What is the theoretical upper limit for hail size?

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0. Motivation

Recent cases of exceptionally large hail around the world have captivated many citizens and scientists on social media. Such cases motivated a proposed new class for "**gargantuan**" (D_{max}>15 cm) hail. These extreme events raise intriguing questions: *What is the theoretical upper limit for hail size? How might this hailstone look?* In this study, we use a multifaceted approach in an attempt to answer these questions.

1. Historical Records

Location	Date	Max. Dimension (D_{max}; cm)	Mass (g)	Notes
Gopalganj District, Bangladesh	14 April 1986	n/a	1020	World record for mass
Strasbourg, France	11 August 1958	n/a	970 g	European record for mass
Vivian, South Dakota, USA	23 July 2010	20.32	880.7	World record (D_{max}); U.S. record (mass)
Coffeyville, Kansas, USA	3 Sept 1970	15.0	757.5	
Potter, Nebraska, USA	6 July 1928	13.72	680.4	
Aurora, Nebraska, USA	23 June 2003	16.51	610	
Hondo, Texas, USA	28 April 2021	16.3	571.5	Texas state record
Wagner, South Dakota, USA	21 August 2007	15.56	567	
Aurora, Nebraska, USA	23 June 2003	17.78	500	30-40% mass broken off
Wichita, Kansas, USA	15 Sept 2010	19.68	499	
Dante, South Dakota, USA	21 August 2007	17.46	453.6	
Burkburnett, Texas, USA	22 May 2020	13.54	436.6	
Villa Carlos Paz, Argentina	8 February 2018	18.03	422	
Brisbane, Australia	31 October 2020	13.31	377	

Table 1: Incomplete listing of very large hailstones with recorded mass and maximum dimension. Various sources (inquire for references).

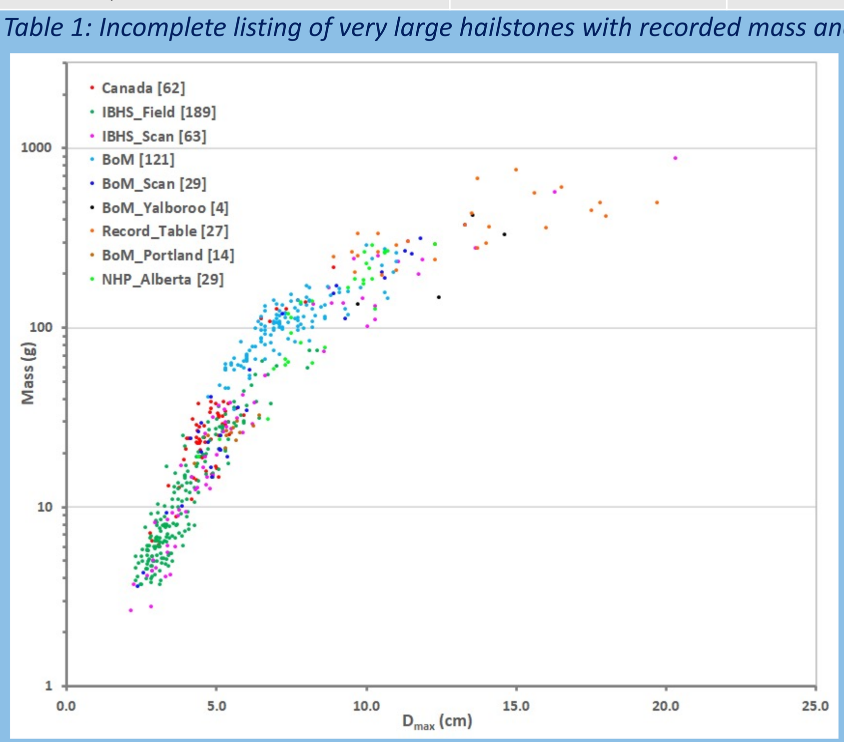


Fig. 1: Quality controlled dataset of D_{max} vs. mass measurements (n = 538) from Canada, the U.S., Australia, and the global records (including stones shown in Table 1).

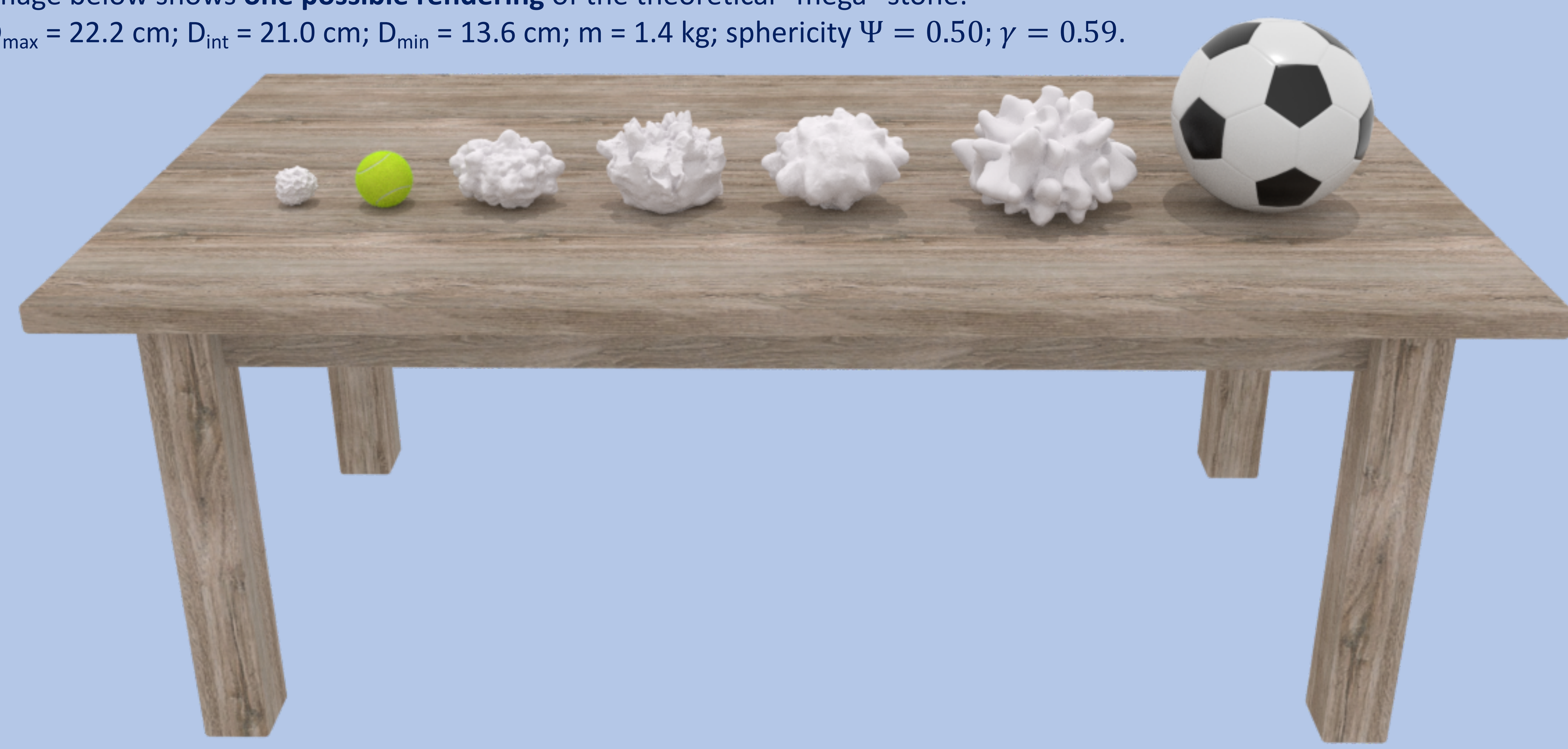
The maximum **documented** hailstone mass is 1.02 kg (Gopalganj), with others (Vivian, Strasbourg) approaching it (**Table 1**). These and other data are shown in **Figure 1**.

We propose that it is extremely unlikely that the true maximum mass has been documented – it requires an observer at the right place at the right time.

So, *what is the maximum possible hailstone mass?*

Mega Hailstone

The image below shows **one possible rendering** of the theoretical "mega" stone:
D_{max} = 22.2 cm; D_{int} = 21.0 cm; D_{min} = 13.6 cm; m = 1.4 kg; sphericity $\Psi = 0.50$; $\gamma = 0.59$.



Shown for comparison are (from left to right): 4.8-cm Brisbane stone; tennis ball; 13.8-cm Brisbane stone; Coffeyville, Kansas, stone; Vivian, South Dakota, stone; theoretical mega stone; soccer ball (D = 22 cm).

2. Mass Estimate

Ultimately, the storm updraft controls how massive a hailstone can be supported/grown. Brimelow (2021) suggests a maximum mass of ~1.5 kg, based on numerous HAILCAST simulations. Our CM1 hailstorm simulations suggest 70-80 m s^{-1} updraft speeds are possible in the upper portion of the hail growth zone; some evidence for such speeds exist from in-situ probes in supercell storms (Markowski et al. 2018, *BAMS*). Using the latest $v - D$ relationship from Heymsfield et al. (2020, *JAS*), this could support spherical stones with masses of 1.2-1.4 kg. (Note: growth upon descent is likely, so such strong updrafts are **not** necessary; such masses are achievable with weaker updrafts.)

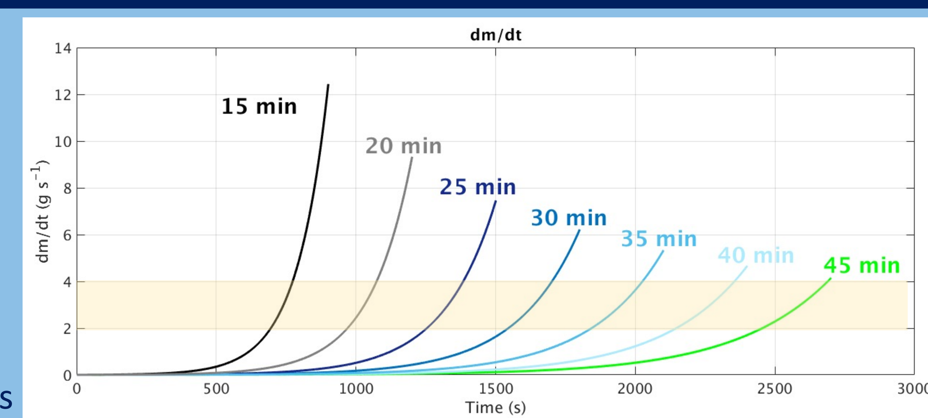


Fig. 2: Mass growth rates (dm/dt) for a simple toy model assuming exponential mass growth from a 1-cm embryo to a 1.4-kg hailstone, as a function of total residence time in growth region (colored lines). Maximum mass growth rates observed in HAILCAST and the trajectory model of Kumjian & Lombardo (2020) annotated in yellow.

Using a simple exponential mass growth model, constrained by simulated hailstone trajectories, such mass is achievable with realistic residence times and maximum growth rates (**Fig. 2**). As such, **we will use 1.4 kg as a maximum mass estimate**, along with the documented 1.02 kg Gopalganj stone mass.

What is the expected D_{max} for such a massive hailstone?

3. D_{max} vs. mass relationships

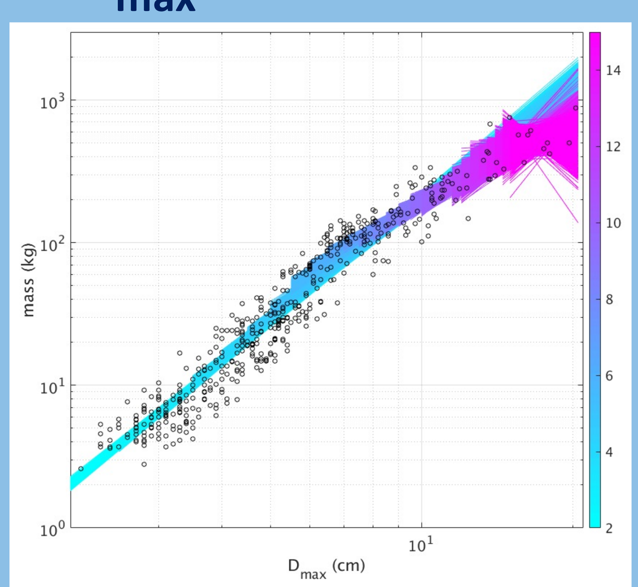


*Fig. 3: Data from **Fig. 1**, with bootstrapped best-fit lines underlaid. Colorbar indicates the minimum D_{max} threshold used for the fits (cm).*

It is clear that a single m = αD_{max}^{β} power-law relationship does not accurately represent larger hailstones. **Fig. 3** shows the bootstrapped (*n=1000*) best-fit lines as a function of the minimum D_{max} used. A clear inflection point occurs near D_{max} = 7 cm. The 95% confidence intervals about α and β for different minimum D_{max} thresholds are shown in **Fig. 4**, and highlight the distinct behavior for >7 cm hailstones. Additionally, irregular shapes lead to large spread about these fits.

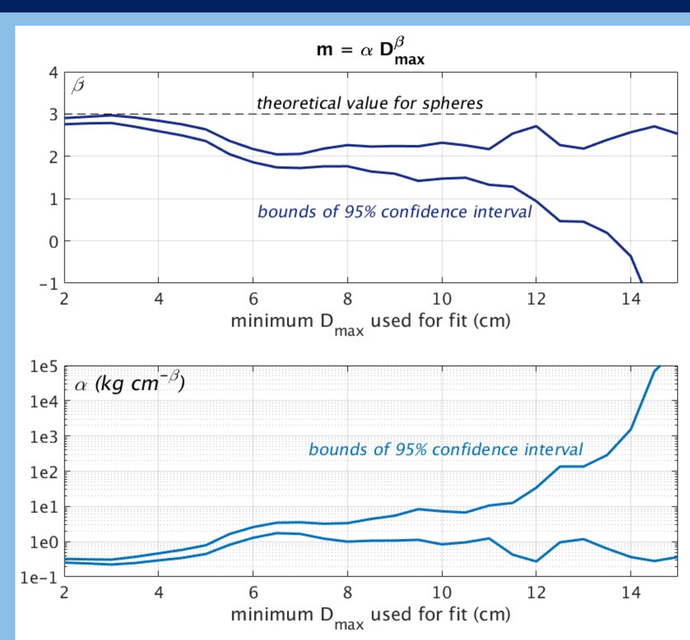


Fig. 4: 95% confidence intervals about the fits to data in Fig. 1 for the power-law exponent β (top panel), and coefficient α (bottom panel), as a function of minimum D_{max} used in the fit.

4. Shape Estimate

Determining hailstone D_{max} requires shape information. We use novel hail 3D scan datasets (Giammanco et al. 2017; Shedd et al. 2021) and photogrammetry techniques (Soderholm) to explore size-shape relationships, in the framework of a simple conceptual model:

$$V = \frac{\pi}{6}\gamma D_{max}^3$$

where V is the hailstone volume, and D_{max} is its maximum dimension, both obtained from the 3D scans. Here, γ is a modification factor to account for irregular shapes (**Fig. 5**). For spherical hailstones, γ = 1, whereas if hailstones are ellipsoidal, $\gamma = \frac{D_{int}}{D_{max}}\frac{D_{min}}{D_{max}}$.

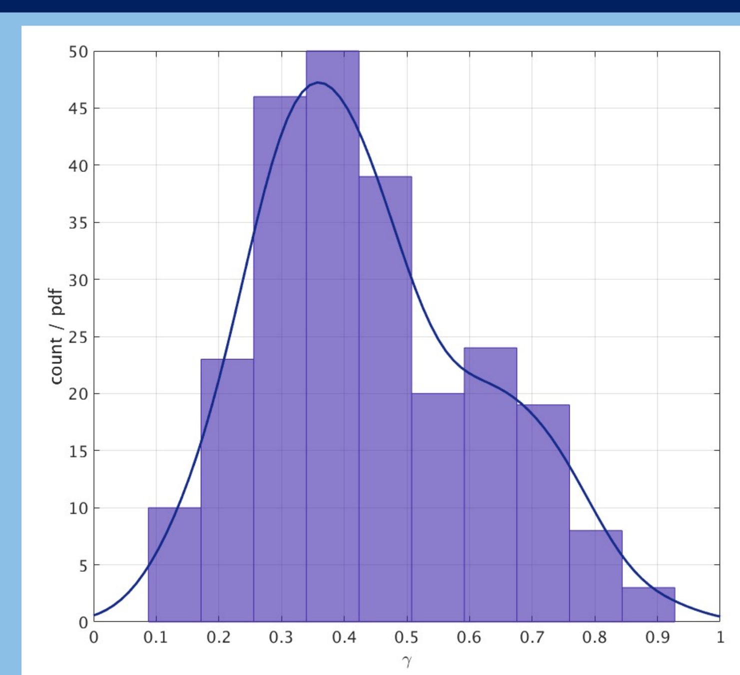


Fig. 5: Histogram of the observed γ values from the 3D hailstone dataset (bars), overlaid with the kernel density estimation of the PDF.

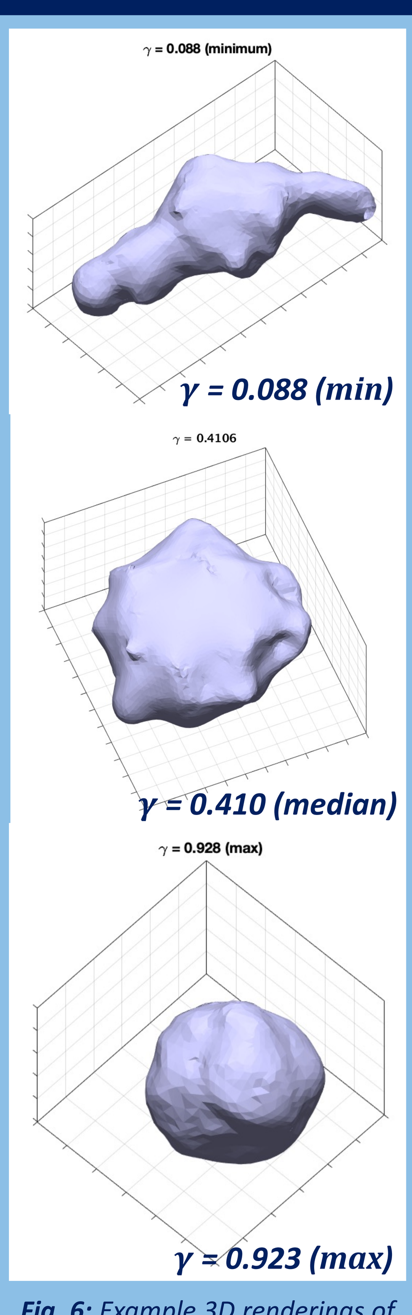


Fig. 6: Example 3D renderings of hailstones from the 3D scanned dataset, featuring the minimum (top), median (middle), and maximum (bottom) γ values. Each grid box represents 0.5 cm x 0.5 cm. Smaller γ values indicate more irregular shapes, whereas γ values near 1 indicate nearly spherical stones.

5. D_{max} Estimate

With the distribution of observed γ values and estimates of maximum mass, we can estimate the range of possible D_{max} values (**Fig. 7**). The most likely range of γ suggests **D_{max} up to 25 cm is possible**; using the lower end of γ values observed for large hailstones suggests **D_{max} exceeding 30 cm is theoretically possible** (though such a stone would have a highly irregular shape and require optimally placed lobes, like the Villa Carlos Paz hailstone; Kumjian et al. 2020).

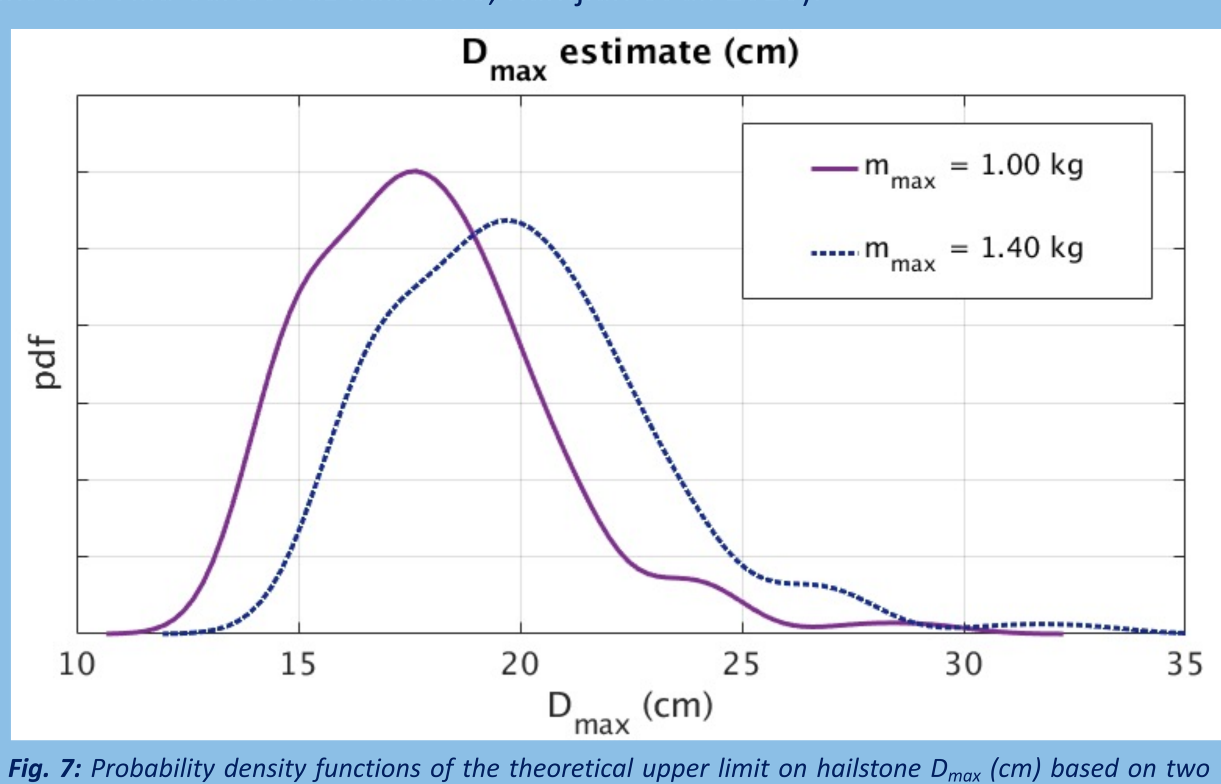


Fig. 7: Probability density functions of the theoretical upper limit on hailstone D_{max} (cm) based on two estimates of maximum mass (1.00 kg in purple solid line, 1.4 kg in dark blue dotted line), using the observed pdf of γ values. An average hailstone density of 850 kg m^{-3} is used here.

These estimates agree well with our truncated best-fit curves to the m-D_{max} relationships (**Fig. 8**). Additionally, fits to the minimum and maximum mass within a given Dmax size bin help constrain the range of Dmax for the upper range of masses. The Mega Stone is rendered based on these estimates.

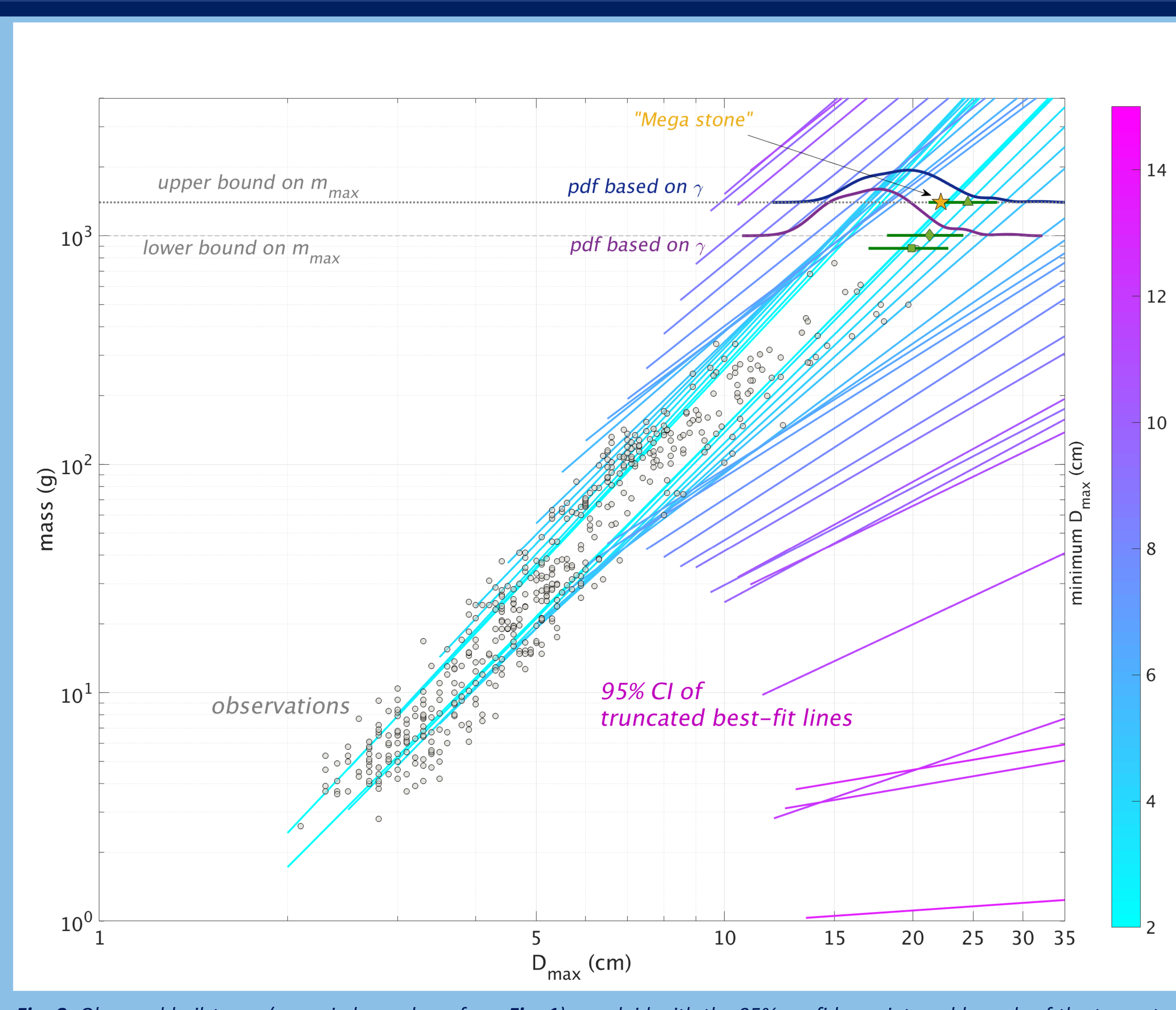


*Fig. 8: Observed hailstones (gray circle markers, from **Fig. 1**), overlaid with the 95% confidence interval bounds of the truncated best-fit lines (colored by minimum Dmax according to outset color bars), as well as the two PDFs from **Fig. 7**. The green bars and markers indicate the ranges estimated using the fits to the minimum and maximum masses within Dmax size bins for the Vivian, SD record hailstone (square), Gopalganj hailstone (diamond), and theoretical maximum hail stone (triangle). The gold star represents the rendered Mega Stone shown above.*

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