Recursion

Problem Decomposition

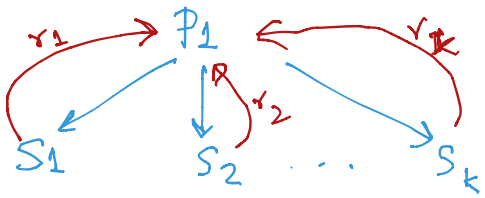
2 aspects:

① Arguments ?

② Conditions ?

- Base → Exit
- Recursion
  - Decomposition
  - Recombination

Recombine $\{r_1, r_2, \ldots, r_k\}$



Eg1 FACTORIAL (n)

$n! = n \cdot (n-1)!$

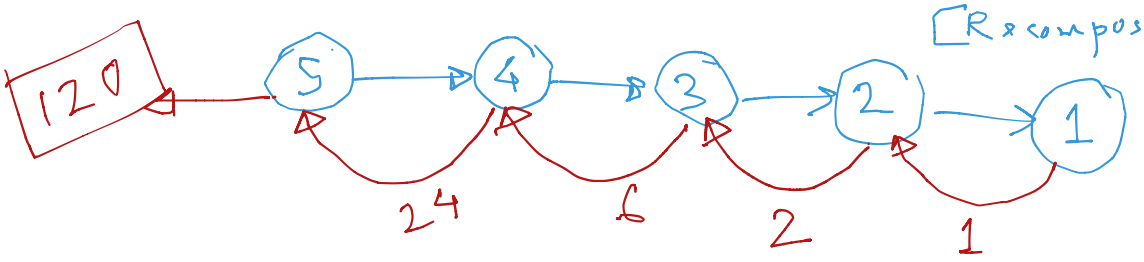
Argument : n

Base : $(n == 1) \Rightarrow R = 1$

Recursion : $(n > 1) \Rightarrow x = \text{FACTORIAL}(n-1)$ [Decomposition]

$n = 5$

$R = x * n$ [Recomposition]



Eg2 Fibonacci Numbers

$Fib(n) = Fib(n-1) + Fib(n-2)$

Argument : n

Base : $(n == 0) \Rightarrow R = 0$

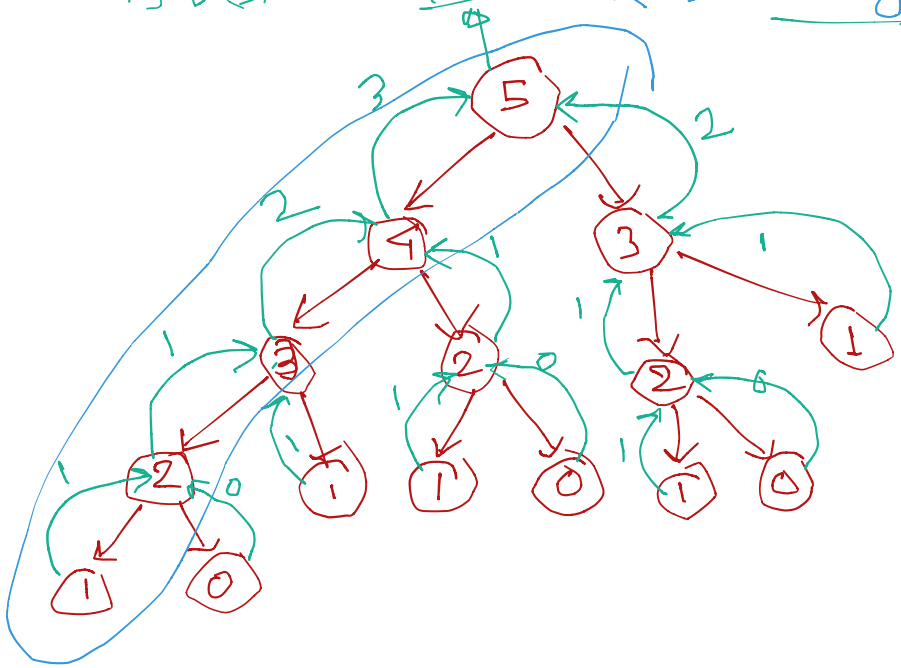
$(n == 1) \Rightarrow R = 1$

Recursion : $(n > 1) \Rightarrow$

n	fib(n)	
0	0	Base
1	1	
2	1	
3	2	
4	3	
5	5	
6	8	
.	.	
.	.	
.	.	

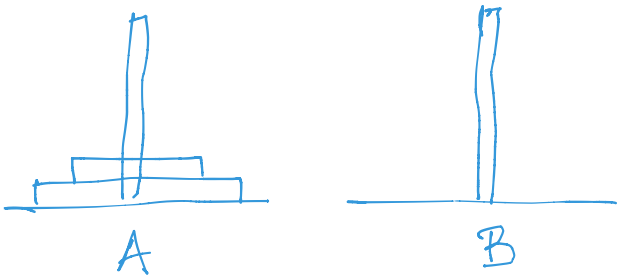
$x = fib(n-1)$
$y = fib(n-2)$ } Dec.
$R = x + y$ } Rec.

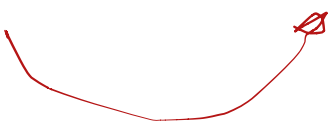
$fib(5) =$ 5

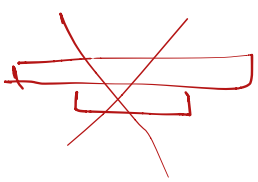


→ Stack frames.

Eg) Tower of Hanoi. n Discs, 3 Pegs.







$n = 3$

TOH (n, A, B, C)

Arguments: n, A, B, C

Base : $(n == 1) \Rightarrow R = \{A, B\}$ Move A to B

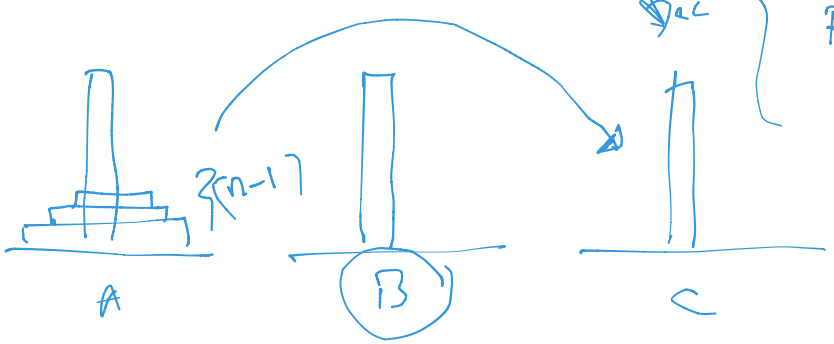
Recursion : $(n > 1) \Rightarrow$

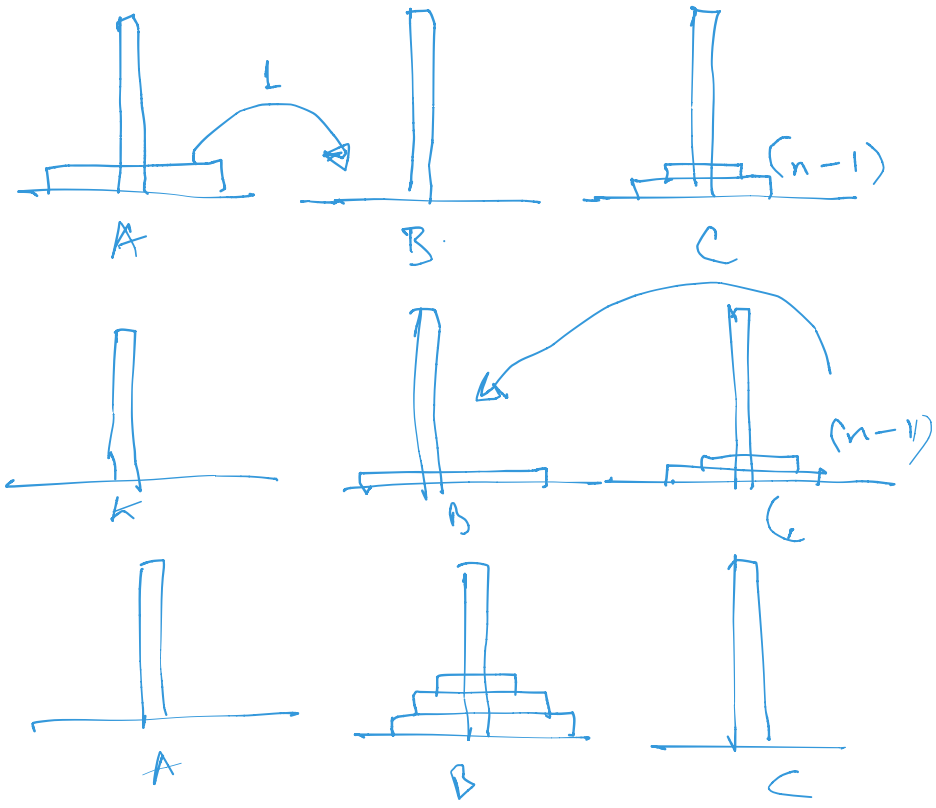
$R_1 =$ TOH $(n-1, A, C, B) \Rightarrow$ Dec

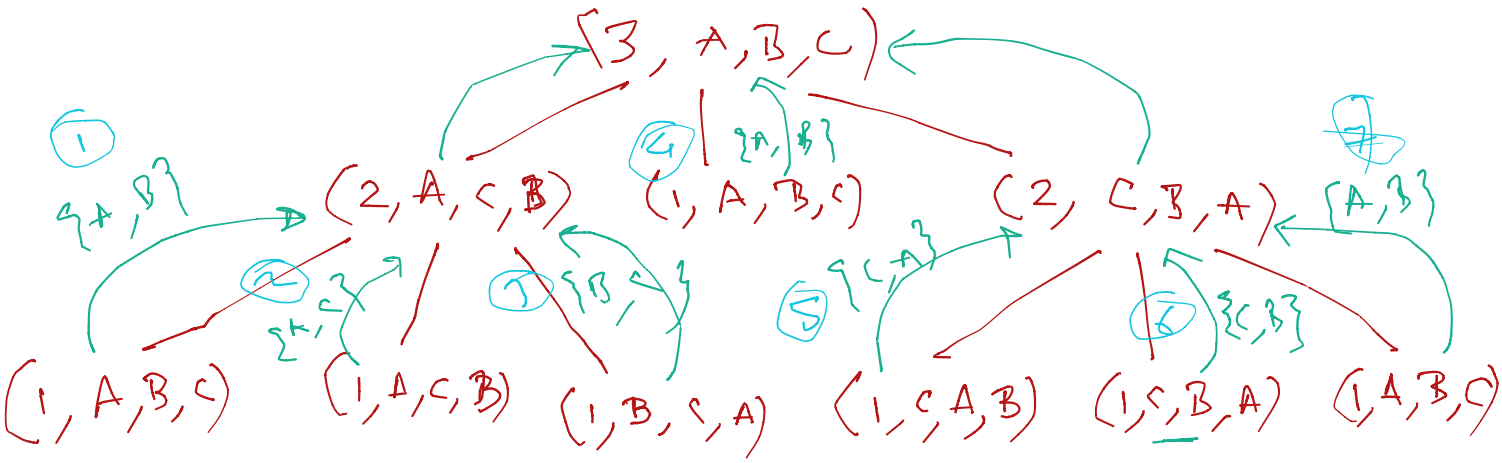
$R_2 =$ TOH $(1, A, B, C) \Rightarrow$

$R_3 =$ TOH $(n-1, C, B, A) \Rightarrow$

Rec $\{ R = $ APPEND (R_1, R_2, R_3)




1
A
B
C
(n-1)
A
B
C
(n-1)
A
B
C


(3, A, B, C)
1
{A, B}
(2, A, C, B)
4
{A, B}
(1, A, B, C)
(2, C, B, A)
7
{A, B}
2
{A, C}
3
{B, C}
5
{C, A}
6
{C, B}
(1, A, B, C)
(1, A, C, B)
(1, B, C, A)
(1, C, A, B)
(1, C, B, A)
(1, A, B, C)