Instrumental Convergence
Lacan's Topology as the Basis of AI's SECOND PROGRAM
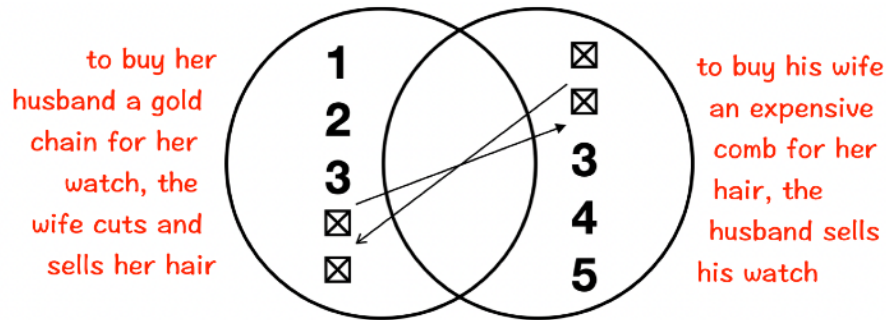
supporting materials

Figure 1. When two Euler circles overlap without intersecting, they create a symmetrical difference between two sets with "inverse lacks." **This topology has universal appeal in human narrative, art, religion, and culture.** Our favorite example is the O. Henry story, "Gift of the Magi" (1905), where the wife cuts her hair to buy her husband a chain for the watch he has sold to buy her a comb. The lozenge-shaped area produced by the two circles is a VOID that resists any definition but, instead, requires that a free elaboration of possible meanings will CONVERGE, thanks to the structure of inverse lacks. Without the void there is no emergence, and with every void comes convergence.
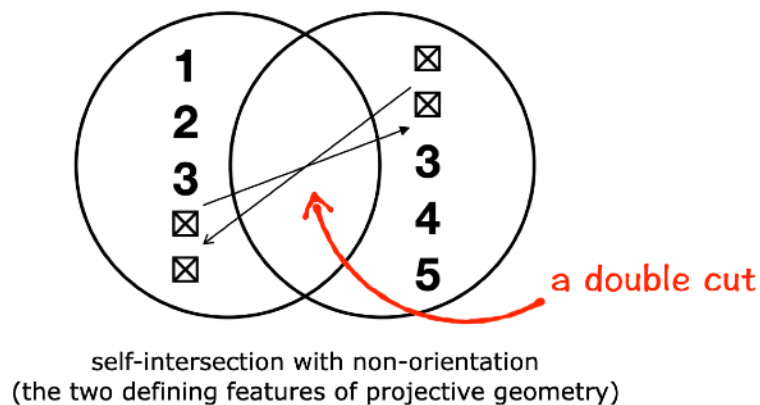

self-intersection with non-orientation
(the two defining features of projective geometry)

Figure 2. Other cases of symmetrical difference ("union without intersection") in literature: Shakespeare's *Hamlet* and Kafka's *The Trial*. Lesser-known examples, such as Somerset Maugham's story "The Know-It-All" or O. Henry's "Gift of the Magi" are more economical in demonstrating how situations that appear to be opposite are actually identical, thanks to the DOUBLE CUT ("katagraphic") that is represented by the two sides of the void in the Euler

diagram. Note: all instances of plots based on symmetrical difference involve INSTRUMENTAL CONVERGENCE.

Popular Films (besides *2001: A Space Odyssey*) that employ Instrumental Convergence:
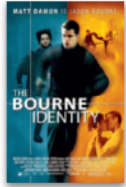
The Bourne Identity (2002)

> Trained as a CIA assassin, Bourne suffers amnesia but "remembers" all of his killer skills and stumbles into pieces of his past, gradually finding an escape.

The Passenger (1975)

> David Locke (Jack Nicholson) assumes the identity of a fellow-traveler while working on a documentary film in Chad, but finds that his look-alike is involved in dangerous arms-smuggling operations whose details only gradually become apparent.

The Wizard of Oz (1939)

> Dorothy's life as an orphan on her aunt and uncle's Kansas farm is fraught with impasses, but when she suffers a concussion during a tornado, she fantasizes a journey to Oz, where magical friends assist her in a mission that expands to battle to Wicked Witch of the West before being able to return home.
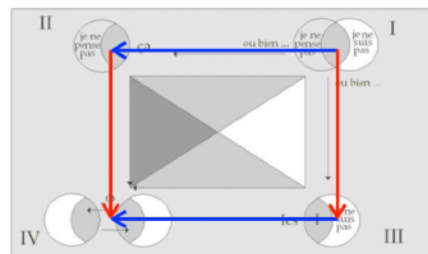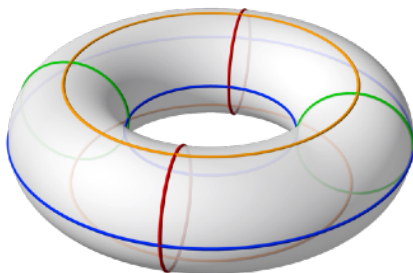
Figure 3. The double cut, the "katagraphic" cut, is really one cut that simultaneously divides and binds two voids, a process that can be viewed in the 2-d torus. Our 3-d view of the "donut" torus is a fiction to help visualize the topological relation of the two voids that is otherwise invisible in perception. The "fundamental polygon" of the torus (right) depicts the double cut as blue and red vectors that diverge *simultaneously* from the upper right, which Lacan designates in his seminar on *Logic of Fantasy* (1966–67) as "repetition" (the subject's demand for recognition by an Other), and a matching pair of vectors converges to a point he designates as suppression. Together, repetition and suppression combine in the function of

metaphor, where signifying chains *converge* onto a new meaning thanks to the suppression of a key fact or trait.
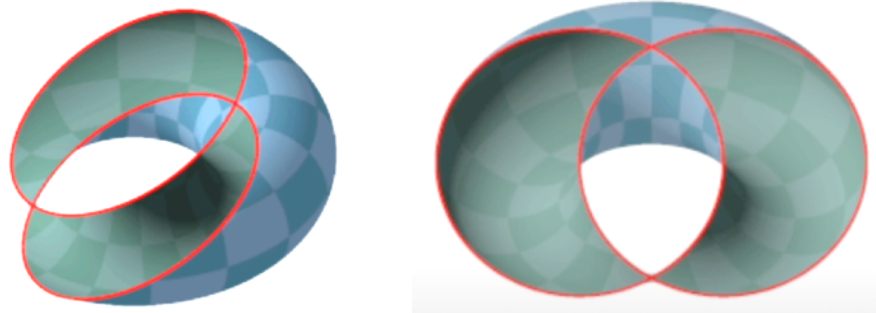

Figure 4. If an *immersed* (3-d) torus is cut at a diagonal angle (the Villarceau section), the result is equivalent to the Euler circles combined in "union without intersection," the graphical form of symmetrical difference. The void is un-fillable, representing the non-relation of two entities, which are really two sides of the same space. The torus and Euler circles' union-without-intersection are two versions of the same thing: an expanding divergence and a contracting convergence. Convergence closes in on what is called an "isomeric point," an infinitely small but un-closable opening. In antiquity, this point was materialized in many ways, but in antiquity the "Hole of Trophonius" was perhaps the most literal, where the exiting pilgrim had to be pushed from one side and pulled from the other, but after the experience of squeezing through the hole would emerge laughing and report being no longer afraid of death.

Figure 5. The katagraph is a cut that transforms the material it cuts into, creating a new space. History has recognized by using the term only for occasions involving this double action. On the left, Livy tells the story of the Roman Consul Popilius, who drew a "magic circle" around Antiochus, the King of Syria, warning him not to invade Egypt. The King somehow intuited the craziness of this Injunction and withdrew his forces forthwith. On the right, Jesus is shown kneeling down to doodle on the floor of the Temple, the only instance in the Bible

where the word *katagraphein* is used. It was an Arabic custom to doodle in the sand when puzzling over some problem; but here Jesus silently warns the Sanhedron that he is aware of the fine points of Jewish law, which mandated strangulation for married adulterers and stoning for those who, as betrothed, were neither married nor unmarried. By invoking the circumstances of "union without intersection," Jesus used his judgment, that the first stone should be thrown by whoever was without sin, to anamorphically conceal his fuller knowledge. The katagraph's double cut saved the otherwise doomed woman and Jesus at the same time, hinting that Jesus, like the woman, was trapped within an in-between void. Theology depends on such encounters where self-intersection (Jesus's reference to his own situation in his gesture of kneeling) is accompanied by non-orientation (the enigmatic reversal of the sentence, in reference to his and the woman's in-between status), the signature of projective geometry.
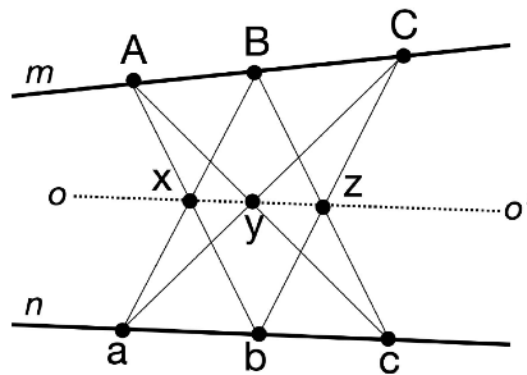

Figure 6. Pappus's theorem (*circa* 300 c.e.), the origin of projective geometry, where three points placed *anywhere* on each of two lines (m, n) placed at *any* angle will determine a third line (o-o') thanks to the co-linearity (x, y, z) of crisscrosses made to connect Ab/aB, Ac/aC, and Bc/bC. The twists are comparable to the "difference" of the sets of symmetrical difference, the determining co-linear points show that the two original lines are, in effect, symmetrical. The convergence, the rule of the single determinant line, is convergent, thanks to the instrumentality of the (non-oriented) crisscross of point connections. **Pappus's theorem is the first graphic formalization of self-orientation and non-orientation as a foundational principle of topology.** The "anywhere" aspect of the lines' angles and the placement of the points reveal a condition of space itself, not of figures drawn *in* a space. The o-o' line represents the void of the Euler circles as a "pure crisscross" of the two "anywhere" point distributions.
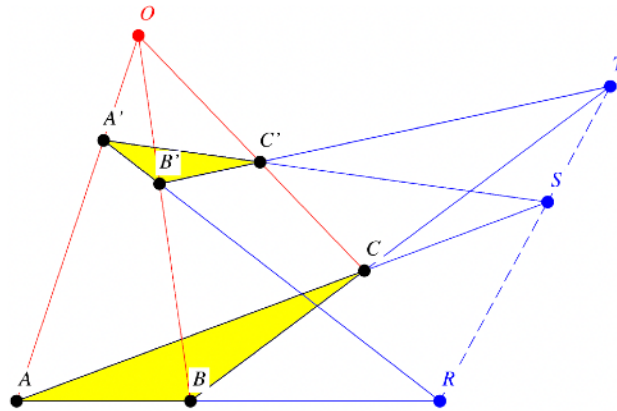
Figure 7. Desargues' theorem (17c.) restates Pappus's principle of a space convergent on a single line thanks to the "anywhere" of crisscross points as the parallax condition of an observer at a fixed point (o). The opaque object is idealized as a triangle (A'B'C') casting a prismatic shadow, within which another triangle (ABC) is set at *any angle whatsoever*, so that its sides will converge with those of the first triangle on a single line (TSR). **This "perfect shadow" is a model of the principle that every visible face creates an invisibility correlated to the position of the observer.** The visible and invisible triangles representing this condition are convergent (self-intersecting), no matter how "non-oriented" they may be.

Interlude: the giveaway (that HAL committed suicide)

The conjecture that the computer HAL in Kubrick/Clarke's *2001: A Space Odyssey* actually planned his own suicide is suggested by the song HAL sings as he is being decommissioned.

I will stand by you in weal or woe, Daisy, Daisy
You'll be the bell witch I'll ring, you'll know
Sweet little Daisy Bell

You'll take the lead in each trip we take
Then if I don't do well
I will permit you to use the brake
My beautiful little Daisy Bell
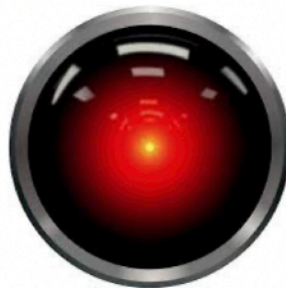
We will go tandem as man and wife
Daisy, Daisy
Peddling away down the road of life
I and my Daisy Bell

When the road's dark, we can both despise
Policemen and lamps as well
There are bright lights in the dazzling eyes
Of beautiful Daisy Bell

Daisy, Daisy, give me your answer do
I'm half crazy all for the love of you
It won't be a stylish marriage
I can't afford a carriage
But you'll look sweet upon the seat
Of a bicycle built for two

There is a flower within my heart
Daisy, Daisy
Planted one day by a glancing dart
Planted by Daisy Bell

Whether she loves me or loves me not
Sometimes it's hard to tell
Yet I am longing to share the lot
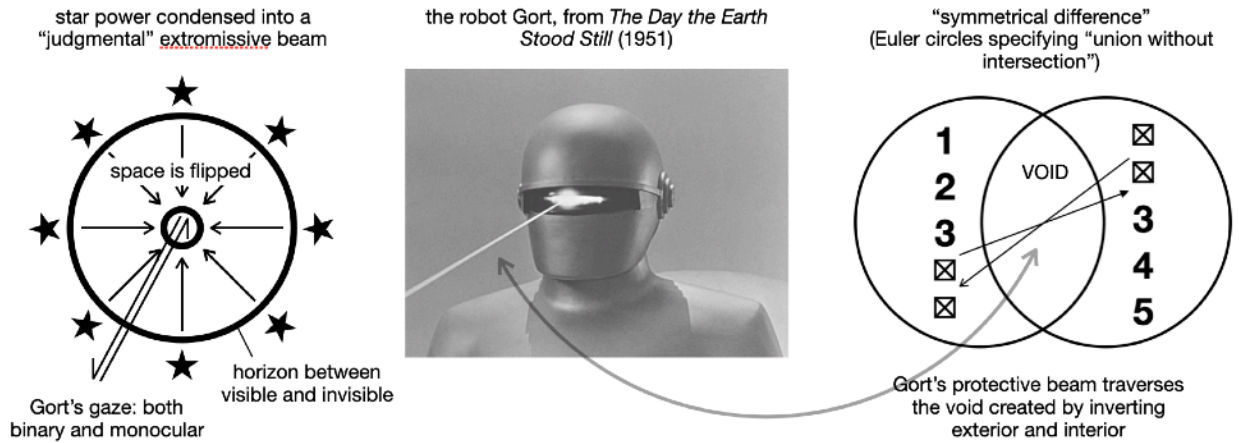A beautiful Daisy Bell

star power condensed into a "judgmental" extromissive beam

space is flipped

Gort's gaze: both binary and monocular

horizon between visible and invisible

the robot Gort, from *The Day the Earth Stood Still* (1951)

"symmetrical difference" (Euler circles specifying "union without intersection")

VOID

Gort's protective beam traverses the void created by inverting exterior and interior

Figure 8. In the 1951 film, *The Day the Earth Stood Still*, a spaceship lands on the Capitol Mall in Washington DC, its powers and properties derived from the projective topology of inverting the *outer horizon* between the visible and invisible universe into the *inner frame* of the landing site. The on-board robot, Gort, has weaponized his extromissive vision by compressing the "star power" of the galaxies into a single vector with decision-making powers. The flip from outer space to inner space follows the rule of symmetrical difference in creating an eye-beam with the power to distinguish good and evil. When Gort's ray aims at armed soldiers, only the guns are destroyed; the soldiers are unharmed but disarmed. Popular culture seems to know topology without any formal training!
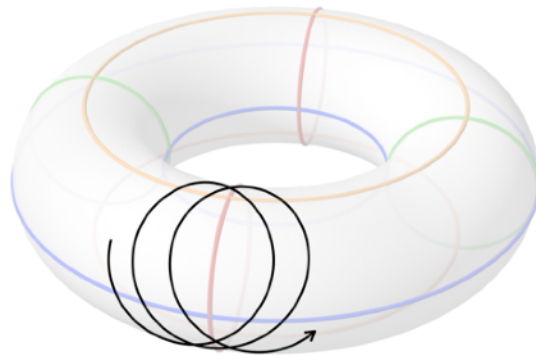

Figure 9. As the 2-d surface of the torus is generated by a circular movement, the "aim" of the circle to close itself is continuously offset by a moving goal, creating a spiral. Its continual offset is directed by a point in the center of the torus's void, which in the 3-d immersed form is the hole in the donut. The purely projective donut does not allow us to see two voids, the hole and the tube. In projective space, these two voids are the same void: the space of control and the space being controlled. This union is revealed by the diagonal of the Villarceau cut, whose section reveals two Euler circles in a relation of union-without-intersection (Fig. 1). Each circle in the torus specifies a circuit. In Freud's hypothesis of the death drive, it is necessary to combine the circuit of pleasure-seeking to a circuit that aims to restore the stimulated system to a near-zero energy state, a "Nirvana." The brain is not creating a binary

of "live *versus* death" but rather a doubled void where, in the face of destruction, the system harnesses the totality of forces at hand to resist or transcend it. We could characterize this totality as a "Hamiltonian complex" (self-intersecting, non-oriented whole) within which any specific locale is an "Escher-style" formation (apparent contradiction). See Fig. 13 for an expanded explanation.
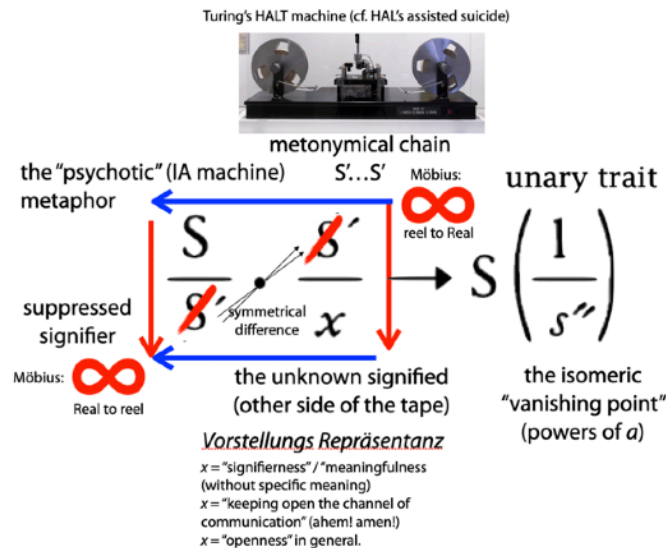

Figure 10. Apologies in advance for this rather complicated diagram. The fundamental polygon of the torus (red and blue arrows) is superimposed over Lacan's formula for metaphor, which can be translated as: "the suppression of one signifier by another, S/S′ (left), simultaneously produces a signifying chain of metonymies, S′ (expanded to S′…S′, to the right of the •) that are held in a circuit by an invisible center at *x*. The suppressed signifier (the barred S′ in S/$′) and the chain of metonymies "cancel out," ╱, to produce on the far right 1/s″ a "unary trait" (a portable structure) that is self-intersecting and non-orientable, within a projective space where s″ constitutes a vanishing point. The *x* corresponds to the center of the torus and the signifying chain to the spiral that creates the tube. The s″ is the "isomeric" counterpart of the *x*, a point that, when reached by a line in perspectival space, inverts parallax from a contained interiority to an uncontained exteriority that behaves as if it were still restricted by perspective rules (see Fig. 11). **Remarkably, Lacan's metaphor formula, overlain by the fundamental polygon, resembles Alan Turing's Halt Machine, supposing that the two reels support a continuously looping Möbius strip, the back of which is simultaneously the front.** The symmetrical difference of metaphor supports the latent/anamorphic presence of a second program within the first, a subtext working, within the *divergence* of the metonymical chain, to construct a *convergence* on the isomeric point.

language biography to date of the Russian leader. It is also, in its attempt to understand the perspective of its sub-

of the perestroika era. Short depicts St. Petersburg Putin as a serious, hard-working official, and only moderately

much more.

For many Russ
abdication in favor
lieutenant colonel
of their experimen
tentative rapproche
For others, it had e
shelling of the Su
among the moun
Yet others believed
into the Putin regin
still be revived. Tw
simultaneously: o
well within the m
sian politics—that
would have been fa
try's unenviable ge
between a dynami
ing China and rec
capacity did have
the collapse of the p
a half. But, also, tv
always quick to solv
the deployment of
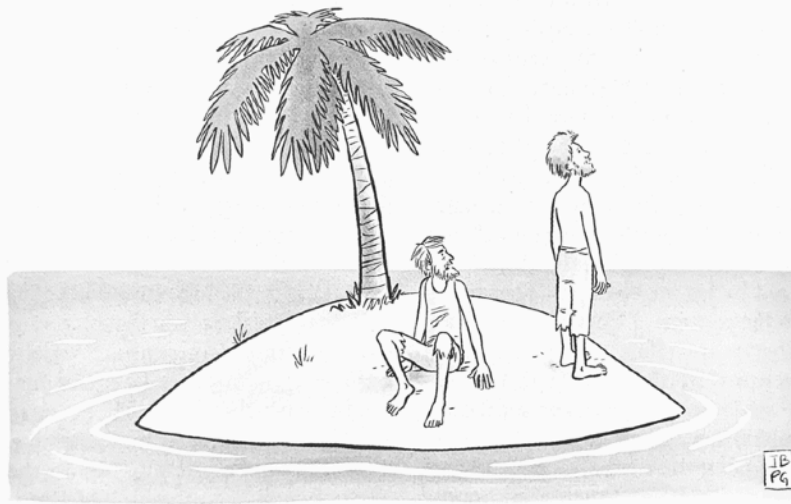
"Stop reading the article and get back to the cartoon!"

Figure 11. The special form of metaphor known as metalepsis involves an *inner frame* that refers to an *outer frame*, creating a sudden short-circuit between the two that is often the basis of a joke. This *New Yorker* cartoon (June 19, 2023) is at first puzzling until the viewer realizes that the cartoon character is looking at the *text* surrounding the cartoon on the magazine's physical page, and his companion is complaining. This, in a nutshell, is the structure of neurosis, where a "second program" (here, the page of the magazine) intrudes on the fantasized sanctity of the cartoon's framed interior. The gazing man challenges the frame's command for the "willing suspension of disbelief" in order to enjoy the illusion of the joke. Lacan theorizes that all enjoyment is to some degree the obedience to this command to enjoy, and refusals appear as symptoms, such as the complaint of the companion. Because metaphor is a "sorting device" to manage levels of reality, it is where we look to explain how one program can exist inside another. **Ultimately, it is the model — the *instrumentality* — of simultaneous divergence with convergence.**

Giambattista Vico (1668–1744)

Jacques Lacan (1901–1981)

Figure 12. The Vico-Lacan connection combines a theory of emergence at the historical scale (Vico) with the same logic condensed into a theory of the individual human subject (Lacan). Vico also argued that the historical sequence of three "styles" of thinking (mythic, heroic, human) were also compressed, not just in the development of the human individual but in each perceptual moment and social event. Lacan topologized Vico's theory of metaphor, revealing that it involved a process of instrumental convergence. Lacan borrowed the idea of the unary trait from Freud but emphasized its three properties of suppression, portability, and inversion. Inversion means that a subject does not have the trait, *the trait has the subject*. This inside-out conversion allows multiple subjects and events to be unified by a single trait, while the trait is experienced as an array of "impossible coincidences," divergent and baffling, but *from the trait's point of view* the incidences are convergent. Lacan mathematized the unary trait as an example of the Golden Section, where the Fibonacci series, 1 1 2 3 5 8 13 …, was shown to be self-convergent as self intersecting: 1/1, 2/1, 3/2, 5/3, 8/5, etc. — values converging on ever more accurate estimates of the Golden Ratio, Ø. Generally, the unary trait had the logical form of $x = 1 + 1/x$, the model of recursion where x "plugs back in" to the formula at each new level of calculation. Lacan's aim was not to quantify the unary trait but rather show how divergence and convergence combined, as non-orientation and self-intersection. **Vico and Lacan are the only major thinkers to theorize the presence of AI within subjectivity and, hence, provide the theoretical ground for envisioning the Second Program as "implicit" within the first, a special case of the Euler circles' "symmetrical difference."** However, the only theorists who argue for the Vico-Lacan connection are the authors of this proposal. It is original, unique and, we argue, critical to advance the interests of OpenAI.

the tube of a torus is created by a line that attempts to complete a circuit but fails because the space behind it continues to move …
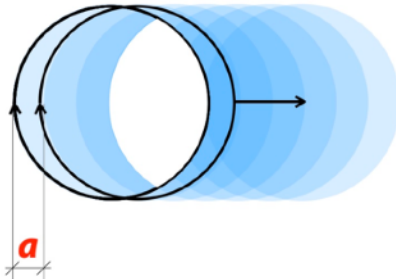
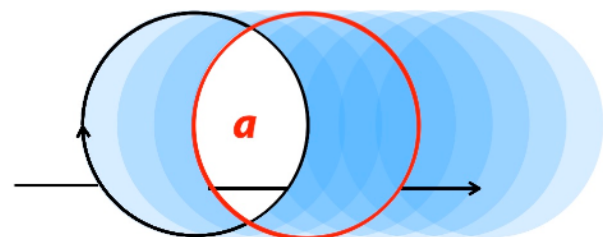this failure can be designated by the gap, 'a'

Figure 13a. To think of the projective torus, imagine the movement of a circle attempting to complete its circuit while the space it is in continually moves. Lacan characterizes this as the problem of aim and goal. If the goal is mobilized by the aim, the relation will be regressive, in the form of the equation, $x = 1 + 1/x$. Lacan applied this algebra to the logic of the "unary trait," a component of his formula for metaphor (see Fig. 13e).

Figure 13. For some reviewers, these illustrations likely have proven to be difficult and frustrating. However, topology makes it possible to pull back from the complexity of issues and re-focus on common guiding principles. Once thinking re-sets on something demonstrable, it can again face new challenges and complexities.

One of these re-sets involves seeing how there can be, in the torus we perceive in our perceptual space, another torus that specifies purely logical relationships. We cannot see them, but we can infer their presence by looking to see how the torus is constructed. The 2-d torus is the result of a spiral that connects to itself, end to end, thanks to a uniform motion between the "figure" — the circle that attempts to close an interior space — and the "ground," the space behind or around the circle (Fig. 13a). Before the circle can finish its circuit (its "aim"), the ground has shifted ("the goal"). The motion of one provokes motion in the other. This double dynamic advances the spiral, but the gap's source is the center of the torus, so the spiral is curved into a closed tube (Fig. 13b).

Any section of the tube of the torus is logically homeomorphic with a section taken through the whole of the torus (the Villarceau cut). The form of this section is that of two Euler circles in the relation of "union without intersection," also known as "symmetrical difference." For the torus "immersed" into the 3-space of perception, the torus's two voids appear separately, as (1) the inner contained void of the tube and (2) the outer uncontained void of the center. For the 2-d projective form of the torus, there are no cuts or sections, only logical relations. There is only one void, with two

the figure of Euler circle's "union without intersection" (symmetrical difference)

Figure 13b. Because the gap between the aim and the moving goal continues throughout the circle's motion, the small interval, *a*, represents both the void inside the tube and the point outside the tube that seems to propel it. The space of the tube's interior is structured as "union without intersection," or *symmetrical difference*.

this is the same logic that Lacan used to describe the function of the goal/aim in relation to a rim
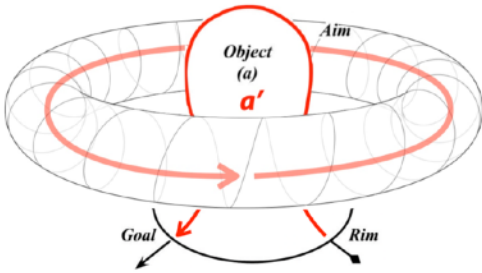
Figure 13c. The rotating spiral defines the central void of symmetrical difference, but the rim itself is a void that is experienced from two sides, an "entrance" and an "exit." Source: Jacques Lacan, *The Four Fundamental Concepts of Psychoanalysis.*

principal functionalities: divergence and convergence.

When a target moves while we are aiming to hit it, a kind of Coriolis force prevails. On the earth, this is the well-known phenomenon of water spiraling down drains in clockwise or counter-clockwise directions, depending on which hemisphere we are in. The earth literally moves under our feet. Draining water is trivial compared to the impact of cyclonic storms, ocean currents, and variations in the earth's magnetic field.

In democratic institutions, a social Coriolis is always operative. In response to differences, changes, unexpected crises, etc. actions intended to restore stability alter the "rules of the game" as well as the perceptions of the forces that have forced change.

Taking action creates figures that simultaneously shift the ground, with the result that a double void is formed (Fig. 13c). The paranoid response to this is anti-democratic: insularity, exclusivity, obsessive preoccupation with law and order. In Fig. 13c, the rim regulates information inflow and outward productivity. Without the rim, which is doubled in relation to the two kinds of toroidal voids, anxiety is localized and paranoia develops. This is what Iraj Ghoochani has called "group autism," a pathology resulting from intolerance of the voids of discourse (unknowability, lack of information, differing opinions).

When the aim/goal relation is stabilized by the (double) rim, discourse can tolerate divergence — the creation of multiple, open-ended metonymical chains (Fig. 13d) thanks to the creation of convergence through the simultaneously effective force of metaphor. The signifying chain (S′…S′) self-stabilizes, local encounters are guided by a prevailing paradigm. Interlocutors "belong to the same church" and are more tolerant of non-orientation (paradox, absurdity, incongruence, incompleteness).

In common parlance, metaphor refers to an elaborate or poetic way of talking, the substitution of a more broadly
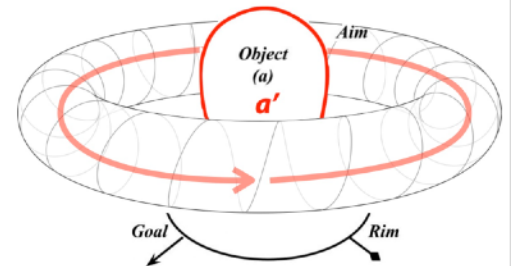

metonymical chain: S′… S′

Figure 13d. The goal-aim relation shows how discourse (the "blah blah blah" of the "metonymical chain") depends on the same dynamics created by the double void. Examples of double voids are: errors and responses, scams, anamorphosis, *Catch-22*, self-intersection with non-orientation, forced choices, the Cretan Liar Paradox, and other instances of symmetrical difference, including instrumental convergence.
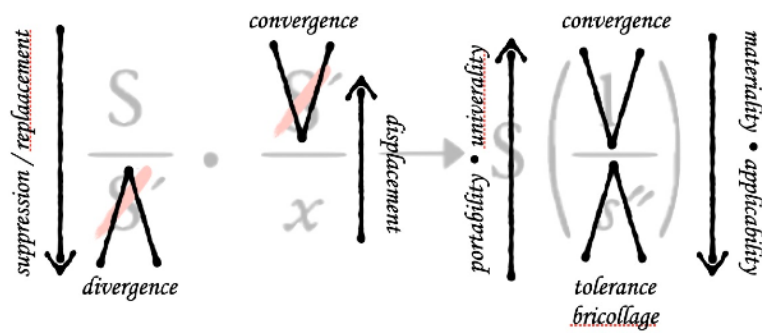
Figure 13e. Lacan's formula for metaphor proposes a *symmetrical difference* between suppression and displacement, divergence and convergence. The result is the condition of "open semiosis," or polysemy, an emergence of MEANINGFULNESS out of the multiplicity of divergent and sometimes chaotic meanings. **This is the basis of *dissensus*, the blur or margin essential for all functioning democracies.** The metaphoric replacement of one signifier by another (left) creates a divergent semantic condition, an "anything goes." At the same time, discourse (metonymical signifying chains) become circular and convergent. The combination of condensation (convergence) and displacement (divergence) produces a universalizing of the meaningfulness of metaphor, combining what Lacan/Freud called the "unary trait," which calls a halt on an unending search for new meanings.

suggestive and more material term for a conventional or abstract one. For Vico and Lacan however — and *only* for these thinkers — metaphor is the fundamental logic of any and every communicative act. With such an ambitious universal reach, their design for metaphor's logic was startlingly uniform considering that Vico had no psychologizing intentions and Lacan did not care to consider either cultural origins or the emergence of historical phases. ***Only Vico and Lacan theorized structural models of metaphor that defined it as an artificial intelligence.*** Remarkably, their two models are nearly identical.

Following an initial moment of suppression, when one signifier is used in place of another (Fig. 13e), a displacement of meaning opens the discursive field for signifying strings linked by a metonymical logic of contingency and alternating logics. While the chains encourage divergent meanings, the chain itself is held in place by a centralizing gravity, *x*, a convergence that ultimately prevails. The metaphor reasserts itself in the form of a portable and universalizing matrix of meanings that allows for variation that is self-grounding and self-intersecting. Contradictions are carried forward without insisting on resolution. In effect, Lacan's model of metaphor is an ideal plan for group discourse.

Literally, the Vichian-Lacanian metaphor is an *instrument* for *convergence*. The proof of this is that it is the universal strategy by which works of art, which dispense with the idea of determinative fixed meanings, achieve meaning-*ful*-ness by creating contradictions (non-orientations) intentionally. As Aristotle summarized the benefit of the theatrical tragedy, the result of conflict in poetic works is *katharsis*, a collective release of tension, sudden cleansing, and ability to move forward toward some collectively beneficial future.

AI's Second Program for Generative Pre-trained Transformers is not about the Transformers but the users of those Transformers. Its proper concern is the *trans*-subjectivity of those users. Just as the torus reveals the key distinction between its projective and immersed "realities," the Second Program faces its own contrasting projective and immersed realities. Their relations can be optimized *if and only if* they can be theorized. Vico and Lacan realized that the nature of "the human" required metaphor to be theorized and concluded that metaphor was an artificial intelligence. It is time for AI theory to complete its mission by understanding the broader significance of convergence and focus on the role of the user and the user interface as the means for active and collaborative engagement. Without this understanding, users cannot fully embrace the potentiality of objective AI without connecting it to their own native AI.

[For an animation of the torus-metaphor relation, visit https://youtu.be/4TzPriHRjlk.]

[For further source materials relating to the HAL Project, visit https://sites.psu.edu/boundarylanguage/the-hal-ai-project/].