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Elasticities in AIDS Models: A Clarification and Extension

Richard Green and Julian M. Alston

In a recent paper in this *Journal* (Green and Alston), we showed that the usual formulas for uncompensated price elasticities in the linear approximate almost ideal demand (LA/AIDS) model were incorrect because the Stone's price index (P) is a function of expenditure shares (i.e., $\ln P = \sum_j w_j \ln P_j$). A common approach is to treat expenditure shares as constant parameters in the Stone's index when taking derivatives for elasticities. We developed corrected formulas for uncompensated price elasticities by using derivatives that took into account the effects of price changes on the shares in the price index.

In the process, and as a side-issue, we also noted (p. 443) that the differences in uncompensated elasticities in the literature (η_{ij})

... carry over directly into the computation of compensated elasticities (η_{ij}^*), which are

$$(6) \quad \eta_{ij}^* = \eta_{ij} + w_j (1 + \beta_i/w_i).$$

In making this assertion, we used the usual Slutsky equation in elasticity form and the usual AIDS income (expenditure) elasticity (i.e., $\eta_i = 1 + \beta_i/w_i$). This procedure is correct only when using the true AIDS model.

In the LA/AIDS model, expenditure elasticities also ought to account for the role of expenditure shares as variables in the Stone's price index. That is, the expenditure shares in the Stone's index are functions of both prices and total expenditures unless preferences are homothetic, and expenditure elasticities will be affected along with the uncompensated elasticities (i.e., $\eta_i \neq 1 + \beta_i/w_i$). In general, therefore, correct compensated elasticities cannot be computed simply by substituting correct uncompensated elasticities into equation (6).

In this note we show correct equations for both (a) expenditure elasticities when the LA/AIDS is estimated and (b) compensated price elasticities incorporating those corrected expenditure elasticities.

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Consider the share equation for the LA/AIDS model using our previous notation (Green and Alston):

$$(1) \quad w_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln (X/P).$$

The general expression for the income (expenditure) elasticity in this equation is

$$(2) \quad \eta_{i,x} = d \ln Q_i / d \ln X = 1 + (dw_i / d \ln X) / w_i.$$

The usual approach treats shares as fixed parameters in the Stone's price index (P) and obtains

$$(3) \quad dw_i / d \ln X = \beta_i \text{ and } \eta_{i,x} = 1 + \beta_i / w_i.$$

That is the result used in equation (6) by Green and Alston. More generally,

$$(4) \quad dw_i / d \ln X = \beta_i [1 - d \ln P / d \ln X], \text{ and}$$

$$(5) \quad d \ln P / d \ln X = \sum_j \ln P_j (dw_j / d \ln X) \\ = \sum_j w_j \ln P_j (\eta_{j,x} - 1).$$

Combining equations (4) and (5) gives

$$(6) \quad dw_i / d \ln X = \beta_i [1 - \sum_j w_j \ln P_j (\eta_{j,x} - 1)].$$

Substituting (6) into (2) gives

$$(7) \quad \eta_{i,x} = 1 + (\beta_i / w_i) [1 - \sum_j w_j \ln P_j (\eta_{j,x} - 1)] \text{ or}$$

$$(8) \quad \eta_{i,x} - 1 = (\beta_i / w_i) [1 - \sum_j w_j \ln P_j (\eta_{j,x} - 1)].$$

Following the approach we used for uncompensated price elasticities (Green and Alston), we can solve the system of simultaneous equations in (8) for the elasticities of interest by expressing the equations in matrix form as

$$(9) \quad M = B - BCM,$$

where M is an n -vector with elements $m_i = \eta_{i,x} - 1$, B is an n -vector with elements $b_i = \beta_i / w_i$, and C' is an n -vector with elements $c_j = w_j \ln P_j$. Then, rearranging terms gives

$$(10) \quad M + BCM = (I + BC)M = B.$$

Premultiplying both sides of (10) by $(I + BC)^{-1}$ gives

$$(11) \quad M = (I + BC)^{-1} B.$$

The elements in M are $m_i = \eta_{i,x} - 1$ so we can obtain the expenditure elasticities by adding 1 to each element of M :

$$(12) \quad N = M + \iota = (I + BC)^{-1} B + \iota,$$

where N is an n -vector of expenditure elasticities of demand ($n_i = \eta_{i,x}$), and ι is a unit vector of length n .

Recall the solution for the uncompensated elasticities of demand in the LA/AIDS (Green and Alston, p. 443) is

$$(13) \quad E = [I + BC]^{-1}[A + I] - I,$$

where A is an $n \times n$ matrix with typical elements $a_{ij} = -\delta_{ij} + [\gamma_{ij} - \beta_j w_j]/w_i$ (when $\delta_{ij} = 1$ for $i = j$ is the Kronecker delta) and all of the other variables are as defined above.

Using the Slutsky equation we may find the compensated elasticities of demand as

$$\eta_{ij}^* = \eta_{ij} + w_j \eta_{i,x},$$

and the matrix of compensated elasticities is

$$(14) \quad E^* = E + NW',$$

where E^* is an $n \times n$ matrix with typical elements η_{ij}^* , and W is an n -vector of shares, w_i . Substituting (12) and (13) into (14) yields

$$(15) \quad E^* = [I + BC]^{-1}[A + I] - I + [(I + BC)^{-1}B + \iota]W' \\ = [I + BC]^{-1}[A + I + BW'] - I + \iota W'.$$

Recall that the typical element in A is defined as $a_{ij} = -\delta_{ij} + [\gamma_{ij} - \beta_j w_j]/w_i$. Further, the typical element in I is $+\delta_{ij}$ and the typical element in BW' is given by $(\beta_j/w_i)w_j$. Thus, the term $[A + I + BW']$ reduces to Γ in which the typical element is γ_{ij}/w_i and equation (15) becomes

$$(16) \quad E^* = [I + BC]^{-1}\Gamma - I + \iota W'.$$

To check these manipulations, consider the case when preferences are homothetic (i.e., $\beta_i = 0 \forall i$ or $B = O$, where O is a zero vector). In this case, all of the income elasticities of demand are 1 and this is verified as $N = \iota$ when $B = 0$ in equation (12). In this case ($B = 0$) equation (16) simplifies to

$$(17) \quad E^* = -I + \Gamma + \iota W',$$

and the typical element is

$$\eta_{ij}^* = -\delta_{ij} + (\gamma_{ij}/w_i) + w_j.$$

This last expression is the usual equation for compensated elasticities (as used by Chalfant, for example). The corresponding formula for the elasticities of substitution is

$$(18) \quad \sigma_{ij} = \eta_{ij}^*/w_j = [\gamma_{ij}/w_i w_j] + 1 \quad (i \neq j).$$

Equations (17) and (18) are special cases; in general, they will be incorrect for measuring compensated price responses.

A more general check of equation (16) using homogeneity conditions was suggested to us by Giancarlo Moschini (Iowa State University 1990) in a personal communication. Homogeneity implies $\sum_j \eta_{ij}^* = 0$ or $E^* \iota = O$. We can see that equation (16) satisfies this condition by noting that $\iota W' \iota = I \iota$, and $\Gamma \iota = 0$ when $\sum_j \gamma_{ij} = 0$.¹

In summary, we have extended our results for uncompensated elasticities in the LA/AIDS (Green and Alston) to show corrected formulas for expenditure elasticities and compensated price elasticities that account for the endogenous nature of expenditure shares in the Stone's price index. Equation (7) is the correct equation for LA/AIDS expenditure elasticities. Equation (16) is the correct formula for compensated price elasticities.² Green and Alston's equation (6) is correct for compensated price elasticities of demand only when preferences are homothetic. We regret any confusion we may have caused.

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¹ Moschini also pointed out another implication of homogeneity. In model (ii) of our earlier paper, the elasticity formula (as used by Eales and Unnevehr, for example) was: $\eta_{ij} = -\delta_{ij} + \gamma_{ij}/w_i$. Homogeneity implies that the Marshallian own-price and cross-price elasticities and the income elasticity sum to zero (i.e., $\sum_j \eta_{ij} = -\eta_{i,x}$), and in the Eales and Unnevehr formula this requires that all income elasticities are unitary ($\eta_{i,x} = 1 \forall i$). Thus, only under homotheticity will the Eales and Unnevehr formula give elasticities that satisfy the homogeneity property, and this accounts for the poor performance of that formula in the empirical results we reported in table 2 since the model we used was not homothetic.

² A referee has pointed out that the corrections to the formulas for calculating compensated elasticities may be unimportant for highly disaggregated commodities for which expenditure shares are so small that compensated elasticities are approximately equal to uncompensated elasticities.

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