Lecture 9

Merge Sort
Quick sort
Graphs.

Heap. $\Theta(n \log n)$ → One entry at a time

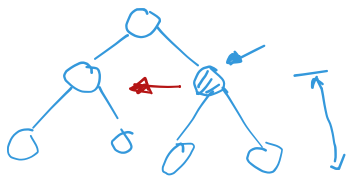
ConstructBottomupHeap()

Lowest Internal node Index: $\left(\frac{n}{2}\right) \rightarrow 0$

→ Bubble Down.

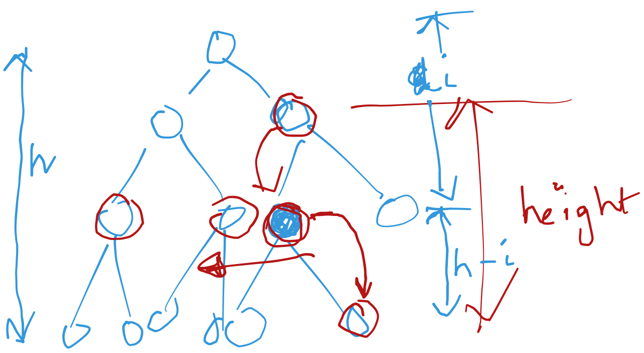
$n = 2^{h+1} - 1$

$\boxed{\Theta(n)}$



d	# nodes at depth d	# total nodes
0	$2^0 = 1$	$2^{0+1} - 1 = 1$
1	$2^1 = 2$	$2^{1+1} - 1 = 3$
2	$2^2 = 4$	$2^{2+1} - 1 = 7$
⋮	⋮	⋮
d	2^d	$\boxed{2^{d+1} - 1}$

# of operations ~ Sum of the heights of all the nodes.



Height of the complete binary tree = h

At depth i, # of nodes = 2^i

Height of a node at depth i
$= h - i$

Sum of heights of all the nodes at depth i
$= 2^i * (h-i)$

Sum of the heights for all nodes

$$S = \sum_{i=0}^{h} 2^i * (h-i)$$

$$- \quad S = h + 2^1 * (h-1) + 2^2 (h-2) + \cdots + 2^{h-1}$$

$$+ \quad 2S = 0 + 2 * h + 2^2 (h-1) + \cdots + 2^h$$

$$S = -h + (2 + 2^2 + \cdots 2^h)$$

$$= -h + 2 (1 + 2 + 2^2 + \cdots + 2^{h-1})$$

$$= -h + 2 * (2^{h} - 1)$$

$$= -h + 2^{h+1} - 2$$

$$= -h + (n+1) - 2$$

$$= (n - h - 1)$$

$$h \rightarrow \Theta(\lg n)$$

$$\boxed{S = \Theta(n)}$$

Number of nodes $= n$

$$\boxed{n = 2^{h+1} - 1}$$

$$\Rightarrow 2^{h+1} = n + 1$$