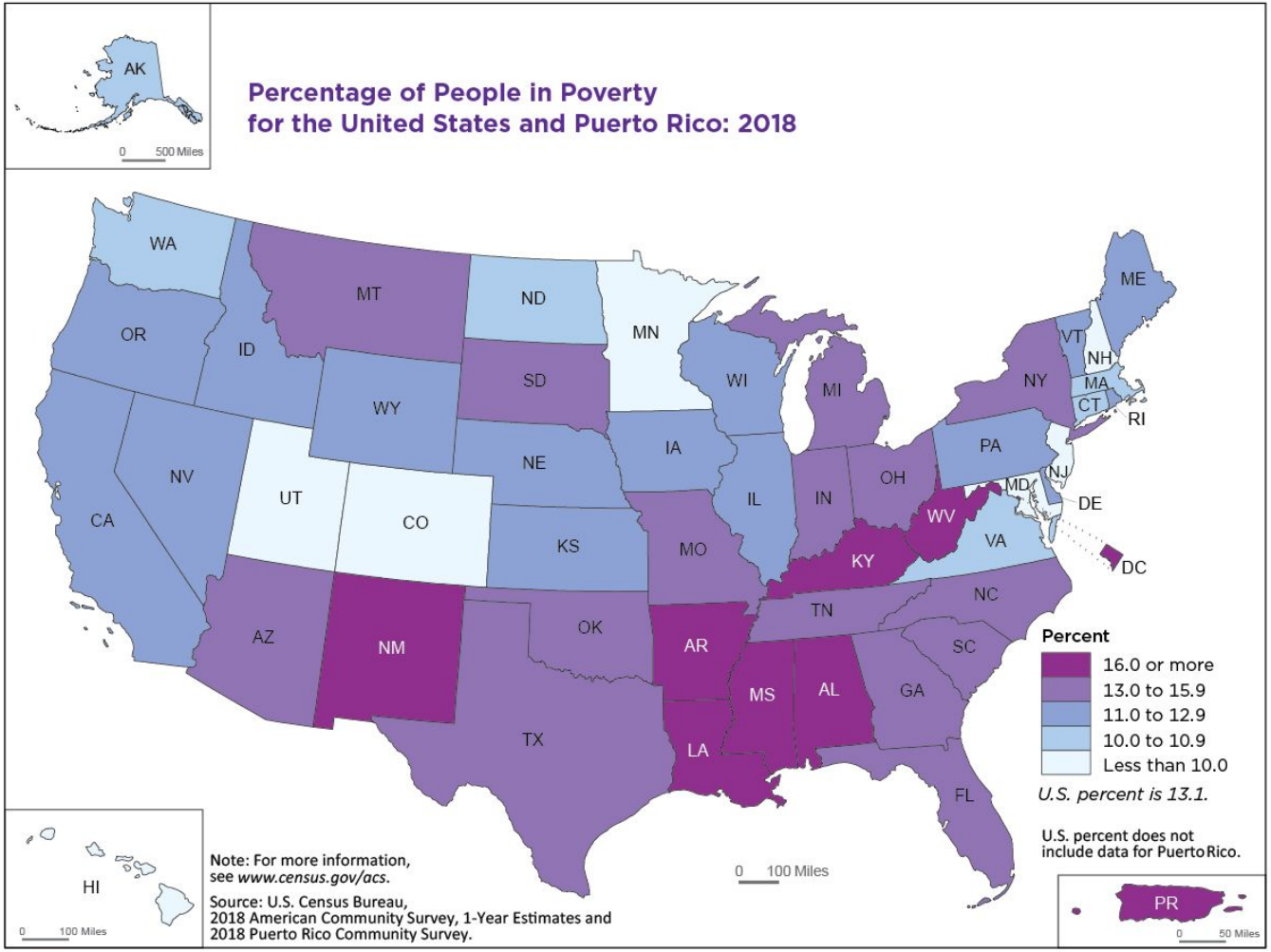
Modeling the Dynamics of Poverty

Rediet Abebe

Harvard University
Society of Fellows


Percentage of People in Poverty
for the United States and Puerto Rico: 2018
AK
0 500 Miles
WA
OR
CA
ID
NV
MT
WY
UT
AZ
CO
NM
ND
SD
NE
KS
OK
TX
MN
IA
MO
AR
LA
WI
IL
MS
MI
IN
KY
TN
AL
OH
WV
GA
FL
SC
NC
VA
PA
NY
MD
DE
NJ
CT
RI
MA
VT
NH
ME
DC
Percent
16.0 or more
13.0 to 15.9
11.0 to 12.9
10.0 to 10.9
Less than 10.0
U.S. percent is 13.1.
U.S. percent does not include data for Puerto Rico.
HI
0 100 Miles
Note: For more information, see www.census.gov/acs.
Source: U.S. Census Bureau, 2018 American Community Survey, 1-Year Estimates and 2018 Puerto Rico Community Survey.
0 100 Miles
PR
0 50 Miles

“official **‘income only’ measurements** of poverty… painted a picture that was too optimistic and didn’t capture the **magnitude of disadvantage**, nor the true struggles New Yorkers face in trying to make ends meet.”

(Wimer at al., 2014)

POVERTY TRACKER

The Role of Shocks in Welfare

- Impact of shocks on housing, health, education
  (Desmond, ’12; Kijima et al., ’06; Kocher et al., ’95; Poverty Tracker, ’14)
- Household consumption dynamics
  (Aiyagari, ’94; Golosov et al., ’06; Kocherlakota, ’04)
- Optimal taxation theory
  (Eaton & Rosen,’80; Farhi & Werning, ’13; Saez and Stantcheva, ’18)

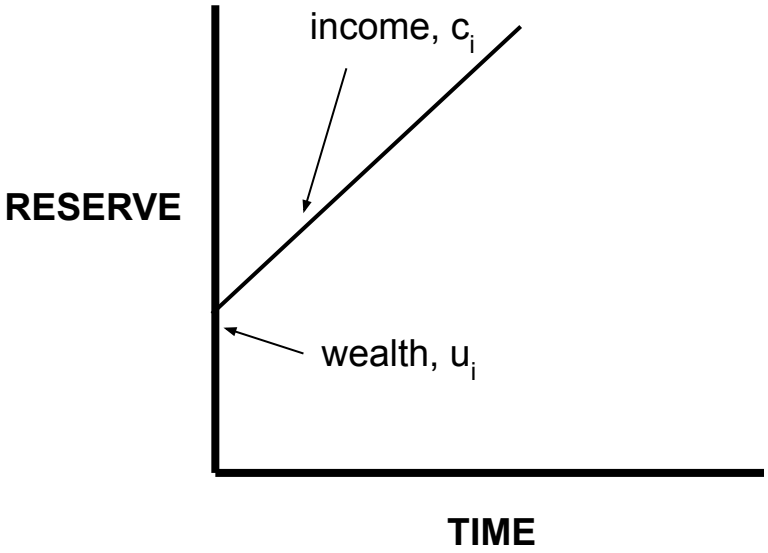
How can we design algorithms to

optimally allocate subsidies

when agents experience income shocks?

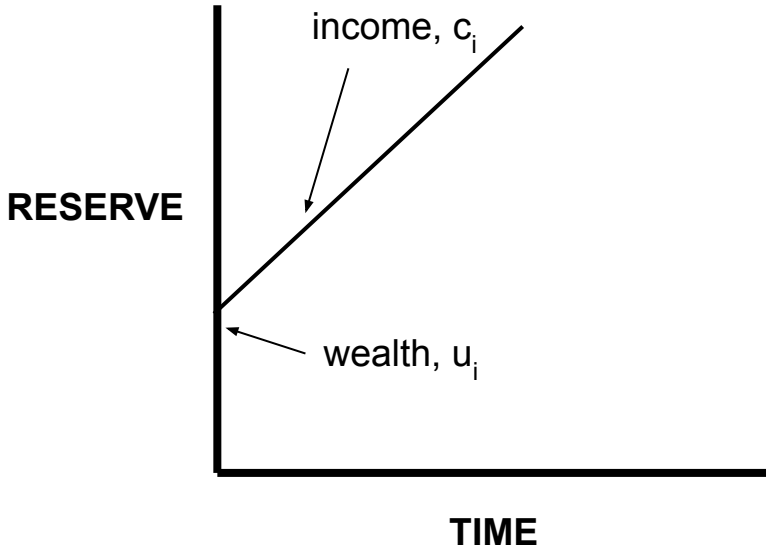
Abebe, Kleinberg, &
Weinberg. (AAAI ’20)

A Model of Welfare



$$N = \{1, 2, \ldots, n\}$$

A Model of Welfare



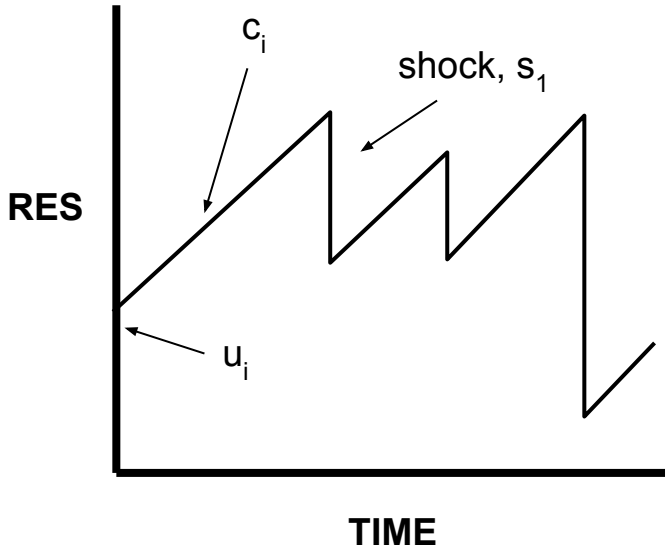
Reserve: no shocks

$$R_i(t) = u_i + c_i t$$

A Model of Welfare

Assumptions:

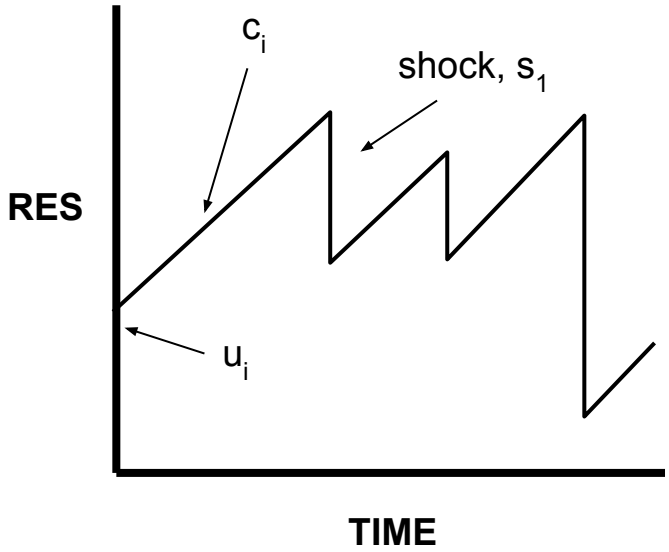
- Shocks: $S_{i1}, S_{i2}, \ldots$ arriving at $T_{i1}, T_{i2}, \ldots$
- Poisson arrival, rate β_i
- Shocks drawn from F_i



A Model of Welfare

Assumptions:

- Shocks: $S_{i1}, S_{i2}, \ldots$ arriving at $T_{i1}, T_{i2}, \ldots$
- Poisson arrival, rate β_i
- Shocks drawn from F_i



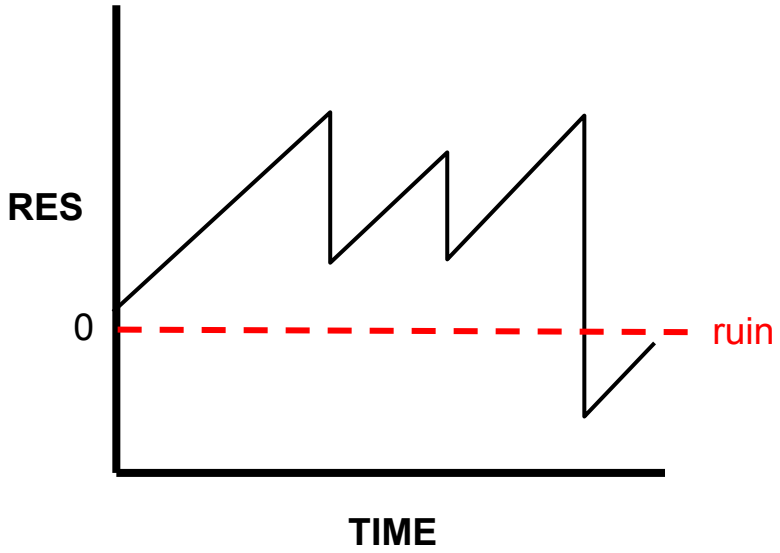
Reserve: with shocks

$$R_i(t) = u_i + c_i t - \sum_{j:T_{ij} \le t} S_{ij}$$

A Model of Welfare

Assumptions:

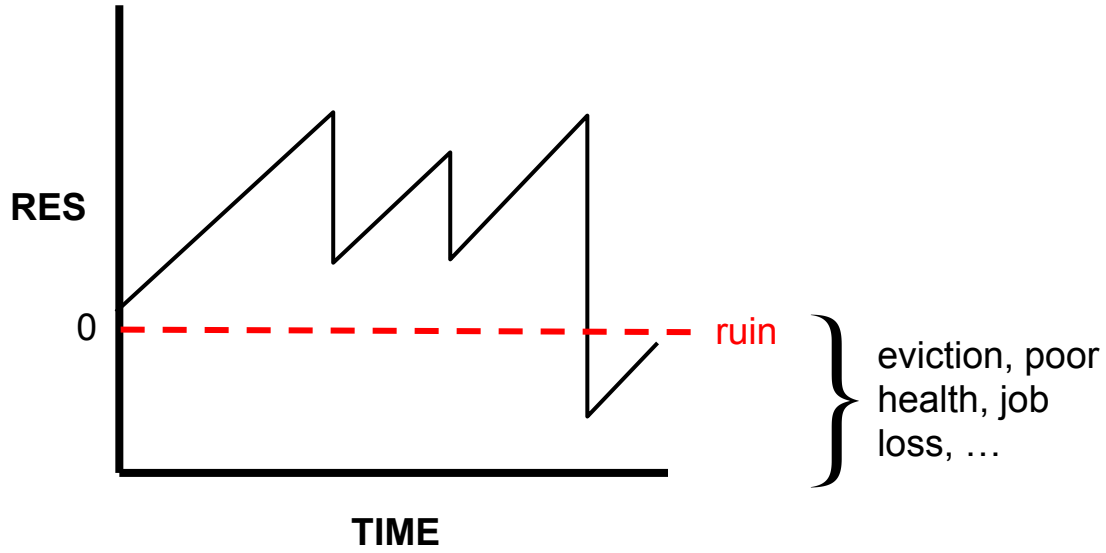
- Shocks: $S_{i1}, S_{i2}, \ldots$ arriving at $T_{i1}, T_{i2}, \ldots$
- Poisson arrival, rate β_i
- Shocks drawn from F_i



A Model of Welfare

Assumptions:

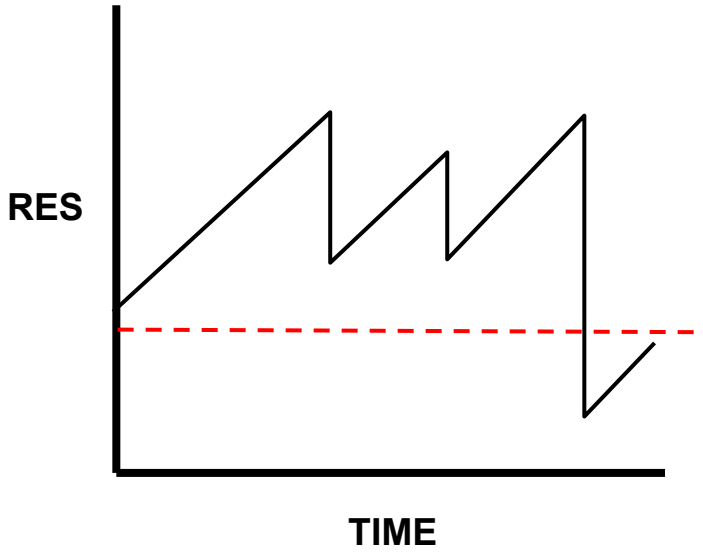
- Shocks: $S_{i1}, S_{i2}, \ldots$ arriving at $T_{i1}, T_{i2}, \ldots$
- Poisson arrival, rate β_i
- Shocks drawn from F_i



The Optimization Problem

Object of Interest: **ruin probability**

$$\psi_i = \psi(c_i, u_i, \beta_i, F_i)$$



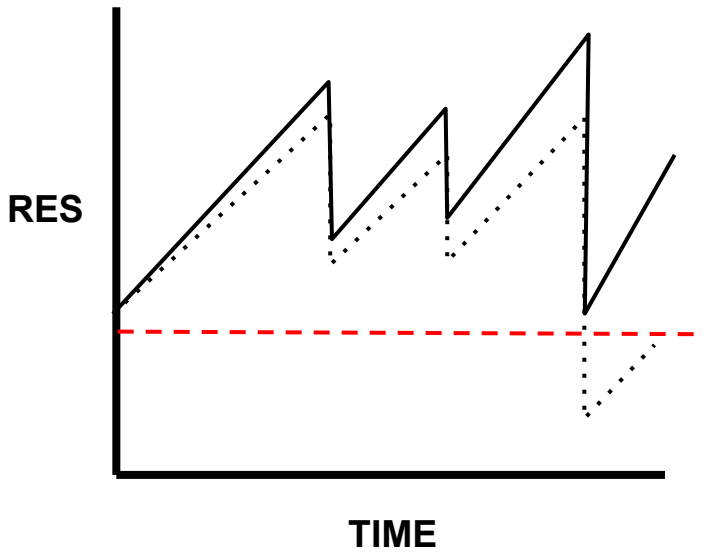
The Optimization Problem

Object of Interest: ruin probability

$$\psi_i = \psi(c_i, u_i, \beta_i, F_i)$$

Intervention: **income subsidy**
subject to budget constraint:

$$c_i \to c_i + x_i$$



The Optimization Problem

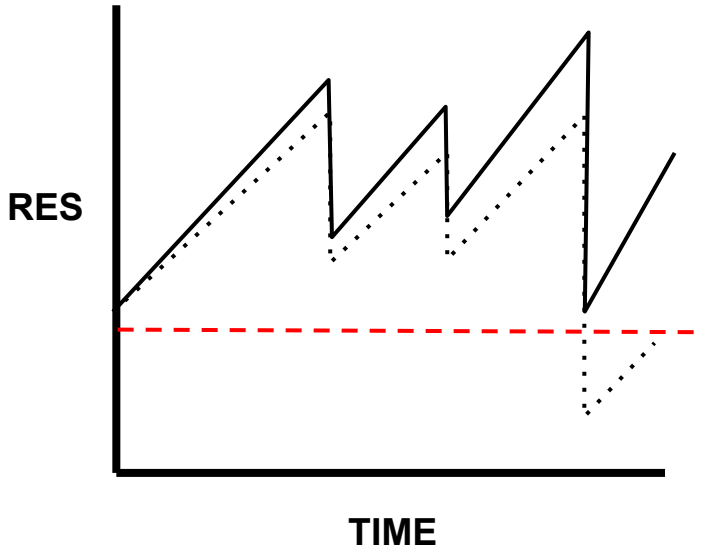
Object of Interest: ruin probability

$$\psi_i = \psi(c_i, u_i, \beta_i, F_i)$$

Intervention: income subsidy subject to budget constraint

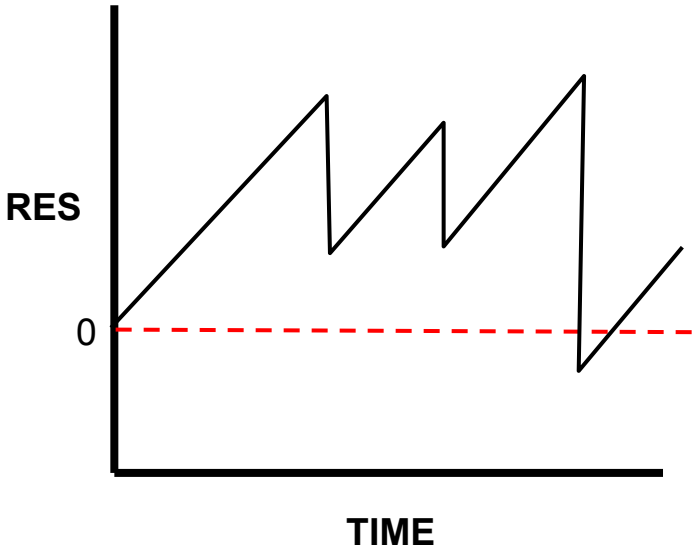
Objective: **min-sum**

$$\min_{x_1+\cdots+x_n=B} \sum_{i=1}^{n} \psi(c_i + x_i, u_i, \beta_i, F_i)$$



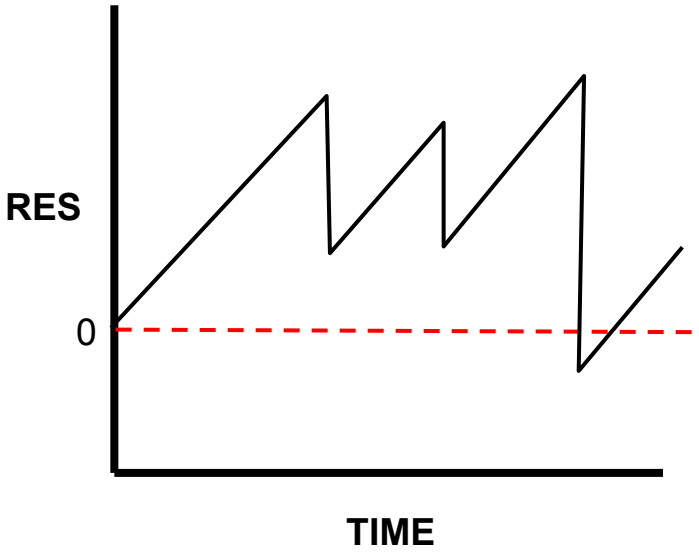
How can we **optimally allocate subsidies**?
Do these solutions give us insights into the **nature of welfare**?

The Case with No Initial Wealth



- Assumptions
  - Zero initial wealth, $u_i = 0$
  - Positive drive, $c_i - \beta_i \mu_i > 0$
  - F_i only give by mean, μ_i

The Case with No Initial Wealth



- Assumptions
  - Zero initial wealth, $u_i = 0$
  - Positive drive, $c_i - \beta_i \mu_i > 0$
  - F_i only give by mean, μ_i

- ψ_i has simple expression

$$\psi_i(c_i, 0, \beta_i, \mu_i) = \frac{\beta_i \mu_i}{c_i}$$

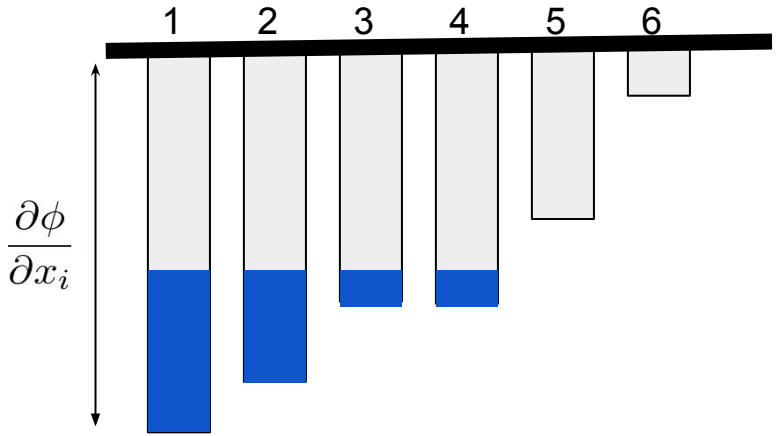
An Optimal Solution: Min-Sum Objective

Objective function

$$\phi(x_1, \ldots, x_n) = \sum_{i=1}^{n} \psi(c_i + x_i, 0, \beta_i, \mu_i)$$

$$\phi(x_1, \ldots, x_n) = \sum_{i} \frac{\beta_i \mu_i}{c_i + x_i}$$

$$\boxed{\frac{\partial \phi}{\partial x_i} = -\frac{\beta_i \mu_i}{(c_i + x_i)^2}}$$



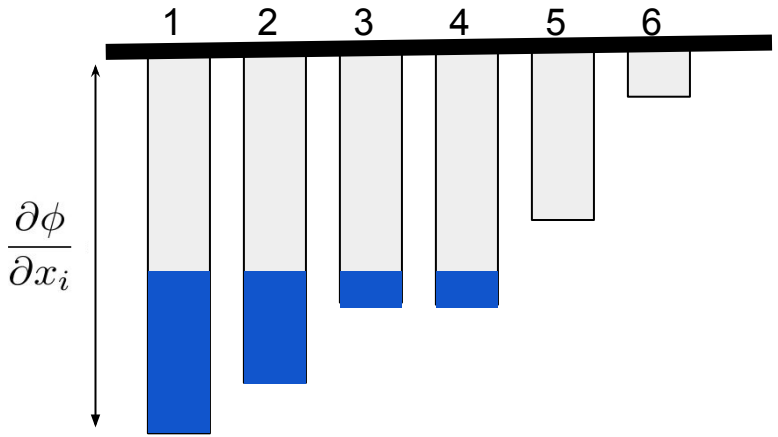
An Optimal Solution: Min-Sum Objective

Objective function

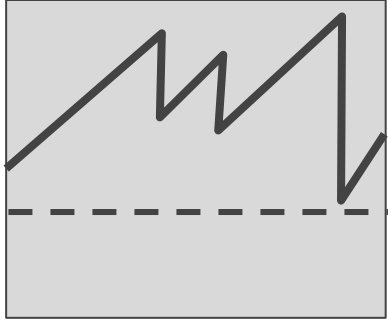
$$\phi(x_1, \ldots, x_n) = \sum_{i=1}^{n} \psi(c_i + x_i, 0, \beta_i, \mu_i)$$

$$\phi(x_1, \ldots, x_n) = \sum_{i} \frac{\beta_i \mu_i}{c_i + x_i}$$

$$\frac{\partial \phi}{\partial x_i} = -\frac{\beta_i \mu_i}{(c_i + x_i)^2}$$

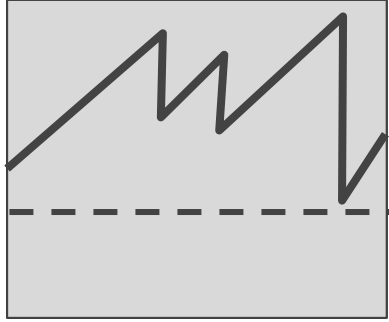


Priority Index



How different can priority indexes be between

income vs. **our priority index**?



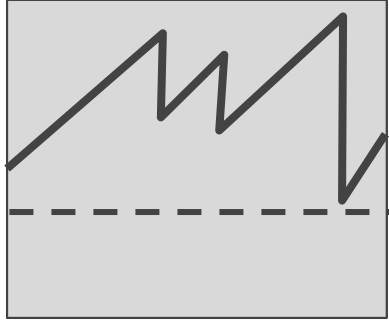
How different can priority indexes be between

income vs. **our priority index**? **fully reversed**

$$c_i = 1 + i\epsilon$$

$$\beta_i = (0.5 + 4i\epsilon)^2$$

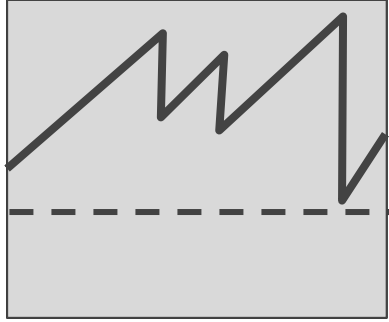
$$\mu_i = 1$$



How different can priority indexes be between

income vs. **our priority index**? **fully reversed**

Lemma: there is a gap of $\Omega(\sqrt{n})$ between our optimal algorithm and any algorithm which only uses income.

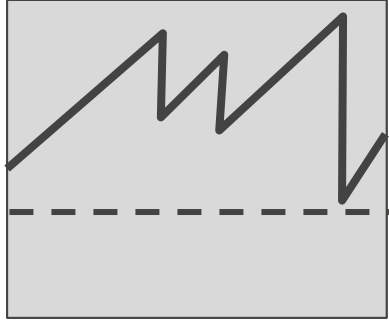


How different can priority indexes be between

income vs. **our priority index**? **fully reversed**

"official **'income only' measurements** of poverty… painted a picture that was too optimistic and didn't capture the magnitude of disadvantage..."

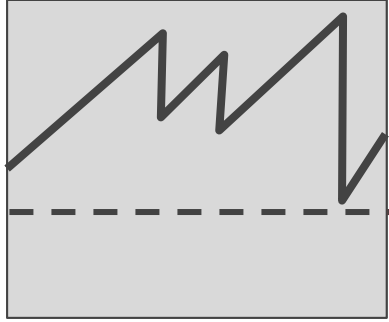
Poverty Tracker Study



How different can priority indexes be between

(1) income vs. our priority index? **fully reversed**

(2) **min-sum** vs. **min-max**?



How different can priority indexes be between

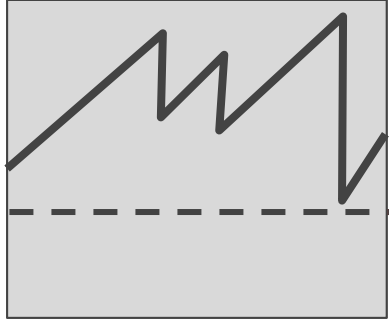
(1) income vs. our priority index? **fully reversed**

(2) **min-sum** vs. **min-max**? **fully reversed**

$$c_i = 1 + i\epsilon$$

$$\beta_i = (0.5 + 4i\epsilon)^2$$

$$\mu_i = 1$$

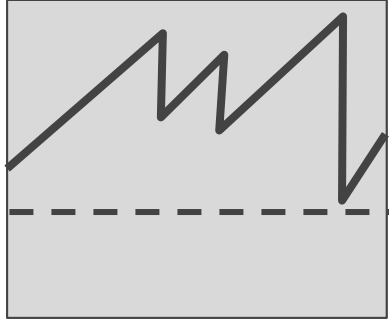


How different can priority indexes be between

(1) income vs. our priority index? **fully reversed**

(2) min-sum vs. min-max? **fully reversed**

(3) **income** vs. **wealth subsidy**?



How different can priority indexes be between

(1) income vs. our priority index? **fully reversed**

(2) min-sum vs. min-max? **fully reversed**

(3) **income** vs. **wealth subsidy**? **fully reversed**

Societal Implications of Results

(1) income vs. our ordering → Information matters

(2) min-sum vs. min-max → Objective functions matter

(3) income vs. wealth subsidy → Intervention types matter

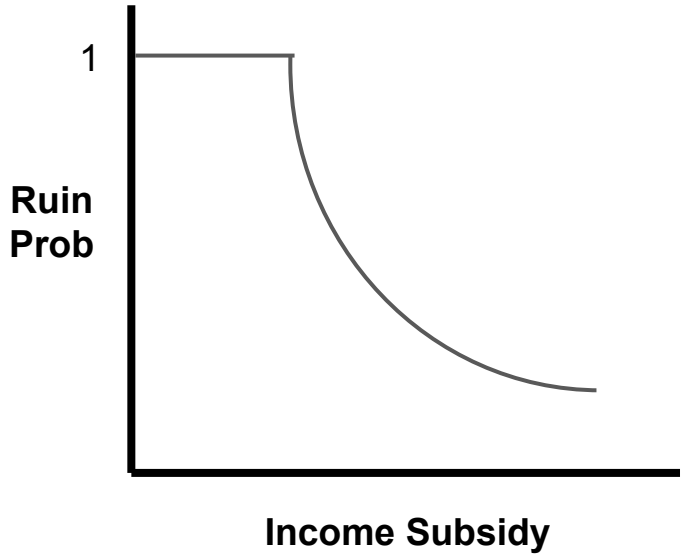
How can we **optimally allocate subsidies** in the general setting?

The General Case

Arbitrary income and wealth

Shocks drawn from general distribution

Can have negative drift, ruin prob. = 1



The General Case

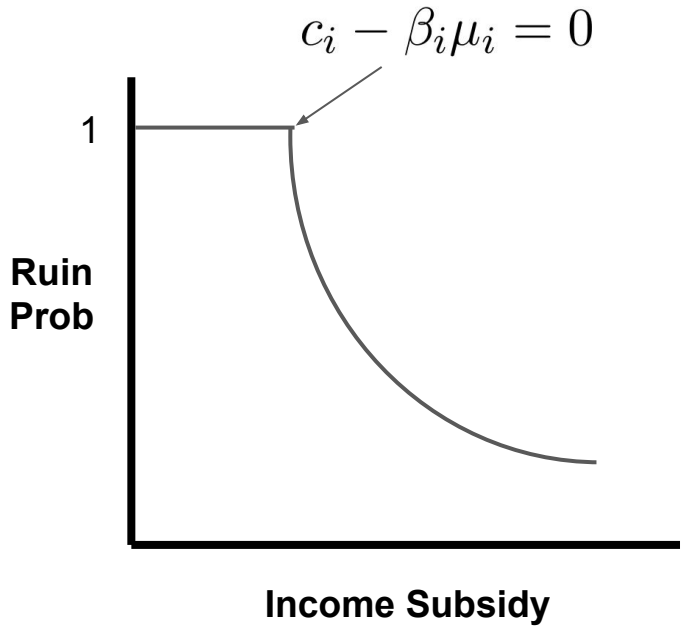
Arbitrary income and wealth

Shocks drawn from general distribution

Can have negative drift, ruin prob. = 1

Objective function is not convex

Optimization problem is NP-hard



FPTAS for General Case

Theorem: we can give a fully-polynomial time approximation scheme (FPTAS) to optimize for the min-sum objective.

FPTAS for General Case

Theorem: we can give a fully-polynomial time approximation scheme (FPTAS) to optimize for the min-sum objective.

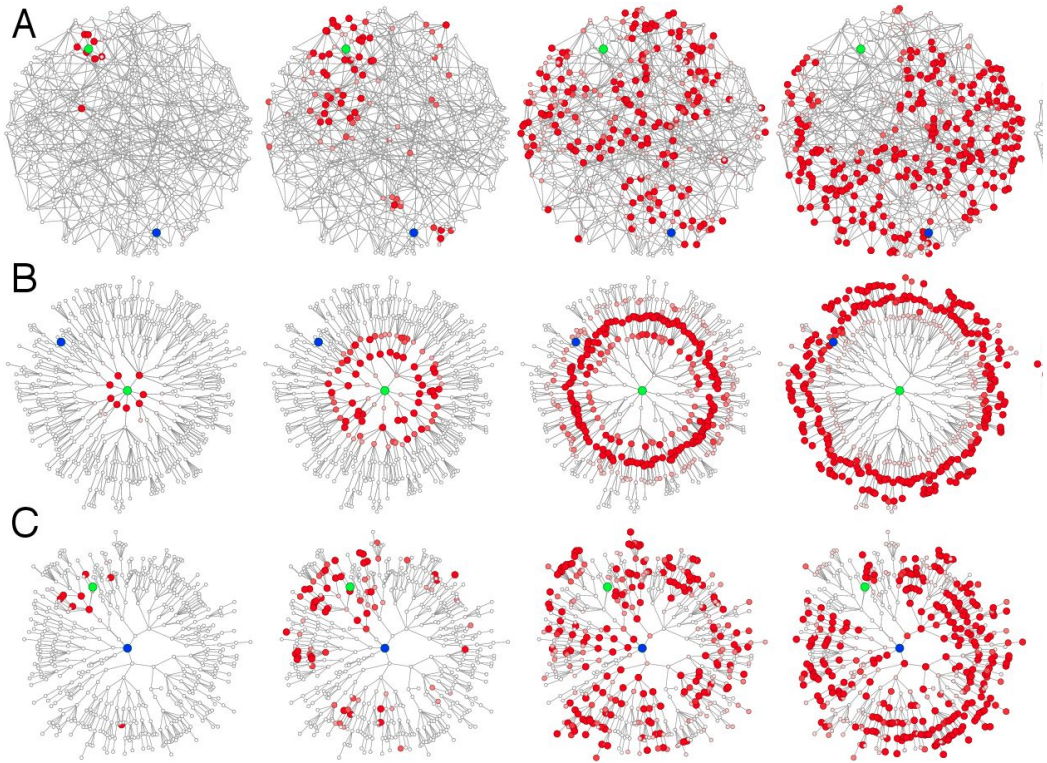
Proof Idea:

Discretize space of possible subsidies and search over this space

Show that this discretization process doesn't lose too much accuracy

Bound the loss of accuracy

Shocks are Networked



Social Insurance Programs: Saving Circles

MD4SG

Mechanism Design for Social Good

algorithmic, optimization, and mechanism design techniques to improve *access to opportunity* for *historically disadvantaged* communities

researchers & practitioners from 100+ institutions in 20 countries




Thank you!

rabebe@fas.harvard.edu

MD4SG.com