Mutant Contagion on Networks

Jonas Juul and Steven Strogatz

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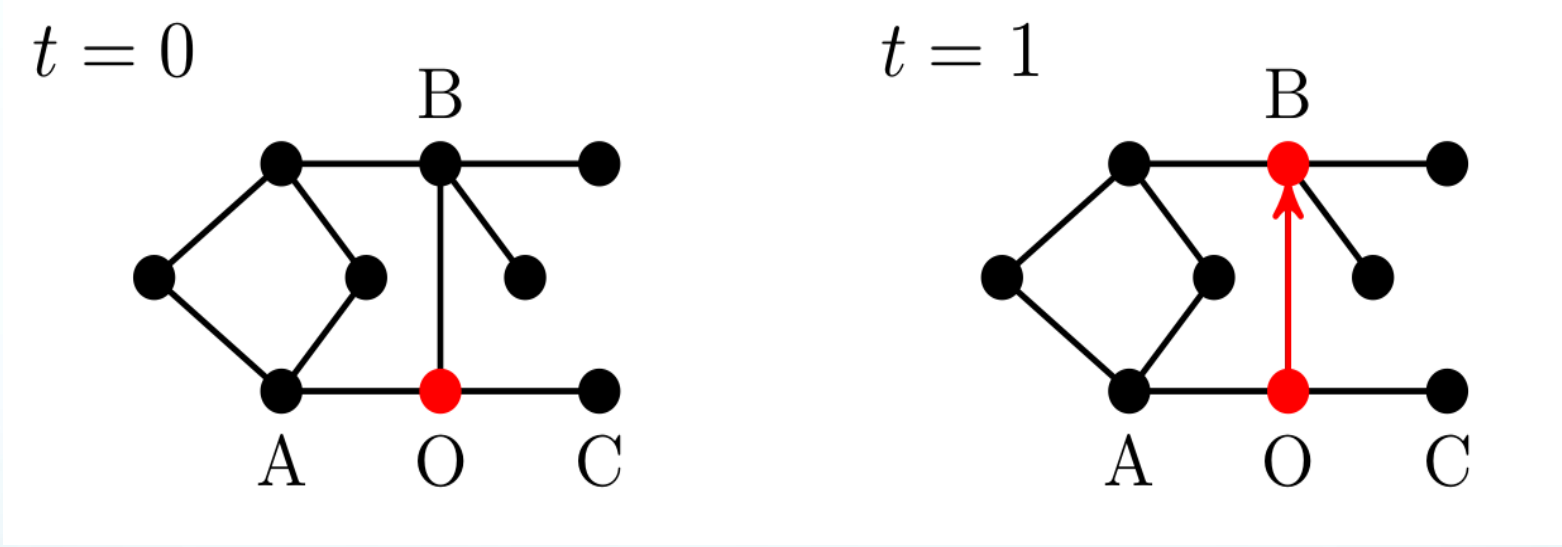
Contagion

- Infectious disease
- Rumors
- Misinformation
- Ideas
- Innovations
- Blackouts
- Bank failures

The question

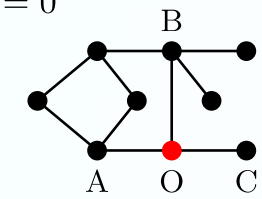
If a contagion *mutates* at some point
as it spreads on a network,
how many nodes downstream
will be confronted by the mutant strain?

Stochastic model of contagion

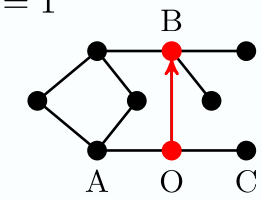


a) **Contagion spreading on contact network**

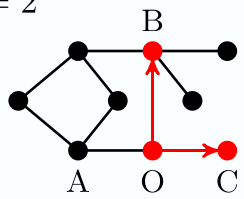
$t = 0$



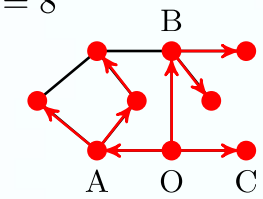
$t = 1$



$t = 2$



$t = 8$



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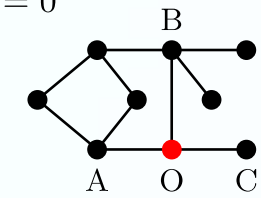
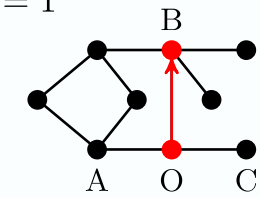
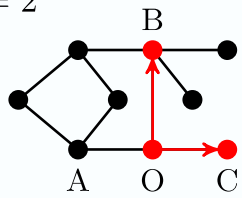


$t = 1$



$t = 2$



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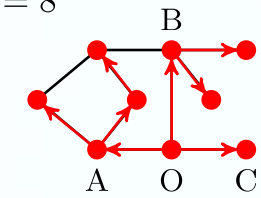
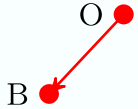


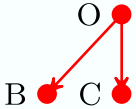
b) **Epidemic tree**

$t = 0$

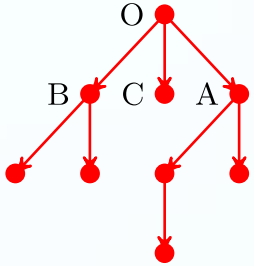
$t = 1$



$t = 2$



$t = 8$



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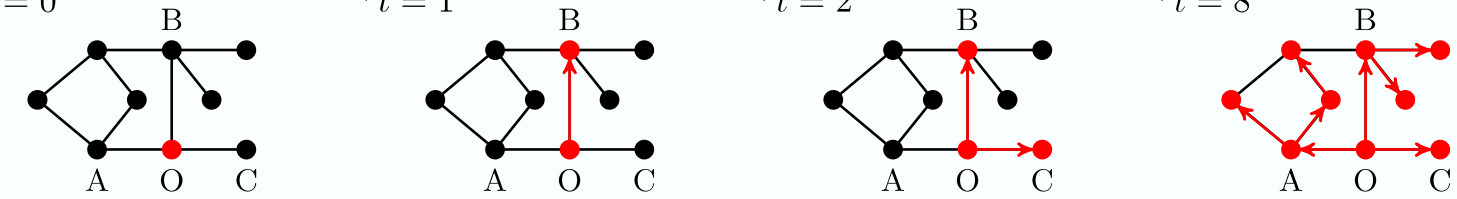
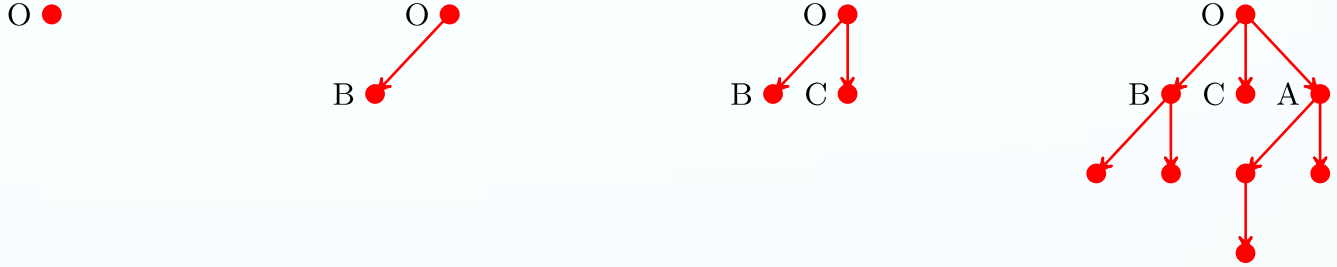


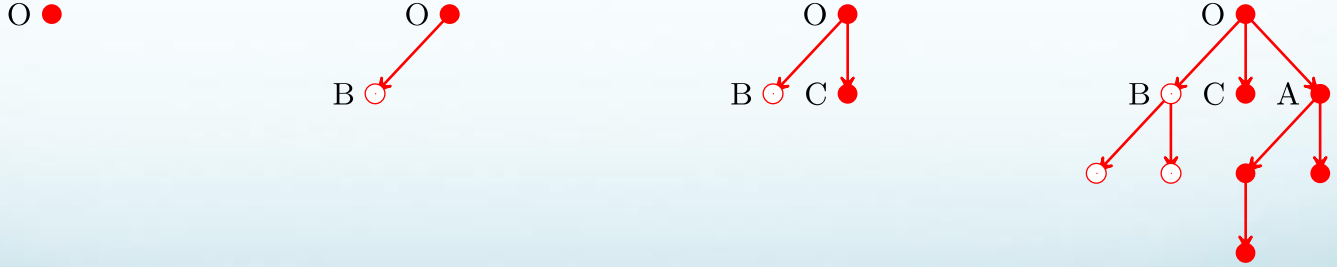
b) **Epidemic tree**

$t = 0$ $t = 1$ $t = 2$ $t = 8$

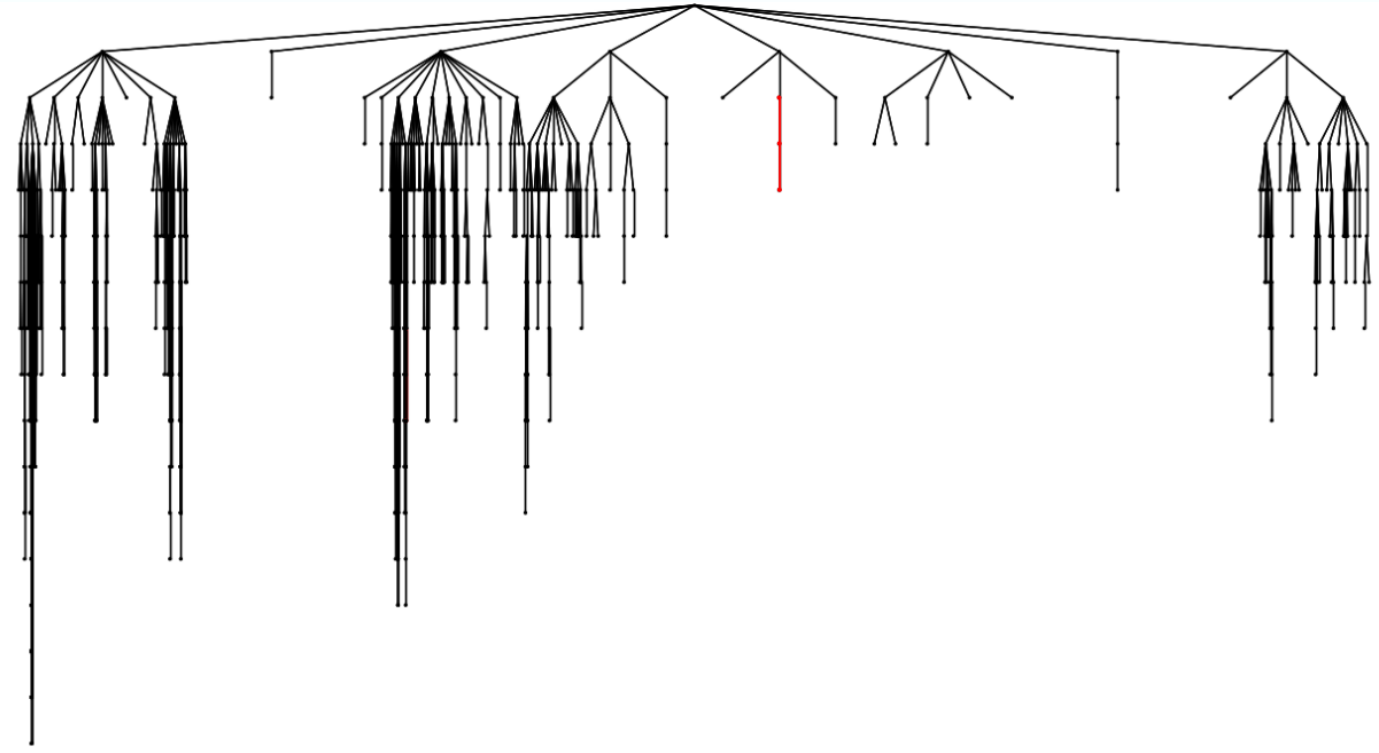


c) **Mutation event at node B**

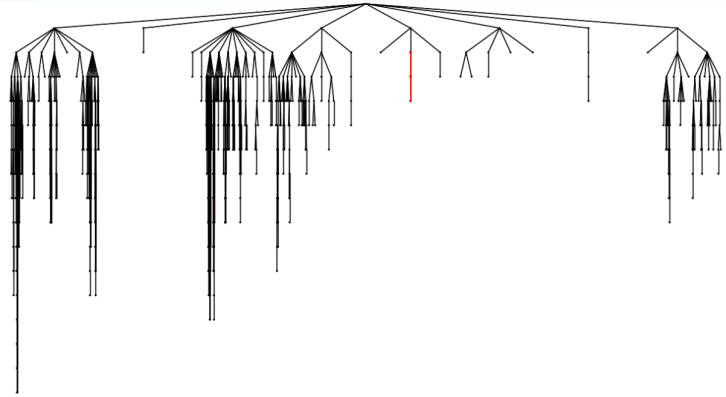
$t = 0$ $t = 1$ $t = 2$ $t = 8$

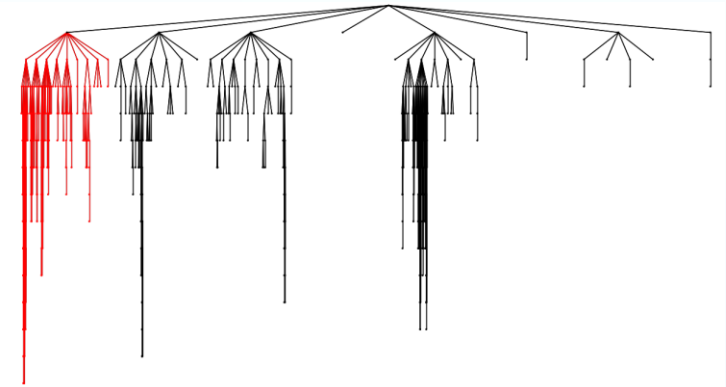


Descendants



Descendants





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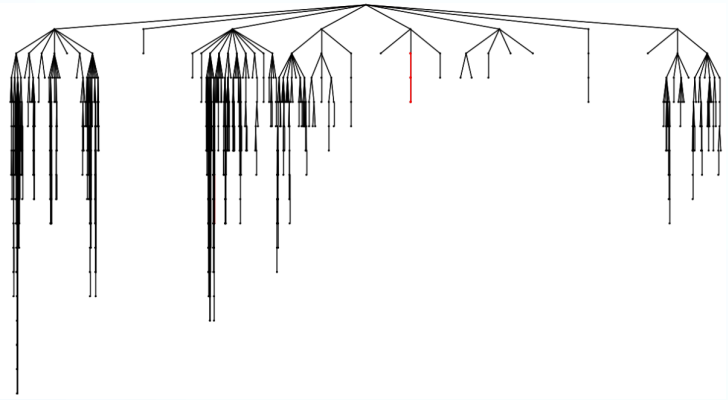

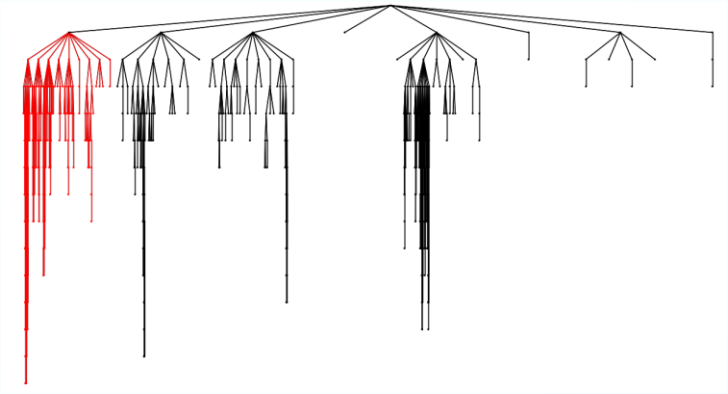

Given a random node,
what is the probability it has *d* descendants?

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Descendant distributions for the impact of mutant contagion on networks

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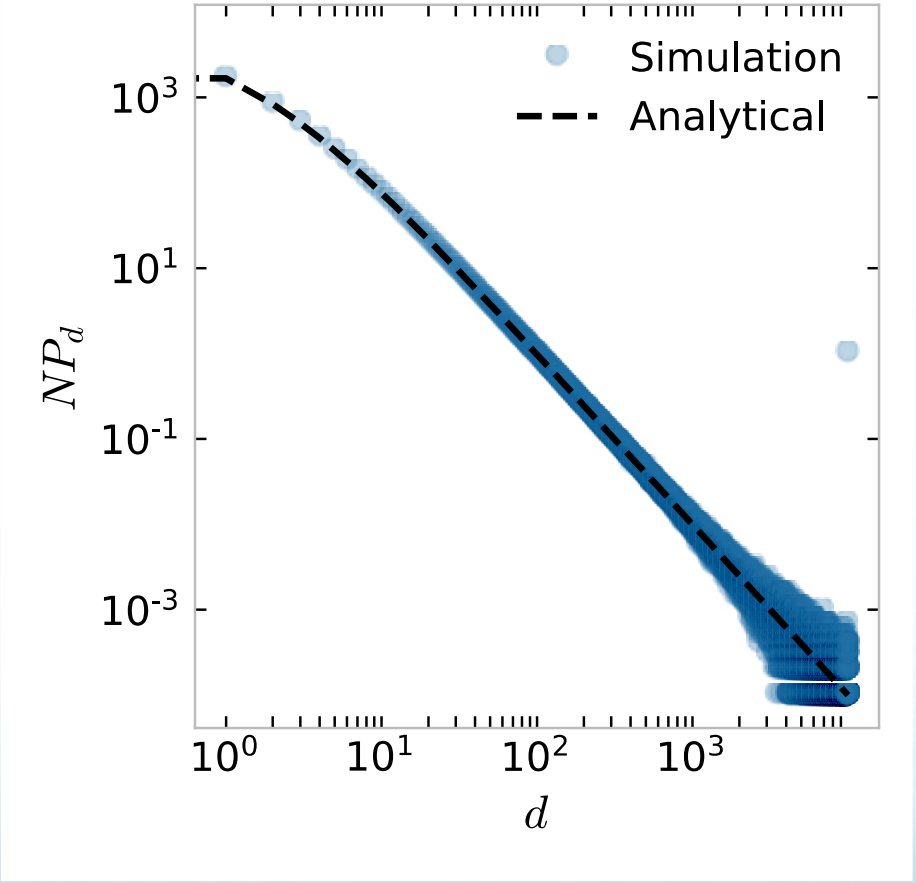
Example: Complete graph



$$P_d = \frac{1}{(d+1)(d+2)}$$

N=10^4 nodes
10^3 simulations

P. L. Krapivsky and S. Redner, Phys. Rev. E 71, 036118 (2005).

Example: Random graphs

- Erdős–Rényi

  each edge (i,j) exists with probability *p*.

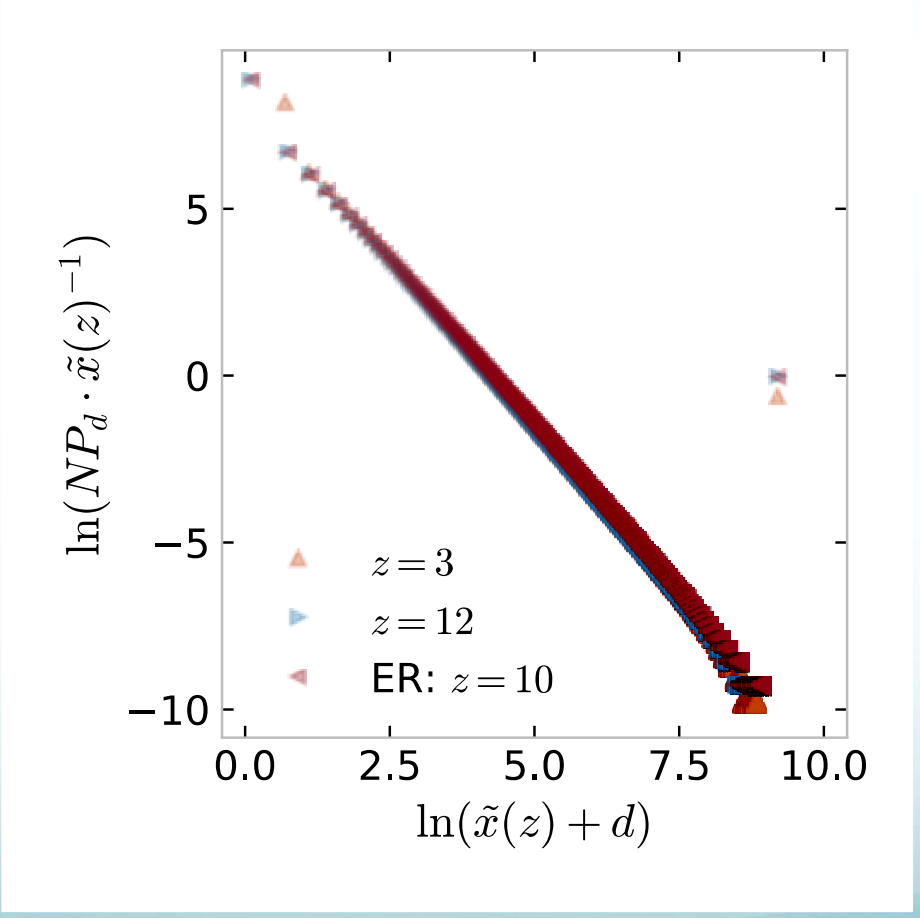
  Then z = (N-1)p is the average degree.

- z-regular configuration model

  create bag of nodes with degree z; then connect their edge ‘stubs’ uniformly at random

Example: Random graphs

(Erdos–Renyi random graphs and z-regular configuration models)



$$P_d = \frac{z-1}{z-2} B\left(\frac{z-1}{z-2}+d, 2\right)$$

$$P_d \propto d^{-2} \text{ for } d \gg 1$$

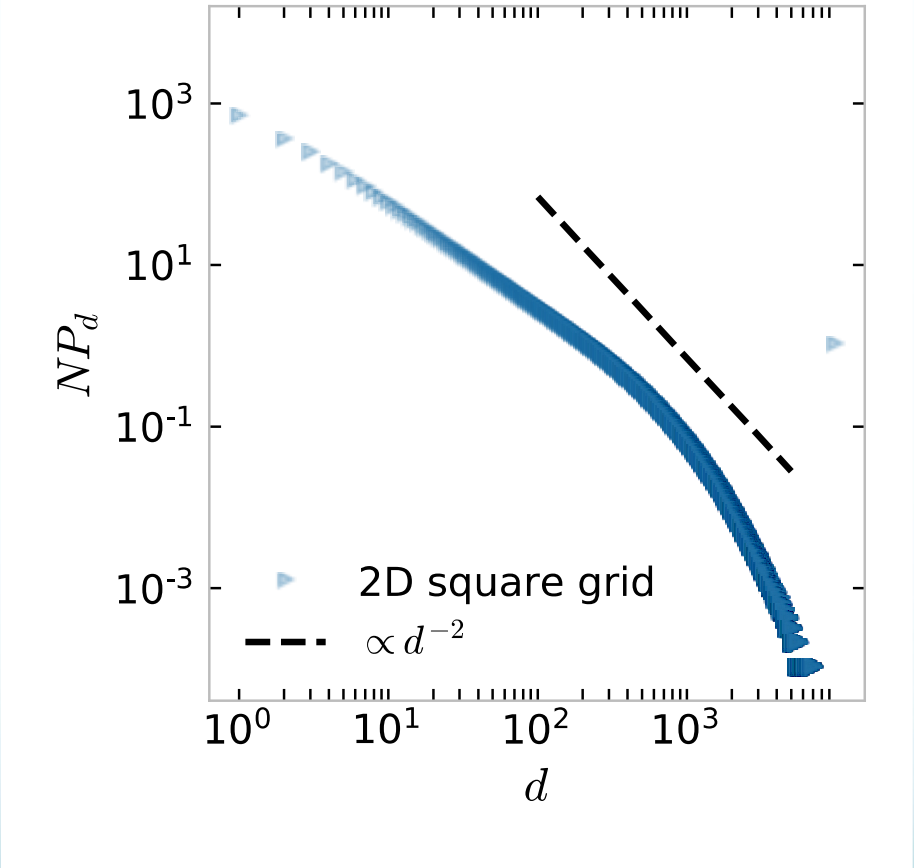
N=10^4 nodes
10^3 simulations

Why d^{-2} ?

True for any “infinite dimensional” network

(infinite dimensional: surface and volume grow equally fast)

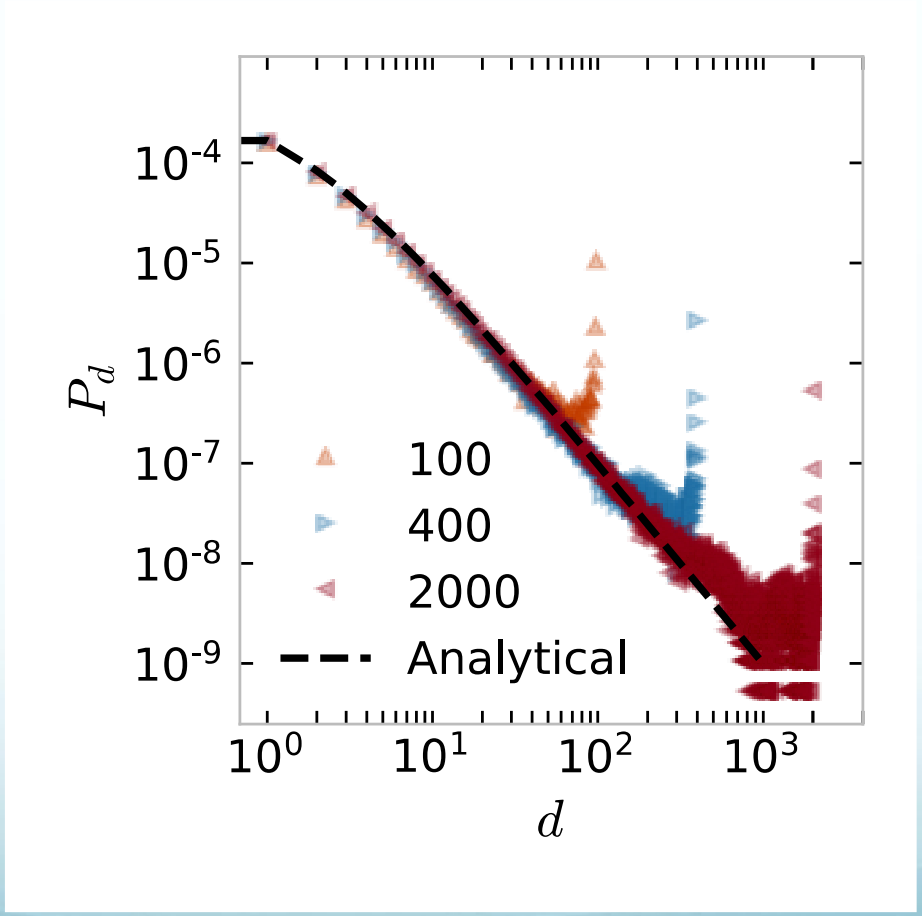
2D square grid behaves differently



Example: Facebook subgraph

4,039 nodes
88,234 edges

J. Leskovec and J. J. McAuley
J. Leskovec and A. Krevl



Facebook study

Information Evolution in Social Networks

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- Adamic et al (2016) studied how memes spread on Facebook
- Memes also *mutate* sometimes
- Users change them to make them funnier or stickier

Adamic et al., 2016

Facebook study

rank	copies	variant
n1	71831	somewhere right now a nurse is getting yelled at for being late with pain meds while holding her bladder because she doesn t have time to pee starving because she missed her break being pooped peed bled on and is missing her family while taking care of yours in the minute you took to read this nurses all over the world are saving lives re post this if you are a nurse love a nurse or appreciate one

rank	copies	variant
n2	18068	somewhere right now a nurse is getting yelled at for being late with meds holding their bladder because they don t have time to pee starving because they missed lunch being peed on puked on pooped on bled on and is missing their family while taking care of yours in the minute it took you to read this nurses all over the world are saving lives repost if you are a nurse love a nurse or appreciate a nurse

- Some memes rarely mutate
- Some do more often

Adamic et al., 201

Facebook study

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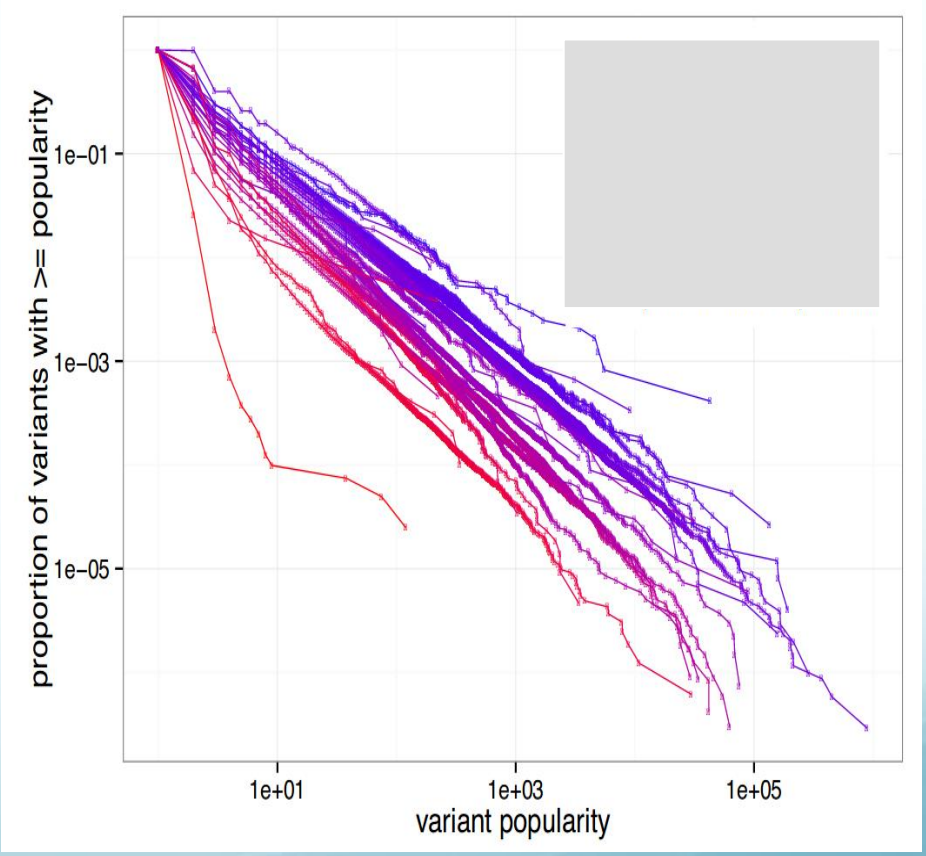

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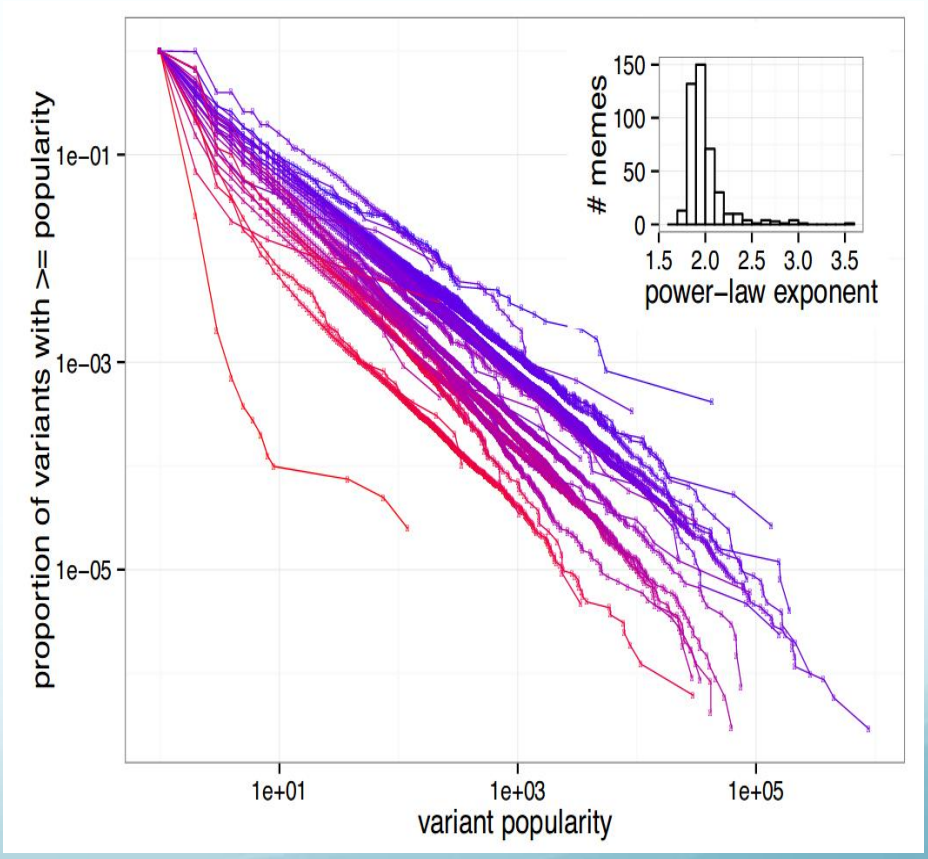

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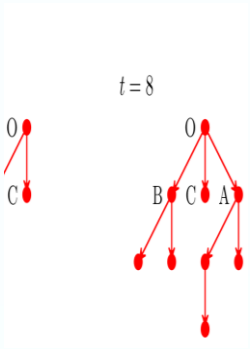
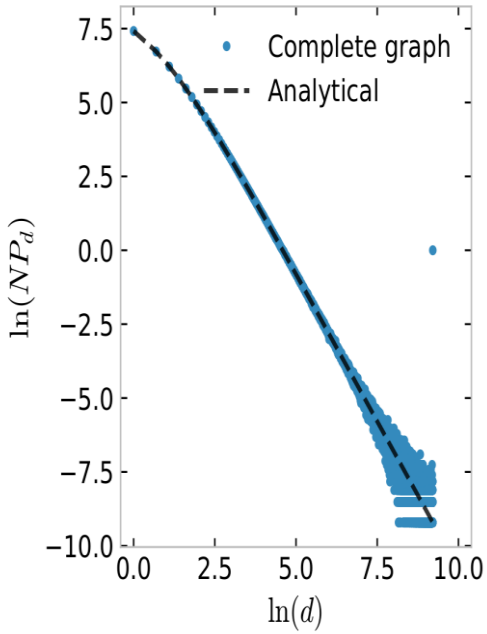
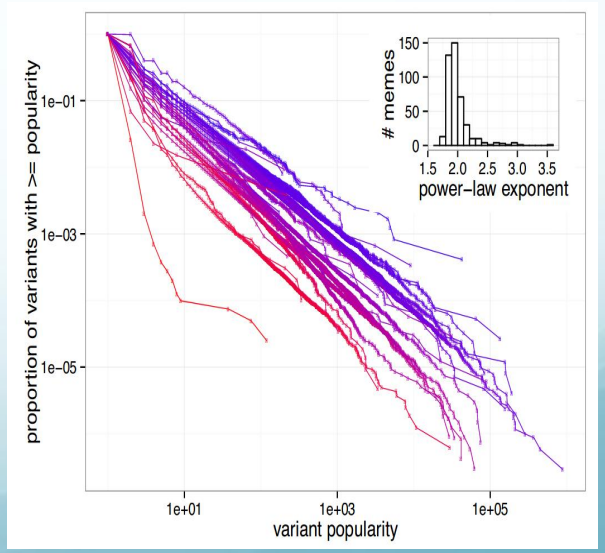
Adamic et al., 2016

Conclusions

- Provided theoretical results for descendant distributions
- Found that many declined as d^{-2}
- Found that this inverse-square law is due to infinite dimensionality of these networks
- Showed that prediction is consistent with data from Facebook







Adamic et al., 2016

Thank you!