example of fragility

Recall that, with full information, our model boils down to the following equation:

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}],$$

where $\delta \equiv \frac{\delta_1}{1-\delta_0}$ and $\mathbb{E}_t$ is the full-information rational expectation. Let us momentarily shut down the fundamentals, assume that $\delta > 1$, and focus on the set of all pure sunspot equilibria:

$$c_t = a \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}, \tag{19}$$

for arbitrary $a \neq 0$. As noted earlier, this can be represented in recursive form as

$$c_t = a\eta_t + \delta^{-1} c_{t-1}. \tag{20}$$

It follows that all sunspot equilibria can be supported with the following "minimal" information set: $\mathbb{I}_{i,t} = \{\eta_t, c_{t-1}\}$. Intuitively, c_{t-1} *endogenously* serves the role of the knife-edge persistent sunspot discussed earlier.

Taken at face value, this challenges our message. But as shown next, this logic, too, can be fragile. Suppose that information is given by

$$\mathbb{I}_{i,t} = \{\eta_t, s_{i,t}\}, \qquad \text{with} \qquad s_{i,t} = c_{t-1} + \varepsilon_{i,t}.$$

Here, $s_{i,t}$ is a private signal of the past aggregate outcome, $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2)$ is idiosyncratic noise, and $\sigma \geq 0$ is a fixed parameter. When $\sigma = 0$, we are back to the case studied above, and the entire set of sunspot equilibria is supported. When instead $\sigma > 0$ but arbitrarily small, agents' knowledge of the past outcome is only slightly blurred by idiosyncratic noise. As shown next, this causes all sunspot equilibria to unravel.

Proposition 3. *Consider the economy described above. For any $\sigma > 0$, no matter how small, and regardless of δ_0 and δ_1, there is a unique equilibrium and it corresponds to the MSV solution.*

The proof is actually quite simple. But we prefer to delegate it to Appendix A, because the present example is still special in two regards: it rules out public signals of c_{t-1}; and it rules out information about longer histories.

The first limitation is easy to address: Proposition 3 readily generalizes to $s_{i,t} = c_{t-1} + v_t + \varepsilon_{i,t}$, where v_t is aggregate noise and $\varepsilon_{i,t}$ is idiosyncratic noise. This can be interpreted as a situation where a publicly available statistic is not only contaminated with measurement error but also observed with idiosyncratic noise due to rational inattention (Sims, 2003) or imperfect cognition (Woodford, 2019). It is only in the knife-edge case in which the statistic is common knowledge that multiplicity survives.[21]

The second limitation is more challenging, because it opens the Pandora's box of signal extraction and infinite regress. In the next subsection, we therefore offer a different approach, which manages to keep this box closed while accommodating direct—and indeed perfect—knowledge of long histories of aggregate output and inflation.

[21] See Appendix B.6. We thank a referee for prompting us to clarify this subtlety.

Breaking the infinite chain even when past outcomes are perfectly observed

In the above exercise, we focused on pure sunspot equilibria. Let us now bring back the fundamental shocks and consider any of the equilibria of the form $c_t^B + ac_t^\eta$, which, recall, were obtained by "solving the model backward" in (15). These can be replicated by letting $\mathbb{I}_{i,t} \supseteq \{\eta_t, c_{t-1}, \theta_{t-1}\}$ and by having each consumer play the following recursive strategy:

$$c_{i,t} = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t. \tag{21}$$

Contrary to the strategy that supported the pure sunspot equilibrium, the above strategy requires that the agents at t know not only c_{t-1} but also θ_{t-1}. Why is knowledge of θ_{t-1} necessary? Because this is what it takes for agents at t to know how to undo the direct, intrinsic effect of θ_{t-1} on the incentives of the agents at $t-1$.

This suggests that the "infinite chain" that supports all backward-looking equilibria—and all sunspot equilibria, as well— breaks if the agents at t do not know what exactly it takes to undo the direct, intrinsic effect of yesterday's fundamental on yesterday's behavior. To make this point crisply, we proceed as follows.

First, we introduce a new fundamental disturbance, denoted by ζ_t; we modify equation (8) to

$$c_{i,t} = E_{i,t}[(1-\delta_0)(\theta_t + \zeta_t) + \delta_0 c_t + \delta_1 c_{t+1}]; \tag{22}$$

and we let ζ_t be drawn independently over time, as well as independently of any other shock in the economy, from a uniform distribution with support $[-\varepsilon, +\varepsilon]$, where ε is positive but arbitrarily small. This lets us parameterize the payoff perturbation by ε, or the size of the support of ζ_t.

Second, we abstract from informational heterogeneity *within* periods, that is, we let $\mathbb{I}_{i,t} = \mathbb{I}_t$ for all i and all t. This guarantees that $c_{i,t} = c_t$ for all i and t, and therefore that we can think of the economy as a sequence of representative agents, or a sequence of players, one for each period. Under the additional, simplifying assumption that $\mathbb{I}_t$ contains both θ_t and ζ_t, we can then write the best response of the period-t representative agent as

$$c_t = \theta_t + \zeta_t + \delta E[c_{t+1}|\mathbb{I}_t]. \tag{23}$$

where $\delta \equiv \frac{\delta_1}{1-\delta_0}$, as always, and $E[\cdot|\mathbb{I}_t]$ is the rational expectation conditional on $\mathbb{I}_t$. This is similar to the standard, full-information benchmark, except that we have allowed for the possibility that today's representative agent does not inherit all the information of yesterday's representative agent: $\mathbb{I}_t$ does not necessarily nest $\mathbb{I}_{t-1}$.

Finally, we let $\mathbb{I}_t$ contain perfect knowledge of arbitrary long histories of the endogenous outcome, the sunspots, and the "main" fundamental; but we preclude knowledge of the past values of the payoff perturbation introduced above. Formally:

Assumption 3. *For each t, there is a representative agent whose information is given by*

$$\mathbb{I}_t = \{\zeta_t\} \cup \{\theta_t, \cdots, \theta_{t-K_\theta}\} \cup \{\eta_t, \cdots, \eta_{t-K_\eta}\} \cup \{c_{t-1}, \cdots, c_{t-K_c}\}$$

for finite but possibly arbitrarily large K_η, K_c, and K_θ.

When $\varepsilon = 0$ (i.e., when the ζ_t shock is absent), Assumption 3 allows replication of all sunspot and backward-looking equilibria with a short memory, namely with $K_\eta = 0$ and $K_\theta = K_c = 1$. This corresponds to the recursive representation reviewed earlier. But there is again a discontinuity: once $\varepsilon > 0$, all the non-fundamental equilibria unravel, no matter how large K_η, K_c, and K_θ are.

Proposition 4. *Suppose that Assumption 3 holds and $\varepsilon > 0$. Regardless of δ, there is unique equilibrium and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same MSV solution as before.*

How does this connect to Proposition 2? Both results introduce a friction in social memory and intertemporal coordination, thus breaking the infinite chain behind all non-fundamental equilibria. But the exact friction is different: whereas in our main result it amounts to *asymptotic* forgetting of the distant past, here it amounts to *immediate* forgetting of a small component of the fundamentals. This also means a change in the formal argument: whereas our main result echoes the global games literature, the present one is more closely connected to Bhaskar (1998) and Bhaskar et al. (2012), which show how the combination of a payoff perturbation and finite social memory can rule out non-Markov perfect equilibria in a certain class of dynamic games. At a high level, related is also a literature that studies how multiplicity in repeated games depends on public versus private monitoring (e.g., Mailath and Morris, 2002; Pęski, 2012). The common thread between all the literature and our results is the role played by the lack of common knowledge. But the precise connections are elusive and deserve further study.

Local vs global determinacy

Throughout, we work with the linearized New Keynesian model and restrict equilibria to be bounded. As previously mentioned, this amounts to focusing on local determinacy around a given steady state (herein normalized to zero). But what about global determinacy?

Let us first address this question within the policy context of interest. To ensure global determinacy, the standard paradigm complements the Taylor principle with an escape clause: to switch from interest-rate setting to a different policy regime, such as money-supply setting or even commodity-backed money, should inflation exit certain bounds.[22] Under the standard approach, the escape clause rules out all unbounded equilibria (i.e., self-fulfilling inflationary and

[22]See, inter alia, Wallace (1981), Obstfeld and Rogoff (1983, 2021), Benhabib et al. (2001, 2002), Christiano and Rostagno (2001), and the discussion of "hybrid" Taylor rules in Atkeson et al. (2010).

deflationary spirals), while the Taylor principle rules out any bounded equilibrium other than the MSV solution. Under our approach, the Taylor principle becomes redundant but the escape clause—or a credible commitment to arrest explosive paths—is still needed.

Consider next other contexts, such as the OLG model of money by Samuelson (1958). This is a non-linear model and it admits two steady-state equilibria: an "autarchic" one, in which the old and the young consume their respective endowment and money is not traded; and a "bubbly" one, in which money facilitates Pareto-improving transfers between the young and the old. In addition, there is a continuum of bounded sunspot equilibria, all of which hover around the first steady state. In this context, we cannot rule out either one of the steady-state equilibria, because our methods presume common knowledge of any given steady state. By extension, we cannot say anything about global determinacy either. But if we linearize that model around each steady state and apply our assumptions and results, we can guarantee local determinacy of *both* steady states, and can therefore rule out the aforementioned sunspot equilibria.[23]

This clarifies the scope of our theoretical contribution. It seems a plausible guess that Proposition 2 extends to a larger class of linear models, such as that considered in Blanchard (1979) and Blanchard and Kahn (1980), provided that these can be recast as dynamic coordination games along the lines we have illustrated here. In non-linear settings, we also expect our results to translate to local determinacy around any given steady state. But we have nothing to say about global determinacy—except for the points made above for the specific context of interest.

Sticky vs flexible prices

Equation (8), the game representation of our baseline model, is valid for any value of κ, the slope of the Phillips curve. The same is true for equation (28), the generalization developed in the next section. This underscores that our game-theoretic prism and, by extension, our main result is not unduly sensitive to the degree of price flexibility. But what if prices are *literally* flexible, or "$\kappa = \infty$"? In this case, aggregate demand ceases to matter for aggregate output and, as a result, the economy can no more be represented as a game among the consumers.

This begs the question of whether a version of our insights applies to flexible-price models. While we will not address this question here, we wish to raise the following flag. In the existing literature, the *real* indeterminacy problem of the New Keynesian paradigm is treated as a direct translation of the *nominal* indeterminacy problem of flexible-price models, which was the domain of Sargent and Wallace (1975). But the two problems turn out to be fundamentally different

[23]We thank the editor for suggesting the link to Samuelson (1958) and a referee for suggesting another non-linear example, which is more directly comparable to our setting. We use that example in Appendix B.7 to further illustrate the issues discussed above.

under our prism. With *any* non-zero degree of nominal rigidity, output and inflation can be understood as the outcomes of a game among the consumers and our results go through. But this game ceases to be well-defined when prices are "truly" flexible.

In our view, this touches on a larger methodological question, whether flexible-price models are proper limits of models with nominal rigidity (Kocherlakota, 2020) or perhaps whether the New Keynesian model itself needs modification. But this is clearly beyond the scope of our paper.

6 Applied Lessons

In this section, we translate our main result to two applied lessons: one regarding the FTPL, and another regarding the Taylor principle. To facilitate these translations, we first illustrate how our main result extends to a larger class of New Keynesian models than that employed thus far.

Nesting a larger class of New Keynesian economies

Borrowing insight from the HANK literature, let us bypass the micro-foundations of consumer behavior and instead assume directly that aggregate demand can be expressed as follows:

$$c_t = \mathscr{C}\left(\{\bar{E}_t\left[y_{t+k}\right]\}_{k=0}^{\infty}, \{\bar{E}_t\left[r_{t+k}\right]\}_{k=0}^{\infty}\right) + \varrho_t, \tag{24}$$

where $r_t \equiv i_t - \pi_{t+1}$ stands for the real interest rate, $\mathscr{C}$ is a linear function, and ϱ_t is an exogenous (and, for simplicity, perfectly observed) aggregate demand shock. This generalizes equation (2) from our baseline model, allowing aggregate consumption to depend on expectations about interest rates and income in all future periods, not just the next period. For instance, Angeletos and Huo (2021) show that, in a perpetual-youth OLG version of the New Keynesian model, equation (24) takes the following form:

$$c_t = \bar{E}_t\left[(1-\beta\omega)\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right\} - \beta\omega\sigma\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k (i_{t+k} - \pi_{t+k+1})\right\}\right] + \varrho_t, \tag{25}$$

where $\omega \in (0,1]$ is the survival rate. This allows us to cast the decay in social memory as the byproduct of individual mortality.[24] But this interpretation is not strictly needed. For the present purposes, we take equation (24) as given and think of it as a linear but otherwise flexible specification of the intertemporal Keynesian cross (Auclert et al., 2018).

Consider next the supply side. We now replace our baseline model's ad hoc, static Phillips

[24] This interpretation restricts $\lambda = 1-\omega$, where $1-\omega$ is the probability of death. But we could have $\lambda < 1-\omega$ if newborn consumers inherit some of the information of the dying consumers. And conversely, we could reconcile $\lambda > 1-\omega$ (e.g., $\omega = 1$) by letting the current generations be altruistic towards future generations (as in Barro, 1974) but let some information be lost across generations.

curve with the standard, micro-founded, and forward-looking New Keynesian Phillips curve:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \xi_t, \tag{26}$$

where $\kappa \geq 0$ and $\beta \in (0,1)$ are fixed scalars and ξ_t is, again, a cost-push shock.[25] Finally, we let the Taylor rule be

$$i_t = z_t + \phi_y y_t + \phi_\pi \pi_t, \tag{27}$$

for some random variable z_t and some fixed scalars $\phi_y, \phi_\pi \geq 0$.[26]

The "famous" three equations are now given by (24), (26) and (27), along with $y_t = c_t$ (market clearing). Solving (26) and (27) for inflation and the interest rate, and replacing these solutions into (24), we can obtain c_t as a linear function of $\{\bar{E}_t[y_{t+k}]\}_{k=0}^{\infty}$, or equivalently of $\{\bar{E}_t[c_{t+k}]\}_{k=0}^{\infty}$. We conclude that a process for c_t is part of an equilibrium if and only if it solves the following:

$$c_t = \bar{E}_t\left[(1-\delta_0)\theta_t + \sum_{k=0}^{+\infty} \delta_k c_{t+k}\right] \tag{28}$$

for some random variable θ_t that is a linear combination of the primitive shocks (z_t, ξ_t, ϱ_t) and some coefficients $\{\delta_k\}_{k=0}^{\infty}$, with $\delta_0 < 1$ and $\Delta \equiv \delta_0 + \sum_{k=1}^{\infty} |\delta_k| < \infty$.[27]

Similar to equation (6) in our baseline model, this equation helps translate the economy to a game among the consumers. Accordingly, the coefficients $\{\delta_k\}_{k=0}^{\infty}$ are transformations of deeper parameters that regulate the relevant GE feedbacks.[28] These feedbacks are now more complicated, and aggregate spending in any given period depends on expectations of economic activity in all future periods as opposed to merely the next period, but the essence is similar.

The overall strategic interdependence, or the analogue of the sum $\delta_0 + \delta_1$ from our main analysis, is now given by Δ. With $\Delta > 1$, multiple self-fulfilling equilibria can be supported under full information, in a similar fashion as in Section 3. But they unravel under Assumption 2, because this again breaks the "infinite chain" behind them. We verify this claim below. The proof is

[25] The micro-foundations of (26) are omitted because they are entirely standard. The only point worth mentioning is that (26) presumes that firms, unlike consumers, have full information. This simplifies the exposition and maximizes proximity to the standard New Keynesian model, without affecting the essence. For, as long as the informational friction is present on the consumer side, it is not necessary to "double" it on the production side.

[26] We can readily accommodate forward-looking terms in the policy rule. This changes the exact values of the coefficients $\{\delta_k\}$ in the upcoming game representation, namely equation (28), but does not affect Proposition 5, because this holds for arbitrary such coefficients. What we cannot readily nest in (28) is a backward-looking Taylor rule, such as $i_t = z_t + \phi_\pi \pi_{t-1}$, or a backward-looking Phillips curve. See, however, Appendix B.5 for an illustration of why this does not upset our result, insofar as, of course, Assumption 2 is maintained.

[27] For instance, when equation (24) specializes to (25), we get $\delta_k \equiv \left(1-\beta\omega-\beta\omega\sigma\phi_y\right)(\beta\omega)^k + \omega\sigma\kappa\left(-\beta\phi_\pi + \left(1-\beta\omega\phi_\pi\right)\frac{1-\omega^k}{1-\omega}\right)\beta^k$, and the restrictions $\delta_0 < 1$ and $\Delta \equiv \delta_0 + \sum_{k=1}^{\infty} |\delta_k| < \infty$ are readily satisfied.

[28] These parameters are: the MPCs out of current and future income, $\{\frac{\partial C}{\partial y_k}\}_{k=0}^{\infty}$; the sensitivities of consumption to current and future real interest rates, $\{\frac{\partial C}{\partial r_k}\}_{k=0}^{\infty}$; the slope, κ, and the forward-lookingness, β, of the NKPC; and the policy coefficients, ϕ_π and ϕ_y.

more tedious than that of Proposition 2 and is delegated to Appendix A, but the basic logic is the same.[29]

Proposition 5 (**Generalized result**)**.** *Consider the above generalization, impose Assumption 1 and 2, and let* $\lambda > 0$*. Whenever an equilibrium exists, it is unique and is given by the MSV solution.*[30]

Feedback rules and Taylor principle: equilibrium selection or stabilization?

Go back to the textbook New Keynesian model. Let $\{i_t^o, \pi_t^o, c_t^o\}$ denote the optimal path for interest rates, inflation, and output, as a function of the underlying demand and supply shocks. And ask the following question: what does it take for the optimum to be implemented as the unique equilibrium? The textbook answer is that, as long as the monetary authority observes the aforementioned shocks, it suffices to follow the following feedback rule, for any $\phi > 1$:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o).$$

This is nested in (5) with $z_t = i_t^o - \phi\pi_t^o$, and is sometimes referred to as the "King rule" (after King, 2000). Note then that ϕ can take any value above 1, and this does not affect the properties of the optimum. That is, the feedback from π_t to i_t serves only the role of equilibrium selection; macroeconomic stabilization is instead achieved via the optimal design of z_t, and in particular via its correlation with the underlying demand and supply shocks.

What if the monetary authority does not observe these shocks? Feedback rules may then help replicate the optimal dependence of interest rates on shocks. But this function could be at odds with that of equilibrium selection; see Galí (2008, p.101) for an illustration with cost-push shocks, and Loisel (2021) for a general formulation. From this perspective, our results help ease the potential conflict between equilibrium selection and stabilization: because feedback rules are no longer needed for equilibrium selection, they are "free" to be used for stabilization.

At the same time, our results help recast the *spirit* of the Taylor principle in a new form. When the equilibrium is unique (whether thanks to our perturbations or otherwise) but the GE feedbacks between spending and income or inflation are sizable, sunspot-like volatility can obtain from overreaction to noisy public news (Morris and Shin, 2002), shocks to higher-order beliefs

[29]It seems a safe guess that the result extends to the multi-variate case, where c_t is a vector and δ_k is a matrix. In other words, our insights apply to a large class of linear, purely forward-looking, rational expectations models, like that studied by Blanchard (1979) under the full-information assumption. Less straightforward is the extension to Blanchard and Kahn (1980), namely to linear models that include payoff-relevant state variables (e.g., capital, habit). Nevertheless, the example in Appendix B.5, which adds such a feature in the form of backward-looking monetary policy, suggests that our insights extend to this case as well. All in all, we therefore see our result as a justification for focusing on the fundamental/MSV solution of linear macroeconomic models.

[30]When θ_t is uncorrelated over time, the MSV solution is again given by $c_t^F = \theta_t$. More generally, it can be solved for in a similar way as in the extension of our baseline model that adds persistent fundamentals (Appendix C).

(Angeletos and La'O, 2013; Benhabib et al., 2015), or related forms of bounded rationality (Angeletos and Sastry, 2021). In this context, ϕ admits a new function: by regulating the strategic complementarity in the economy, it also regulates the magnitude of such sunspot-like fluctuations along the unique equilibrium path.

Finally, our results reduce the need for communicating either a "threat to blow up the economy" (Cochrane's provocative interpretation of the Taylor principle) or the kind of "sophisticated" off-equilibrium strategies articulated in Atkeson et al. (2010). Provided that expectations are anchored in the narrow sense that private agents are confident that fluctuations in inflation and output gaps won't explode away from the steady state,[31] it suffices for the monetary authority to communicate what she plans to do *on equilibrium only*.

On the Fiscal Theory of the Price Level (FTPL)

We now turn to how our paper relates to the FTPL. To this goal, let us momentarily go back to the basics: the textbook, three-equation New Keynesian model. Add now a fourth equation, written compactly (and in levels) as follows:

$$\frac{I_{t-1}B_{t-1}}{P_t} = PVS_t, \tag{29}$$

where B_{t-1} denotes the amount of one-period nominal bonds issued by the government at $t-1$,[32] $I_{t-1}B_{t-1}/P_t$ is the real debt burden at t (inclusive of interest payments), and PVS_t is the real present discounted value of primary surpluses.[33]

As long as the Taylor principle holds, (29) plays no role in the determination of inflation, output and interest rates: these variables are pinned down by the MSV solution of the model's other three equations. By the same token, P_t can be treated as an exogenous variable in equation (29) and the latter can be read as a constraint on fiscal policy: the government must adjust PVS_t so as to make sure that (29) holds at the price level dictated by the monetary authority.

The FTPL turns this logic upside down: it lets the government choose PVS_t as if (29) were not a binding constraint and, instead, requires that P_t itself adjusts so as to satisfy (29). This is a coherent alternative as long as an equilibrium *other* than the MSV solution is selected, which in turn is possible in the standard paradigm if and only if the Taylor principle is violated. But

[31] One may of course question the credibility and effectiveness of the escape clauses or other commitments that rule out "unbounded" equilibria (e.g., Wallace, 1981; Christiano and Rostagno, 2001; Atkeson et al., 2010). In fact, one could even say that, by reducing the importance of the Taylor principle for equilibrium selection, our results help redirect the question of time inconsistency from this principle (Neumeyer and Nicolini, 2022) to the aforementioned escape clauses. But these issues are beyond the scope of our paper.

[32] These bonds ("treasury bills") are perfect substitutes to the nominal claims held against the central bank ("reserves").

[33] We continue to work with a cashless economy, which explains the absence of seigniorage in (29).

we already argued that the MSV solution is the only possible equilibrium under Assumption 2, regardless of whether the Taylor principle is satisfied or not. It follows that the FTPL finds no place under our perturbations.

To illustrate this point, abstract from discount-rate and cost-push shocks ($\varrho_t = \xi_t = 0$), let $\phi = 0$, and suppose the government runs a higher deficit today and commits to adjust neither taxes nor spending in the future. According to the FTPL, (29) can be satisfied *without* monetary policy accommodation: there is an equilibrium in which interest rates remain unchanged ($i_t = z_t = 0$ for all t) and nevertheless the price level increases, and the real debt burden falls, so as to offset the deficit. But since $\varrho_t = \xi_t = z_t = 0$ imply $c_t^F = \theta_t = 0$, this scenario is clearly inconsistent with the MSV solution and is therefore ruled out by our perturbations. To put it differently, for the aforementioned fiscal policy to be feasible under our perturbation, it now *has* to be that it is supported by sufficient monetary accommodation (i.e., that interest rates fall enough to allow for the necessary increase in the price level to obtain along the MSV solution itself).[34]

One may of course question the plausibility of our perturbations. Indeed, if we depart from Assumption 2 and let consumers condition their spending on noisy signals of the aggregate quantity of public debt, it is possible to resurrect the FTPL equilibrium.[35] Furthermore, and more importantly, public debt and deficits naturally enter the fundamentals of the economy, and thereby its MSV solution, once markets are incomplete (think HANK). Additional work is therefore necessary before one can evaluate the theoretical robustness and empirical plausibility of the FTPL. Still, by offering a possible way out of the equilibrium selection conundrum, we hope to redirect attention to the following, arguably more interesting, question: can an increase in the deficit be self-financed by an FTPL-like adjustment in the price level and the real debt burden (or adjustment in aggregate income and the tax base) *along* the MSV solution, instead of outside of it? We explore this question in ongoing work (Angeletos et al., 2023).

[34]This example illustrates, not only the policy implications of our uniqueness result, but also the following point: while assumptions about off-equilibrium threats are untestable, the difference between the FTPL and MSV equilibria described above are clearly testable: the two types of equilibria impose different restrictions on the joint paths of deficits, inflation and interest rates. A separate question, which we explore in ongoing work (Angeletos et al., 2023), is how the MSV solution itself is modified in economies where Ricardian equivalence breaks because of finite horizons/incomplete markets.

[35]As long as consumers have infinite horizons (which is the case of interest in the present context), public debt is payoff irrelevant in the game among the consumers. In other words, public debt is effectively an endogenous sunspot. This circles back to the discussion in Section 5. But the endogenous sunspot is now different, so the specific results of that section are no more applicable.

7 Conclusion

In this paper, we revisited the indeterminacy issue of the New Keynesian model. We highlighted how all sunspot and backward-looking equilibria hinge on a delicate, infinite, self-fulfilling chain between current and future behavior. And we showed how to break this chain, and guarantee that the model's fundamental or MSV solution is the unique rational expectations equilibrium regardless of monetary or fiscal policy, by appropriately perturbing the model's assumptions about social memory and dynamic coordination.

We thus provided a rationale for why the monetary authority may be able to regulate aggregate demand and inflation without a strict reliance on the Taylor principle or any other off-equilibrium threat; indeed, our perturbations recovered a unique equilibrium even under an interest rate peg. But we also discussed how one could reconcile our determinacy result with sunspot-like volatility in the form of shocks to higher-order beliefs; and we highlighted that a more hawkish monetary policy can contain such volatility, similarly to the case of traditional demand and supply shocks. More succinctly, we dispense with the Taylor principle as a form of equilibrium selection but recast its spirit as a novel form of macroeconomic stabilization.

We offered a similar two-sided approach to the FTPL. By removing the need for equilibrium selection via either the non-Ricardian assumption or the Taylor principle, our paper suggests new avenues for studying the relationship between fiscal policy and inflation. For instance, consider the question of whether a higher deficit can be self-financed by an increase in inflation and a reduction in the real debt burden, holding constant interest rates. The existing version of the FTPL offers a positive answer to this question by selecting an equilibrium other than the MSV solution. But if debt and deficits enter aggregate demand because of incomplete markets, the same answer may be possible along the MSV solution (Angeletos et al., 2023).

As another example, consider the question of whether US fiscal policy will eventually force the Fed's hand towards more lax monetary policy, or the related question of which authority is "dominant." Such questions seem to call for modeling the interaction between the two authorities as that of two players in a game of chicken (Sargent and Wallace, 1981; Canzoneri et al., 2010). But for such a game to be well defined, there must exist a unique mapping from the two players' actions (government deficits and interest rates)—to market outcomes (output, inflation, etc) and thereby to the players' payoffs. Such a unique mapping is missing in the standard paradigm, because of the equilibrium determinacy problem. By providing a possible fix to this "bug," or at least a formal justification for bypassing it, our paper may pave the way to new research on these important policy questions.

Appendices

A Proofs

Proof of Proposition 1

Part (i) follows directly from the fact that $c_t^F \equiv \theta_t$ satisfies (10).

Consider part (ii). Let $\{c_t\}$ be any equilibrium and define $\hat{c}_t = c_t - c_t^F$. From (10),

$$\hat{c}_t = \delta \mathbb{E}_t[\hat{c}_{t+1}]. \tag{30}$$

From Definition 1,

$$\hat{c}_t = \sum_{k=0}^{\infty} \hat{a}_k \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}_k \theta_{t-k},$$

with $|\hat{a}_k| \leq \hat{M}$ and $\left|\hat{\gamma}_k\right| \leq \hat{M}$ for all k, for some finite $\hat{M} > 0$. From Assumption 1, we have

$$\mathbb{E}_t[\hat{c}_{t+1}] = \sum_{k=0}^{+\infty} \hat{a}_{k+1} \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}_{k+1} \theta_{t-k}.$$

The equilibrium condition (30) can thus be rewritten as

$$\sum_{k=0}^{\infty} \hat{a}_k \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}_k \theta_{t-k} = \delta \left(\sum_{k=0}^{+\infty} \hat{a}_{k+1} \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}_{k+1} \theta_{t-k} \right).$$

For this to be true for all t and all states of nature, the following restrictions on coefficients are necessary and sufficient:

$$\hat{a}_k = \delta \hat{a}_{k+1} \ \forall k \geq 0 \qquad \text{and} \qquad \hat{\gamma}_k = \delta \hat{\gamma}_{k+1} \ \forall k \geq 0.$$

When the Taylor principle is satisfied ($\phi > 1$ and $\delta < 1$), $\hat{a}_k$ and $\hat{\gamma}_k$ explodes unless $\hat{a}_0 = 0$ and $\hat{\gamma}_0 = 0$. We know that the only bounded solution of (30) is $\hat{c}_t = 0$. As a result, c_t^F is the unique equilibrium.

Finally, consider part (iii). $c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}$ and $c_t^{\eta} \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$ are bounded in the sense in Definition 1 when the Taylor principle is violated ($\phi \in [0,1)$ and $\delta \in (1, +\infty)$). c_t^B satisfies (10). So does $c_t = (1-b) c_t^F + b c_t^B + a c_t^{\eta}$ for arbitrary $b, a \in \mathbb{R}$.

Proof of Proposition 2

Consider an equilibrium taking the form of (9). From (6), Assumption 1, and Assumption 2, we know:

$$\sum_{k=0}^{+\infty} a_k\eta_{t-k} + \sum_{k=0}^{\infty}\gamma_k\theta_{t-k} = (1-\delta_0)\theta_t + \bar{E}_t\left[\delta_0\left(\sum_{k=0}^{+\infty} a_k\eta_{t-k} + \sum_{k=0}^{\infty}\gamma_k\theta_{t-k}\right) + \delta_1\left(\sum_{k=0}^{+\infty} a_k\eta_{t+1-k} + \sum_{k=0}^{\infty}\gamma_k\theta_{t+1-k}\right)\right]$$
$$= (1-\delta_0)\theta_t + \bar{E}_t\left[\sum_{k=0}^{+\infty}(\delta_0 a_k + \delta_1 a_{k+1})\eta_{t-k} + \sum_{k=0}^{\infty}\left(\delta_0\gamma_k + \delta_1\gamma_{k+1}\right)\theta_{t-k}\right], \tag{31}$$

and

$$\bar{E}_t[\eta_{t-k}] = \begin{cases}\mu_k\eta_{t-k} & \text{if } k\ge 0\\ 0 & \text{otherwise}\end{cases} \quad\text{and}\quad \bar{E}_t[\theta_{t-k}] = \begin{cases}\mu_k\theta_{t-k} & \text{if } k\ge 0\\ 0 & \text{otherwise}\end{cases},$$

where $\mu_k = (1-\lambda)^k$ is the measure of agents who remember the sunspot and the fundamental realized k periods earlier. Since (31) is true for all states of nature, we can compare coefficient in front of η_{t-k} and use the facts that each sunspot is orthogonal to all fundamentals:

$$a_k = \mu_k(\delta_0 a_k + \delta_1 a_{k+1}) = \frac{\mu_k\delta_1}{1-\mu_k\delta_0}a_{k+1} \quad \forall k\ge 0,$$

where we use the fact that $\delta_0 < 1$ for the second equality. Because $\delta_1 > 0$ and $\mu_k \to 0$ as $k\to\infty$, $|a_k|$ explodes to infinity, and hence a bounded solution does not exist, unless $a_0 = 0$. But $a_0 = 0$ implies $a_k = 0\ \forall k$.

We now compare coefficients on each θ_{t-k} in (31):

$$\gamma_0 = (1-\delta_0) + \delta_0\gamma_0 + \delta_1\gamma_1 = 1 + \frac{\delta_1}{1-\delta_0}\gamma_1 \tag{32}$$

$$\gamma_k = \left(\delta_0\gamma_k + \delta_1\gamma_{k+1}\right)\mu_k = \frac{\mu_k\delta_1}{1-\mu_k\delta_0}\gamma_{k+1} \quad \forall k\ge 1. \tag{33}$$

Because $\delta_0 < 1$, $\delta_1 > 0$, and $\mu_k \to 0$ as $k\to\infty$, $|\gamma_k|$ explodes to infinity, and hence a bounded solution does not exist, unless $\gamma_1 = 0$. $\gamma_1 = 0$ implies $\gamma_k = 0\ \forall k\ge 1$. Using (32), we know $\gamma_0 = 1$. Together, this means that the equilibrium is unique and is given by $c_t = c_t^F$, where $c_t^F = \theta_t$.

Proof of Proposition 3

Since information sets are given by $I_{i,t} = \{\eta_t, s_{i,t}\}$, any (stationary) strategy[36] can be expressed as

$$c_{i,t} = a\eta_t + bs_{i,t},$$

[36] That the strategy has to be stationary follows from the stationarity of the equilibrium, as defined in Definition 1. To see this, note that, as long as (9) holds, the projection of c_{t+1} on the information set $\{\eta_t, s_{i,t}\}$ yields $E_{i,t}[c_{t+1}] = \tilde{a}\eta_t + \tilde{b}s_{i,t}$ for some coefficients $\tilde{a}, \tilde{b}$. By the individual best response (8), it then also follows that $c_{i,t} = a\eta_t + bs_{i,t}$, for some coefficients a, b.

for some coefficients a and b. Then, $c_{t+1} = a\eta_{t+1} + bc_t$; and since agents have no information about the *future* sunspot, $E_{i,t}[c_{t+1}] = bE_{i,t}[c_t]$. Next, note that $E_{i,t}[c_t] = a\eta_t + b\chi s_{it}$, where

$$\chi = \frac{Var(c_{t-1})}{Var(c_{t-1}) + \sigma^2} \in (0,1].$$

Combining these facts, we infer that condition (8), the individual best response, reduces to

$$c_{i,t} = E_{i,t}[\delta_0 c_t + \delta_1 c_{t+1}] = (\delta_0 + \delta_1 b)E_{i,t}[c_t] = (\delta_0 + \delta_1 b)\left(a\eta_t + b\chi s_{i,t}\right).$$

It follows that a strategy is a best response to itself if and only if

$$a = (\delta_0 + \delta_1 b)a \qquad \text{and} \qquad b = (\delta_0 + \delta_1 b)b\chi. \tag{34}$$

Clearly, $a = b = 0$ is always an equilibrium, and it corresponds to the MSV solution. To have a sunspot equilibrium, on the other hand, it must be that $a \neq 0$ (and also that $|b| < 1$, for it to be bounded). From the first part of condition (34), we see that $a \neq 0$ if and only if $\delta_0 + \delta_1 b = 1$, which is equivalent to $b = \delta^{-1}$. But then the second part of this condition reduces to $1 = \chi$, which in turn is possible if and only if $\sigma = 0$ (since $Var(c_{t-1}) > 0$ whenever $a \neq 0$).

Proof of Proposition 4

Given Assumption 3, a possible equilibrium takes the following form:[37]

$$c_t = \sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k \theta_{t-k} + \chi\zeta_t. \tag{35}$$

From (23), we have that

$$\begin{aligned}
\sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k \theta_{t-k} + \chi\zeta_t &= \theta_t + \zeta_t + \delta\mathbb{E}[\sum_{k=0}^{K_\eta - 1} a_{k+1}\eta_{t-k} + \sum_{k=0}^{K_\beta - 1} \beta_{k+1} c_{t-k} + \sum_{k=0}^{K_\theta - 1} \gamma_{k+1}\theta_{t-k} | \mathbb{I}_t] \\
&= \theta_t + \zeta_t + \delta\left[\sum_{k=0}^{K_\eta - 1} a_{k+1}\eta_{t-k} + \sum_{k=1}^{K_\beta - 1} \beta_{k+1} c_{t-k} + \sum_{k=0}^{K_\theta - 1} \gamma_{k+1}\theta_{t-k}\right] \\
&\quad + \delta\beta_1\left[\sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k \theta_{t-k} + \chi\zeta_t\right]
\end{aligned}$$

[37] Similarly to footnote 36, the stationarity of the coefficients in (35) follows directly from the stationarity of the respective coefficients in Definition 1.

where we use Assumption 1 and the fact that ζ_t is drawn independently over time. For this to be true for all states of nature, we can compare coefficients:

$$a_k = \delta a_{k+1} + \delta\beta_1 a_k \quad \forall k \in \{0, \cdots, K_\eta - 1\} \quad \text{and} \quad a_{K_\eta} = \delta\beta_1 a_{K_\eta} \tag{36}$$

$$\beta_k = \delta\beta_{k+1} + \delta\beta_1\beta_k \quad \forall k \in \{1, \cdots, K_\beta - 1\} \quad \text{and} \quad \beta_{K_\beta} = \delta\beta_1\beta_{K_\beta} \tag{37}$$

$$\gamma_k = \delta\gamma_{k+1} + \delta\beta_1\gamma_k \quad \forall k \in \{1, \cdots, K_\theta - 1\} \quad \text{and} \quad \gamma_{K_\theta} = \delta\beta_1\gamma_{K_\theta} \tag{38}$$

$$\gamma_0 = 1 + \delta\gamma_1 + \delta\beta_1\gamma_0 \quad \text{and} \quad \chi = 1 + \delta\beta_1\chi. \tag{39}$$

First, from the second equation in (39), we know $\delta\beta_1 \neq 1$. Then, from the second parts of (36)–(38), we know $a_{K_\eta} = 0$, $\beta_{K_\beta} = 0$, and $\gamma_{K_\theta} = 0$. From backward induction on (36)–(39), we know that all a, b, γ are zero except for the following:

$$\gamma_0 = 1.$$

From the second equation in (39), we then know $\chi = 1$. We conclude that the unique solution is

$$c_t = c_t^F + \zeta_t,$$

where $c_t^F = \theta_t$.

Proof of Proposition 5

We first note that, with Assumption 1, the MSV solution of (28) is still given by $c_t^F = \theta_t$. Consider an equilibrium taking the form of (9). From (28), Assumption 1, and Assumption 2, we know:

$$\sum_{l=0}^{+\infty} a_l \eta_{t-l} + \sum_{l=0}^{\infty} \gamma_l \theta_{t-l} = (1-\delta_0)\theta_t + \bar{E}_t\left[\sum_{k=0}^{+\infty} \delta_k \left(\sum_{l=0}^{+\infty} a_l \eta_{t+k-l} + \sum_{l=0}^{\infty} \gamma_l \theta_{t+k-l}\right)\right], \tag{40}$$

and

$$\bar{E}_t[\eta_{t-l}] = \begin{cases} \mu_l \eta_{t-l} & \text{if } l \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \bar{E}_t[\theta_{t-l}] = \begin{cases} \mu_l \theta_{t-l} & \text{if } l \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

where $\mu_l = (1-\lambda)^l$ is the measure of agents who remember the sunspot and the fundamental realized l periods earlier, as in the proof of Proposition 2. Comparing coefficient in front of η_{t-l} and using the facts that each sunspot is orthogonal to all fundamentals:

$$a_l = \mu_l \sum_{k=0}^{+\infty} \delta_k a_{k+l} \quad \forall l \geq 0. \tag{41}$$

Because $\lim_{l\to\infty} \mu_l = 0$, there necessarily exists an $\hat{l}$ finite but large enough $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$.[38]

Since we are focusing bounded equilibria as in Definition 1, there exists a scalar $M > 0$, arbi-

[38] $\sum_{k=0}^{\infty} |\delta_k| < \infty$ because $\Delta < \infty$.

trarily large but finite, such that $|a_l| \leq M$ for all l. From (41), we then know that, for all $l \geq \hat{l}$,

$$|a_l| \leq \mu_{\hat{l}} M \sum_{k=0}^{+\infty} |\delta_k|, \tag{42}$$

where we also use the fact that the sequence $\{\mu_l\}_{l=0}^{\infty}$ is decreasing. Now, we can use (41) and (42) to provide a tighter bound of $|a_l|$. That is, for all $l \geq \hat{l}$,

$$|a_l| \leq \left(\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k|\right)^2 M.$$

We can keep iterating. Since $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$, we then have $a_l = 0$ for all $l \geq \hat{l}$. Using (41) and doing backward induction, we then know $a_l = 0$ for all l, where we use the fact that $\delta_0 < 1$.

Now, (40) can be simplified as

$$\begin{aligned} \sum_{l=0}^{\infty} \gamma_l \theta_{t-l} &= (1-\delta_0)\theta_t + \bar{E}_t\left[\sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{\infty} \gamma_l \theta_{t+k-l}\right]. \\ &= (1-\delta_0)\theta_t + \sum_{k=0}^{+\infty} \delta_k \gamma_k \theta_t + \bar{E}_t\left[\sum_{l=1}^{+\infty}\left(\sum_{k=0}^{+\infty} \delta_k \gamma_{k+l}\right)\theta_{t-l}\right]. \end{aligned} \tag{43}$$

For this to be true for all states of nature, we can compare coefficients on each θ_{t-l}:

$$\gamma_0 = 1 - \delta_0 + \sum_{k=0}^{+\infty} \delta_k \gamma_k \tag{44}$$

$$\gamma_l = \mu_l \sum_{k=0}^{+\infty} \delta_k \gamma_{k+l} \quad \forall l \geq 1. \tag{45}$$

The above two equations can be re-written as:

$$\gamma_0 = (1-\delta_0)^{-1}\left(1 - \delta_0 + \sum_{k=1}^{+\infty} \delta_k \gamma_k\right) \tag{46}$$

$$\gamma_l = \left(1 - \mu_l \delta_0\right)^{-1}\left(\sum_{k=1}^{+\infty} \delta_k \gamma_{k+l}\right) \quad \forall l \geq 1, \tag{47}$$

where we use $\delta_0 < 1$ and $\mu_l < 1$.

From Definition 1, we know that there is a scalar $M > 0$ such that $\left|\gamma_l\right| \leq M$ for all $l \geq 0$. From (45), we know, for all $l \geq 1$

$$\left|\gamma_l\right| \leq \mu_l \left(\sum_{k=0}^{+\infty} |\delta_k|\right) M. \tag{48}$$

Because $\lim_{l \to \infty} \mu_l = 0$, there necessarily exists an $\hat{l}$ finite but large enough such that $\left(\sum_{k=0}^{+\infty} |\delta_k|\right) \mu_{\hat{l}} < 1$. We then know that, for all $l \geq \hat{l}$,

$$\left|\gamma_l\right| \leq \mu_{\hat{l}} \left(\sum_{k=0}^{+\infty} |\delta_k|\right) M.$$

Now, we can use the above formula and (45) to provide a tighter bound of $|\gamma_l|$: for all $l \geq \hat{l}$,

$$|\gamma_l| \leq (\mu_{\hat{l}})^2 \left(\sum_{k=0}^{+\infty} |\delta_k| \right)^2 M.$$

We can keep iterating. Since $\left(\sum_{k=0}^{+\infty} |\delta_k|\right) \mu_{\hat{l}} < 1$, we then have $\gamma_l = 0$ for all $l \geq \hat{l}$. Using (47) and doing backward induction, we then know $\gamma_l = 0$ for all $l \geq 1$ and, from (46),

$$\gamma_0 = 1.$$

Together, this means that the equilibrium is unique and is given by $c_t = c_t^F = \theta_t$. This proves the Proposition.

B Additional Material for Sections 2-5

This Appendix corroborates various claims made in the main text. First, we show how our analysis goes through if consumers observe perfectly their own income and own interest rate, provided that we abstract from signal-extraction about payoff- irrelevant histories. Second, we show how to fill in the missing details about labor supply. Third, we explain why the simplification of infinite histories and stationary equilibria is non-essential. Fourth, we formalize the sense in which Assumption 2 is compatible with nearly perfect information of both exogenous shocks and endogenous outcomes. Fifth, we illustrate how our result extends to variants of Taylor rules, whereby monetary policy responds to either past inflation or its expected future value. Sixth, we generalize Proposition 3 to the case with correlated private noise. Finally, we discuss how our result may translate in non-linear settings featuring multiple steady states.

B.1 A Variant with Observation of Current Outcomes

Our baseline model abstracts from idiosyncratic shocks. It also lets consumers be inattentive to, or face uncertainty about, their current income and interest rates. We now relax these assumptions and show how to nest the economy in the same game form as that in the main text, modulo an inconsequential adjustment in the coefficients δ_0 and δ_1.

For any aggregate variable $x_t \in \{y_t, i_t, \pi_t, \varrho_t\}$, let $x_{i,t}$ be the corresponding individual-level variable. Suppose further that future idiosyncratic shocks are unpredictable, so that $E_{i,t}[x_{i,t+1}] =$

$E_{i,t}[x_{t+1}]$ for any such variable, and that consumer i observes $(y_{i,t}, i_{i,t}, \pi_{i,t}, \varrho_{i,t})$ when setting $c_{i,t}$. Then, the optimal consumption function of a young consumer, equation (2) in the main text, is modified as follows:[39]

$$c^1_{i,t} = -\tfrac{\beta}{1+\beta}\sigma(i_{i,t} - E_{i,t}[\pi_{t+1}] - \varrho_{i,t}) + \tfrac{1}{1+\beta}y_{i,t} + \tfrac{\beta}{1+\beta}E_{i,t}[y_{i,t+1}]$$

Aggregating the above and using the fact, explained in the main text, that aggregate consumption equals the average consumption of the young, we infer that

$$c_t = -\tfrac{\beta}{1+\beta}\sigma(i_t - \varrho_t) + \tfrac{1}{1+\beta}y_t + \bar{E}_t\left[\tfrac{\beta}{1+\beta}y_{t+1} + \tfrac{\beta}{1+\beta}\sigma\pi_{t+1}\right].$$

Combining this with market clearing ($y_t = c_t$ and $y_{t+1} = c_{t+1}$), and solving out for c_t we get

$$c_t = -\sigma\left(i_t - \bar{E}_t[\pi_{t+1}] - \varrho_t\right) + \bar{E}_t\left[c_{t+1}\right].$$

That is, the DIS curve is now the same as in the representative-agent benchmark, modulo the replacement of that agent's full-information expectation with the average, incomplete-information expectation in the population. By the same token, once we substitute out the interest rate and inflation, our game representation becomes

$$c_t = \theta_t + \delta\bar{E}_t[c_{t+1}].$$

That is, the game representation is even simpler than that in the main text.

Clearly, Proposition 5 continues to hold, provided that consumers form expectations in the manner implied by Assumption 2. But now there is a tension between this assumption and the assumption made above that consumers observe their own income and interest rates. By invoking Assumption 2, we have effectively abstracted from the possibility that consumers extract information about payoff-irrelevant aggregate histories from their own individual wealth, income and interest rates. This seems realistic, especially given that the idiosyncratic fluctuations are much larger than the aggregate ones. But it also brings to the forefront the technical complications that our analysis has painstakingly tried to bypass, either by abstracting from signal extraction (here) or by allowing it but introducing different perturbations (in Section 5).

With signal extraction, there might exist non-fundamental equilibria in which the observation of own income and own interest rates may reveal information about past sunspots, and such endogenous information may not necessarily satisfy Assumption 2. Such signal extraction is bound to confound sunspots with idiosyncratic fundamentals, even if there are no aggregate fundamental shocks. Such confounding can itself be the source of multiple equilibria (Benhabib et al., 2015; Gaballo, 2017; Acharya et al., 2021), albeit of a different kind than those obtained in the full-information benchmark. All in all, we are unsure what it takes for our uniqueness result

[39] On equilibrium, it is always that $\tau_{t+1} = 0$ as in the main text.

to be robust to such signal extraction—and we cannot really address the issue because of the severe technical complications introduced by signal-extraction and infinite-regress problems.

This circles back to our discussion of why our main approach treats information as exogenous. That said, it should be clear from the above that the observability of current income and interest rates is not relevant *per se*: if consumers observe these objects but their expectations of future aggregate outcomes continue to satisfy Assumption 2, then the MSV solution remains the unique equilibrium. Finally, if one insists that young consumers not only observe these objects but also freely condition their expectations of future outcomes on them, then our main argument no more applies, but uniqueness can still be obtained via the alternative perturbation considered in Section 5.

B.2 Flexible Labor Supply

In the main text, we sidestepped labor supply and production. We now show how to fill in the missing details, without affecting our results.

Production is given by

$$Y_t = N_t, \tag{49}$$

where N_t is total employment. Since output is demand-determined, labor demand is given by $N_t = Y_t = C_t$, where C_t is aggregate spending. Conditional on C_t, the specification of labor supply therefore matters only in the determination of the real wage and the split of total income between labor income and firm profits. What needs to be shown, however, is that C_t is determined in the same way as in the main text.

As in the main text, there are overlapping generations of consumers, each living for two periods. But unlike the main text, consumers choose not only how much to spend but also how much to work. Accordingly, the complete preferences are given by

$$u(C^1_{i,t}) - v\left(N^1_{i,t}\right) + \beta e^{-\varrho_t}\left(u(C^2_{i,t+1}) - v\left(N^2_{i,t+1}\right)\right),$$

and the complete budgets in the two periods of life are given by

$$P_t C^1_{i,t} + B_{i,t} = P_t\left(W_t N^1_{i,t} + D^1_t\right) \qquad \text{and} \qquad P_{t+1}C^2_{i,t+1} = P_{t+1}\left(W_{t+1}N^2_{i,t+1} + D^2_{t+1} - T_{t+1}\right) + I_t B_{i,t},$$

where $v(N) \equiv \frac{1}{1+\psi}N^{1+\psi}$, $N^1_{i,t}$ and $N^2_{i,t+1}$ are the amounts of labor supplied when young and old, respectively, W_t is the real wage, D^1_t and D^2_t are the real profits distributed to young and old agents, and all other variables are the same as in Section 1. Also as in the main text, the central bank clears any non-zero aggregate claims with taxes on the old, $P_t T_t = I_{t-1}\int B_{i,t-1}di$. Finally, we let $\tilde{B}_{i,t} = B_{i,t}/P_t$ denote real saving/borrowing.

To simplify, we assume that old consumers choose consumption and labor supply under full information.[40] After the usual log-linearization, this translates to the following optimal rules for the old consumers:

$$n_{i,t}^2 = \frac{1}{\psi}\left(w_t - \frac{1}{\sigma}c_{i,t}^2\right),$$

$$c_{i,t}^2 = \tfrac{1}{\beta}\tilde{b}_{i,t-1} + \Omega\left(w_t + n_{i,t}^2\right) + (1-\Omega)\,d_t^2 - \tau_t,$$

where Ω is the ratio of labor income to total income in steady state.[41] Young consumers, on the other hand, are subject to the informational friction of interest, so that their optimal rules are given by the following:

$$n_{i,t}^1 = \frac{1}{\psi}\left(E_{i,t}[w_t] - \frac{1}{\sigma}c_{i,t}^1\right),$$

$$c_{i,t}^1 = E_{i,t}\left[\tfrac{1}{1+\beta}\left(\Omega\left(w_t + n_{i,t}^1\right) + (1-\Omega)\,d_t^1\right) + \tfrac{\beta}{1+\beta}\left(\Omega\left(w_{t+1} + n_{i,t+1}^2\right) + (1-\Omega)\,d_{t+1}^2 - \tau_{t+1}\right) - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right],$$

where $E_{i,t}$ is the rational expectation conditional on a young consumer's information set, whatever that might be.[42]

So far, we have taken no stand on how firm profits are distributed between the young and the old. To map exactly to the analysis in Section 1, we henceforth let

$$D_t^1 = Y_t - W_t \int N_{i,t}^1 di \quad \text{and} \quad D_t^2 = Y_t - W_t \int N_{i,t}^2 di. \tag{50}$$

This can be justified by having the government tax all firm profits and redistribute them according to the above rule. Alternatively, we can assume that firms live for two periods; and that young (respectively, old) firms are owned exclusively by young (old) consumers and employ exclusively young (old) workers. Either way, the key is that the average income of the young is the same as that of the old (and hence they are both equal to Y_t), just as in Section 1. Relaxing this assumption complicates the game representation but does not change the essence.

Using the above, we infer that

$$E_{i,t}\left[\Omega\left(w_t + n_{i,t}^1\right) + (1-\Omega)\,d_t^1\right] = E_{i,t}\left[\Omega\left(w_t + n_t^1\right) + (1-\Omega)\,d_t^1\right] = E_{i,t}\left[y_t\right]$$

$$E_{i,t}\left[\Omega\left(w_{t+1} + n_{i,t+1}^2\right) + (1-\Omega)\,d_{t+1}^2\right] = E_{i,t}\left[\Omega\left(w_{t+1} + n_{t+1}^2\right) + (1-\Omega)\,d_{t+1}^2\right] = E_{i,t}\left[y_{t+1}\right],$$

where $n_t^1 = \int n_{i,t}^1 di$ and $n_{t+1}^2 = \int n_{i,t+1}^2 di$.

Because the central bank clears any non-zero aggregate claims with taxes on the old, $P_t T_t =$

[40] This simplification is not essential but mirrors the main text's treatment of the old consumers as "robots" and, as it will be shown below, helps reduce the economy to *exactly* the same game as that obtained in the main text.

[41] Consistent with footnote 12, we have let $\tilde{b}_{i,t} \equiv \tilde{B}_{i,t}/Y^{ss}$, because $B^{ss} = 0$.

[42] Similar to the main text, the above allows the young consumers to be uncertain about, or inattentive to, current income (here, wages and dividends) and current interest rates. But as discussed there, such inattention is vanishingly small when $\lambda \to 0^+$, and can be dispensed with along the lines spelled out in Appendix B.

$I_{t-1}\int B_{i,t-1}di$ and therefore $\int C_{i,t}^2 di = Y_t$. By market clearing in the goods market, $C_t \equiv \frac{1}{2}\int C_{i,t}^1 di + \frac{1}{2}\int C_{i,t}^2 di = Y_t$. Combining, we infer that $\int C_{i,t}^1 di = Y_t = C_t$ and, by direct implication, $\int B_{i,t} di = 0$ and $T_{t+1} = 0$. As in the main text, in effect, the net supply of bonds is zero.

As a result, the young consumer's optimal consumption can be written as

$$c_{i,t}^1 = E_{i,t}\left[\tfrac{1}{1+\beta}y_t + \tfrac{\beta}{1+\beta}y_{t+1} - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right].$$

Aggregating the above equation, we get

$$\int c_{i,t}^1 di = \bar{E}_t\left[\tfrac{1}{1+\beta}y_t + \tfrac{\beta}{1+\beta}y_{t+1} - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right].$$

Since the average private saving of the young has to be zero in equilibrium, similar to the main analysis, we still have that

$$\int c_{i,t}^1 di = \int c_{i,t}^2 di = c_t = y_t.$$

Putting everything together, we arrive at the same DIS equation as in the main text:

$$c_t = \bar{E}_t\left[\tfrac{1}{1+\beta}c_t + \tfrac{\beta}{1+\beta}c_{t+1} - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right].$$

By direct implication, the rest of the analysis remains the same as well.

B.3 Time 0 and Non-stationary Equilibria

In the preceding analysis, we let histories be infinite and restricted equilibria to be stationary. To understand what exactly this simplification does, abstract from fundamentals (this is without any loss), let calendar time start at $t = 0$, and modify (9) as follows:

$$c_t = \bar{c}_t + \sum_{k=0}^{t} a_{t,k}\eta_{t-k},$$

where $\{a_{t,k}\}$ and $\{\bar{c}_t\}$ are uniformly bounded deterministic coefficients. Note that this allows for (i) a time-varying, non-zero deterministic intercept and (ii) the equilibrium load of a sunspot to be a function of not only its age (k) but also the calendar time.

It is straightforward to show that Assumption 2 continues to rule out sunspot fluctuations, that is, $a_{t,k} = 0$ for all t, k.[43] But it does not immediately rule a deterministic, time-varying inter-

[43] To illustrate, consider the simplified case (16) studied in the main text. Then, condition (17) becomes $a_{t,k} = \mu_k \delta a_{t+1,k+1}$, and similarly condition (18) becomes

$$a_{t+k,k} = \frac{a_{t,0}}{\delta^k(1-\lambda)^{\frac{k(k-1)}{2}}},$$

which once again explodes unless $a_{t,0} = 0$ for all t. That is, we now have $a_{t,k} = 0$ for all t and all k, which means that sunspot equilibria are ruled out even if we do not impose stationarity.

cept. In particular, c_t is now an equilibrium if and only if

$$c_t = \bar{c}_t = \delta^{-t}\bar{c}_0, \tag{51}$$

for arbitrary $\bar{c}_0 \in \mathbb{R}$. At first glance, this appears to contradict our claim of equilibrium uniqueness. But this is only an artifact of introducing infinite social memory "through the back door."

Let us explain. Clearly, (51) is exactly the same as the following sunspot equilibrium:

$$c_t = \delta^{-t}\eta_0,$$

with the constant $\bar{c}_0$ in place of the sunspot η_0. That is, all the "deterministic" equilibria obtained above are really sunspot equilibria in disguise. But by treating $\bar{c}_0$ (equivalently, c_0) as a deterministic scalar instead of a random variable, we have artificially bypassed the friction of interest: we have effectively imposed that the initial sunspot can never be forgotten.

To sum up, insofar as one remains true to the spirit of Assumption 2, one must treat any initial sunspot as a random variable rather than a deterministic constant. And provided that this is done, our result goes through.

B.4 Indirect Knowledge about Endogenous Outcomes

Although Assumption 2 excluded direct observation of endogenous aggregate outcomes, such as output and inflation, our main result can be said to compatible with nearly perfect knowledge of such outcomes, in the following sense:

Proposition 6 (Nearly perfect information about endogenous outcomes). *Suppose $Var(c_t) < +\infty$.*[44] *For any given mapping from h^t to c_t as in Definition 1, any $K < \infty$ arbitrarily large but finite, and any $\epsilon, \epsilon' > 0$ arbitrarily small but positive, there exists $\hat{\lambda} > 0$ such that: whenever $\lambda \in (0, \hat{\lambda})$, $Var\left(E_t^i[c_{t-k}] - c_{t-k}\right) \leq \epsilon$ for all $k \in \{0, 1, \cdots, K\}$, for at least a mass $1-\epsilon'$ of agents and for every t. (And the same is true if we replace c_{t-k} with π_{t-k}, i_{t-k}, or any linear combination thereof.)*

Proof: Consider a candidate equilibrium c_t in Definition 1. We first use I_t^s to denote the information set of the period-t agent with memory length s:

$$\mathbb{I}_t^s = \left\{\eta_{t-s}, \cdots, \eta_t, \theta_{t-s}, \cdots, \theta_t\right\}.$$

From Definition 1, we know that c_t can be written as

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k}.$$

[44] This slightly strengthens the notion of boundedness relative to Definition 1.

From the law of total variances, we have

$$Var\left(E_t\left[c_t|I_t^s\right]-c_t\right)\leq Var\left(\sum_{k=s+1}^{\infty}a_k\eta_{t-k}+\sum_{k=s+1}^{\infty}\gamma_k\theta_{t-k}\right).$$

Since η_t and θ_t are independent of each other as well as independent over time, the finiteness of $Var\left(c_t\right)$ implies that

$$\lim_{s\to+\infty}Var\left(\sum_{k=s+1}^{\infty}a_k\eta_{t-k}+\sum_{k=s+1}^{\infty}\gamma_k\theta_{t-k}\right)=0.$$

As a result, for any $\epsilon>0$ arbitrarily small but positive, there exists $\hat{s}_0$, such that

$$Var\left(E_t\left[c_t|\mathbb{I}_t^s\right]-c_t\right)\leq\epsilon$$

for all $s\geq\hat{s}_0$ and every t. Similarly, for each $k\leq K$, there exists $\hat{s}_k$, such that

$$Var\left(E_t\left[c_{t-k}|\mathbb{I}_t^s\right]-c_{t-k}\right)\leq\epsilon$$

for all $s\geq\hat{s}_k$ and every t. Now, for any $\epsilon'>0$ arbitrarily small but positive, we can find $\hat{\lambda}>0$ such that $\left(1-\hat{\lambda}\right)^{\hat{s}_k}\geq 1-\epsilon'$ for all $k\in\{0,\cdots,K\}$. Together, this means that, whenever $\lambda\in(0,\hat{\lambda})$, we have $Var\left(E_t^i\left[c_{t-k}\right]-c_{t-k}\right)\leq\epsilon$ for all $k\leq K$, for at least a fraction $1-\epsilon'$ of agents, and for every period t. □

The following important qualification, however, applies. The above result allows the mapping from h^t to c_t to be arbitrary but treats this mapping as fixed when λ is lowered towards 0. But the *equilibrium* mapping from h^t to c_t may well vary with λ, upsetting the result. In Section 5 we therefore present two alternative information structures, which allow for direct observation of past outcomes and properly deal with this endogeneity.

B.5 Alternative Monetary Policies

In the main analysis, we let monetary policy respond to the *current* rate of inflation. Here, we illustrate how our result extends to variants of such Taylor rules, whereby monetary policy responds to either past inflation or its expected future value.

Consider first the following forward-looking rule:

$$i_t=z_t+\phi\mathbb{E}_t\left[\pi_{t+1}\right],\tag{52}$$

where $\phi\geq 0$. In this case, the economy still reduces to a game as in (6), albeit for different values for δ_0 and δ_1. But since our result does not depend on the values of these coefficients, Proposition 5 directly extends.

Suppose next the following backward-looking rule:

$$i_t = z_t + \phi \pi_{t-1}, \tag{53}$$

where $\phi \geq 0$. Even though this case is not directly nested in (6), a version of our argument still goes through.

Proposition 7 (**Alternative monetary policies**). *Suppose that Assumption 2 holds, that there are no shocks to fundamentals, and monetary policy takes the form of (53). The equilibrium is unique and is given by the MSV solution.*

Proof: From (3), (4), and (53), we have that any equilibrium must satisfy

$$c_t = \bar{E}_t \left[\tfrac{1}{1+\beta} c_t - \tfrac{\beta}{1+\beta} \sigma \phi \kappa c_{t-1} + \tfrac{\beta}{1+\beta} (1+\sigma\kappa) c_{t+1} \right]; \tag{54}$$

and since there are no shocks to fundamentals, we search for solutions of the form $c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k}$. The goal is to verify that $a_k = 0$ for all k.

By Assumption 2, we have that, for all $k \geq 0$,

$$\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$$

where $\mu_k \equiv (1-\lambda)^k$ measures the fraction of the population at any given date that know, or remember, a sunspot realized k periods earlier. Future sunspots, on the other hand, are known to nobody. It follows that, in any candidate solution, average expectations satisfy

$$\bar{E}_t[c_t] = \sum_{k=0}^{+\infty} a_k \mu_k \eta_{t-k}$$

and similarly

$$\bar{E}_t[c_{t-1}] = \sum_{k=1}^{+\infty} a_{k-1} \mu_k \eta_{t-k}.$$

$$\bar{E}_t[c_{t+1}] = \sum_{k=0}^{+\infty} a_{k+1} \mu_k \eta_{t-k}$$

For condition (54) to be true for all sunspot realizations, it is necessary and sufficient that

$$a_0 = (1+\sigma\kappa) a_1$$

and, for $k \geq 1$,

$$a_k = \mu_k \left(\frac{1}{1+\beta} a_k - \tfrac{\beta}{1+\beta} \sigma \phi \kappa a_{k-1} + \tfrac{\beta}{1+\beta} (1+\sigma\kappa) a_{k+1} \right).$$

We hence have, for $k \geq 1$,

$$a_{k+1} = \frac{\frac{1}{\mu_k} - \frac{1}{1+\beta}}{\frac{\beta}{1+\beta}(1+\sigma\kappa)} a_k + \frac{\sigma\phi\kappa}{1+\sigma\kappa} a_{k-1}. \tag{55}$$

Since $\frac{1}{\mu_k} - \frac{1}{1+\beta} > 0$, we know that, all $\{a_k\}_{k=0}^{+\infty}$ have the same sign if $a_0 \neq 0$. But because $\mu_k \to 0$, we have that $|a_k|$ explodes to infinity as $k \to \infty$ from (55) unless $a_0 = 0$. But $a_0 = 0$ implies $a_k = 0$ for all k. We conclude that the unique bounded equilibrium is $a_k = 0$ for all k, or equivalently $c_t = 0$ for all t and h^t, which herein corresponds to the MSV solution. $\square$

B.6 Extending Proposition 3 to Correlated Signals

Here, we generalize Proposition 3 by letting information be given by

$$\mathbb{I}_{i,t} = \{\eta_t, s_{i,t}\}, \qquad \text{with} \qquad s_{i,t} = c_{t-1} + v_t + \varepsilon_{i,t}. \tag{56}$$

where $v_t \sim \mathcal{N}(0, \sigma_v^2)$ is an aggregate noise and $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2)$ is idiosyncratic noise. They are independent of each other, other shocks, and across time. This can be interpreted as a situation where a publicly available statistic is not only contaminated with measurement error but also observed with idiosyncratic noise due to rational inattention (Sims, 2003) or imperfect cognition (Woodford, 2019). The case studied in main text can be nested by letting $\sigma_v^2 = 0$.

Corollary 1. *Proposition 3 continues to hold when information is given by (56).*

Proof: Since information sets are given by $\mathbb{I}_{i,t} = \{\eta_t, s_{i,t}\}$, any (stationary) strategy can be expressed as

$$c_{i,t} = a\eta_t + b s_{i,t},$$

for some coefficients a and b. Then, $c_{t+1} = a\eta_{t+1} + b(c_t + v_{t+1})$; and since agents have no information about the *future* η_{t+1} and v_{t+1}, $E_{i,t}[c_{t+1}] = bE_{i,t}[c_t]$. Next, note that $E_{i,t}[c_t] = a\eta_t + b\chi s_{i,t}$, where

$$\chi = \frac{Var(c_{t-1}) + \sigma_v^2}{Var(c_{t-1}) + \sigma_v^2 + \sigma^2} \in (0, 1].$$

Combining these facts, we infer that condition (8), the individual best response, reduces to

$$c_{i,t} = E_{i,t}[\delta_0 c_t + \delta_1 c_{t+1}] = (\delta_0 + \delta_1 b) E_{i,t}[c_t] = (\delta_0 + \delta_1 b)\left(a\eta_t + b\chi s_{i,t}\right).$$

It follows that a strategy is a best response to itself if and only if

$$a = (\delta_0 + \delta_1 b)a \qquad \text{and} \qquad b = (\delta_0 + \delta_1 b) b\chi. \tag{57}$$

Clearly, $a = b = 0$ is always an equilibrium, and it corresponds to the MSV solution. To have a sunspot equilibrium, on the other hand, it must be that $a \neq 0$ (and also that $|b| < 1$, for it to be bounded). From the first part of condition (57), we see that $a \neq 0$ if and only if $\delta_0 + \delta_1 b = 1$, which is equivalent to $b = \delta^{-1}$. But then the second part of this condition reduces to $1 = \chi$, which in turn is possible if and only if $\sigma = 0$ (since $Var(c_{t-1}) > 0$ whenever $a \neq 0$). $\square$

B.7 Non-linearities and Multiple Steady States

Here we use an example, suggested by a referee, to clarify that our result speaks only to local determinacy around a given steady state: global indeterminacy may still be possible, at least when non-linearities support multiple steady-state equilibria.

Suppose that an agent's best response is given by

$$c_{i,t} = \delta \mathbb{E}_{i,t}[c_{t+1}] - a\mathbb{E}_{i,t}[c_t^3], \tag{58}$$

for some scalars δ, a. When $a = 0$, this reduces back to our baseline, linear model and our main result applies. The point here is to understand what happens when $a \neq 0$. Let us focus in particular on how a matters when $\delta > 1$.

When $a \leq 0$, there is a unique steady state and is given by $c_{i,t} = 0$. When instead $a > 0$, (58) admits *three* steady states. These are given by

$$c_{i,t} = -\bar{c}, \qquad c_{i,t} = 0, \qquad \text{and} \qquad c_{i,t} = \bar{c},$$

where $\bar{c} \equiv \sqrt{\frac{\delta-1}{a}}$. If we linearize (58) around any of these steady states, we can apply our result to the corresponding linearized model. In this sense, our approach guarantees local determinacy around all three steady states regardless of their eigenvalues. But our approach does not guarantee global determinacy.

This should not be totally surprising. In our baseline model, the unique steady state, which is given by $c_{i,t} = 0$, serves as an anchor for expectations of future outcomes, in a similar way that the common prior serves as an anchor for higher-order beliefs in the static games of Morris and Shin (1998, 2002). When there are multiple steady states, each one of them can play this kind of anchoring role locally, helping guarantee local determinacy. But our approach is silent about global dynamics, such as jumps from one steady state to another.

To illustrate what we mean, consider the following example, which was proposed by a referee. Suppose there exists a sunspot following a two-state Markov chain with values $\eta_t \in \{-1, +1\}$ and transition probability $1 - \pi$. Suppose next that all agents coordinate on playing the following strategy, which requires knowledge only of the concurrent sunspot realization:

$$c_{i,t} = \omega \eta_t,$$

for some $\omega \neq 0$. This means, more simply, that all agents coordinate on playing the same action, and that this action follows a two-state Markov chain with values $c_{i,t} \in \{-\omega, +\omega\}$ and with the probability of staying in the same state equals to π.

It is straightforward to check that this strategy constitutes an equilibrium if and only if $\omega = \sqrt{\frac{\delta(2\pi-1)-1}{a}}$, which in turn is well defined if and only if $\pi \in \left(\frac{1+\delta^{-1}}{2}, 1\right)$. Also, as $\pi \to 1$, we have that

$\omega \to \bar{c}$, that is, this type of equilibrium translates to infrequent jumps across the two outer steady states. Finally, this type of equilibrium is robust to imperfect knowledge of the distant past in the following sense: it suffices to have common knowledge of the current realization of the sunspot (which itself is persistent as long as $\pi \neq \frac{1}{2}$) and of the parameters π, a, and δ.

It is important to recognize that the equilibrium constructed above is *not* memoryless: the restriction $\pi > \frac{1+\delta^{-1}}{2}$ implies $\pi > \frac{1}{2}$, which means that the sunspot itself *has* to be persistent. This example therefore links to our discussion of persistent sunspots discussed in Section 5. But there is a key difference: whereas there was a unique value for the persistence parameter ρ that supported multiplicity in our linear setting, now there is a whole range of values for the corresponding parameter π that supports multiplicity in the present example.

Does this upset our main message? Not necessarily. First of all, we have been upfront that our paper is ultimately only about local determinacy, and from this perspective, our result is still valid: if we linearize the present example around any of the three steady states, we still have local determinacy. Second, and related, the above example is not a "perturbation" of our original setting: as the non-linearity gets smaller (in the sense that a converges to 0 from above), the outer two steady states diverge to plus/minus infinity, and so do the values of c_t in the equilibrium constructed above; that is, this equilibrium becomes unbounded. Last but not least, even though it is robust to Assumption 2, this equilibrium still assumes a significant degree of dynamic coordination: to jump from one steady state to another, or more precisely between the two points of the Markov chain, agents must be confident not only that other agents will do the same today but also that future generations will stay at the new point with sufficient probability.

This begs the question of how sensitive the type of equilibrium constructed above is to perturbations of intertemporal common knowledge, albeit of a different form from those considered in this paper. But our methods are not equipped to answer this question. At the end of the day, we thus prefer to iterate our "real" take-home lesson: our contribution is not to argue that all kinds of dynamic indeterminacy are gone, but rather to shed new light on the (local) determinacy problem of the New Keynesian model, to provide a formal justification for treating this problem as a bug, and to set the foundations for re-thinking both the Taylor principle and the FTPL.

C Persistent Fundamentals and Persistent Sunspots

In this Appendix, we first extend Proposition 2 to a more general specification for the fundamentals and make clear that this only changes the nature of the MSV solution. We then verify that our uniqueness argument is robust to persistent sunspots, except for a knife-edge case. We deal with each separately only to maximize clarity.

C.1 General Fundamentals

Consider the baseline model but modify the specification of the fundamental as follows:

Assumption 4 (***General Fundamentals***)**.** *The fundamental admits the following representation:*

$$\theta_t = q' x_t \qquad with \qquad x_t = R x_{t-1} + \varepsilon_t^x, \tag{59}$$

where $q \in \mathbb{R}^n$ is a vector, R is an $n \times n$ matrix of which all the eigenvalues are within the unit circle (to guarantee stationarity), $\varepsilon_t^x \sim \mathcal{N}(\mathbf{0}, \Sigma_\varepsilon)$, and Σ_ε is a positive definite matrix. The sunspot η_t is i.i.d. over time with mean zero.

This directly nests the case in which (ϱ_t, ξ_t, z_t) follows an arbitrary VARMA. It also allows x_t to contain "news shocks," or forward guidance about future monetary policy. We henceforth refer to x_t as the *fundamental state*. The economy's history is now given by $h^t = \{x_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$, the infinite history of the fundamental state and the sunspot.

Definition 1 and Assumption 2 adapt to this generalization as follows.

Definition 2 (**Equilibrium**)**.** *An equilibrium is any solution to equation (6) along which: expectations are rational, although potentially based on imperfect and heterogeneous information about h^t; and the outcome is given by*

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k' x_{t-k} \tag{60}$$

where $a_k \in \mathbb{R}$ and $\gamma_k \in \mathbb{R}^n$ are known and uniformly bounded coefficients.[45]

Assumption 5 (**Social memory**)**.** *In every period t, a consumer's information set is given by*

$$\mathbb{I}_{i,t} = \{(x_t, \eta_t), \cdots, (x_{t-s_{i,t}}, \eta_{t-s_{i,t}})\},$$

where $s_{i,t} \in \{0, 1, \cdots\}$ is drawn from a geometric distribution with parameter λ, for some $\lambda \in (0,1]$.

With these minor adjustments in place, we can readily extend our main result. As anticipated in the main text, the only subtlety regards the existence and characterization of the MSV solution.

Because equation (10) is purely forward looking and x_t is a sufficient statistic for both the concurrent θ_t and its expected future values, it is natural to look for a solution in which c_t is a function of x_t alone, and this defines the MSV solution. Thus guess $c_t = \gamma' x_t$ for some $\gamma \in \mathbb{R}^n$; use this to compute $\mathbb{E}_t[c_{t+1}] = \gamma' R x_t$; and substitute into (10) to get $c_t = \theta_t + \delta\gamma' R x_t = [q' + \delta\gamma' R] x_t$. Clearly, the guess is verified if and only if γ' solves $\gamma' = q' + \delta\gamma' R$, which in turn is possible if and only if $I - \delta R$ is invertible (where I is the $n \times n$ identity matrix) and $\gamma' = q'(I - \delta R)^{-1}$. We conclude that the following assumption is necessary and sufficient for the existence of the MSV solution:

[45] This means that there exists a scalar $M > 0$ such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k, where $\|\cdot\|_1$ is the L^1-norm.

Assumption 6. *The matrix $I - \delta R$ is invertible.*

This is the analogue of the restriction $\delta\rho \neq 1$ in the main text, where θ_t follows an AR(1). Like before, this restriction is used to guarantee the existence of the MSV solution but, as it is evident from the proof of the next proposition, it has no bite on the question of whether *other* solutions exist. For the latter, what is key is the information agents have about payoff-irrelevant histories, which is where Assumption 5 comes in.

The following generalizes our main result to the present context:

Proposition 8. *Under Assumptions 5 and 6, Proposition 2 continues to hold, modulo the following adjustment of the MSV solution:*

$$c_t^F \equiv q' (I - \delta R)^{-1} x_t. \tag{61}$$

Proof. Since the sunspots $\{\eta_{t-k}\}_{k=0}^{\infty}$ are orthogonal to the fundamental states $\{x_{t-k}\}_{k=0}^{\infty}$, the same argument as that used in Proposition 2 still proves that $a_k = 0$ for all k. We can thus focus on solutions of the following form:

$$c_t = \sum_{k=0}^{\infty} \gamma_k' x_{t-k}. \tag{62}$$

And the remaining task is to show that $\gamma_0' = q'(I - \delta R)^{-1}$ and $\gamma_k' = 0$ for all $k \geq 1$, which is to say that only the MSV solution survives.

To start with, note that, since x_t is a stationary Gaussian vector given by (59), the following projections apply for all $k \geq s \geq 0$:

$$\mathbb{E}[x_{t-k} | \{x_t, \cdots, x_{t-s}\}] = W_{k,s} x_{t-s},$$

where

$$W_{k,s} \equiv \mathbb{E}\left[x_{t-k} x_{t-s}'\right] \mathbb{E}\left[x_t x_t'\right]^{-1} = \mathbb{E}\left[x_t x_t'\right] \left(R'\right)^{k-s} \mathbb{E}\left[x_t x_t'\right]^{-1}$$

is an $n \times n$ matrix capturing the relevant projection coefficients.

Next, note that

$$\| W_{k,s} \|_1 \leq \| \mathbb{E}\left[x_t x_t'\right] \|_1 \| \left(R'\right)^{k-s} \|_1 \| \mathbb{E}\left[x_t x_t'\right]^{-1} \|_1, \tag{63}$$

where $\| \cdot \|_1$ is the 1-norm. Since all the eigenvalues of R are within the unit circle, we know its spectral radius is less than one: $\rho(R) = \rho\left(R'\right) < 1$. From Gelfand's formula, we know that there exists $\bar{\Lambda} \in (0,1)$ and $M_1 > 0$ such that

$$\| \left(R'\right)^{k-s} \|_1 \leq M_1 \bar{\Lambda}^{k-s},$$

for all $k \geq s \geq 0$. Together with the fact that $E\left[x_t x_t'\right]$ is invertible (because Σ_ε is positive definite and $\rho(R) < 1$), we know that there exists $M_2 > 0$ such that

$$\|W_{k,s}\|_1 \leq M_2 \bar{\Lambda}^{k-s}, \tag{64}$$

for all $k \geq s \geq 0$. Now, from Assumption 5, we know that

$$\bar{E}_t[x_{t-k}] = (1-\lambda)^k x_{t-k} + \sum_{s=0}^{k-1} \lambda (1-\lambda)^s \mathbb{E}[x_{t-k}|\{x_t, \cdots, x_{t-s}\}] \equiv \sum_{s=0}^{k} V_{k,s} x_{t-s}, \tag{65}$$

where, for all $k \geq 0$,

$$V_{k,k} \equiv (1-\lambda)^k I_{n\times n} \quad \text{and} \quad V_{k,s} \equiv \lambda (1-\lambda)^s W_{k,s} \quad s \in \{0, \cdots, k-1\},$$

Together with (64), we know that there exists $M_3 > 0$ and $\Lambda = \max\left\{1-\lambda, \bar{\Lambda}\right\} \in (0,1)$ such that, for all $k \geq s \geq 0$,

$$\|V_{k,s}\|_1 \leq M_3 \Lambda^k. \tag{66}$$

Now consider an equilibrium in the form of (62). From equilibrium condition (6), we know

$$\begin{aligned}
\sum_{k=0}^{+\infty} \gamma'_k x_{t-k} &= (1-\delta_0)\theta_t + \delta_0 \bar{E}_t\left[\sum_{k=0}^{+\infty} \gamma'_k x_{t-k}\right] + \delta_1 \bar{E}_t\left[\sum_{k=0}^{+\infty} \gamma'_k x_{t+1-k}\right] \\
&= \left((1-\delta_0) q' + \delta_0 \gamma'_0 + \delta_1 \gamma'_0 R + \delta_1 \gamma'_1\right) x_t + \bar{E}_t\left[\sum_{k=1}^{+\infty} \left(\delta_0 \gamma'_k + \delta_1 \gamma'_{k+1}\right) x_{t-k}\right] \\
&= \left((1-\delta_0) q' + \delta_0 \gamma'_0 + \delta_1 \gamma'_0 R + \delta_1 \gamma'_1\right) x_t + \sum_{k=1}^{+\infty} \left(\delta_0 \gamma'_k + \delta_1 \gamma'_{k+1}\right)\left(\sum_{s=0}^{k} V_{k,s} x_{t-s}\right).
\end{aligned}$$

For this to be true for all states of nature, it has to be that the load of x_{t-k} on the left hand side coincides with that on the right hand side, for all $k \geq 0$. That is, the $\{\gamma_k\}_{k=0}^{\infty}$ coefficients must solve the following system:

$$\begin{aligned}
\gamma'_0 &= (1-\delta_0) q' + \delta_0 \gamma'_0 + \delta_1 \gamma'_0 R + \delta_1 \gamma'_1 + \sum_{l=1}^{+\infty} \left(\delta_0 \gamma'_l + \delta_1 \gamma'_{l+1}\right) V_{l,0} \\
\gamma'_k &= \sum_{l=k}^{+\infty} \left(\delta_0 \gamma'_l + \delta_1 \gamma'_{l+1}\right) V_{l,k} \quad \forall k \geq 1.
\end{aligned} \tag{67}$$

From the boundedness property in Definition 2, we know that there is a scalar $M > 0$ such that $\|\gamma'_k\|_1 \leq M$ for all $k \geq 0$, where $\|\cdot\|_1$ is the 1-norm. Using this fact along with (66) and (67), we can then infer that, for all $k \geq 1$,

$$\|\gamma'_k\|_1 \leq (|\delta_0| + |\delta_1|) \sum_{l=k}^{+\infty} \|V_{l,k}\|_1 M \leq (|\delta_0| + |\delta_1|) M_3 \frac{\Lambda^k}{1-\Lambda} M. \tag{68}$$

Because $\lim_{k\to\infty} \Lambda^k = 0$, there necessarily exists an $\hat{k}$ finite but large enough such that

$$(|\delta_0| + |\delta_1|) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1. \tag{69}$$

From (68), for all $k \geq \hat{k}$,

$$\|\gamma'_k\|_1 \leq (|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} M.$$

Now, we can use the above formula and (67) to provide a tighter bound for $\|\gamma'_k\|_1$: for all $k \geq \hat{k}$,

$$\|\gamma'_k\|_1 \leq \left((|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^2 M.$$

And then we can keep iterating the same argument to get the following: for all $k \geq \hat{k}$ and $l \geq 0$,

$$\|\gamma'_k\|_1 \leq \left((|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^l M.$$

And since the term in the parenthesis is less than 1, we conclude that any non-zero value for γ'_k can be ruled out, for all $k \geq \hat{k}$. Using (67) and doing backward induction, we conclude that $\gamma'_k = 0$ for all $k \geq 1$, where I use $\delta_0 < 1$.

We are then left with a single equation for γ'_0:

$$\gamma'_0 = (1-\delta_0)\, q' + \delta_0 \gamma'_0 + \delta_1 \gamma'_0 R.$$

Under Assumption 6, the above reduces to $\gamma'_0 = q' (I - \delta R)^{-1}$, which corresponds to the MSV solution. And since we have already proved that $\gamma_k = 0$ for all $k \geq 1$ and $a_k = 0$ for all $k \geq 0$, we conclude that the MSV solution is the unique equilibrium. □

We conclude with the following remarks. In the very last step of the above proof, we invoke Assumption 6 to prove the existence of a solution for γ_0. But this assumption was not used in any previous step: to prove $a_k = 0$ for all k and $\gamma_k = 0$ for all $k \neq 1$, we only had to invoke Assumption 5. This verifies the claim made earlier that Assumption 5 alone is sufficient for ruling out all equilibria other than the MSV solution; and Assumption 6 is used only to guarantee the existence of the MSV solution. Finally, note that Assumption 6 necessarily holds if $\sum_{h=0}^{\infty} (\delta R)^h$ is finite, but the converse is not true. That is, the MSV solution may fail to obtain from forward iteration, and when this is the case the MSV solution may feature counter-intuitive comparative statics. This mirrors our earlier discussion of the neo-Fisherian property: in the main analysis, $\frac{\partial c_t^F}{\partial \theta_t} = \frac{1}{1-\delta\rho}$ switches sign and is discontinuous at $\rho = \delta^{-1}$. But again, these "pathologies" have nothing to do with our own argument, which instead regards the non-existence of solutions other than the MSV.

C.2 Persistent Sunspots

We now modify the baseline model by letting the sunspot be persistent.

Assumption 7 (***Persistent sunspots***)**.** *The sunspot η_t follows an AR(1) process*

$$\eta_t = \rho\eta_{t-1} + \varepsilon_t^\eta, \tag{70}$$

where $\rho \in [0,1)$ and $\varepsilon_t^\eta \sim \mathcal{N}\left(0,\sigma_\eta^2\right)$. The fundamental θ_t is still i.i.d. over time with mean zero.

The economy's history, the definition of the equilibrium, and Assumption 2 remain exactly the same as in the baseline model. The following generation of our main result then applies:

Proposition 9. *Proposition 2 continues to hold, provided that $\rho \neq \delta^{-1}$.*

Proof. Since sunspots $\{\eta_{t-k}\}_{k=0}^\infty$ are orthogonal to fundamentals $\{\theta_{t-k}\}_{k=0}^\infty$, the same argument as that used in Proposition 2 still proves that $\gamma_k = 0$ for all $k \geq 1$ and $\gamma_k = 1$. We can thus shut down fundamentals (set $\theta_t = 0$ for all t) and focus on ruling out sunspot equilibria of the following form:

$$c_t = \sum_{k=0}^\infty a_k \eta_{t-k}. \tag{71}$$

To start with, note that, since η_t is a stationary Gaussian random variable given by (70), the following projections apply for all $k \geq s \geq 0$:

$$\mathbb{E}\left[\eta_{t-k}|\{\eta_t,\cdots,\eta_{t-s}\}\right] = \rho^{k-s}\eta_{t-s},$$

Now, from Assumptions 2, we know that

$$\bar{E}_t\left[\eta_{t-k}\right] = (1-\lambda)^k\eta_{t-k} + \sum_{s=0}^{k-1}\lambda(1-\lambda)^s\mathbb{E}\left[\eta_{t-k}|\{\eta_t,\cdots,\eta_{t-s}\}\right] \equiv \sum_{s=0}^{k} v_{k,s}\eta_{t-s}, \tag{72}$$

where, for all $k \geq 0$,

$$v_{k,k} \equiv (1-\lambda)^k \quad \text{and} \quad v_{k,s} \equiv \lambda(1-\lambda)^s\rho^{k-s} \quad s \in \{0,\cdots,k-1\}.$$

We know that there exists $M_3 > 0$ and $\Lambda = \max\left\{1-\lambda,\rho\right\} \in (0,1)$ such that for all $k \geq s \geq 0$,

$$0 \leq v_{k,s} \leq M_3\Lambda^k. \tag{73}$$

Now consider an equilibrium in the form of (71). From equilibrium condition (6) and the fact that we set $\theta_t = 0$, we know

$$\begin{aligned}
\sum_{k=0}^{+\infty} a_k\eta_{t-k} = \delta_0\bar{E}_t\left[\sum_{k=0}^{\infty} a_k\eta_{t-k}\right] + \delta_1\bar{E}_t\left[\sum_{k=0}^{\infty} a_k\eta_{t+1-k}\right] \\
= \left[\delta_0 a_0 + \delta_1\left(a_0\rho + a_1\right)\right]\eta_0 + \bar{E}_t\left[\sum_{k=1}^{\infty}(\delta_0 a_k + \delta_1 a_{k+1})\eta_{t-k}\right] \\
= \left[\delta_0 a_0 + \delta_1\left(a_0\rho + a_1\right)\right]\eta_0 + \sum_{k=1}^{\infty}(\delta_0 a_k + \delta_1 a_{k+1})\left(\sum_{s=0}^{k} v_{k,s}\eta_{t-s}\right).
\end{aligned}$$

For this to be true for all states of nature, it has to be that the load of η_{t-k} on the left hand side coincides with that on the right hand side, for all $k \geq 0$. That is, the $\{\eta_k\}_{k=0}^\infty$ coefficients must solve the following system:

$$a_0 = \delta_0 a_0 + \delta_1 \left(a_0 \rho + a_1 \right) + \sum_{l=1}^{+\infty} \left(\delta_0 a_l + \delta_1 a_{l+1} \right) v_{l,0}$$

$$a_k = \sum_{l=k}^{\infty} \left(\delta_0 a_l + \delta_1 a_{l+1} \right) v_{l,k} \quad \forall k \geq 1. \tag{74}$$

From the boundedness property in Definition 1, we know that there is a scalar $M > 0$ such that $|a_k| \leq M$ for all $k \geq 0$. Using this fact along with (73) and (74), we can then infer that, for all $k \geq 1$,

$$|a_k| \leq \left(|\delta_0| + |\delta_1| \right) \sum_{l=k}^{+\infty} v_{l,k} M \leq \left(|\delta_0| + |\delta_1| \right) M_3 \frac{\Lambda^k}{1-\Lambda} M. \tag{75}$$

Because $\lim_{k \to \infty} \Lambda^k = 0$, there necessarily exists an $\hat{k}$ finite but large enough such that

$$\left(|\delta_0| + |\delta_1| \right) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1. \tag{76}$$

From (75), for all $k \geq \hat{k}$,

$$|a_k| \leq \left(|\delta_0| + |\delta_1| \right) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} M.$$

Now, we can use the above formula and (74) to provide a tighter bound for $|a_k|$: for all $k \geq \hat{k}$,

$$|a_k| \leq \left(\left(|\delta_0| + |\delta_1| \right) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^2 M.$$

And then we can keep iterating the same argument to get the following: for all $k \geq \hat{k}$ and $l \geq 0$,

$$|a_k| \leq \left(\left(|\delta_0| + |\delta_1| \right) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^l M.$$

And since the term in the parenthesis is less than 1, we conclude that any non-zero value for a_k can be ruled out, for all $k \geq \hat{k}$. Using (74) and doing backward induction, we conclude that $a_k = 0$ for all $k \geq 1$, where I use $\delta_0 < 1$.

We are then left with a single equation for a_0 :

$$a_0 = \left(\delta_0 + \delta_1 \rho \right) a_0$$

Recall that $\delta \equiv \frac{\delta_1}{1-\delta_0}$. The restriction $\rho \neq \delta^{-1}$ translates to $\delta_0 + \delta_1 \rho \neq 1$. As a result, we also have $a_0 = 0$. We conclude that, as long as $\rho \neq \delta^{-1}$, the MSV solution is the unique equilibrium. □

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