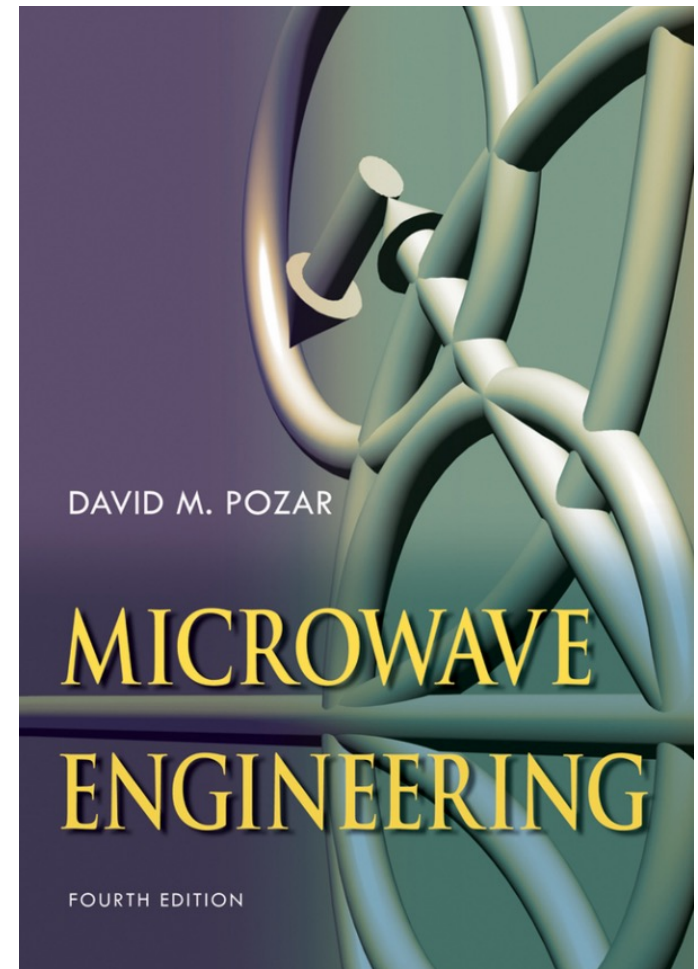


Microwave Engineering Study Group Chapter 3: Transmission Lines and Waveguides

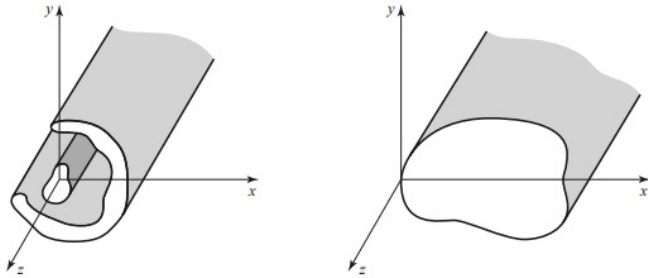
Ian Philippi



Overview

- General solutions for TEM, TE, and TM waves
- Waveguides
 1. Parallel Plate
 2. Rectangular
 3. Circular
 4. Grounded Dielectric Sheet
- Transmission Lines
 1. Coaxial
 2. Stripline
 3. Microstrip

General Solution



For a wave propagating along the z-axis:

$$\vec{E}(x, y, z) = [\vec{e}(x, y) + \hat{z}e_z(x, y)]e^{-j\beta z}, \quad (3.1a)$$

$$\vec{H}(x, y, z) = [\vec{h}(x, y) + \hat{z}h_z(x, y)]e^{-j\beta z}, \quad (3.1b)$$

Write Maxwell's equations as:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \quad (3.2a)$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E}. \quad (3.2b)$$

The three components of each vector quantity can be reduced to obtain the following:

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x, \quad (3.3a)$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y, \quad (3.3b)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z, \quad (3.3c)$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x, \quad (3.4a)$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y, \quad (3.4b)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z. \quad (3.4c)$$

Which can then be solved for the transverse components in terms of the longitudinal components:

$$H_x = \frac{j}{k_c^2} \left(\omega\epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x} \right), \quad (3.5a)$$

$$H_y = \frac{-j}{k_c^2} \left(\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} \right), \quad (3.5b)$$

$$E_x = \frac{-j}{k_c^2} \left(\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} \right), \quad (3.5c)$$

$$E_y = \frac{j}{k_c^2} \left(-\beta \frac{\partial E_z}{\partial y} + \omega\mu \frac{\partial H_z}{\partial x} \right), \quad (3.5d)$$

Where the cutoff wave number k_c is defined as

$$k_c^2 = k^2 - \beta^2 \quad \text{and} \quad k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$$

Transverse Electromagnetic (TEM) Waves

Characterized by the lack of longitudinal field components:

$$E_z = H_z = 0.$$

Solving via elimination:

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x, \quad (3.3a)$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y, \quad (3.4b)$$

We obtain:

$$\beta^2 E_y = \omega^2 \mu \epsilon E_y,$$

$$\beta = \omega \sqrt{\mu \epsilon} = k,$$

$$k_c^2 = k^2 - \beta^2$$

TEM waves behave like a plane wave inside the material filling the line. No cutoff frequency, nondispersive propagation, and a defined voltage and current.

$$\vec{e}(x, y) = -\nabla_t \Phi(x, y), \quad (3.13)$$

$$\text{where } \nabla_t = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$$

$$\nabla_t^2 \Phi(x, y) = 0, \quad (3.14)$$

To obtain the voltage, current, and impedance, apply the following:

$$V_{12} = \Phi_1 - \Phi_2 = \int_1^2 \vec{E} \cdot d\vec{\ell}, \quad (3.15)$$

$$I = \oint_C \vec{H} \cdot d\vec{\ell}, \quad (3.16)$$

$$Z_{\text{TEM}} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} = \eta, \quad (3.17a)$$

$$Z_0 = V/I \quad \text{Where V and I are the amplitudes of the incident voltage and current waves}$$

$$\vec{h}(x, y) = \frac{1}{Z_{\text{TEM}}} \hat{z} \times \vec{e}(x, y). \quad (3.18)$$

Transverse Electric (TE) Waves

For the case where $E_z = 0$ and $H_z \neq 0$, Eq. 3.5 can be reduced to the following:

$$H_x = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial x}, \quad (3.19a)$$

$$H_y = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial y}, \quad (3.19b)$$

$$E_x = \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y}, \quad (3.19c)$$

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x}. \quad (3.19d)$$

First solving for H_z with the Helmholtz wave equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) H_z = 0, \quad (3.20)$$

Reducing to a 2d wave equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) h_z = 0, \quad (3.21)$$

3.21 must be solved subject to the boundary conditions of the specific geometry of the guide

The TE wave impedance can be found as:

$$Z_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}, \quad (3.22)$$

TE waves can be supported inside closed conductors, as well as between two or more conductors.

Transverse Magnetic (TM) Waves

For the case where $H_z = 0$ and $E_z \neq 0$, Eq. 3.5 can be reduced to the following:

$$H_x = \frac{j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial y}, \quad (3.23a)$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial x}, \quad (3.23b)$$

$$E_x = \frac{-j\beta}{k_c^2} \frac{\partial E_z}{\partial x}, \quad (3.23c)$$

$$E_y = \frac{-j\beta}{k_c^2} \frac{\partial E_z}{\partial y}. \quad (3.23d)$$

First solving for E_z with the Helmholtz wave equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) E_z = 0, \quad (3.24)$$

Reducing to a 2d wave equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) e_z = 0, \quad (3.25)$$

3.25 must be solved subject to the boundary conditions of the specific geometry of the guide

The TM wave impedance can be found as:

$$Z_{\text{TM}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}, \quad (3.26)$$

TM waves can be supported inside closed conductors, as well as between two or more conductors.

Solution Summary

TEM waves

1. Solve Laplace's equation for the potential difference between the conductors. The solution will contain unknown constants.
2. Apply boundary conditions for the known voltages of the conductors to find the constants.
3. Compute $\bar{E}$ and $\bar{H}$
4. Compute voltage and current
5. Calculate the propagation constant and the characteristic impedance Z_0

TE and TM waves

1. Solve the reduced Helmholtz equation for h_z or e_z . The solution will contain unknown constants and the unknown cutoff wave number k_c .
2. Use the reduced form of the general solution for that system to find the transverse fields.
3. Apply the boundary conditions of your system to the appropriate field components to find the constants and k_c .
4. Calculate the wave impedance and propagation constant.

Attenuation Due to Dielectric Loss

So far, we have mostly assumed the dielectric filling the transmission line or waveguide is lossless. In reality, dielectric materials are not perfect. Some of the electromagnetic energy is absorbed by the material and converted into heat. When the propagation is complex, it can be written as follows

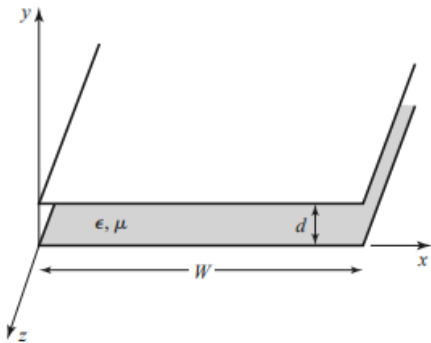
$$\gamma = \frac{k^2 \tan \delta}{2\beta} + j\beta, \quad (3.28)$$

Where $\tan \delta$ is the loss tangent. While the attenuation constant due to dielectric loss is given by the following

$$\alpha_d = \frac{k^2 \tan \delta}{2\beta} \text{ Np/m (TE or TM waves).} \quad (3.29)$$

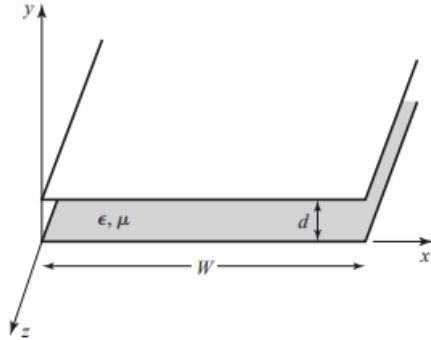
$$\alpha_d = \frac{k \tan \delta}{2} \text{ Np/m (TEM waves).} \quad (3.30)$$

Parallel Plate Waveguide



- The parallel plate waveguide is the simplest waveguiding structure. It consists of two conducting plates separated by a dielectric, with the assumption that the plate width is much larger than the spacing so that edge effects can be ignored. This structure can support all three mode types: TEM, TE, and TM. The TEM mode has no cutoff and behaves like an ordinary transmission line wave, while the higher order TE and TM modes appear when the fields form standing waves in between the plates.

Parallel Plate TEM solution



First solve Laplace's equation for the electrostatic potential:

$$\nabla_t^2 \Phi(x, y) = 0, \quad \text{for } 0 \leq x \leq W, \quad 0 \leq y \leq d. \quad (3.31)$$

Apply boundary conditions:

$$\Phi(x, 0) = 0, \quad (3.32a)$$

$$\Phi(x, d) = V_o. \quad (3.32b)$$

Create general solution and solve:

$$\Phi(x, y) = A + By,$$

$$\Phi(x, y) = V_o y/d. \quad (3.33)$$

Use solution to solve for the transverse and total fields (electric and magnetic):

$$\vec{e}(x, y) = -\nabla_t \Phi(x, y) = -\hat{y} \frac{V_o}{d}, \quad (3.34)$$

$$\vec{E}(x, y, z) = \vec{e}(x, y) e^{-jkz} = -\hat{y} \frac{V_o}{d} e^{-jkz}, \quad (3.35)$$

$$\vec{H}(x, y, z) = \vec{h}(x, y) e^{-jkz} = \frac{1}{\eta} \hat{z} \times \vec{E}(x, y, z) = \hat{x} \frac{V_o}{\eta d} e^{-jkz}, \quad (3.36)$$

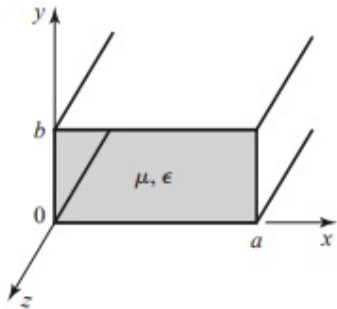
Then solve for voltage, current, and impedance:

$$V = -\int_{y=0}^d E_y dy = V_o e^{-jkz}, \quad (3.37)$$

$$I = \int_{x=0}^W \vec{J}_s \cdot \hat{z} dx = \frac{W V_o}{\eta d} e^{-jkz}. \quad (3.38)$$

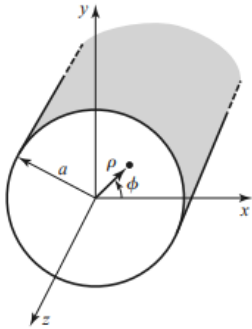
$$Z_0 = \frac{V}{I} = \frac{\eta d}{W}, \quad (3.39)$$

Rectangular Waveguide



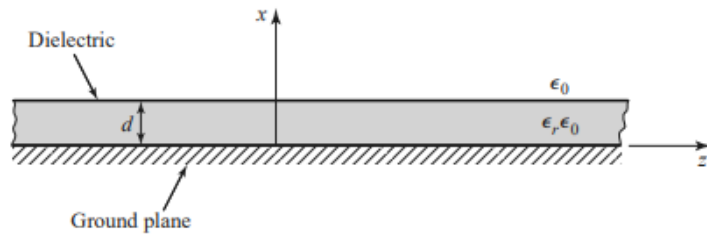
- The rectangular waveguide is a metal pipe with a rectangular cross section, usually with one dimension wider than the other. Unlike the parallel plate guide, a hollow rectangular waveguide cannot support a TEM mode because it has only one conductor. Instead, it supports TE and TM modes, which only propagate above their cutoff frequencies. The most important mode is the TE₁₀ mode because it has the lowest cutoff frequency and is therefore the first mode to propagate. This makes the rectangular waveguide useful for low loss and high power microwave transmission, but it is also larger and less convenient to integrate into compact circuits.

Circular Waveguide



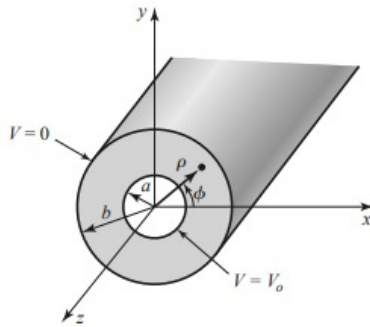
- The circular waveguide is similar to the rectangular waveguide as it is a single conductor that encloses a dielectric, but its cross section is circular instead of rectangular. It also cannot support a TEM mode, so its propagating fields are TE and TM modes. The main difference is that the circular geometry leads to field patterns described by circular symmetry, so the allowed modes are determined differently than in rectangular waveguide, as it requires a conversion to cylindrical coordinates. Circular waveguide can be useful when symmetry is important, and some of its modes can have very low loss.

Grounded Dielectric Sheet



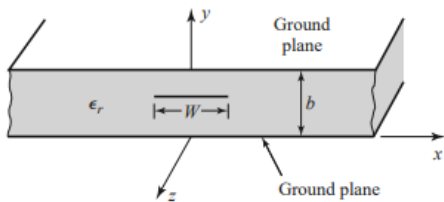
- A grounded dielectric sheet is a dielectric layer placed on top of a conducting ground plane. This structure can support surface waves, where the fields travel along the dielectric while decaying away from the surface. These surface waves can have TM and TE modes. This is different from a hollow metal waveguide because the wave is guided partly by the dielectric material rather than only by conducting walls. Surface waves can be excited on some types of planar transmission lines, like microstrip and slotline.

Coaxial Line



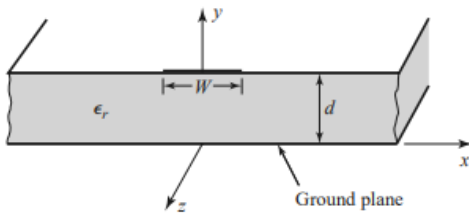
- The coaxial line is a two conductor transmission line made from an inner conductor surrounded by an outer conducting shield. Because it has two conductors, it supports a TEM mode. This means the electric and magnetic fields are completely transverse to the direction of propagation, and the line has a well defined voltage and current. The TEM mode has no cutoff frequency, so coaxial lines can operate over a broad frequency range. Similar to the parallel plate waveguide, it also supports TE and TM modes. In practice, coax is useful because it is shielded and easy to connect to microwave equipment, but at high enough frequencies higher order modes can appear, which limits its useful frequency range.

Stripline



- Stripline is a planar transmission line where a conducting strip is embedded between two ground planes inside a dielectric material. Since the fields are contained in the dielectric region and the structure has more than one conductor, the dominant mode is TEM. This makes stripline behave very much like an ideal transmission line, with no cutoff for the main mode and little dispersion. Stripline is useful in microwave circuits because it has well confined fields, which helps reduce radiation. Its main drawback is that the signal conductor is buried inside the structure, so it is less convenient for mounting components or making adjustments compared with microstrip.

Microstrip Line



- Microstrip is one of the most common transmission lines used in microwave integrated circuits. It consists of a conducting strip on top of a dielectric substrate with a ground plane underneath. Unlike stripline, the fields are partly in the dielectric and partly in the air, so the mode is not exactly TEM. Instead, it is called quasi TEM, meaning it behaves approximately like a TEM wave but with dispersion and radiation effects. Microstrip is popular because it is easy to fabricate, easy to connect to components, and well suited for compact microwave circuits. The tradeoff is that it generally has more loss, more radiation, and less ideal field confinement than stripline.

TABLE 3.6 Comparison of Common Transmission Lines and Waveguides

Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred	TEM	TE ₁₀	TEM	Quasi-TEM
Other	TM,TE	TM,TE	TM,TE	Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration with	Hard	Hard	Fair	Easy