Efficient Distributed MAC for Dynamic Demands: Congestion and Age Based Designs

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Abstract—Future generation wireless technologies are expected to serve an increasingly dense and dynamic population of users that generate short bundles of information to be transferred over the shared spectrum. This calls for new distributed and low-overhead Multiple-Access-Control (MAC) strategies to serve such dynamic demands with spectral efficiency characteristics. In this work, we address this need by identifying and developing two fundamentally different MAC paradigms: (i) congestion-based paradigm that estimates the congestion level in the system and adapts to it; and (ii) age-based paradigm that prioritizes demands based on their ages. Despite their apparent differences, we develop policies under each paradigm in a generic multi-channel access scenario that are provably throughput-optimal when they employ any asymptotically-efficient channel encoding/decoding mechanism. We also characterize the stability regions of the two designs, and investigate the conditions under which one design outperforms the other. We perform extensive simulations to validate the theoretical claims and investigate the non-asymptotic performances of our designs.

Index Terms—Distributed MAC, throughput-optimal scheduling, IoT, 5G, network stability, delay and age analysis.

I. INTRODUCTION

Older generation wireless technologies are the emerging explosion in the scale and requirements of the diverse mobile devices that will need to be supported over an ultra-wide frequency spectrum (including the 57–71 GHz in the U.S. [1], [2]). For example, new services in diverse domains, including health, transportation, energy, and entertainment, within the broad framework of the so-called Internet-of-Things (IoT) present scenarios where many mobile devices intermittently generate small bundles of information to transmit to a backbone wireless server (such as a base station or router). The distributed, large-scale, dynamic, and intermittent nature of these new service requirements call for the design of new spectrum access strategies. In this work, we aim to respond to this need by developing low-overhead and provably-efficient distributed medium-access-control (MAC) strategies aimed at serving a large (possibly unbounded) population of dynamic users with intermittent service demands.

In a related thread of important works (e.g., [3]–[7]), adaptive random access solutions for single channel slotted-Aloha are proposed to address the instability problem. These works guarantee stability when arrival rates are less than e−1 per slot. While resolving the stability problem under dynamic demands, these designs have been facilitated by the relatively tractable nature of the slotted-Aloha protocol, but also have been constrained by its relatively low efficiency of around ∼37%.

To increase channel efficiency, uncoordinated distributed MAC has been proposed and has a large number of interesting variants (e.g., [8]–[12] to name a few). The ideas employed in these works range from developing random access strategies to avoid collisions over a subset of channels (e.g., [8], [12]), to considering advanced signaling capabilities, such as multi-packet reception (e.g., [9], [10]) or orthogonal-frequency-division-multi-access (OFDMA) capabilities (e.g., [13]). In recent years, there have been exciting advances in the design and analysis of the so-called ‘Modern Random Access’ strategies (e.g., [14]–[16]) that yield provably efficient random access schemes by efficiently encoding/decoding over a given number of (virtual) channels (in frequency and/or in time). Among them, one spectrally-efficient strategy is based on Successive Interference Cancellation (SIC) and irregular repetition slotted Aloha (IRSA) (e.g., [17], also see [18] and references therein).

However, these and a large portion of other works in this domain either build on sophisticated capabilities of the devices (such as the aforementioned advanced signaling and sensitive timing capabilities), and/or consider static user populations under which efficiency characteristics are investigated. As such, there is a need for developing efficient Multi-Channel MAC solutions that account for these user dynamics and are also well-suited to distributed operation.

Prior studies in [12], [13] stabilize the Multi-Channel MAC under dynamic arrivals by a similar method proposed in [7], but under the assumption that those (virtual) channels are independent with each other, which does not fit the setups in modern random access area. [19] developed a policy which uses the time elapsed and the number of repeated transmissions to control the transmission probability of users, and is agnostic to the number of users. [20], and a recent work, [21], provided...
two probability adjustment strategies based on estimating the network state by both assuming no new arrivals during the period when the estimation is performed. However, these works do not provide theoretical guarantees and exhibit degrading performance under highly loaded conditions. This differs from our first approach in that we use simple feedback to track the network state and offer provable good performance even in highly loaded conditions.

Above mentioned works only investigated age-independent randomized policies in which all demands in the system are treated as homogeneous demands and follow the same activating policy. These works aim at optimizing those conventional metrics such as throughput or average delay, but didn’t consider how much delay each demand will experience in the system from destination side. The concept of this new metric Age of Information (AoI) was first introduced in [22] and has attracted huge attention recently in variety networking areas ([23]–[25]).

To name a few age based activating strategies: the work in [26] designed a round-robin scheme for AoI minimization, however, it is incapable of dealing with the change of number of nodes and thus is not suitable for applications with dynamic arrivals; the works [27]–[29] proposed different age-dependent strategies including a learning based strategy for dynamic arrivals, but all under the assumption that all demands implement a single channel slotted ALOHA decoding protocol; the work in [30] evaluated the AoI evolution by means of a Markovian analysis under SIC scheme, but under the assumption that there are always a fixed number of demands in the system; Those previous works on age based activating strategies are either under centralized control, or under a relative static setup, or have an easier decoding protocol; the work in [30] evaluated the AoI evolution by means of a Markovian analysis under SIC scheme, but under the assumption that there are always a fixed number of demands in the system: Those previous works on age based activating strategies are either under centralized control, or under a relative static setup, or have an easier decoding protocol.

motivated by them, we design a provable good or under a relative static setup, or have an easier decoding based activating strategies are either under centralized control, or under a relative static setup, or have an easier decoding channel model. Motivated by them, we design a provable good age driven distributed random activation strategy for dynamic arrivals under simple channel feedback. We then compare two proposed policies, conclude that neither can totally outperform the other and which one is better depends on the properties of the arrival distributions.

Our work makes the following contributions:

- We consider a threshold-based success model which can capture basic characteristics of many spectrally-efficient multi-channel uplink modern MAC schemes, such as many SIC-based schemes ([16], [17]) and network-coded diversity protocol (NCDP) based on physical layer network coding (PNC) (see [18] Fig 3.18 and 3.19). The channel performance experiences a linear increase up to a point and then collapses completely (see [31] Ch.5 and Fig. 5.25 for more references). Under that, we develop provably-stabilizing distributed Multi-Channel MAC strategies for dynamic users with a novel low-overhead estimation and feedback mechanism which solves the instability problem of static MAC solutions for the setting of dynamic users.

- We design two fundamentally different and asymptotically-optimal types of activation strategies that are inspired by the use of congestion dynamics of the system and the age dynamics of the demands.

We first propose a congestion based feedback and activation mechanism, aiming to estimate the network state and probabilistically transmit based on the estimate. We second propose an age based feedback and activation mechanism, aiming at always serving the demands with oldest age first. This idea was commonly used in previous works (see [28], [32]–[34]), but they are assuming either a more static arrival or implement a single channel ALOHA protocol. We characterize the stability region of traffic (Theorem 1, 2 and 3) and show both maximum rates are asymptotically-optimal in fully utilizing the threshold-based success model as the scales grow.

- By investigating particular success/failure characteristics of the SIC operation, we also extend (in Section VII) our design for the threshold-based success model to the IRSA encoding/decoding scheme described in [17]. Using a simple feedback strategy, this design closes the loop by utilizing the principles and operations developed under the approximate model to be applied successfully to the SIC setting. We validate our designs by performing extensive numerical studies (in Section VI) to investigate the their stability and delay characteristics.

We note that our focus in this work is on achieving network stability under dynamic users under the preferred MAC strategy. While we take IRSA in [17] as an instance of a spectrally-efficient strategy, our approach can be extended to other MAC solutions with possibly different spectral-efficiency characteristics.

II. PROBLEM FORMULATION

We consider the operation of a distributed spectrum access system that evolves over discrete-time, whereby dynamic demands arrive and exit after successful transmission over orthogonal time-frequency channels (see Fig.1). Demands can be individual users or simply packets created by IoT (Internet of Things) devices. Our goal is to maximize the throughput, as well as, keep the system stable and achieve better delay performance.

A. Network Dynamics

1) Dynamic Demand Arrival Model: In each time slot \( \tau = 1, 2, 3, \ldots \), a random number \( R(\tau) \) of new demands
and identically distributed random variables with mean \( \lambda \).

At the beginning of each frame, each demand in the system has to decide whether to be active (i.e., transmit) and how to encode its content over the physical frequency bands, where \( W \), the physical frequency band is 3). Components of Spectrum Access Policy: At the beginning of each frame, each demand in the system has to decide whether to be active (i.e., transmit) and how to encode its content over the physical frequency bands, where \( W \), the physical frequency band is

\[ P_\pi[t] = \sum_{t \geq 1} \mathbb{P}(N_\pi[t] = t) \]

where \( \mathbb{P}(N_\pi[t] = t) \) gives the total number of demands that leave the system at the end of frame \( t \). We say that a policy, say \( \pi \), stabilizes the system if

\[ \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[N_\pi[t]] < \infty. \]

We assume that all demands in the system can track their own waiting time/age \( W_\pi[t] = 0, 1, 2, \ldots \), which is defined as the number of time slots that have passed since demand \( d \) arrived at the system at time slot \( \tau \).

In this paper, our goal is to design stabilizing policies with low-overhead feedback structures that provide high throughput guarantees for the distributed and dynamic spectrum access setting described above. Next, we describe the space of policies over which we will pursue this goal.

**B. Distributed MAC Strategy Space II**

As briefly introduced in section II-A, a distributed spectrum access Policy \( \pi \) in the feasible policy space II is described by a triplet: (i) the common feedback \( F_\pi \); (ii) the activation strategy \( X_\pi \); and (iii) the encoding/decoding strategy governing the service process \( S_\pi \). Next, we clarify these components.

1) Common Feedback \( F_\pi \): At the end of each frame \( t \), the base station broadcasts a common feedback \( F_\pi[t + 1] \) to all demands in the system to guide their decision in frame \( t + 1 \). Although the exact nature of this common feedback depends on the Policy \( \pi \), all common feedback designs must fulfill two goals: (a) demands that made transmissions in frame \( t \)

2Occasionally, we will drop this superscript to simplify the notation when the policy we refer to is clear from the context.
can infer their success/failure status from the feedback and leave the system if successful; and (b) the remaining demands (including the newcomers) can use the feedback to decide their actions in the next frame, \( t + 1 \).

2) Demand Activation Strategy \( X^\pi = (X^\pi_d)_d \): At the beginning of frame \( t \), each demand \( d \) in the system distributedly uses the common feedback \( F^\pi[t] \) to decide its activation state, \( X^\pi_d[t] \in \{0, 1\} \), as determined by the activation policy we will design. In particular, \( X^\pi_d[t] = 1 \) indicates that the demand will be active and make a transmission attempt in frame \( t \) over one or multiple of the \( C \) channels, while \( X^\pi_d[t] = 0 \) indicates that it will be silent during the frame.

3) Multi-Channel Success Model Governing \( S^\pi \): Given the activation vector \( X^\pi \), the next step in the description of the policy \( \pi \) is the success model for the \( C[t] = M F[t] \) channels, over which the active demands will attempt transmissions. Clearly, diverse MAC and coding schemes may be employed for such distributed uplink transmission. These can range from the simplest collision-based Aloha-like success/failure dynamics to more sophisticated space-time encoding/decoding methods.

While the performance of these schemes may vary, we aim to consider a common success model that can capture their basic characteristics while also providing a tractable model. Accordingly, many multi-channel uplink MAC schemes will exhibit an activation-success performance that on average grows linearly with increasing number of active demands until the number becomes too high for the strategy to support them. After this critical load level, which we denote as \( \eta(C) \) for \( C \) channels, the channels are overloaded and the success rate Given the \( C[t] = M F[t] \) channels, over which the active demands will attempt transmissions. Clearly, diverse MAC and coding schemes may be employed for such distributed uplink transmission. These can range from the simplest collision-based Aloha-like success/failure dynamics to more sophisticated space-time encoding/decoding methods.

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\[
S^\pi[t] = \begin{cases} 
X^\pi[t], & X^\pi[t] \leq \eta(C) \\
0, & X^\pi[t] > \eta(C),
\end{cases}
\]

whereby \( \eta(C) \in \{0, 1, \ldots, C\} \) is a MAC-strategy-dependent parameter that measures the efficiency of the strategy. Accordingly, the success status of demand \( d \in N^\pi[t] \) is given by

\[
S^\pi_d[t] = \mathbb{1}(X^\pi_d[t] \leq \eta(C)) X^\pi_d[t]. \tag{2}
\]

Figure 3 demonstrates the validity of this threshold-based success model for the case of the SIC-based strategy (see [17], [18]). We will describe the details of the encoding/decoding process of the SIC strategy in Section VII. Fig. 3 shows how the threshold-based success model (dashed) closely follows the simulated success performance of the SIC-based scheme in [17] (solid) as the number of active demands increases. We note that SIC-based and IRSA schemes are of particular interest due to its distributed encoding/decoding procedures. Moreover, the IRSA scheme in [17] is known to be asymptotically efficient in that the fraction of successful transmissions per channel, \( \eta(C)/C \), converges to its maximum possible level of 1 as \( C \to \infty \) (see [17], [18]). After developing our activation policies under the threshold-based success model in Sections IV-A and IV-B, we will return to the specific case of SIC-based strategy in Section VII to show how our designs can be adapt to that strategy.

C. Designing Activation Policies

Our main goal is to design low overhead feedback mechanisms and distributed activation policies for dynamic demands that have desirable stabilization characteristics. In particular, we are interested in designs that can support a wider set of arrival processes \( \{R(\tau)\}_{\tau} \) and achieve a higher throughput (as close to \( M \) demands per slot as possible) under stability condition 1.

Based on the channel success model introduced in Fig. 3, we can see that activating too little or too many demands leads to either a sparse or an overloaded network situation, resulting in low service rate and low throughput. So a total activation \( X[t] \) which is close to but smaller than \( \eta(C[t]) \) is desired. Our goal now becomes activating a target number of demands in a distributed manner with possibly simple low overhead channel feedback and information. Also, noticing that in most asymptotically efficient encoding/decoding schemes, the \( \eta(C)/C \) ratio increases as the number of virtual channels \( C \) increases, thereby increasing the spectral efficiency. However, such increased efficiency typically yields higher delay since it requires a larger frame size \( F \).

Next, we introduce two main design paradigms with different strengths and weaknesses, which will be analyzed and compared in the rest of this paper.

III. CONGESTION AND AGE-BASED DESIGN PARADIGMS

Within the general framework of low-overhead distributed multi-channel MAC strategies, we will comparatively investigate two fundamentally different and asymptotically-optimal types of activation strategies that are inspired by the use of congestion dynamics of the system and the age dynamics of the demands. In this section, we introduce the structure, the guiding principles, and the statistical characteristics of these so-called congestion-based and age-based design paradigms, which will later be used in Sections IV and V to develop provably efficient policies in each design class.
A. Congestion-Based Design Paradigm

The main idea behind the congestion-based design paradigm is to: (i) maintain an estimate of the number of demands in the system; and (ii) announce this number to the demands so that each demand independently decides to transmit or not with a probability of \( r/N \). Under this design, the number of active demands is distributed as \( \text{Bin}(N, r/N) \). Note that, if our estimate \( \hat{N} \) stays close to the true value \( N \), then \( \text{Bin}(N, r/N) \) approaches the Poisson distribution for large \( N \). This allows us to optimize the parameter \( r \) independently from the congestion-level in order to maximize the service rate for a given number of virtual channels \( C \).

In Section IV, we will build on this foundation to develop the required estimator and the optimal parameter choice with provably good performance characteristics. Due to the Poisson nature of the activation number, our analysis will also reveal that the stability region of our congestion-based design depends only on the mean \( \lambda \) of the arrival process \( R(\tau) \).

B. Age-Based Design Paradigm

In contrast to the congestion-based design that uses the congestion-level \( N \) to determine the activation probability of all demands, the age-based design aims to: (i) maintain an age-threshold level; and (ii) announce this level to the demands at the start of each frame so that only those demands with ages greater than the threshold transmit. In particular, the age-threshold is selected so that \( K \) slots of demands with oldest ages are activated in frame \( t \), where \( K \) is a design parameter. Under this design, the number \( X \) of active users is the sum of i.i.d. arrivals over \( K \) slots. Accordingly, by the central limit theorem, the distribution of \( X \) can be approximated by \( \mathcal{N}(K\lambda, K\sigma^2) \) for large enough \( K \).

In Section V, we will build on this foundation to develop the required threshold update policy and the service mechanism in the event of failures with provably good performance characteristics. Due to the Gaussian nature of the activation number, our analysis will also reveal that the stability region of our age-based design depends on both the mean \( \lambda \) and the standard deviation \( \sigma \) of the arrival process \( R(\tau) \).

We note that neither of the two design paradigms outperform the other under all arrival processes. This is illustrated in Fig. 4 using the Poisson and Gaussian approximations that govern the active demands under the two paradigms. In particular, for arrival processes with a common mean \( \lambda \), as the variance \( \sigma^2 \) is reduced from \( \sigma^2 > \lambda \) in Fig. 4(a) to \( \sigma^2 < \lambda \) in Fig. 4(b), the Poisson approximation becomes narrower while the Gaussian approximation remains unchanged. This allows the Gaussian distribution to be centered at a higher level to increase the success rate. We shall confirm this intuition in Section VI to show that congestion-based design will be preferable for highly variable arrival processes with respect to their mean, while the age-based design will be preferable for less variable arrival processes.

IV. Congestion-Based Design and Analysis

In this section, we detail the design and analysis of a provably efficient distributed-MAC strategy based on the congestion-based design paradigm outlined in Section III-A. In particular, we first address the design and analysis assuming that the network state \( N^\pi(t) \) is perfectly known (Section IV-A). Then, we build on this basis to include the design of the estimator \( \hat{N}^\pi \) that can track the actual state \( N^\pi(t) \) (Section IV-B).

A. With Perfect Network State

In this section, we design and analyze a provably efficient distributed-MAC strategy with perfect network state, which describes how the base station and each distributed demand must act in each frame.3 At the beginning of each transmission frame under \( \pi \), every demand \( d \in N^\pi(t) \) transmits with probability \( \min\{\frac{r^\pi}{\eta^\pi}, 1\} \), where,

\[
\frac{r^\pi}{\eta^\pi} = \arg\max_{r \geq 0} \mathbb{E}[Y(r) I(Y \leq \eta(C))],
\]

and \( Y(r) \) is a Poisson random variable with mean \( r \). At the end of frame \( t \), the base station sets \( Z[t] = 1(X[t] \leq \eta(C)) \), where \( X[t] = \sum_{d \in N^\pi[t]} X_d[t] \); demand \( d \in N^\pi[t] \) is successful (i.e., \( S_d[t] = 1 \)) and leaves the system if \( X_d[t] = 1 \) and \( Z[t] = 1 \). Then the base station sets \( F[t + 1] = (Z[t], N[t + 1]) \).

Note that feedback \( F[t + 1] \) of Policy \( \pi \) for frame \( t + 1 \) is composed of two components: the binary variable \( Z[t] \) ∈

\[ \text{This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.} \]
\{1, 0\}, indicating whether the decoding operation at the end of frame \(t\) was successful or not; and the number of demands in the system \(N^\pi[t+1]\) at the beginning of frame \(t+1\). Under the threshold-based success model, the binary feedback simply becomes \(Z^\pi[t] = \mathbb{1}(X^\pi[t] \leq \eta(C))\). And, under the perfect network state information assumption, \(N^\pi[t+1]\) is known perfectly by the base station.

At beginning of frame \(t\), demands who have made a transmission in the previous frame leave or stay (depending on the binary feedback). Those \(N^\pi[t]\) demands that are still in the system independently decide to become active with probability \(\max\{r^*/N^\pi[t], 1\}\). Here, \(r^*\) is optimally selected according to an easily solvable maximization (3). This optimization is based on a Poisson approximation of the random demand activations and aims to optimize spectral efficiency. Those demands that decide to transmit then utilize the SIC-based encoding/decoding procedure described in Section II-B(iii).

2) Performance Analysis of Policy \(\tilde{\pi}\): For each positive integer \(n\), define \(S^\pi(n)\) as the number of demands successfully served in a frame under \(\tilde{\pi}\) when \(N[t] = n\). Define \(r^*_n = \min\{r^*, n\}\) for all \(n \in \{1, 2, 3, \ldots\}\), where \(r^*\) is defined in (3).

\[
P(S^\pi(n) = i) = \binom{n}{i} \left(\frac{r^*_n}{n}\right)^i \left(1 - \frac{r^*_n}{n}\right)^{n-i},
\]

for all \(i \in \{1, \ldots, \min\{n, \eta(C)\}\}\). This is a truncated Binomial\((n, r^*_n/n)\) distribution. Consequently, its limit is a truncated Poisson distribution from the Poisson approximation result of binomials. In particular, as \(n \to \infty\), \(S^\pi(n)\) converges in distribution to \(Y(r^*)\mathbb{1}(Y(r^*) \leq \eta(C))\), where \(Y(r^*)\) is a Poisson random variable with mean \(r^*\). Accordingly, we define \(\rho^*(C)\) as

\[
\rho^*(C) = \lim_{n \to \infty} \mathbb{E} \left[ S^\pi(n) \right]
\]

as

\[
= \mathbb{E} \left[ Y(r^*)\mathbb{1}(Y(r^*) \leq \eta(C)) \right] = \sum_{i=1}^{\eta(C)} \binom{\eta(C)}{i} \frac{(r^*)^i}{i!} e^{-r^*},
\]

where (5) follows from the boundedness of the range \(\{0, \ldots, \eta(C)\}\) of \(S^\pi(n)\) and its convergence in distribution.

Theorem 1: Fix frame size \(F\). Assume arrivals \(\{R(t)\}_{t=0}^\infty\) are i.i.d. with mean \(\lambda\) and variance \(\sigma^2\). Suppose \(\lambda < \rho^*(C)/F\), where \(C = MF\) and \(\rho^*(C)\) is defined in (4). Then, Policy \(\tilde{\pi}\) stabilizes the network. Moreover, the resulting average demand delay satisfies (in units of slots):

\[
\mathbb{E} \left[ W^\pi \right] \leq \frac{m\rho^*(C) + F\lambda}{\lambda\rho^*(C) - F\lambda} \left(\frac{\lambda}{\rho^*(C) - F\lambda}\right),
\]

where \(m \triangleq \lfloor (\rho^*(C) + F\lambda)/2 \rfloor \in \{0, \ldots, \lfloor \rho^*(C) \rfloor\}\).

Furthermore, noting that \(\rho^*(C)/\eta(C) \to 1\), and \(\eta(C)/C \to 1\) as \(C \to \infty\) under the IRSA scheme in [17], we get \(\rho^*(C)/F \to M\) as \(F \to \infty\). Since the network cannot be stabilized for any \(\lambda > M\), this shows that the Policy \(\tilde{\pi}\) fully utilizes the threshold-based model and is \textit{asymptotically throughput-optimal} under the specific IRSA scheme.

Proof: Please refer to [35], which is an extended version of our accepted but unpublished paper.

In addition to providing a delay bound, Theorem 1 shows that near-perfect utilization of all \(M\) channels can be achieved (as we extend the frame duration \(F \to \infty\)). To appreciate this, consider a simpler scenario with exactly \(M\) demands that never leave. We could assign each demand to one of the \(M\) channels, each demand could send one packet per slot with no contentions, and \(100\%\) throughput would be achieved. Theorem 1 shows the remarkable fact that similar throughput can also be achieved when demands dynamically arrive and depart and make randomized transmission decisions. Since each demand can only send one packet, there is limited time for learning or coordination. This is the strength of the SIC-based coding combined with the simple randomized MAC scheme (Policy \(\tilde{\pi}\)). Recall that Policy \(\tilde{\pi}\) requires perfect state information \(N[t]\), which is difficult to realize in practice. The next section shows that similar near-perfect throughput can be achieved without knowledge of \(N[t]\).

B. With Estimated Network State

Our distributed MAC Policy \(\hat{\pi}\) in the previous section has asymptotic optimality characteristics as well as large performance improvements compared to those schemes designed for static arrivals in the non-asymptotic regime. However, it relies on a perfect network state \(N[t]\) to be known at the base station. This section develops a new distributed-MAC policy that does not require knowledge of \(N[t]\) but achieves the same throughput. The technique is inspired by an estimation method developed for basic slotted Aloha in [7].

1) Distributed-MAC Policy Design: \(\hat{\pi}\): In Algorithm \(\hat{\pi}\), we propose a new policy \(\hat{\pi} \in \Pi\), that uses an estimate \(\hat{N}[t]\) of the (unknown) \(N[t]\) value to guide the distributed spectrum access strategy.

Note that the feedback structure \(F^\pi\) and the demand activations \(X^\pi\) of Policy \(\hat{\pi}\) have the same form as that of Policy \(\tilde{\pi}\), except for the difference that the exact network state \(N^\pi[t]\) is replaced with the estimated \(\hat{N}^\pi[t]\) in both the feedback content and the activation decision.

The key modification in the tracking operations of the network state. The algorithm does not assume perfect knowledge of the arrival rate \(\lambda\) and therefore takes as input an estimated arrival rate \(\hat{\lambda}\). We show later that this estimate can take a range of values, and can also be fixed to an upper-limit that will be given in Theorem 2. At the end of each frame, the network state estimate \(\hat{N}^\pi\) is incremented by the estimated arrival rate of \(\hat{F}\lambda\), regardless of the decoding operation outcome. However, depending on the success/failure indicator \(Z^\pi[t]\) of the decoding operation, the estimate of \(\hat{N}^\pi[t]\) is: decremented by \(r^*\) defined in (3) if \(Z[t] = 1\), and incremented by the new parameter \(r^+\) defined in (7) if \(Z[t] = 0\). Both of these parameters can be easily calculated for the given number of channels \(C\). Their values are carefully selected to guarantee that the estimate \(\hat{N}^\pi[t]\) can track the true value \(N[t]\) under the system dynamics. An interesting feature of Policy \(\hat{\pi}\) is that, despite its lack of knowledge of the system state \(N[t]\), its feedback has the same low-overhead structure as Policy \(\tilde{\pi}\).

2) Performance Analysis: In this section, we analyze the stability characteristics and prove asymptotic efficiency of our proposed policy, \(\hat{\pi}\).
Algorithm $\hat{\pi}$: Spectrum-Access Policy With Estimated $\hat{N}[t]$

Input: An estimate $\hat{\lambda}$ of $\lambda = E[R(\tau)]$

Initialize: $F[0] = (Z[0] = 0, \hat{N}[0] \in \mathbb{N})$

repeat For $t = 1, 2, \ldots$

Each demand $d$: At the beginning of frame $t$ do:

if $d \in N[t-1], X_d[t-1] = 1$, and $Z[t-1] = 1$ then

Set $D_d[t-1] = 1$ and demand $d$ leaves the system
endif

if $d \in N[t]$ then

Set $X_d[t] = \begin{cases} 1, \text{w.p. min}\{\frac{r^*}{N[t]}, 1\} \\ 0, \text{otherwise}, \end{cases}$

where $r^*$ is defined in (3)
endif

if $X_d[t] = 1$ then

demand $d$ performs SIC-based encoding in frame $t$
endif

Base station: At the end of frame $t$ do:

Set $Z[t] = 1(X[t] \leq \eta(C))$

where $X[t] = \sum_{d \in N[t]} X_d[t]$

if $Z[t] = 1$ then

Set $N[t] = \max(N[t] - 1 + K\hat{\lambda} - r^*, r^*)$
else if $Z[t] = 0$ then

Set $N[t] = N[t] + 1 + K\hat{\lambda} + r^*$

where $r^*$ is the solution to

$$r^* = \frac{\mathbb{E}[[r^* - \Upsilon(r^*)]1(\Upsilon(r^*) \leq \eta(C))]}{\Pr(\Upsilon(r^*) > \eta(C))}$$ (7)
endif

Set $\mathbb{P}[t] = (Z[t], \hat{N}[t])$
endif

end repeat

Theorem 2: Assume arrivals $\{R(\tau)\}^\infty_{r=0}$ are i.i.d. with mean $\lambda$ and variance $\sigma^2$. For any $\lambda \in [0, \rho^*(C)/F)$, the Policy $\hat{\pi}$ stabilizes the system for any choice of $\hat{\lambda} \in [\lambda, \rho^*(C)/F]$, where $\rho^*(C)$ is defined in (4), $F$ is the number of slots in a frame, and $C = MF$ is the number of frequency-time channels in a frame. Consequently, as with Policy $\pi$, Policy $\hat{\pi}$ is asymptotically throughput-optimal.4

Proof: Please refer to [35].

This theorem establishes that the stability region of the practical $\hat{\pi}$ policy that works with an estimated arrival rate $\hat{\lambda}$ and estimated network state $\hat{N}[t]$ is the same as the stability region of Policy $\pi$ that requires knowledge of $N[t]$. It is also worth noting that the arrival rate estimate can be fixed to its upper limit $\rho^*(C)/F$ if there is no good prior knowledge of the true arrival rate. Of course, more accurate levels of $\lambda$ can affect the delay performance, but as long as it is greater than $\lambda$, the policy is guaranteed to stabilize the network.

4We can also establish a delay bound on Policy $\hat{\pi}$, with asymptotic $\mathbb{W} \leq O(F/(\rho^*(C) - F\lambda))$ similar to (6), but the expression is more involved and we omit that due to space limitation (see final comment in proof).

V. AGE-BASED DESIGN AND ANALYSIS

As outlined in Section III, the main idea of age-based design is to sequentially activate and serve a group of the oldest demands that are in the system. Our strategy to achieve this is to activate a group of oldest demands by announcing an age-threshold at the beginning of new frames so that all demand with ages exceeding the threshold transmit. Next, we will describe our proposed age-based distributed MAC strategy $\pi \in \Pi$.

A. Age-Based Policy $\pi$ Design

Let us call each group of demands that are activated at the same time a cohort. Once the $i^{th}$ cohort is formed, only this cohort will be active until the whole cohort succeeds and leaves the system. Our strategy to achieve this efficiently is to select appropriate age threshold levels and virtual frame sizes based on the channel feedback. On one hand, the age threshold will be selected to assure that a good number of demands becomes active. In particular, each cohort under our policy consists of a random number of demands that have arrived over $K$ consecutive time slots. On the other hand, the virtual frame sizes will be adjusted in the event of failure to increase the probability of success. In particular, the frame sizes $\{F_{i,j}\}_{j \geq 1}$ for cohort $i$ will increase after $j^{th}$ attempt, until success, at which time it resets to $F_{i+1,1}$ (see Figure 5).

To express this system dynamics rigorously, consider the service of a newly formed cohort $i$. In the first attempt, we choose a frame size of $F_{i,1}$ slots, which leads to $C_{i,1} = MF_{i,1}$ virtual channels. This attempt, according to our success model, ends in one of two outcomes. If it succeeds, we move to the formation and service of cohort $i+1$ as described above. If it fails for the $j^{th}$ time, then we continue serving the same cohort $i$ over increasingly larger frame sizes $\{F_{i,j}\}_{j \geq 2}$ until all demands in cohort $i$ are served. To express this, let $Z_{i,j}$ be the binary channel feedback to represent whether the $i^{th}$ cohort of demands is successfully transmitted in the $j^{th}$ attempt, then the evolution of the Threshold $T_{i,j}$ for cohort $i$ and attempt $j$ will be

$$T_{i,j+1} = T_{i,j} + F_{i,j},$$ (8)

when $Z_{i,j} = 0$, i.e., if $j^{th}$ attempt of cohort $i$ is unsuccessful. Notice that, we are adjusting the Threshold $T_{i,j}$ to keep the
same demands in cohort $i$ active. If instead the $j^{th}$ attempt succeeds, i.e., $Z_{i,j} = 1$, then age threshold for cohort $i+1$ is set as

$$T_{i+1,1} = \max\{T_{i,j} + F_{i,j} - K, 0\}. \quad (9)$$

The $\max\{}$ operator in this equation captures the scenario when there are less than $K$ slots of demands in the system, and all the pending demands will be a member of the $i+1^{th}$ cohort. Within this structure of age-based design, we select the same parameters $\{F_{i,j}\}_{i,j} \geq 1$ for all cohorts $i = 1, 2, \ldots$ as

$$F_{i,j} = F_1^* + (j - 1)\Delta^*(F_1),$$

with

$$\Delta^* = \Delta^*(F_1) = \arg\min_{\Delta \in \mathbb{N}} \left\{ (F_1 + \Delta)(1 + \frac{\sqrt{K} \pi}{2 \eta (M\Delta)}) + \frac{K \sigma^2 \Delta}{\eta^2 (M\Delta)} \right\},$$

$$F_1^* = \arg\min_{F_1 : K \lambda < \eta (M F_1)} \left\{ (F_1 + \Delta^*)(1 + \frac{\sqrt{K} \pi}{2 \eta (M\Delta^*)}) + \frac{K \sigma^2 \Delta^*}{\eta^2 (M\Delta^*)} \right\}, \quad (10)$$

where $\lambda, \sigma$ are the mean and standard deviation of the arrival process $R(\tau)$, and $K$ and $M$ are fixed. Define $I_i$ to be the number of transmissions used for cohort $i$, define $Y_i = \sum_{j=1}^{I_i} F_{i,j}$. The complete algorithm is given below.

### B. Age-Based Policy $\pi$ Analysis

In this section, we analyze the stability characteristics and prove asymptotic efficiency of our proposed policy, $\pi$.

**Theorem 3:** For $M$ physical channels per slot and a given $K$ value, a sub-Gussian arrival distribution with mean $\lambda$ and variance $\sigma^2$ is supportable, i.e., the $(\lambda, \sigma)$ pair is in the stability region of the age-based algorithm $\pi$ if:

$$\min_{F_1, \Delta \in \mathbb{N} : K \lambda < \eta (M F_1)} \left\{ (F_1 + \Delta)(1 + \frac{\sqrt{K} \pi}{2 \eta (M\Delta)}) + \frac{K \sigma^2 \Delta}{\eta^2 (M\Delta)} \right\} < K. \quad (11)$$

where $\{F_{i,j}\}$ will be chosen as $F_{i,j} = F_1^* + (j - 1)\Delta^*$. $F_1^*$ and $\Delta^*$ are the optimizer of equation 11, and are defined in equation 10.

Also, the oldest age equals to the average delay, and satisfies:

$$E[\mathcal{W}] \leq \frac{\sigma^2}{2(K - E[Y])} + \frac{3K - 1}{2} + \frac{\sigma^2}{2E[Y]},$$

where $\sigma^2$ is the variance of $Y_i$.

**Proof:** Please refer to Appendix VIII.

A random variable $X \in \mathbb{R}$ is said to be sub-Gaussian with variance proxy $\sigma^2$ if $E [X^2] = 0$ and its moment generating function satisfies: $E[\exp(sX)] \leq \exp \left( \frac{s^2 \sigma^2}{2} \right)$, $\forall s \in \mathbb{R}$.

**Corollary 4:** The stability region of the age-based policy $\pi$ will be asymptotically efficient as $K$ goes to infinity, i.e., for all $\lambda < M$, the system will be stable for large enough $K$ value.

**Proof:** To show that the age-based algorithm is asymptotically optimal, it is sufficient to show that for all $\lambda < M$, for $K$ large enough, there exists $F_1 : \eta (M F_1) > K \lambda$ and $\Delta > 0$, such that:

$$\frac{F_1}{K} + \exp \left( \frac{-(\eta (M F_1) - K \lambda)^2}{2K \sigma^2} \right) \cdot \left( \frac{F_1 + \Delta}{K} \right) \left( 1 + \frac{\sqrt{K} \pi}{2 \eta (M\Delta)} + \frac{K \sigma^2 \Delta}{\eta^2 (M\Delta)} \right) < 1$$

Since $\lambda < M$, so $\lambda + \varepsilon < 1$, so there exists $\varepsilon$, such that: $\frac{\lambda + \varepsilon}{M} < 1$, ($\varepsilon$ is independent of $M$.) Observing that $\eta (m) \to \infty$ as $m \to \infty$. Let $F_1$ be the first positive integer to let $\frac{\eta (M F_1)}{MK} \geq \frac{\lambda + \varepsilon}{M}$, then, $\eta (M F_1) > K \lambda + \varepsilon M K > \lambda K$ meets the condition. Since $\frac{\eta (M F_1)}{MK} = \frac{\eta (M F_1)}{MF_1} \cdot F_1 \geq \frac{\lambda + \varepsilon}{M} + \varepsilon < 1$, and when $K \to \infty$, $F_1$ also must goes to infinity, so $\frac{\sigma^2}{MF_1} \to 1$, and $\frac{\lambda}{K} \to \frac{\lambda}{M} + \varepsilon < 1$. Now take $\Delta = \sqrt{K} > 0$, we want to
show that for large enough $K$:

$$
\frac{F_1}{K} + \exp\left(-\frac{\varepsilon^2 M^2}{2\sigma^2} \cdot K \right) \left[ \left( \frac{F_1}{K} + \frac{1}{\sqrt{K}} \right) \left( 1 + \sqrt{\frac{K\pi}{2}} \cdot \frac{\sigma}{\eta(M\sqrt{K})} \right) + \frac{\sqrt{K\pi}}{2} \cdot \frac{\sigma}{\eta(M\sqrt{K})} \right] < 1
$$

Notice that when $K \to \infty$,

$$
\frac{F_1}{K} + \frac{1}{\sqrt{K}} < 1,
\sqrt{\frac{K\pi}{2}} \cdot \frac{\sigma}{\eta(M\sqrt{K})} < \frac{\pi}{2} \cdot \frac{\sigma}{M^2},
\sqrt{K\sigma^2} \cdot \frac{\sigma^2}{\eta^2(M\sqrt{K})} \to 0.
$$

also, $\exp\left(-\frac{\varepsilon^2 M^2}{2\sigma^2} \cdot K \right) \to 0$ since $\varepsilon$ is independent of $K$, so the limit of the left hand side will converge to $\frac{\lambda}{M} + \varepsilon < 1$, when $K$ goes to infinity, which implies that for large enough $K$, the left hand side will be always smaller than 1. This conclude our claim that the algorithm will be asymptotically optimal.

VI. COMPARISON OF CONGESTION AND AGE-BASED POLICY PERFORMANCES

We have performed extensive numerical simulations under congestion-based and age-based policies. In this section, we present some of these results, both to validate the theoretical claims, and to investigate and compare their throughput and delay characteristics. In addition to confirming the asymptotic throughput-optimality of both designs, we shall observe that neither can outperform the other one in all situations. Our investigations also lead to insights on when to prefer one design over the other.

We start with investigating the stabilizable arrival processes of two policies in Figure 6. We take the case of $M = 10$ channels, use $\eta(C) = C(1 - \frac{1}{\lambda/\sigma^2 + 1})$ that formulates the simulated performance in Figure 3, and further assume that the per-slot arrival process $R(\tau)$ is $\sigma$-SubGaussian distributed with mean $\lambda$. We plot the common theoretically-guaranteed stability region of $R(\tau)$ for both Congestion-Based policies $\hat{\pi}, \hat{\tau}$ with different $F$ choices. We see that these regions are insensitive to the value of $\sigma$ and are limited by vertical lines that is governed by the arrival rate limit we characterized in Theorems 1,2. These regions reveal that as the frame size $F$ increases the stability region expands, and in the limit will converge to the maximum possible value of $M = 10$. In the same figure, we also plot the theoretically-guaranteed stability region of $R(\tau)$ for the Age-Based policy $\tau$ derived in Theorem 3 for different $K$ choices. Different from the congestion-based designs, this stability region is sensitive to the standard deviation $\sigma$ of the arrival process as $\sigma$ increases, the stabilizable rate $\lambda$ must decrease. Yet, we also see that as the activating slots $K$ increase, this stability region also expands and converges to the vertical line $M = 10$ as we established in our analysis. These observations conclude that all our congestion and age-based designs are asymptotically-efficient, but exhibit different characteristics in non-asymptotic regimes. In Figure 7, we compare the achieved performance of the Congestion and Age-Based policies $\hat{\pi}, \hat{\tau}, \tau$ to the theoretically-guaranteed regions from the previous figure. We can see that the stabilizable arrival processes are closely captured by the theoretical regions.

One important insight that emerges from both of these investigations is that our age-based policy can support higher arrival rates when the variance is small while the congestion-based policy can support arrival process with any variance as long as the arrival rate is below the limit. As a simple rule-of-thumb, if the choices of $K$ and $F$ parameters under the two designs that result in the use of approximately equal number of virtual channels $C$ (on average), the age-based design is preferable when $\lambda/\sigma^2 > 1$, whereas congestion-based design is preferable when $\lambda/\sigma^2 < 1$. This insight agrees perfectly with the Poisson and Gaussian approximation par-

6More specifically, this is a close functional approximation to $\eta(C)$ for IRSA scheme under a Soliton-based transmission strategy, which we describe in further detail in Section VII.
adigms underlying the operation of the congestion and age-based designs, as introduced in Section III-B. In particular, for the two distributions with the same mean $\lambda$, the Gaussian distribution (approximating age-based policy activations) is more concentrated (and hence better for activation) when its variance $\sigma^2$ is less than $\lambda$. Otherwise, the Poisson distribution (approximating congestion-based policy activations) is preferable.

In Figure 8, we move on from supportable arrival processes to average delay and oldest age performances of the proposed two policies $\hat{\pi}$ and $\overline{\pi}$. We set the number of physical channels at $M = 10$, $\eta(C) = C(1 - 1/M)$, and fix the arrival rate $\lambda = 7.8$ with Poisson distributed arrivals in (a) and $\sigma-$SubGaussian arrivals with $\beta \triangleq \sigma^2/\lambda = 0.5$ in (b). The figure plots the average delay and average maximum-age performance under two policies, for increasing $F$ and $K$ values.7 The plots show that: (i) if $F$ and $K$ are not large enough, the number of channels is not high enough to stabilize the arrival rate $\lambda$; (ii) after $F,K$ exceeds this stabilizing level, we see that the delay performance decreases for a while for both policies, and then starts to increase. This happens because initially increasing $F$ and $K$ increases the service rate with small additional cost incurred from waiting for the frame to complete. However, as $F$ and $K$ increases the delay from the increased duration of the frame starts to dominate the marginal gains received from the increase in the service rate. This reveals that finding the best choice of frame duration as a function of the arrival process $\{R(t)\}$ can yield non-negligible delay reductions while maintaining stability guarantees.

We also notice that two policies yield different delay and maximum-age performances by choosing different $F$ or $K$ parameters. To further compare the performance of two proposed policies, in Fig. 9, we take the same channel setup as in Fig. 8, but we fix the $\sigma-$SubGaussian arrivals with $\lambda = 7.8$, and plot the minimized average delay and oldest age for different $\beta$ values. First we can see that the age-based policy always outperforms the congestion-based policy in terms of average maximum age, which is reasonable since our age-based policy always serves older demands first while congestion-based policy serves all demands with equal probability and very unlucky demands may experience very large waiting time. On the average delay comparison, we can see that when $\beta$ is relatively small or large, the age-based policy has better performance, when $\beta$ is in between, two policies achieve similar performances. This reveals that although the congestion-based policy is more robust to the variance of the arrival rate, age-based design with optimized parameters will be more preferable in achieving low average delay and maximum-age performance.

VII. APPLICATION TO SIC-BASED ENCODING/DECODING

Until now we have developed congestion and age-based distributed MAC policies $\hat{\pi}$ and $\overline{\pi}$ which we have specifically designed for the threshold-based success model given in (2). As illustrated in Fig. 3, such a model only approximately captures the performance of the specified IRSA policy. In this section, we build on the mechanisms of these designs to develop two new policies, $\hat{\pi}^*$ and $\overline{\pi}^*$ that are adapted to operate efficiently under the actual SIC-based success model for congestion-based and age-based design. We show via simulation (cf. Section VII-C) that our Policy policies, $\hat{\pi}^*$ and $\overline{\pi}^*$ closely match and can even outperform the policies $\hat{\pi}$ and $\overline{\pi}$ that assumed the threshold-based success model.

A. Encoding/Decoding Process of SIC

Under the SIC processing, each active demand $d$ distributes selects a subset (to be described below) of the available $C$ channels to transmit the same copy of its packet in frame $t$. Then, at the receiver-end (base station), the decoding algorithm works recursively as follows: at each stage all channels with a single packet are decoded, and their copies transmitted over the other channels8 are cancelled. This process is repeated until no channels with a single packet are left in the frame. The work [17] shows that: (i) by letting each active demand $d$ choose the number of copies $g_d \in \{1, \ldots, C\}$ according to the truncated Soliton distribution; and (ii) selecting $g_d$ channels uniformly out of $C$ channels, the number of successfully

---

7When comparing the delay performance, please notice that the delay is measured in slots, the demands that arrive will be ready to transmit at the next transmission frame and successes happen at the end of each transmission frame.

8Each packet include a small header to their packets declaring all the packets over which its copies are transmitted, which is negligible compared to packet size.
decoded packets can get arbitrarily close to 1 packet per channel as $C$ increases. Due to this asymptotic efficiency guarantee, we also assume in our investigation that this Soliton-based SIC encoding/decoding processing is used by the activated users.

### B. Success/Failure/Idle Characteristics of SIC-Based Schemes

Under the above SIC scheme, we can no longer observe such a drastic drop in decodability that is assumed in the threshold-based success model (2) when the number of active demands $X[t]$ increases from $\eta(C)$ to $\eta(C) + 1$. While the relatively gradual drop in performance is desirable in terms of improving throughput, it makes base station feedback more challenging since a simple collective success or failure condition no longer exists. Therefore, the base station needs to make a different decision based on other decoding metrics inherent to the SIC-based success model. We depict several of these decoding metrics available to the base station in Fig. 10 under the SIC-based scheme for a given $C = 500$: the number of decodable, undecodable, and idle channels. In particular, we observe that the optimal utilization of the $C$ channels (i.e., the point corresponding to $\eta(C)$ in the threshold-based success model) occurs almost precisely at the point where the number of idle and undecodable channels meet. This is sensible, since many idle channels indicate under-utilization while many undecodable channels indicate over-congestion. This observation forms the key in capturing the earlier success/failure signals under the threshold-based model through the number of Idle/Undecodable channels under the SIC operation.

### C. Policy Design and Performance Under the SIC Operation

Inspired by the above key observation, our proposed new Policies $\hat{\pi}^*$ and $\hat{\pi}^*$ under the SIC-based scheme inherits the structure of our previous designs $\hat{\pi}$ and $\hat{\pi}$ with two crucial differences: (i) the update condition at the base station, and (ii) the feedback signal. Here, we point to these differences and omit the full definition of the activation and estimation steps, which are the same as in the Policies $\hat{\pi}$ and $\hat{\pi}$.

(i) Regarding the update condition, instead of using the indicator $Z[t] = \mathbb{I}(X \leq \eta(C))$ that Policies $\hat{\pi}$ and $\hat{\pi}$ use, Policies $\hat{\pi}^*$ and $\hat{\pi}^*$, use, for each transmission frame $t$,

$$ Z'[t] = \mathbb{I}(\# \text{ idle channels} > \# \text{ undecodable channels}). $$

This is based on the observations from the previous subsection showing how the number of idle and decodable channels reveal the effectiveness of the SIC operation.

(ii) Regarding the feedback, Policies $\hat{\pi}^*$ and $\hat{\pi}^*$ includes in their feedback the $C$-bit signal $D[t] \in \{0, 1\}^C$ with its $k^{th}$ dimension indicating whether the content in the corresponding $k^{th}$ virtual channel was successfully decoded at the end of frame $t - 1$. This low overhead signal allows active users in frame $t - 1$ to know whether their transmission has been successful or not. This is different from the one-bit feedback $Z[t]$ under Policies $\hat{\pi}$ and $\hat{\pi}$ since demands do not succeed or fail all together under the SIC-based scheme.

With these modifications to the original algorithms, we next check their performance compared to the earlier threshold-based model. In Fig. 11, we plot the samples of the maximum throughputs of Policies $\hat{\pi}$ and $\hat{\pi}^*$ for different values of $C$ under a Poisson arrival process. In Fig. 12, we plot the samples of the maximum throughputs of Policies $\hat{\pi}$ and $\hat{\pi}^*$ for different values of $K$ when $M = 10$ and the arrival process has $\beta = \sigma^2/\lambda = 0.5$. The maximum stability levels of these policies are virtually indistinguishable as the parameters increase, supporting the efficiency of the SIC-based designs. We also see that all the throughputs increase towards the maximum limit of 1 packet/channel, as predicted by our Theorems 1 and 3. We also note that as the parameters increase, Policies $\hat{\pi}^*$ and $\hat{\pi}^*$ can even outperform the benchmarks $\pi$ and $\pi$. This is due
to the fact that the approximation to the IRSA performance in [17] (see Fig. 3) is typically an underestimate of the capacity of SIC encoding/decoding process.

VIII. CONCLUSION

In this paper, we proposed two fundamentally different Multiple-Access-Control (MAC) strategies to serve increasingly dynamic users with minimal coordination and low-complexity for future wireless networks. The congestion-based design estimates the number of demands in the system with a simple binary feedback and adapts the transmission probability to it. The age-based design prioritizes demands based on their ages and flexibly chooses proper virtual channel size to serve them. We considered a threshold-based success model that can capture essential characteristics of many spectrally-efficient multi-channel uplink MAC schemes. Under that model, we characterized the stability region of our designs with respect to $(\lambda, \sigma)$ pair, which both fully utilize the threshold-based success model as the scales grow. We also point out that under non-asymptotic regime, we prefer age-based design when the arrival distribution is less variate. Considering the spectrally-efficiency SIC-based IRSA mechanism as a particular protocol, we extend our design to incorporate the characteristics of the SIC operation. We also provided a delay analysis and various simulation results that confirm their efficiency.

APPENDIX A

PROOF OF 3

As can be seen from equation 9.8 and the algorithm described above, we will have the following evolution of the broadcast threshold at the beginning of each cohort $\{T_{i,1}\}$:

$$T_{i+1,1} = \max(0, T_{i,1} + Y_i - K).$$

The evolution of $T_{i,1}$ thus can be viewing as a queuing system, where there is a deterministic service rate equals to $K$, and an arrival distribution follows the distribution of $Y_i$, which is the total number of slots we need to spend till the current cohort $i$ successfully transmits. So by applying Foster-Lyapunov Theorem we can know that $T_{i,1}$ will be positive recurrent if $\mathbb{E}[Y_i] < K$.

$$\mathbb{E}[Y_i] \triangleq s \sum_{c=1}^{\infty} \mathbb{P}(M F_{i,c} < X_i \leq \eta(M F_{i,c})) \cdot \sum_{j=1}^{c} F_{i,j}$$

$$= \sum_{c=1}^{\infty} \{ \Phi(\eta(M F_{i,c})) - \Phi(\eta(M F_{i,c-1})) \} \cdot \sum_{j=1}^{c} F_{i,j}$$

where $\Phi$ is the cumulative density function of $X_i$.

$$\mathbb{E}[Y_i] = \sum_{c=1}^{\infty} F_{i,c} \cdot \sum_{j=c}^{\infty} \{ \Phi(\eta(M F_{i,j})) - \Phi(\eta(M F_{i,j-1})) \}$$

$$= \sum_{c=1}^{\infty} F_{i,c} [1 - \Phi(\eta(M F_{i,c-1}))]$$

Since we will select the same parameters $\{F_{i,j}\}_{j \geq 1}$ for all cohorts $i = 1, 2, \ldots$ as

$$F_{i,j} = F_1 + (j - 1) \Delta,$$

then $\mathbb{E}[Y_i]$ is reduced to $\mathbb{E}[Y(F_1, \Delta)]:$

$$\mathbb{E}[Y(F_1, \Delta)] = F_1 + \sum_{c=1}^{\infty} (F_1 + c \Delta)[1 - \Phi(\eta(M F_1 + M(c - 1) \Delta))].$$

Since the arrival distribution is assumed to be a sub-gaussian distribution, so $\mathbb{E}[Y(F_1, \Delta)]$, as shown at the bottom of the page.

Notice that we assume that $\eta(C)/C$ is monotone increasing as $C$ increasing, so, third inequality holds since $\eta(a + b) = 1/a(1 + b/a) + b(a + b) \geq a(a + b) + b(a + b) = \eta(a) + \eta(b)$ and the fifth inequality holds since $\eta(ab) \geq \eta(a) \geq \eta(b)$ for any $a, b > 0$. And since for $A > 0$:

$$\int_{-\infty}^{\infty} e^{-x^2} \cdot dx = \sqrt{\pi}, \quad \int_{0}^{\infty} e^{-Ax^2} \cdot dx = \frac{1}{2} \sqrt{\frac{\pi}{A}},$$

and,

$$\int_{0}^{\infty} x e^{-Ax^2} \cdot dx = \int_{0}^{\infty} \frac{1}{2} e^{-\frac{1}{2} A u} \cdot du = \frac{1}{A}.$$
so, we will have:
\[
\sum_{n=0}^{\infty} \exp(-An^2) \leq 1 + \sum_{n=1}^{\infty} \exp(-An^2) \leq 1 + \frac{1}{2} \sqrt{\frac{\pi}{A}}
\]
and,
\[
\sum_{n=1}^{\infty} n a \exp\left(-\frac{1}{2} A a^2, 2\right) \leq \frac{1}{a} \int_{0}^{\infty} xe^{-\frac{1}{2} A x^2} dx = \frac{1}{aA}
\]
So,
\[
E[Y(F_1, \Delta)] \leq F_1 + \exp\left(-\frac{(\eta(MF_1) - K\lambda)^2}{2K\sigma^2}\right) \cdot \left[\frac{(F_1 + \Delta)}{2} \sqrt{\frac{\pi}{2 \eta(M\Delta)}} + \frac{\Delta K\sigma^2}{\eta^2(M\Delta)}\right]
\]
To minimize \(E[Y(F_1, \Delta)]\), we choose the optimal \(F_1\) and \(\Delta\) as defined in equation 10, and any \((\lambda, \sigma)\) pair that satisfies condition 11 lies in the stability region of policy \(\pi\).
Also, since \(T_{i+1} = \max(0, T_{i+1} + Y_i - K)\), so based on the average delay bound for a \(G[G[1\text{ queue}], E[T_{i+1}] \leq \frac{\sigma_Y^2}{2(K-E[Y])] + \frac{K-E[Y]}{2}}\). Notice that the expected number of demands in the system equals to average waiting times arrival rate, also equals to arrival rate times average oldest age \(O\) in our age based design, so,
\[
E[W] \leq E[T_{i+1}] + K + \frac{\frac{1}{2}E[Y(Y-1)]}{E[Y]} \leq \frac{\sigma_Y^2}{2(K-E[Y])] + \frac{3K}{2} + \frac{\sigma_Y^2}{2E[Y]} - \frac{1}{2}
\]
the first inequality is because the oldest age at the beginning of transmission \(i\) always smaller or equal to the threshold \(T_{i,j} + K\), and there are no successes in the slots after \(T_{i,1}\) and before \(T_{i+1}\), so the oldest age will be incremented by 1 in those slots.

References


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