Solving Traveling Salesman Problems by D-Wave Hybrid

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***Abstract*—We report outstanding results for solving four asymmetric 10-city traveling salesman problems by the D-Wave quantum/classical software Kerberos. The problems are too large to embed on the D-Wave quantum computer. Our test made four executions for each problem. Each of the Kerberos solutions is an optimal tour. This means 100% of the 16 solutions are shortest length loops through all cities.**

***Keywords*—traveling salesman problem, D-Wave hybrid, quantum/classical solver, Kerberos, optimal tour**

1. Introduction

This paper is a continuation of the study [1-3] of solving small traveling salesman problems (TSPs) on a D-Wave quantum computer. Small TSPs are those on the border for embedding in the current D-Wave hardware. For the current quantum processor DW_2000Q_5 that has 2030 qubits, our experience is that 6-city TSPs can be embedded and 10-city TSPs cannot be embedded. Embedding problem variables to qubits can be very difficult due to lack of connections between qubits.

Given n cities and the distance between city i and city j, the symmetric TSP asks for a shortest route through the n cities visiting each city once and returning to the originating city. When the distance from city i to city j is allowed to be different than the distance from city j to city i, it is called an asymmetric TSP (ATSP). The adaption of the TSP to a quantum annealer in [4, 5] is for the ATSP. This is natural and prevents subloops through proper subsets of the n cities. Therefore, we advocate ATSPs in this paper. A tour for the salesman is one loop through all the cities, i.e., a cyclic permutation of the cities. The length of a tour is the distance traveled through the loop. An optimal tour is a shortest tour.

2. Related Work

In [1] we show the expected quantum output for 5,000 ATSPs on 6, 8 and 10-cities whose distances between cities are random integers in the interval [1, 21]. The results were obtained classically. In [2] we study three software programs that approximately solve small, symmetric TSPs. Paper [3] is a model for the current paper, since it examines four, 6-city ATSPs by making four quantum executions of each. The results in the current paper were obtained in a hybrid manner using both classical and quantum means.

Quantum annealing is a new method of computing that has the potential to solve optimization problems faster than classical means. The theory for quantum annealing implies that the qubits will achieve an optimal state of low energy when super cooled. This is represented by (1) where an initial Hamiltonian H_o evolves to its low energy state in a final Hamiltonian H_p according to

$$H(t) = \left(1 - s\left(\frac{t}{T}\right)\right) H_o + s\left(\frac{t}{T}\right) H_p \text{ for } 0 \leq t \leq T \quad (1)$$

as $s(\tau)$ increases from $s(0) = 0$ to $s(1) = 1$ and if H_o and H_p do not commute. In theory, T is the time imposed by the Schrödinger equation for the initial Hamiltonian to evolve to its low energy state. On a D-Wave computer, time T is in microseconds. The Hamiltonian H_o is established by D-Wave for all problems. The Hamiltonian H_p represents the combinatorial problem to be solved and is an input. Besides H_p, other inputs include the number of samples of the problem, the time T within a given range, and scaling factors. Essentially, a result $H(t)$ is a sample from a Boltzmann distribution. A physical implementation of (1) does not strictly meet the conditions for quantum annealing. In addition, values loaded by the user may differ slightly from the machine interpretation of the numbers. These difficulties are partially overcome by taking multiple samples and choosing a valid solution that has minimum energy.

3. Results for Four TSPs

Kerberos is a hybrid, asynchronous sampler [6] that decomposes problems that are too large to embed on a D-Wave processor. Kerberos finds best samples by running in parallel tabu search, simulated annealing, and D-Wave quantum subproblem sampling on problem variables that have high-energy impact. After generating a QUBO Q for the TSP according to [4, 5], we implemented Kerberos with the commands:

```
import hybrid
sampler = hybrid.KerberosSampler()
result = sampler.sample_qubo(Q)
print(result.first.sample)
```

The output from Kerberos is one sample that has lowest energy. The output from D-Wave is all samples, their energy, and their chain break fraction.

We used the default settings for all Kerberos parameters. The tunable parameter gamma for controlling constraints was set to 500 for all executions.

Table 1 lists characteristics of the four TSPs that we originated, studied and solved. Table 2 shows optimal tours for all results.

TABLE 1. FOUR ASYMMETRIC 10-CITY TSPs

TSP Reference in myScripts	**Number of Optimal Tours**	**Length of Optimal Tour**	**Length of Shortest Non-Optimal Tour**	**Number of Shortest Non-Optimal Tours**	**Distance from City A to City B**	**Distance Matrix**	**Optimal Tour(s)**
One10City Ex1 One10City Ex11	24	325	380	480	5 or 60	-- 60 60 60 60 5 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 5 -- 60 60 60 60 60 60 60 60 60 5 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 --	(1 6 10 8 7 4 3 2 9 5)
One10City Ex2 One10City Ex22	6	270	325	108	5 or 60	-- 60 60 60 60 5 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 5 -- 60 60 60 60 60 60 60 60 60 5 -- 60 60 60 60 60 60 60 60 60 60 -- 60 5 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 --	(1 6 4 3 2 9 5 7 8 10)
One10City Ex4 One10City Ex44	2	160	215	14	5 or 60	-- 60 60 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 5 -- 60 60 60 60 60 60 60 60 60 5 -- 60 60 60 60 60 60 60 60 60 60 -- 60 5 60 60 60 60 60 5 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 5 60 60 60 60 60 60 60 60 60 -- 60 5 60 60 60 60 5 60 60 60 -- 60 5 60 60 60 60 60 60 60 60 --	The optimal tours are (1 6 4 3 2 9 5 7 8 10) and (1 4 6 3 2 9 5 7 8 10)
One10City Ex3 One10City Ex33	1	105	215	36	5 or 60	-- 60 60 60 60 5 60 60 60 60 60 -- 60 60 60 60 60 60 5 60 60 5 -- 60 60 60 60 60 60 60 60 60 5 -- 60 60 60 60 60 60 60 60 60 60 -- 60 5 60 60 60 60 60 60 60 60 -- 60 60 60 60 60 60 60 60 60 60 -- 5 60 60 60 60 60 60 60 60 60 -- 60 5 60 60 60 60 5 60 60 60 -- 60 5 60 60 60 60 60 60 60 60 --	The optimal tour is (1 6 4 3 2 9 5 7 8 10)

TABLE 2. KERBEROS RESULTS

TSP Reference in myScripts	Number of Executions	Number of Optimal Tours Found	Number of Distinct Tours Found	Percent of Output that is Optimal
One10City Ex1 One10City Ex11	4	4 (1 per execution)	4	100
One10City Ex2 One10City Ex22	4	4 (1 per execution)	2	100
One10City Ex4 One10City Ex44	4	4 (1 per execution)	2	100
One10City Ex3 One10City Ex33	4	4 (1 per execution)	1	100

4. Conclusion

The solution quality recorded in this work contributes evidence that the D-Wave hybrid heuristic Kerberos is a valid approach to accurately solve some computational problems on the border for embedding on the D-Wave quantum processor DW_2000Q_5.

5. Next Steps

We recommend that the 14-city TSP Burma and the 16-city TSP Ulysses in the TSP library [7] be solved with Kerberos using the commands and settings in Section 3. They are the smallest, symmetric TSPs in the TSP Library and have been studied extensively. Also, asymmetric TSPs [8] from the TSP Library should be included.

Ten asymmetric TSPs that are reported in [9, Table 2] will be a challenge for the next generation D-Wave processor and hybrid solvers.

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Declarations

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