The Legacy of Ramanujan 2024

Celebrating the 85th Birthdays of George Andrews and Bruce Berndt

Penn State University
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Talk Titles and Abstracts

Plenary Talks

Krishna Alladi **(University of Florida)**

Second order duality between prime factors and prime numbers in arithmetic progressions

Abstract: In 1977, I noticed a Duality between the largest and smallest prime factors of the integers involving the Moebius function, and used this to establish the following result as a consequence of the Prime Number Theorem for Arithmetic Progressions: If k and ℓ are positive integers, with $1 \leq \ell \leq k$ and $(\ell, k) = 1$, then

$$\sum_{n \geq 2,\, p(n) \equiv \ell (mod\, k)} \frac{\mu(n)}{n} = \frac{-1}{\phi(k)}, \tag{1}$$

where $\mu(n)$ is the Moebius function, $p(n)$ is the smallest prime factor of n, and $\phi(k)$ is the Euler function. In the last decade, several authors have obtained analogues of (1) in the setting of algebraic number fields by using the Chebotarev Density Theorem.

Also in 1977, I proved higher order duality identities involving the k^{th} largest and smallest prime factors, facilitated by the Moebius function and $\omega(n)$, *the number of distinct prime factors of* n. In this talk we will exploit the second order duality between the second largest

prime factor and the smallest prime factor, to show that if ℓ and k are as above, then

$$\sum_{n\geq 2,\, p(n)\equiv \ell (mod\, k)} \frac{\mu(n)\omega(n)}{n} = 0. \tag{2}$$

The proof of (2) is more complicated owing to the weight $\omega(n)$, and also because it relies on the distribution of the second largest prime factor which is more subtle compared to the distribution of the largest prime factor. This is joint work with my PhD student Jason Johnson.

Towards the end of the talk, we will briefly mention further joint work with another of my PhD students Sroyon Sengupta on consequences of such dualities involving the k^{th} largest and smallest prime factors, when $k \geq 3$, as well as Sengupta's work on algebraic analogues of (2) utilizing the Chebotarev Density Theorem.

George Andrews (The Pennsylvania State University)

A Lifetime Journey with P.A. MacMahon

Abstract: This talk spans more than half a century of interactions with the work of P.A. MacMahon. The journey began in the 1960s and continues today. The talk begins with an account of how I wound up editing the Collected Papers of MacMahon. Much of the journey relates to 15+ papers on MacMahon's Partition Analysis (joint with Peter Paule). Most recently (jointly with Simon Rose, Tewodros Amdeberhan and Roberto Tauraso) I have been enmeshed in MacMahon's papers from the 1920's on generalizations of divisor sums in the theory of partitions.

Bruce Berndt (University of Illinois Urbana-Champaign)

Finite Trigonometric Sums

Abstract: Several kinds of finite trigonometric sums are evaluated in closed form. Reciprocity and/or three-sum relations are established. Upper bounds and conjectured lower bounds for sums with multiple sines or cosines are given. We evaluate and find reciprocity relations for sums appearing in the transformation formulae of theta functions and Eisenstein series. We evaluate analogues of Ramanujan sums, and for one of these, we obtain a theorem similar to the Franel–Landau criterion for the Riemann Hypothesis. Several open problems will be discussed.

Howard Cohl (NIST)

Dick Askey (1933-2019) and what I've learned about him and his life

Abstract: Richard (Dick) Allen Askey was born in 1933 in Saint Louis, Missouri and passed away in 2019 in Madison, Wisconsin. Dick had a monumental influence on the mathematics associated with special functions and orthogonal polynomials. Dick influenced many people through his important contributions, as well as through suggestions and encouragement in the continuing advancement of this ever evolving field. In 2016, Robert Lange interviewed Richard Askey in the University of Wisconsin-Madison Oral History program over the course of a six hour interview over three days. A group of Dick's friends, colleagues and students contributed towards a full transcription of this UW-Madison interview which we have been editing. There was also a two hour interview (with transcription) with Askey performed by Frank B. Allen of the R. L. Moore Collection in 1999. I have also gained access to a redacted copy of Askey's faculty file at UW-Madison and as well other important documents which shed information on Dick's important passage among us. In connection with the (1) Askey Liber Amicorum assembled by myself and Mourad Ismail; (2) its recent and impending publication of an Askey collection on the Celebratio Mathematica website; and (3) a planned Springer Askey Volume which will contain a collection of historical vignettes, remembrances and research papers with strong connections to Richard Askey which will be co-edited by George Andrews, Mourad Ismail, Luc Vinet and myself, I will try to present some of this material and hopefully shed some light on Dick's amazing life story.

Amanda Folsom **(Amherst College)**

Conjectures of Andrews and Berndt

Abstract: George Andrews and Bruce Berndt are contemporary experts and mathematical leaders in the areas of analytic number theory, q-series, mock theta functions, special functions, combinatorics and partitions, applications, and more. Their combined 700+ articles and books have established research directions that have influenced all of us and many more. In this talk, we will discuss some of their contributions and conjectures related to work of Ramanujan, including joint works of the author with Bringmann, Males, Rolen and Storzer.

Frank Garvan **(University of Florida)**

Mock Atkin-Lehner Symmetry

Abstract: We re-examine identities for the third order mock theta functions in Ramanujan's last letter to Hardy. We use Atkin-Lehner involutions to obtain new congruences for Andrews $\overline{EO}$ partition function and Ramanujan's $\omega(q)$.

Christian Krattenthaler **(Universität Wien)**

Proofs of Borwein Conjectures

Abstract: The (so-called) "Borwein Conjecture" arose around 1990 and states that the coefficients in the polynomial

$$(1-q)(1-q^2)(1-q^4)(1-q^5)\cdots(1-q^{3n-2})(1-q^{3n-1})$$

have the sign pattern $+--+--\ldots$. This innocent looking prediction has withstood all proof attempts until five years ago when Chen Wang found a proof that combines asymptotic estimates with a computer verification for "small" n.

However, Borwein made actually in total three sign pattern conjectures of similar character - with the previously mentioned conjecture being just the first one -, and recently Wang discovered a further one. It seemed unlikely that Wang's proof could be adapted to work for these other conjectures since it crucially used identities that are only available for the "First Borwein Conjecture".

I shall start by presenting these conjectures and then review the history of the conjectures and the various attempts that have been made to prove them - as a matter of fact, these attempts concerned exclusively the "First Borwein Conjecture", while nobody had any idea how to attack the other conjectures.

I shall then outline a proof plan that is (in principle) applicable to all these conjectures. Indeed, this leads to a new proof of the "First Borwein Conjecture", the first proof of the "Second Borwein Conjecture", and to a proof of "two thirds" of Wang's conjecture. We are convinced that further work along these lines will lead to - at least - a partial proof of the "Third Borwein Conjecture".

I shall close with further open problems in the same spirit.

This is joint work with Chen Wang.

Ken Ono **(University of Virginia)**

Diophantine equations in partition functions

Abstract: This talk presents "partition theoretic" analogs of the classical work of Matiyasevich that resolved Hilbert's Tenth Problem in the negative. The Diophantine equations we consider involve equations of MacMahon's partition functions and their natural generalizations. This is joint work with Will Craig and Jan-Willem van Ittersum.

Peter Paule **(RISC at J. Kepler University Linz and TCAM at Tianjin University)**

Ramanujan-type Formulae for 1 over π: A New Algorithmic Wind?

Abstract: In 1914 Ramanujan recorded a list of 17 series for 1 over π. In his 2008 article "Ramanujan-type formulae for 1 over π: A second wind?" Wadim Zudilin surveyed methods of proof of Ramanujan's formulae and discussed various generalizations and new

discoveries. Since then Zudilin's prediction of a "second wind" has been proven by many further developments. In this talk, which arose from joint work with Ralf Hemmecke (RISC) and Cristian-Silviu Radu (RISC), we present new computer algebra tools in the hope to contribute to a "third wind" of research in this area. This talk is the long version of my presentation at the "Modular Forms and q-Series" conference in Cologne, March 2024.

Ole Warnaar (The University of Queensland)

The A_2 Bailey lemma

Abstract: The Bailey lemma is one of the most powerful tools for proving q-series identities of Rogers–Ramanujan type. In 1999 George Andrews, Anne Schilling and the speaker found a generalisation of the Bailey lemma for the root system A_2, which resulted in the discovery of analogues of the Rogers–Ramanujan and Andrews–Gordon identities for the affine Lie algebra $\mathrm{A}_2^{(1)}$. The results of Andrews et al., however, were far from complete, and recently Kanade and Russell conjectured what they referred to as the "complete set of ASW identities". In this talk I will explain how the A_2 Bailey lemma of ASW can be generalised to an A_2 Bailey tree and how this tree leads to a proof of the Kanade–Russell conjectures.

Doron Zeilberger (Rutgers University)

Early masterpieces by 3 of my great heroes

Abstract: The three heroes, in chronological order of birth, are Srinivasa Ramanujan (born Dec. 22, 1887), Bruce Berndt (born March 13, 1938), and George Andrews (born Dec. 4, 1938). The three masterpieces were written when they were 24, 28, and 22 years-old, respectively.

Invited Talks

Abdulaziz Alanazi (University of Tabuk)

Alternative Combinatorial Interpretations of Göllnitz-Gordon Identities and Little Göllnitz Identities

Abstract: In this talk, we will describe new signed partition and overpartition interpretations for the first and second Göllnitz-Gordon identities, as well as for the first and second little Göllnitz identities. Both generating functions and bijective proofs will be provided.

This is a joint work with Andrew Sills and Augustine Munagi.

Cristina Ballantine (College of the Holy Cross)

Inequalities for the number of partitions with parts separated by parity

Abstract: Using asymptotics, Bringmann, Craig and Nazaroglu proved inequalities between different numbers of partitions with parts separated by parity. We use combinatorial injections to provide new proofs of some of the inequalities. We also prove an inequality conjectured by Fu and Tang between certain numbers of partitions with parts separated by parity with additional parity restrictions for the multiplicity of parts. This is joint work with Amanda Welch.

Alex Berkovich (University of Florida)

Extension of Bressoud's Generalization of Borwein's Conjecture and Some Exact Results

Abstract: In my talk I conjecture an extension to Bressoud's 1996 generalization of Borwein's famous 1990 conjecture. Using certain positivity-preserving transformations for q-binomial coefficients, I prove certain cases of this new conjecture. This talk is based on my recent joint work with Aritram Dhar.

Walter Bridges (University of Cologne)

Enumeration and statistics for certain Lie algebras

Abstract: The partition enumeration function $p(n)$ also counts the number of n-dimensional representations of the Lie algebra $\mathfrak{sl}_2(\mathbb{C})$, and $\mathfrak{sl}_3(\mathbb{C})$-representations also have a product generating function, namely,

$$\prod_{j,k\geq 1} \frac{1}{1-q^{\frac{jk(j+k)}{2}}}.$$

In 2017, Romik proved an asymptotic for the number of n-dimensional $\mathfrak{sl}_3(\mathbb{C})$-representations, in the process developing a statistical mechanics-type probabilistic model and studying the Witten zeta function for $\mathfrak{sl}_3(\mathbb{C})$.

I will discuss a general asymptotic expansion for such products, as well as the distribution of several statistics for representations of $\mathfrak{sl}_3(\mathbb{C})$ under the uniform measure on representations of dimension n, as $n \to \infty$. This is joint work with Benjamin Brindle, Kathrin Bringmann, Johann Franke and Caner Nazaroglu.

Hannah Burson (University of Minnesota)

Counting Numerical Semigroups via Integer Partitions

Abstract: Numerical semigroups are cofinite additive submonoids of the natural numbers motivated by the study of linear Diophantine equations. Through a simple injection to Young diagrams, researchers have used known results about numerical semigroups to answer questions about core partitions.

In this talk, we will explore this connection between integer partitions and numerical semigroups with a focus on counting the partitions that appear in the image of the injection from numerical semigroups. This talk is based on joint work with Hayan Nam and Simone Sisneros-Thiry.

Song Heng Chan (Nanyang Technological University)

Finite versions of some Andrews-Gordon type identities

Abstract: We begin with a brief review of the Andrews-Gordon identity and Bressoud's identity. Inspired by Bressoud's ideas in *An easy proof of the Rogers-Ramanujan identities*, we present several finite versions of Andrews-Gordon type identities which are discovered and proved in similar spirit. These include finite versions of the Ramanujan-Gordon-Göllnitz identities, finite identities linked to the works of Andrews, Kurşungöz, S. Kim, and A.J. Yee, and a new companion identity. As immediate consequences, we obtain finite versions of Rogers-Ramanujan type identities, some of which are new. We provide brief sketches of the proofs for these identities.

This talk is based on joint work with Heng Huat Chan.

Shane Chern (Dalhousie University)

Linked partition ideals: Combinatory Analysis meeting Computer Algebra

Abstract: The framework of linked partition ideals, which serves as an important tool for integer partition identities, was introduced by George Andrews in the 1970s. One main objective of this framework concerns the construction of Andrews–Gordon type generating functions for partition sets under certain gap conditions. However, while utilizing linked partition ideals, we eventually encounter the problem of solving a system of q-difference equations, making this framework ineffective in the traditional pencil-and-paper mode. In this talk, I will discuss how a computer algebraic procedure works for such q-difference systems, thereby enhancing the power of Andrews' linked partition ideals in the modern study of partition identities.

William Craig (University of Cologne)

Partitions with Parts Separated by Parity and Ramanujan's Sigma-function

Abstract: Recently, Andrews initiated the study of partitions with parts separated by parity in connection with Ramanujan's mock theta functions. Various families of partitions built from Andrews' ideas have very similar hypergeometric constructions but wildly differing modular structures. We study eight such examples and give their relations to various types of modular objects, using this relationship to compute the asymptotic growth of these partition families. We give special emphasis to an example connecting to the Ramanujan sigma function, whose modular structure is tied to Maass forms by famous work of Andrews-Dyson-Hickerson and Cohen. In this special setting, we use a new approach towards the error to modularity of false indefinite theta functions to compute a full Rademacher-type expansion for this special family. The techniques critically involve the relationship between false indefinite theta functions and mock Maass forms as defined by Zwegers.

Atul Dixit (Indian Institute of Technology Gandhinagar)

Functional equations for Herglotz-type integrals

Abstract: In his 1925 work on the Kronecker limit formula for real quadratic fields, Herglotz explicitly evaluated a certain integral whose evaluation by elementary means, and likewise, of similar such integrals, has evaded mathematicians since then. Recently, Radchenko and Zagier gave explicit evaluations of several analogous integrals using algebraic and analytic methods in number theory. Recently, Choie and Kumar showed that these integrals satisfy elegant functional equations. In this talk, we will give a grand generalization of such functional equations involving Fekete polynomials associated with Dirichlet characters as well as character polylogarithms. We will also discuss the framework involving a generalization of the character analogue of the Mordell-Tornheim zeta function in which such equations lie. This is joint work with Sumukha Sathyanarayana and N. Guru Sharan.

Dennis Eichhorn (University of California, Irvine)

Identities and Inequalities for the Rank and Cranks

Abstract: Dyson famously conjectured that his rank statistic witnesses Ramanujan's first two congruences for $p(n)$, and that there exists a "crank" statistic that witnesses Ramanujan's congruence modulo 11 in a similar fashion. He also conjectured many other identities between the rank classes of partitions outside of the arithmetic progressions $5n + 4$ and $7n + 5$, and he conjectured that, once discovered, the crank would have similar relations outside of the arithmetic progression $11n + 6$. All of these conjectures have been shown to be true, by Atkin and Swinnerton-Dyer, Andrews and Garvan, and Garvan. Furthermore, in addition to all of these identities, there are now many known inequalities between rank classes and crank classes. As it turns out, these two phenomena, identities and inequalities involving congruence-witnessing statistics, also occur in other contexts within partition theory. In this talk, we explore these two phenomena in several of these other contexts.

Shishuo Fu (Chongqing University)

Sequences of odd length in strict partitions

Abstract: In this talk, we enumerate strict partitions with respect to the size, the number of parts, and the number of sequences of odd length. We write this trivariate generating function as a double sum q-series, which on one hand gives partition theoretical interpretations to the sum side of several identities obtained previously by Cao-Wang and Wang-Wang via other methods, on the other hand motivates us to look for further refinements of Euler's famed odd vs. strict partition theorem. A close relation to the 2-measure of partitions, a notion introduced by Andrews et al., will also be mentioned.

Ankush Goswami (University of Texas Rio Grande Valley)

Congruences for coefficients in expansions of certain q-hypergeometric series

Abstract: In 2014, Andrews and Sellers discovered remarkable congruences for the *Fishburn numbers*, defined as the coefficients in the $(1-q)$-expansion of $F(q) = \sum_{n\geq 0}(q)_n$. This function, an element of the Habiro ring, is one of the most influential examples of quantum modular forms that satisfy a "strange" identity. Inspired by Andrews and Sellers' work, many subsequent papers expanded on their findings. Notably, Garvan (2015), Straub (2015), and Ahlgren–Kim–Lovejoy (2018) made significant contributions by addressing several important questions posed by Andrews and Sellers.

Following their work, various researchers have applied and extended these ideas to prove congruences associated with various q-series, some in connection with different families of torus knots. Interestingly, all existing examples of q-series satisfying the aforementioned congruences belong to the Habiro ring.

In this talk, I will report on an ongoing joint work with Jeffery Opoku (UTRGV), where we obtain prime power congruences for certain q-series not in the Habiro ring. If time permits, I will also discuss a generalization whereby we extend these congruences (under suitable assumptions) to a general family of q-series that do not belong to the Habiro ring.

Timothy Huber (University of Texas Rio Grande Valley)

Congruences for Quotients of Rogers-Ramanujan Functions

Abstract: This lecture will survey preliminary work on coefficient congruences for quotients of the form

$$\left(\prod_{n=1}^{\infty} 1-q^{5n}\right)^{a_0}\left(\prod_{n=0}^{\infty}(1-q^{5n+1})\prod_{n=0}^{\infty}(1-q^{5n+4})\right)^{a_1}\left(\prod_{n=0}^{\infty}(1-q^{5n+2})\prod_{n=0}^{\infty}(1-q^{5n+3})\right)^{a_2},$$

where $(a_0, a_1, a_2) \in 2\mathbb{Z} \times \mathbb{Z}^2$. The focus will be on congruences involving prime powers for primes greater than 5. This is joint work with Maria Del Rosario Valencia Arevalo (University of Oklahoma), Jeffery Opoku (University of Texas Rio Grande Valley), and Dongxi Ye (Sun Yat-sen University).

Shashank Kanade (University of Denver)

CMPP meet GOW

Abstract: A few years ago, in a sequence of two papers, S. Capparelli, A. Meurman, A. Primc, M. Primc and then M. Primc proposed three remarkable sets of combinatorial conjectures regarding coloured integer partitions. On the other hand, about a decade ago, M. Griffin, K. Ono and S. O. Warnaar proved their much celebrated Rogers–Ramanujan-type q-series identities related to the characters of certain affine Lie algebras. In this talk, I'll present connections between these two worlds. This is a joint work with M. C. Russell, S. Tsuchioka and S. O. Warnaar.

Soon-Yi Kang (Kangwon National University)

On d-distinct partitions

Abstract: Partitions of n into parts at least d apart is one of the oldest subjects in partition theory. Euler, Rogers-Ramanujan, and Schur established partition identities for d-distinct partitions when $d = 1, 2, 3$, respectively. These identities were extended to a partition inequality for d-distinct partition for $d > 2$ by Alder, Andrews, Yee and more. In recent years, numerous researchers have developed analogous and generalized versions of Alder-type partition inequalities. Furthermore, the modularity of the generating functions for d-distinct partitions has been revealed by remarkable discoveries made by Zagier and Folsom. In this talk, we try to provide a comprehensive survey of the findings pertaining to d-distinct partitions.

William Keith (Michigan Technological University)

Reciprocals of False Theta Functions

Abstract: We give a variety of results concerning reciprocals of false theta functions, such as congruences and an interesting relationship to the truncated pentagonal number theorem of Andrews and Merca.

Byungchan Kim (SeoulTech)

Reciprocal Sums of Parts of Integer Partitions

Abstract: Integer partitions have been extensively studied with various constraints on the parts. However, conditions on the reciprocal sum of the parts, denoted by $\mathrm{srp}(\lambda)$ for the partition λ,

remain relatively unexplored. R. Graham proved that there is a partition λ of n into distinct part with $\mathrm{srp}(\lambda) = 1$ if $n \geq 78$, but little is known beyond this. In this talk, we introduce recent results on how $\mathrm{srp}(\lambda)$ is distributed and exhibit a numerical approach to find a partition λ into speific parts with $\mathrm{srp}(\lambda) = 1$.

Brandt Kronholm (University of Texas Rio Grande Valley)

Congruences and cranks for partitions bounded by part size and number

Abstract: We establish infinite families of cranks witnessing infinite families of congruences for the function $p(n, m)$ which enumerates partitions of n into at most m parts. We show that Dyson's rank witnesses infinitely many of these congruences.

We will discuss recent results on infinite families of congruences for $p(n, m, N)$, the function enumerating partitions into at most m parts, no part larger than N. For small values of m, we will establish cranks.

These results come from joint work with Joselyne Aniceto, Dennis Eichhorn, Lydia Engle, and Acadia Larsen.

Rahul Kumar (Penn State)

Period function from Ramanujan's Lost Notebook

Abstract: The Lost Notebook of Ramanujan contains a number of beautiful formulas involving certain interesting functions, one of which we denote as $\mathcal{F}_1(x)$. In this talk, we show that $\mathcal{F}_1(x)$ belongs to the category of period functions as it satisfies the period relations of Maass forms in the sense of Lewis and Zagier (2001). Hence, we refer to $\mathcal{F}_1(x)$ as the Ramanujan period function. We also establish that $\mathcal{F}_1(x)$ appears in a Kronecker limit formula of a certain zeta function. If time permits, we will also discuss further properties of $\mathcal{F}_1(x)$. This talk is based on the joint paper with Professor YoungJu Choie.

Runqiao Li (Pennsylvania State University)

Partition Analysis and the Little Göllnitz Identity

Abstract: The summation side of Little Göllnitz identity generates the partitions with certain gap condition, while a companion of it, provided by Savage and Sills, generates partitions restricted by position parity. In this talk, we apply MacMahon's Partition Analysis to study those partitions and provide generating functions in the general form, which gives refinements and Schmidt-type identities. We will also discuss the connection between Little Göllnitz identity and another partition identity with modulus 8 by Andrews.

Örs Rebák (University of Tromsø – The Arctic University of Norway)

Special values of Ramanujan's theta function $\varphi(q)$

Abstract: In his notebooks, Ramanujan determined some values for his theta function $\varphi(q)$. In his lost notebook, Ramanujan provided an incomplete value for $\varphi(e^{-7\pi\sqrt{7}})$, which was recently completely evaluated. We present a sketch of the proof. In joint work with Berndt, we develop general cubic and quintic analogues of Ramanujan's now completed septic formula. It turns out that some of the values are expressible in terms of trigonometric function values. As corollaries, we are able to determine several new values of $\varphi(e^{-\pi\sqrt{n}})$.

Matthew Russell (University of Illinois Urbana-Champaign)

A refinement of, and a companion to, MacMahon's partition identity

Abstract: We provide a refinement of MacMahon's partition identity on sequence-avoiding partitions, and use it to produce another mod 6 partition identity. In addition, we show that our technique also extends to cover Andrews's generalization of MacMahon's identity. Our proofs are bijective in nature, exploiting a theorem of Xiong and Keith.

Michael Schlosser (University of Vienna)

An intrinsic congruence modulo the square of a prime and its q-analogue

Abstract: Congruences and q-series are both objects Ramanujan was very much interested in, and very central to number theory. We shall present an intrinsic congruence, which we coin "Dissection–Dilution Lemma", valid in general for Laurent series over the ring of p-adic integers, modulo the square of an odd prime. An easy application of our Dissection–Dilution Lemma is Babbage's congruence for the binomial coefficients, which we effortlessly generalize to a congruence between convoluted sums of binomial coefficients. Other applications of the Dissection–Dilution Lemma are congruences of various classes of partitions. We also provide a q-analogue of the Dissection–Dilution Lemma, valid in general for Laurent series over the ring of rational functions in q over the integers, modulo the square of any cyclotomic polynomial. This gives rise to q-analogues of the aforementioned applications, including Clark's q-analogue of Babbage's congruence and refined congruences for partitions.

Robert Schneider (Michigan Technological University)

Partition-theoretic divergent series

Abstract: In two recent papers, Ono, Wagner and I proved families of formulas for arithmetic densities of subsets of $\mathbb{N}$, computed using q-series as $q \to 1^-$ (instead of the usual Dirichlet series as $s \to 1^+$ employed in density computations); these papers are inspired by work of Alladi. In a nutshell, certain q-series which diverge in the limit, when multiplied by $(q;q)_\infty$, yield arithmetic densities as $q \to 1^-$.

In this talk, I show the more general backdrop against which such computations emerge naturally. Using partition generating function techniques, we prove q-series analogues of a formula

of Frobenius generalizing Abel's convergence theorem for complex power series. Frobenius' result states that for $|q| < 1$, $\lim_{q\to 1}(1-q)\sum_{n\geq 1} f(n)q^n$ is equal to the average value $\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} f(k)$ of the sequence $\{f(n)\}$ as $n \to \infty$, if the average value exists.

This work fits in nicely with the theme of this conference: divergent series were of strong interest to Ramanujan and Hardy. Moreover, my methods were influenced by conversations with G. E. Andrews, who provided key insights related to complex analysis for which I am very grateful.

James Sellers (University of Minnesota Duluth)

Congruences for k-elongated Partition Diamonds

Abstract: In 2007, George Andrews and Peter Paule published the eleventh paper in their series on MacMahon's partition analysis, with a particular focus on broken k-diamond partitions. On the way to broken k-diamond partitions, Andrews and Paule introduced the idea of k-elongated partition diamonds. In 2022, Andrews and Paule revisited the topic of k-elongated partition diamonds in a paper that appeared in the Journal of Number Theory. Using partition analysis and the Omega operator, they proved that the generating function for the partition numbers $d_k(n)$ produced by summing the links of k-elongated plane partition diamonds of length n is given by $\frac{(q^2;q^2)_\infty^k}{(q;q)_\infty^{3k+1}}$ for each $k \geq 1$. A significant portion of their 2022 paper involved proving several congruence properties satisfied by d_1, d_2 and d_3, using modular forms as their primary proof tool. Nicolas Smoot extended the work of Andrews and Paule, refining one of their conjectures and proving an infinite family of congruences modulo arbitrarily large powers of 3 for the function d_2; his paper appeared in the Journal of Number Theory in 2023.

In this talk, our goal is to discuss extensions of some of the results proven by Andrews and Paule. First, we will highlight proofs of infinitely many congruence properties satisfied by the functions d_k for an infinite set of values of k that employ elementary proof techniques, relying on generating function manipulations and classical q-series results. This is joint work with Robson da Silva and Mike Hirschhorn which appeared in Discrete Math in 2022. We will then close with more recent work, joint with Nicolas Smoot, on an infinite family of congruences modulo powers of 8 for d_7 which appeared in the International Journal of Number Theory in 2024.

Andrew Sills (Georgia Southern University)

Building a database of Rogers-Ramanujan-Slater type identities

Abstract: The two Rogers-Ramanujan identities, a pair of "q-series = infinite product" identities, originally discovered by L. J. Rogers in 1894 and later rediscovered independently by S. Ramanujan and I. Schur, are central results in the theory of q-series. Their standard combinatorial interpretations provide partition identities in the spirit of Euler's "odd/distinct theorem." In the early twentieth century, practitioners such as F. H. Jackson and G. W. Starcher, along with Rogers and Ramanujan, discovered additional identities of a similar type. During World War II, W. N. Bailey (who knew Ramanujan at Cambridge during the former's undergraduate years) undertook a study of identities of Rogers-Ramanujan type and discovered that these identities were essentially limiting cases of q-hypergeometric transformations with the method that we now call inserting "Bailey pairs" into limiting cases of "Bailey's lemma." Bailey's student L. J. Slater in 1952 published a list of 130 identities of Rogers-Ramanujan type. This list, along with various extended lists that include

additional identities discovered after Slater's time, has been a mainstay of q-series research, which has extended beyond analysis, number theory, and combinatorics to include applications in physics, Lie theory and vertex operator algebras, automated proof theory, knot theory, etc. However, no single printed list of identities will ever fully serve the community of students and researchers due to the simple fact that the series and infinite products represented do not have a single, canonical "simplest" form. One may be looking for a certain "mod 8" identity and fail to find it on Slater's list because it is actually "hiding" among, say, the mod 16 or mod 32 identities, where the equivalent product appears in a "disguised" form. Similarly, there may be many different ways of presenting a given q-series sum.

Accordingly, it seems only fitting that in the 21st century, we should build an online searchable database of Rogers-Ramanujan type identities, with multiple modes of search and lookup, references, and links to related identities, analogous to the LMFDB (L-functions and modular forms database) and the Atlas of finite group representations. We are in the early stages of building such a database, that we propose to name the "RRSDB"—Rogers-Ramanujan-Slater database. We wish to demonstrate the current early state, discuss where we hope to go in the future, and to solicit input from the community.

This is joint work with lead programmers Hunter Waldron and C. McCarthy of Michigan Tech. Additional volunteers are most welcome! We appreciate the encouragement provided by Lucy Slater Library Project at Michigan Tech, led by Robert Schneider. We thank George Andrews for his support and encouragement.

Nicolas Smoot (University of Vienna)

Congruence Families and Automorphisms

Abstract: Ramanujan's classic congruence families were the first important arithmetic properties that were discovered for the partition function p(n). It is now known that similar properties are exhibited by the Fourier coefficients of various different modular forms. Some of these are much harder to prove than others. We will show a new proof method that was recently applied to one of the more difficult congruence families, exhibited by a generalized Frobenius partition function, associated with the modular curve $X_0(20)$. The idea is that one constructs an automorphism on a certain free $Z[t]$-module R of functions which permutes the generators of R while fixing the functions on the curve $X_0(5)$. To our knowledge, this is an altogether new approach to the problem of proving p-adic convergence of modular function sequences, and the implications for future work are enormous. This is joint work with Frank Garvan and James A. Sellers.

Armin Straub (University of South Alabama)

An invitation to constant term sequences

Abstract: Many sequences in combinatorics and number theory can be represented as constant terms of powers of multivariate Laurent polynomials and, therefore, as diagonals of multivariate rational functions. On the other hand, it is an open question, raised by Don Zagier, to classify those diagonals which are constant terms. We provide such a classification in the case of sequences satisfying linear recurrences with constant coefficients. Various related examples, applications and open problems will be given as time permits. This talk is based on joint work with Alin Bostan and Sergey Yurkevich.

Ali Kemal Uncu **(Austrian Academy of Sciences RICAM & University of Bath)**

Factorial Basis Method for q-Series Applications

Abstract: Many combinatorics questions trickle down to finding a formula for a particular solution of some recurrence relation. This is often called the inverse Zeilberger problem. The Factorial Basis method provides solutions to linear recurrence equations in the form of definite sums. We will demonstrate the q-analog of this method and apply this extended technique to automatically prove identities and unveil novel ones, particularly some associated with the Rogers-Ramanujan identities.

This is joint work with Antonio Jimenez-Pastor.