

Math for Allied Health Fields at Penn State University

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In the summer of 2019 a group of mathematics and nursing faculty collaborated to design a mathematics course intended for Nursing majors, spurred by the announcement that the standard Intermediate Algebra course would no longer meet general education requirements.

Course Offering and Current Status

Penn State University Park ran a pilot version of the course in Fall 2020 and Spring 2021, both run remotely with 75-minute synchronous lecture sessions twice weekly. The Fall 2020 section had thirty students and the Spring 2021 section had thirteen. All but one student were majors in the College of Nursing in their first or second year. The course was intended to replace the Intermediate Algebra course that majors and many other students were taking for their quantification course requirements.

This course will not be offered in Fall 2021 due to limited teaching faculty and other priorities for our campus' Math department. It is expected to return in Spring 2022. See "Administrative Challenges" below.

Overarching Course Goals and Themes

We had three broad primary design goals for the general education math course.

1. All topics can be motivated by putting them in the context of Allied Health to enforce relevance and make connections clear.
2. Strengthen student's fluency with quantity, operations, proportional reasoning, and plotting data, for both precise calculations and quick estimates.
3. Cultivate self-efficacy and a productive disposition toward mathematics.

In the Spring and Summer of 2019 we met with faculty from a variety of departments to ask what they would want to see in a course tailored to Allied Health. We chose three units for the course: Measurement, Covariation, and Uncertainty.

1. **Measurement.**

The unit on Measurement focuses on unit conversion, dimensional analysis, mental floating point arithmetic and scientific notation, absolute and relative comparisons, working with concentrations for dosage and mixing, and logarithmic scales. Measurement discussions also include elementary discrete probability from a frequentist viewpoint in analogy with concentration of successes.

2. **Covariation.**

The Covariation unit focuses on linear, exponential, and power functions. Extra time is devoted to reading plots of data using both logarithmic and linear axes and verbal descriptions of those relationships. Students also spend time with computer spreadsheets to investigate sequences and simple discrete dynamical systems.

3. **Uncertainty.**

The Uncertainty unit focuses on accuracy and precision in measurements and error propagation in calculations, including a discussion of false precision and significant figures. Prior topics are revisited with the inclusion of error bars, and Bayesian viewpoints of probability can be introduced.

There was an additional theme of **Context** that threaded throughout the course above, where mathematics could give insight to broader social and ethical concerns such as public health or racial inequity in healthcare. For example, the problem set on Measurement centers on lead poisoning in the water supply for Flint MI. For the Uncertainty unit in Spring 2021 the COVID-19 vaccine effectiveness and safety provided context for probability-based decision-making.

Challenges

We encountered several challenges in creating and offering this course. The ongoing COVID-19 pandemic and the consequent changes to instruction were one obvious challenge. Both semesters were done remotely with synchronous lectures and breakout room activities, but these were not ideal circumstances for gauging students' responses. Also, as the instructor I can say that the in-progress nature of the course plus the challenges of juggling multiple courses in a still-unfamiliar format each semester reduced my teaching effectiveness.

Another ongoing challenge is the course approval process. The course has yet to be inducted as a permanent addition to the course catalog, with the bottleneck at the department level. There are so few analogues of this course elsewhere that there is some concern regarding transfer credit. Furthermore, many math faculty perceive proportional reasoning as elementary mathematics, and most of the course is grounded in proportional reasoning. This perception raises doubts that the course is “rigorous” enough to merit college math credit, ignoring the reality that many students enter college without the ability to apply proportional reasoning.

Third, our team has been unable to find a textbook that addresses all of the topics we wish to address. This had led me to the laborious process of creating texts and other handouts to formalize the ideas and provide examples.

Last, this course faces similar challenges as other “boutique” courses. Any academic department has goals that pull its limited resources in many directions, pushing smaller-scale courses aside. For example, the first-year calculus sequence involves well over a thousand students each semester while any Allied Health math course would involve less than a tenth of that.

Additional Details for Learning Objectives

What follows is a more detailed list of course learning objectives.

Measurement Objectives

1. Work fluently within and across measurement systems, including the metric system, imperial system, apothecary system, and ad hoc systems (e.g. “half the width of a human hair”). Students are expected to both estimate the order of magnitude of a result and perform a precise calculation.
 - a. Convert a measurement given in one measurement system into a different unit in that same measurement system (e.g., meters to kilometers, cubic meters to liters).
 - b. Convert a measurement given in one measurement system into a unit in a different measurement system (e.g., meters to feet, cubic centimeters to drams).
 - c. Convert a measurement given in compound units, such as rates and concentrations, into a specified unit (e.g., determine mass of glucose in 1 pint of a solution with glucose concentration 5g/dL, determine

- mass-per-volume of a solute in solution given the solute's molarity and atomic weight of the solute).
- d. Perform dosage calculations that vary according to patient mass, medicine concentration, dosage over time, IV drip rates, and timing of delivery for split doses.
2. Gain fluency in scientific notation and floating point arithmetic.
 - a. Convert into and out of scientific notation.
 - b. Perform floating point arithmetic without converting to fixed-point representations.
 - c. Read and reason about measurements made on logarithmic scales, such as pH, perceived loudness, and minuscule probabilities.
 3. Compute and reason about concentrations as rates and proportions.
 - a. Determine the resulting concentration when mixing two solutions with known volumes and concentrations.
 - b. Determine the amount of a solution with known concentration to a solution with known concentration and volume in order to increase or decrease the mixture's concentration.
 - c. Perform titration calculations (determine the concentration of a solution knowing that a given amount of solution of known concentration was added and the result had a known concentration)
 - d. Complete the preceding mixture problems with pH of an acid based on the corresponding hydrogen ion concentration.
 4. Compare two or more quantities or rates in a variety of ways and determine which is most appropriate.
 - a. Given two measurements, determine their absolute difference, relative difference, and percentage difference.
 - b. Given one measurement, determine the other measurement so that the absolute difference, relative difference, and percentage difference is a specified amount.
 5. Convert fluently between representations of probability, including fractions, percentages, odds, the relative frequency of success, and the expected number of successes for a given sample.

Covariation Objectives

6. Read and translate between verbal, tabular, algebraic, and graphical representations of two-variable relationships.
 - a. Identify relationships of the following types in any of the above representations: Direct proportion ($y=kx$), inverse proportion ($xy = k$), linear ($y=k(x-a)$), exponential ($y=a k^x$), logarithmic ($y = a \log(x) + k$), and power ($y=a x^k$)
7. Create and analyze exponential models from a variety of different given information.
 - a. Create an algebraic expression for an exponential model, where the initial reference value is given together with a second reference value, doubling time, half-life, annual relative growth rate, and initial reference value.
 - b. Given an algebraic expression for an exponential model, determine the doubling time, half-life, annual relative growth rate, the output at a specific input, and the input to generate a specific output.
 - c. Given a “line” plotted on a combination of logarithmic and linear axes describe the implied relationship between absolute/relative changes in input and absolute/relative changes in output.
 - d. Medical contexts include exponential population growth, epidemiology, half-life for radioactive isotopes and drug concentrations.
8. Generate terms of a recursive time-series with specified recurrence relation.
 - a. Use computer spreadsheet software to quickly generate one-hundred terms and plot accordingly.

Uncertainty Objectives

9. Recognize that all measurements carry a margin of error. Reason effectively and take these errors into account.
 - a. Compute absolute, relative, and percentage error given a reported measurement and the true value.
 - b. State an interval for the true value when given a reported measurement and its absolute, relative, or percentage error.
 - c. Given two measurements and their error bounds, state an interval for the arithmetic combination of those measurements along with corresponding error bound.

- d. Use the rule-of-thumb provided by significant figures to deduce precision by reported measurements and to report results that imply appropriate levels of precision.
10. Incorporate error bounds for calculations based on linear and exponential models where parameters have given error bounds.
11. Compute joint and conditional probabilities of nominal data, particularly in medical statistics.
- a. Use contingency tables to evaluate conditional probability in medical contexts such as comorbidity, risk factors, and medical testing.
 - b. Compute and interpret various comparisons of probability, including number needed to treat, odds ratio, and relative risk.
 - c. Execute and explain chi-squared tests to evaluate independence.
12. Identify and describe common probability fallacies and paradoxes. These include the Gambler's Fallacy (assuming independent events are dependent), the Prosecutor's Fallacy (reversing conditional probability in cases of multiple testing), Berkson's paradox, Simpson's paradox, and the limitations of arguments based on correlation.

Sample Tasks

What follows is a small sampling from in-class problems, homework, and summative assessments. Additional tasks are available by emailed request at amb69@psu.edu.

IV Drip Settings. Suppose you come on your shift and a patient is 90 minutes into an IV drip with prescription 500 mL over 4 h. The bag looks to have 250 mL remaining. Is the machine's drop rate correct? If not, should you increase it or decrease it? What rate ensures the remaining 250 mL is distributed evenly over the remaining time? Describe that new drip rate as a multiple of the original drip rate (e.g. "the new drip rate should be two-thirds of the original drip rate" or "the new drip rate should be 50% more than the old drip rate").

Mixing Acid. Acid A has pH 3.5 and Acid B has pH 4.2. Determine the amount of acid A that must be added to 50 mL of Acid B so that the resulting mixture has pH

4.0. Then, determine the resulting pH if you were to add *twice* as much Acid A as needed.

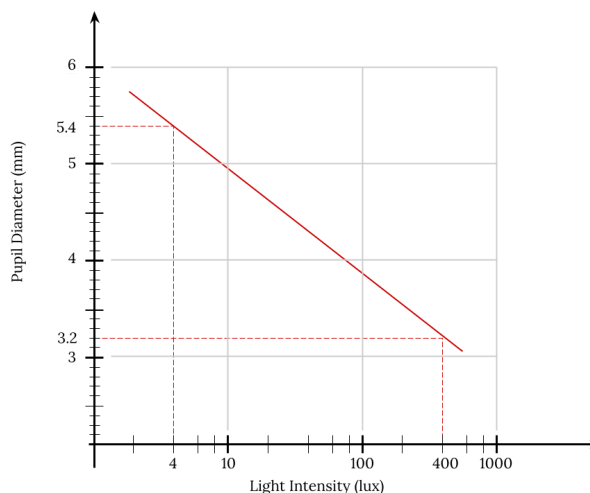
Exponential Growth. The COVID-19 pandemic has presented an immediate example of exponential growth as it spreads. The *daily incidence rate*, DIR, is the number of new cases per day, which grows relative to the number of already-infected individuals. The exact rate depends on the mitigation efforts in place (e.g., mask mandates, social distancing, and more).

This problem is interested in two rates in particular. An article gives the estimates that the doubling rate for the DIR in Pennsylvania before mitigations averaged **3.67 days**, while during the stay-at-home orders in April 2020 the DIR in Pennsylvania showed an average doubling rate of **18.34 days**. We are going to see how well these measures “flattened the curve” by looking at what the DIR *would have been* without the stay-at-home order.

- Create an exponential model for the DIR for the no-mitigation doubling time of 3.67 days, beginning with an estimated DIR of 800 cases/day on March 31 (use $t=0$ to be March 31, so April 1 is $t=1$). Then use your model to predict the DIR for April 30.
- Create an exponential model for the DIR for the stay-at-home doubling time of 18.34 days, beginning with the same estimated DIR of 800 cases/day on March 31 (use $t=0$ to be March 31, so April 1 is $t=1$). Then use your model to predict the DIR for April 30.
- Determine how many days it would take under the stay-at-home doubling time to reach the no-mitigation April 30 DIR that you predicted in part a.

Semilog Plot. Consider the following semilog plot showing the diameter of an eye’s pupil (in millimeters) when exposed to light of varying brightness (in lux). Write an accurate description of the relationship.

Fully correct response: “Increasing the intensity by a factor of 100 leads to an absolute decrease in the diameter by 2.2 mm.”



Half-Life with Error Bounds. According to the manufacturer, the recommended adult dose of Exampometicin is 10 mg for every 35 kg of body weight, and the typical half-life in the body is 2 h with a relative error of $\pm 25\%$ to account for variation between patients.

1. Determine the dose for a patient you estimate weighs 185 lbs with an absolute margin of error of ± 20 lbs. Report the minimum and maximum dosages implied by this range of weights. Round dosages to the nearest whole milligram.
2. Suppose you give the patient a 20 mg dose. Create two models for the amount that remains in the bloodstream: one for the maximal half-life and one for the minimal half-life within the given error bounds. Compare the drug's mass in the bloodstream at the 4-hour point as predicted by the two models. Also compare the time each model predicts for the drug to reach 4 mg. Plot these two models on log-linear axes and mark where your answers are visible.

Comorbidity. Consider two conditions which we will label “A” and “B” which may be correlated. Condition A is a very rare but serious condition. Condition B, on the other hand, is a more common and more easily treated condition. What follows is a variety of ways that one can measure the predictive power that having condition B has for predicting whether the patient has condition A.

1. Complete the contingency chart below based on the following data. Condition A occurs in only 1 out of every 8000 people, while Condition B occurs in 12% of the general population. In a survey of 60 people with Condition A, only 3 *did not* have condition B as well.

	Has condition A	Not condition A	Total
Has condition B	57		
Not condition B	3		
Total	60		

2. Fill in the blank: In a randomly chosen group of ____ patients with condition B we can expect to see 1 patient who also has condition A.
3. Determine the probability that a randomly chosen group of 10 patients with condition B has *at least one* person with condition A as well.
4. Compare the probability a patient who has condition B also has condition A to the probability that any patient has condition A. Use both the absolute risk reduction and the relative risk reduction, writing each comparison in a sentence.

Conditional Probability. In clinical trials, twenty-thousand subjects were given an experimental vaccine and twenty-thousand were given a placebo as a control. Of the subjects who received the vaccine, 8 had contracted the disease it was meant to prevent. Of the subjects who received the placebo, 86 had contracted the disease.

- a. Compute the probability that a subject (in either group) contracted the disease during the study.
- b. Compute the Absolute Risk of contracting the disease given the vaccine.
- c. Compute the Relative Risk of contracting the disease given the vaccine.
- d. How many people would need to receive the vaccine to prevent one additional occurrence of the disease?