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The Stability of Democracies

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Stability of democracies: a complex systems perspective

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Abstract

The idea that democracy is under threat, after being largely dormant for at least 40 years, is looming increasingly large in public discourse. Complex systems theory offers a range of powerful new tools to analyse the stability of social institutions in general, and democracy in particular. What makes a democracy stable? And which processes potentially lead to instability of a democratic system? This paper offers a complex systems perspective on this question, informed by areas of the mathematical, natural, and social sciences. We

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What science can do for democracy – A complexity science approach

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Abstract

Political scientists have conventionally assumed that achieving democracy is a one-way ratchet. Only very recently has the question of 'democratic backsliding' attracted any research attention. We argue that democratic instability is best understood with tools from complexity science. The explanatory power of complexity science arises from several features of complex systems. Their relevance in the context of democracy is discussed. Several policy recommendations are offered to help (re)stabilize current systems of representative democracy.

Introduction

The *Economist* recently identified 80 countries whose democracy score declined during the last decade, including the U.S. and some consolidated European democracies (*The Economist Intelli-*

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Available at http://eliassi.org/pubs.html

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Measuring Tie Strength in Implicit Social Networks

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Measuring Tie Strength in Implicit Social Networks*

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ABSTRACT

Given a set of people and a set of events attended by them, we address the problem of measuring *connectedness* or *tie strength* between each pair of persons. The underlying assumption is that attendance at mutual events gives an implicit social network between people. We take an axiomatic approach to this problem. Starting from a list of axioms, which a measure of tie strength must satisfy, we characterize functions that satisfy all the axioms. We then show that there is a range of tie-strength measures that satisfy this characterization.

A measure of tie strength induces a ranking on the edges of the social network (and on the set of neighbors for every person). We show that for applications where the ranking, and not the absolute value of the tie strength, is the important thing about the measure, the axioms are equivalent to a natural partial order. To settle on a particular measure, we must make a non-obvious decision about extending this partial order to a total order. This decision is best left to particular applications. We also classify existing tie-strength measures according to the axioms that they satisfy; and observe that none of the "self-referential" tie-strength measures satisfy the axioms. In our experiments, we demonstrate the efficacy of our approach; show the completeness and soundness of our axioms, and present Kendall Tau Rank Correlation between various tie-strength measures.

Author Keywords

Social networks, tie strength, axiomatic approach

ACM Classification Keywords

J.4 Computer Applications: Social and Behavioral Sciences [Sociology]

General Terms

Theory, Measurement, Experimentation

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INTRODUCTION

Explicitly declared friendship links suffer from a low signal to noise ratio (e.g. Facebook friends or LinkedIn contacts). Links are added for a variety of reasons like reciprocation, peer-pressure, etc. Detecting which of these links are important is a challenge.

Social structures are implied by various interactions between users of a network. We look at event information, where users participate in mutual events. Our goal is to infer the strength of ties between various users given this event information – i.e. the implicit social networks.

There has been a surge of interest in implicit social networks. We can see anecdotal evidence for this in startups like COLOR (`http://www.color.com`) and new features in products like Gmail. COLOR builds an implicit social network based on people's proximity information while taking photos.[1] Gmail's *don't forget bob* feature uses an implicit social network to suggest new people to add to an email given a existing list [16].

Consider people attending different events with each other. We define an *event* by the set of people that attend it. An event can represent the set of people who took a photo at the same place and time, like COLOR, or a set of people who are on an email, like in Gmail. Given the set of events, we would like to infer how *connected* two people are – i.e. we would like to measure the *strength of the tie* between people. All that is known about each event is the list of people who attended it. People attend events based on an implicit social network (with ties between pairs of people). We want to solve the inference problem of finding the weighted social network that gives rise to the set of observed events.

Given a bipartite (a.k.a. two-mode) graph, with people as one set of vertices and events as the other set, we want to infer the tie-strength between the set of people. Hence, in our problem, we do not even have access to any directly declared (i.e., explicit) social network between people. We want to infer the *implicit* social network based on the set of people who interact together at different events.

We start with a set of axioms and find a characterization of functions that could serve as a measure of tie strength, just given the event information. We do not end up with a single function that works best under all circumstances. In fact, we show that there are non-obvious decisions that need to be made to settle down on a single measure of tie strength.

[1] `http://mashable.com/2011/03/24/color/`

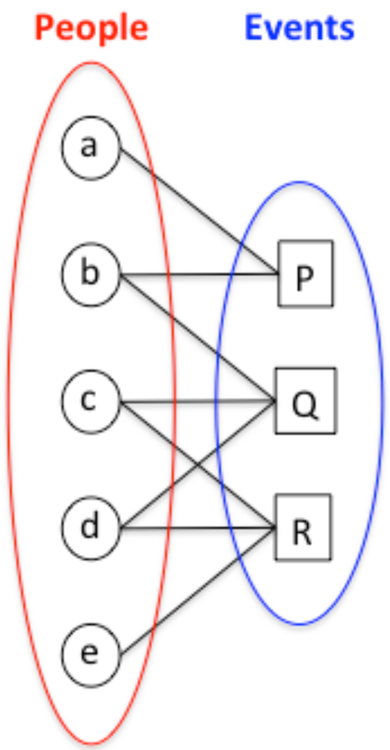
Mangesh Gupte

Available at http://eliassi.org/papers/gupte-websci12.pdf

Problem Definition

- Given a bipartite graph with people as one set of vertices and events as the other set, measure *tie strength* between each pair of individuals

- Assumption
  - Attendance at mutual events implies an implicit weighted social network between people



Motivation

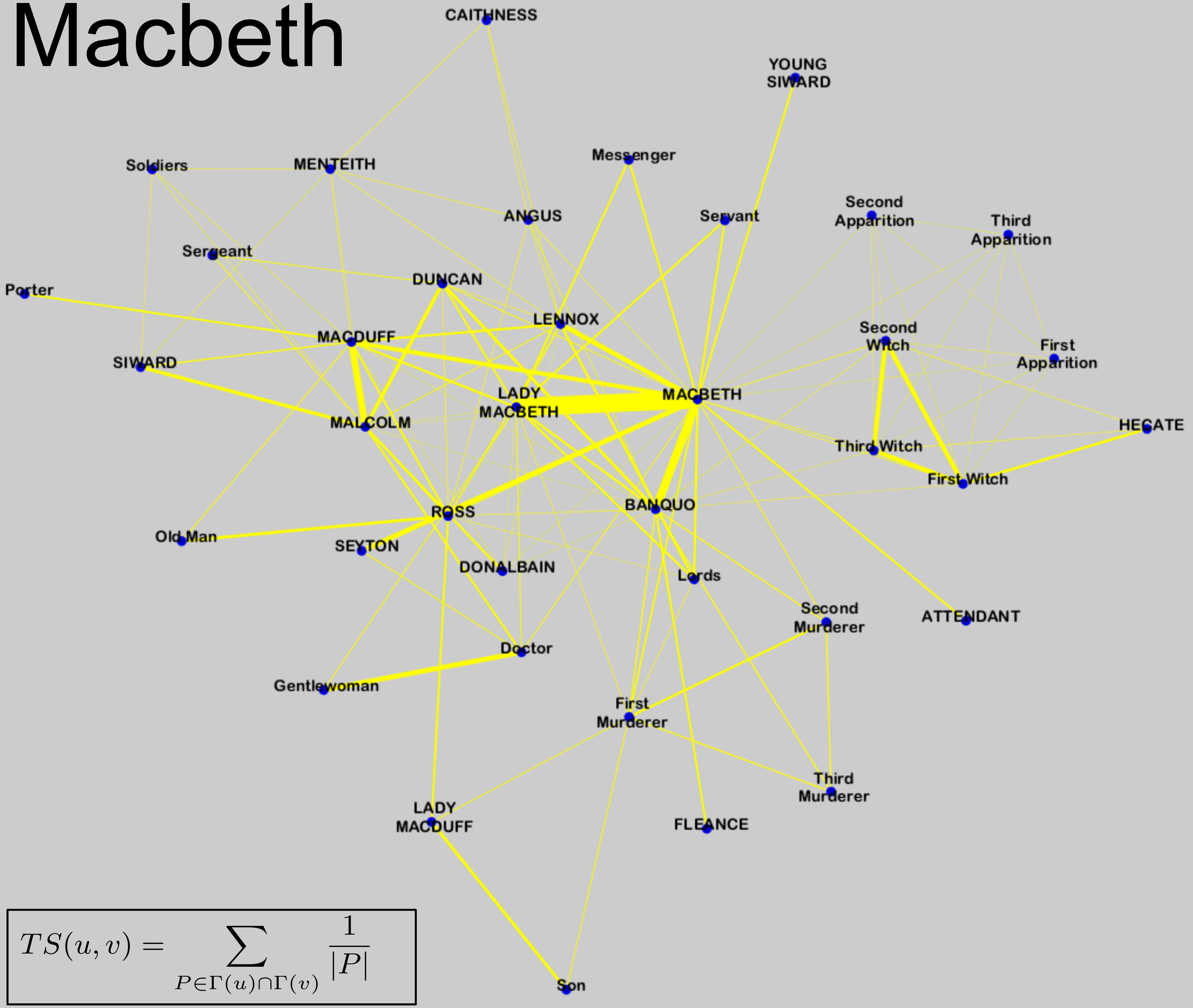
- Most real-world networks are bipartite and are converted to unipartite (e.g., via AA^T)
- Explicitly declared friendship links can suffer from a low signal-to-noise ratio (e.g., Facebook friends)
- **Challenge:** Detect which of links in the unipartite graph are important
- **Goal:** Infer the implicit weighted social network from people's participation in mutual events

Tie Strength

- A measure of tie strength induces
  - a ranking on all the edges, and
  - a ranking on the set of neighbors for every person

- Example of a simple tie-strength measure
  - Common neighbor measures the total number of common events to a pair of individuals

Macbeth



$$TS(u,v) = \sum_{P \in \Gamma(u) \cap \Gamma(v)} \frac{1}{|P|}$$

Decisions, Decisions

There are many different measures of tie-strength

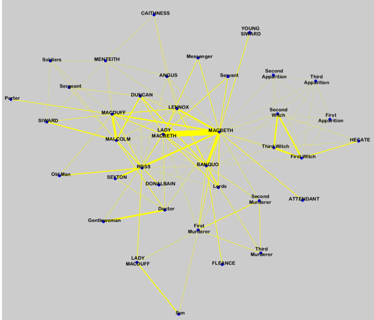
1. Common neighbor
2. Jaccard index
3. Max
4. Linear
5. Delta
6. Adamic and Adar
7. Preferential attachment
8. Katz measure
9. Random walk with restarts
10. Simrank
11. Proportional
12. …

Which one should you choose?

Roadmap

- An axiomatic approach to the problem of inferring implicit social networks by measuring tie strength
- A characterization of functions that satisfy all our axioms
- Classification of prior measures according to the axioms that they satisfy
- Experiments
- Summary

Running Example

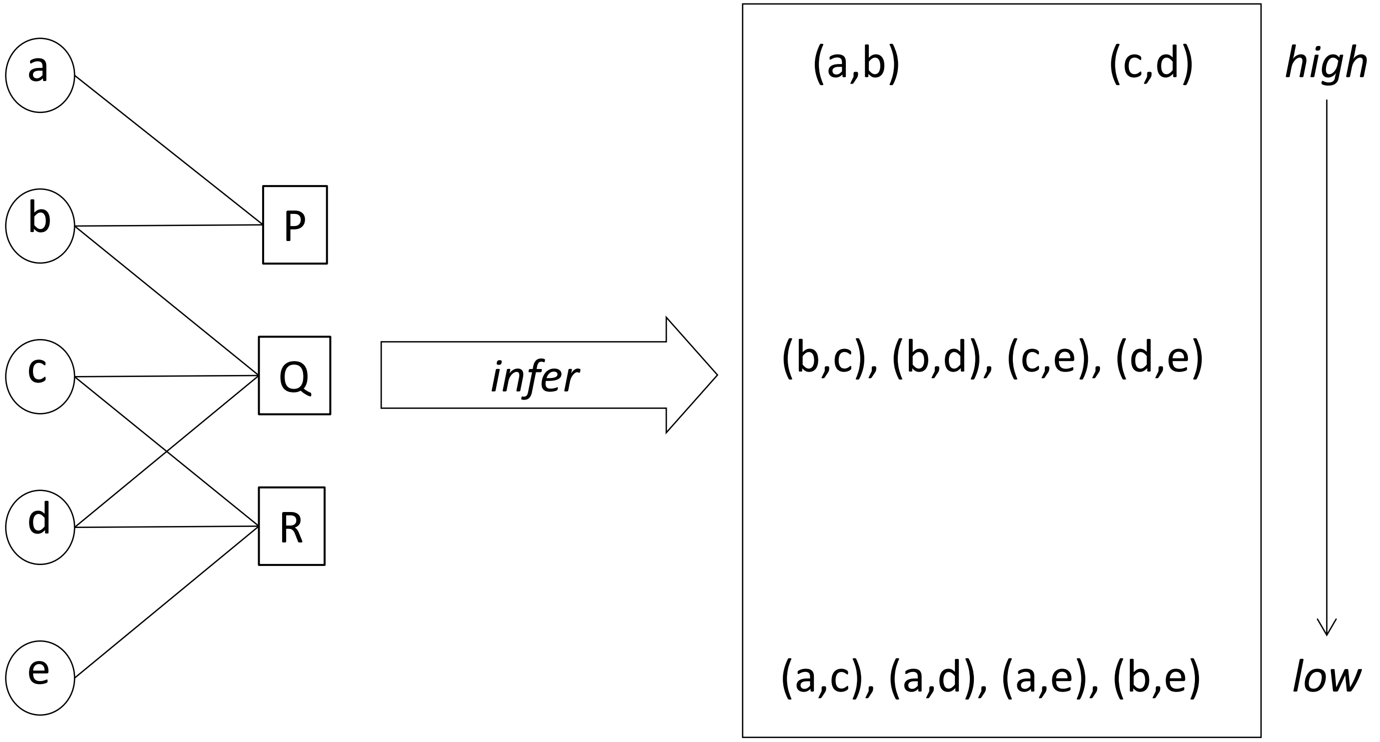


Input

People × Event Bipartite Graph

Output

Partial Order of Tie Strength among People

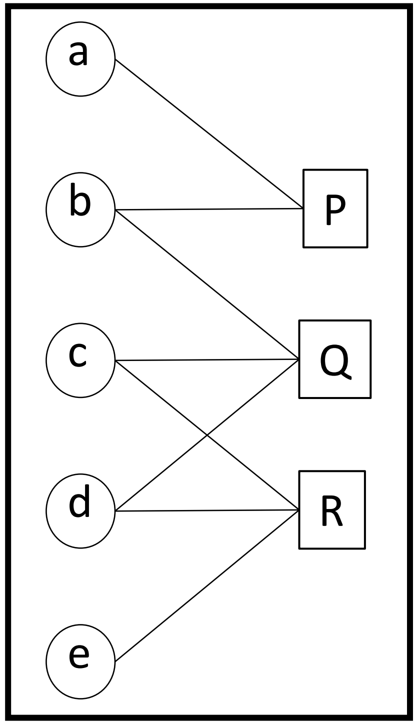


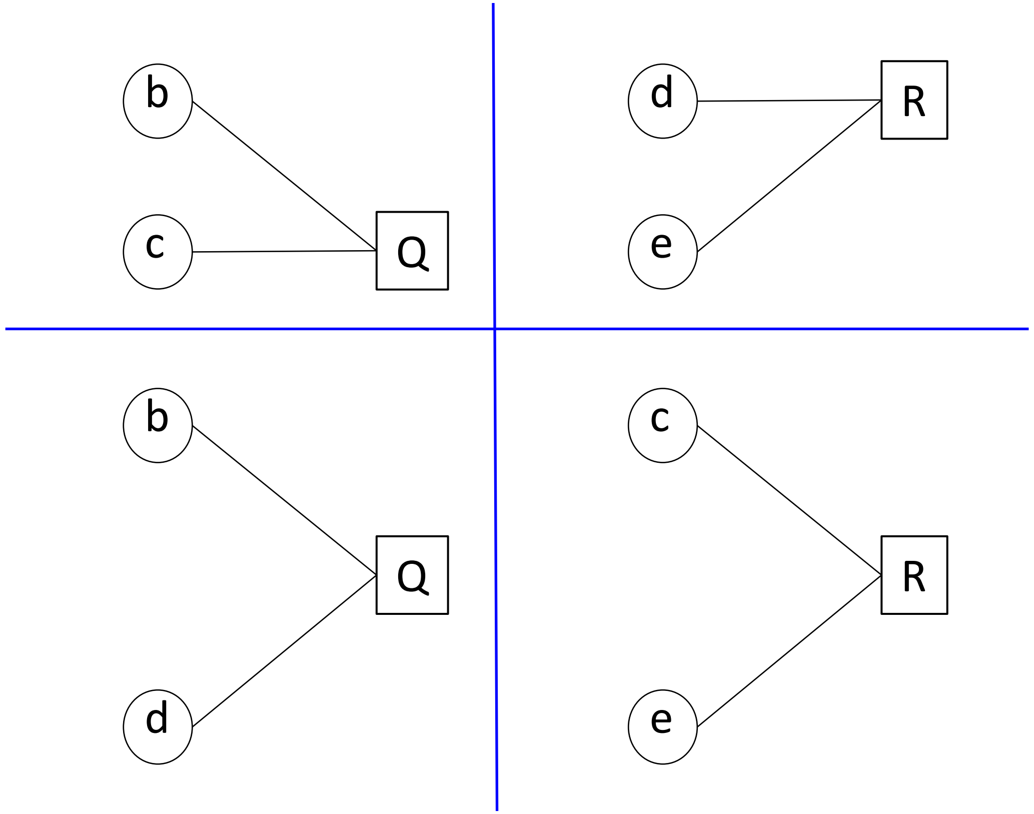
Axioms

- Axiom 1: Isomorphism
- Axiom 2: Baseline
- Axiom 3: Frequency
- Axiom 4: Intimacy
- Axiom 5: Popularity
- Axiom 6: Conditional Independence of People
- Axiom 7: Conditional Independence of Events
- Axiom 8: Submodularity

Axiom 1: Isomorphism

- Tie strength between *u* and *v* is independent of the labels of *u* and *v*





Axiom 2: Baseline

- If there are no events, then tie strength between each pair *u* and *v* is 0

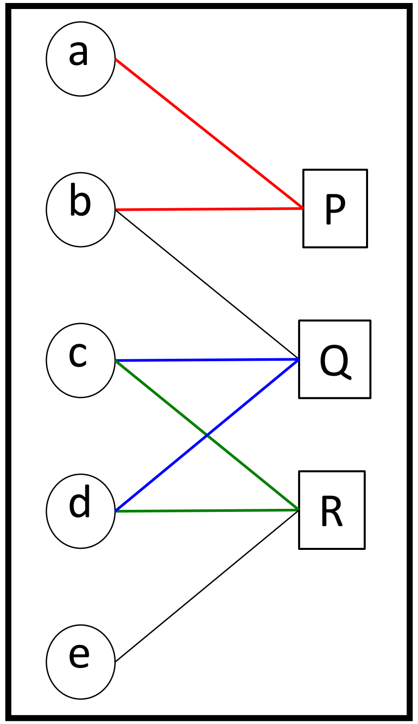
$$TS_{\varnothing}(u, v) = 0$$

- If there are only two people *u* and *v* and a single event *P* that they attend, then their tie strength is at most 1

$$TS_P(u, v) \leq 1$$

  - Defines an upper-bound for how much tie strength can be generated from a single event between two people

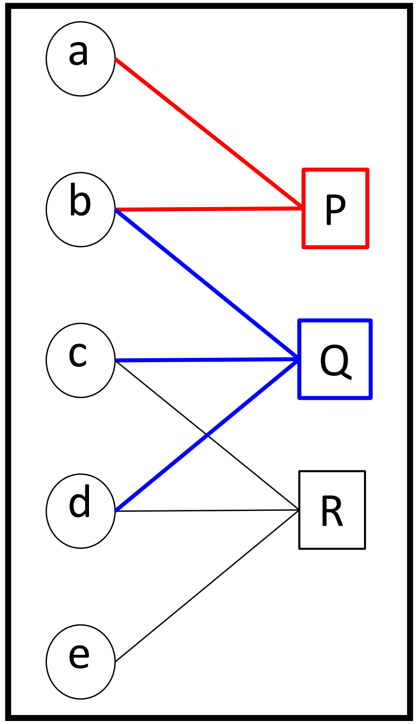
Axiom 3: Frequency & Axiom 4: Intimacy



- Axiom 3 (**Frequency**)
  - More events create stronger ties
  - All other things being equal, the more events common to *u* and *v*, the stronger their tie-strength
- Axiom 4 (**Intimacy**)
  - Smaller events create stronger ties
  - All other things being equal, the fewer invitees there are to any particular event attended by *u* and *v*, the stronger their tie-strength

Axiom 5: Popularity

- Larger events create more ties
- Consider two events *P* and *Q*
- If $|Q| > |P|$, then the total tie strength created by *Q* is more than that created by *P*



Axioms 6 & 7: Conditional Independence of People and of Events

- Axiom 6: **Conditional Independence of People**
  - A node *u*'s tie strength to other people does **not** depend on events that *u* does **not** attend
- Axiom 7: **Conditional Independence of Events**
  - The increase in tie strength between *u* and *v* due to an event *P* does **not** depend on other events, just on the existing tie strength between *u* and *v*
  - $TS_{(G+P)}(u, v) = g(TS_G(u, v), TS_P(u, v))$
    - where *g* is some monotonically increasing function

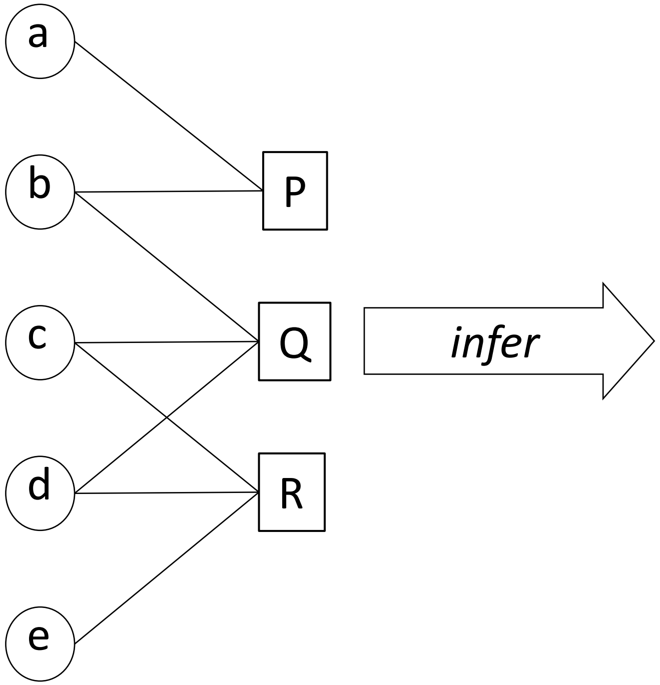
Axiom 8: Submodularity

- The marginal increase in tie strength of *u* and *v* due to an event *Q* is at most the tie strength between *u* and *v* if *Q* was their only event
- If *G* is a graph and *Q* is a single event, then $TS_{(G+Q)}(u, v) - TS_G(u, v) \leq TS_Q(u, v)$

Example – Mapping to Axioms

Input

People × Event Bipartite Graph



Output

Partial order of Tie Strength

Tie strength	Pairs	Axioms
high	(a,b) (c,d)	*Axiom 4 (Intimacy) & Axiom 3 (Freq)*
	(b,c), (b,d), (c,e), (d,e)	*Axiom 1 (Isomorphism)*
low	(a,c), (a,d), (a,e), (b,e)	*Axiom 2 (Baseline) & Axiom 6 (Cond. Indep. of Vertices) & Axiom 7 (Cond. Indep. of Events)*

Observations on the Axioms

- Our axioms are fairly intuitive

A1: Isomorphism	A2: Baseline	A3: Frequency	A4: Intimacy
A5: Popularity	A6: Cond. Indep. of people	A7: Cond. indep. of events	A8: Submodularity

- But, several previous measures in the literature break some of these axioms
- Satisfying all the axioms is not sufficient to uniquely identify a measure of tie strength
  - One reason: inherent tension between Axiom 3 (Frequency) and Axiom 4 (Intimacy)

Inherent Tension Between Frequency & Intimacy

- Scenario #1 (intimate)
  - Mary and Susan go to 2 parties, where they are the only people there.
- Scenario #2 (frequent)
  - Mary, Susan, and Jane go to 3 parties, where they are the only people there.
- In which scenario is Mary’s tie to Susan stronger?

Observations on the Axioms (cont.)

A1: Isomorphism	A2: Baseline	A3: Frequency	A4: Intimacy
A5: Popularity	A6: Cond. Indep. of people	A7: Cond. indep. of events	A8: Submodularity

- Axioms are equivalent to a natural partial order on the strength of ties
  - Pertinent to ranking application
- Choosing a particular tie-strength function is equivalent to choosing a particular linear extension of this partial order
  - Non-obvious decision
  - Details in paper: http://eliassi.org/papers/gupte-websci12.pdf

Preamble to the Characterization Theorem

- Let *f*(*n*) = total tie strength generated in a single event with *n* people
- If there is a single party with *n* people, the tie strength of each tie is $\frac{f(n)}{\binom{n}{2}}$
    - Based on Axiom 1 (Isomorphism)
- The total tie strength created at an event *P* with *n* people is a monotone function *f*(*n*) that is bounded by $1 \leq f(n) \leq \binom{n}{2}$
    - Based on Axiom 2 (Baseline) and Axiom 4 (Intimacy) and Axiom 5 (Popularity)

Characterizing Tie Strength

Theorem. Given a graph $G = (L \cup R, E)$ *and two vertices u and v, if* the tie-strength function *TS* follows Axioms (1-8), then the function has to be of the form

$$TS_G(u, v) = g(h(|P_1|), h(|P_2|), \ldots, h(|P_k|))$$

- $\{P_i\}_{1 \leq i \leq k}$ are the events common to both *u* and *v*
- h is a monotonically decreasing function bounded by $1 \geq h(n) \geq \frac{1}{\binom{n}{2}}, n \geq 2; h(1) = 1; h(0) = 0.$
- g is a monotonically increasing submodular function

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Many Measures of Tie Strength

1. Common neighbor
2. Jaccard index
3. Max
4. Linear
5. Delta
6. Adamic and Adar
7. Preferential attachment
8. Katz measure
9. Random walk with restarts
10. Simrank
11. Proportional

$$TS(u,v) = |\Gamma(u) \cap \Gamma(v)|$$

$$TS(u,v) = \frac{|\Gamma(u) \cap \Gamma(v)|}{|\Gamma(u) \cup \Gamma(v)|}$$

$$TS(u,v) = \max_{P \in \Gamma(u) \cap \Gamma(v)} \frac{1}{|P|}$$

$$TS(u,v) = \sum_{P \in \Gamma(u) \cap \Gamma(v)} \frac{1}{|P|}$$

$$TS(u,v) = \sum_{P \in \Gamma(u) \cap \Gamma(v)} \frac{1}{\binom{|P|}{2}}$$

$$TS(u,v) = \sum_{P \in \Gamma(u) \cap \Gamma(v)} \frac{1}{\log |P|}$$

$$TS(u,v) = |\Gamma(u)| \cdot |\Gamma(v)|$$

$$TS(u,v) = \sum_{q \in \text{ path between } u,v} \gamma^{-|q|}$$

$$TS(u,v) = \begin{cases} 1 & \text{if u = v} \\ \gamma \cdot \frac{\sum_{a \in \Gamma(u)} \sum_{b \in \Gamma(v)} TS(a,b)}{|\Gamma(u)| \cdot |\Gamma(v)|} & \text{otherwise} \end{cases}$$

$$TS(u,v) = \sum_{P \in \Gamma(u) \cap \Gamma(v)} \frac{\epsilon}{|P|} + (1-\epsilon) \frac{TS(u,v)}{\sum_{w \in \Gamma(u)} TS(u,w)}$$

Non Self-Referential Tie Strength Measures

- **Common neighbor**
  - The total # of common events that both u and v attended
- **Jaccard Index**
  - Similar to common neighbor
  - Normalizes for how “social” *u* and *v* are
- **Adamic and Adar** [2003], **Delta**, and **Linear**
  - Tie strength increases with the number of events
  - Tie strength is 1 over a simple function of event size
- **Max**
  - Tie strength does not increase with the number of events
  - Tie strength is the maximum tie strength from all common events

Self-Referential Tie-Strength Measures

- **Katz measure** [Katz,1953]
  - Tie strength is the number of paths between u and v, where each path is discounted exponentially by the length of the path
- **Random walk with restarts**
  - A non-symmetric measure of tie strength
  - Tie strength is the stationary probability of a Markov chain process
  - With probability α, jump to a node u; and with probability 1-α, jump to a neighbor of a current node.
- **Simrank** [Jeh & Widom, 2002]
  - Tie strength is captured by recursively computing the tie strength of neighbors
- **Proportional**
  - Tie strength increases with # of events
  - People spend time proportional to their tie-strength at a party

Measures of Tie-Strength that Satisfy All the Axioms

A1: Isomorphism	A2: Baseline	A3: Frequency	A4: Intimacy
A5: Popularity	A6: Cond. indep. of P	A7: Cond. indep. of E	A8: Submodularity

	A1	A2	A3	A4	A5	A6	A7	A8	$g(a_1, \ldots, a_k)$ $h(\lvert P_i \rvert) = a_i$
Common Neighbors	✓	✓	✓	✓	✓	✓	✓	✓	$g(a_1, \ldots, a_k) = \Sigma a_i$ $h(n) = 1$
Delta	✓	✓	✓	✓	✓	✓	✓	✓	$g(a_1, \ldots, a_k) = \Sigma a_i$ $h(n) = 2(n(n-1))^{-1}$
Adamic & Adar	✓	✓	✓	✓	✓	✓	✓	✓	$g(a_1, \ldots, a_k) = \Sigma a_i$ $h(n) = (\log(n))^{-1}$
Linear	✓	✓	✓	✓	✓	✓	✓	✓	$g(a_1, \ldots, a_k) = \Sigma a_i$ $h(n) = n^{-1}$
Max	✓	✓	✓	✓	✓	✓	✓	✓	$g(a_1, \ldots, a_k) = \max\{a_i\}$ $h(n) = n^{-1}$

Measures of Tie-Strength that Do **Not** Satisfy All the Axioms

A1: Isomorphism	A2: Baseline	A3: Frequency	A4: Intimacy
A5: Popularity	A6: Cond. indep. of P	A7: Cond. indep. of E	A8: Submodularity

	A1	A2	A3	A4	A5	A6	A7	A8	$g(a_1, \ldots, a_k)$ $h(\|P_i\|) = a_i$
Jaccard Index	✓	✓	✓	✓	✓	✗	✗	✗	✗
Katz Measure	✓	✗	✓	✓	✓	✓	✗	✗	✗
Preferential Attachment	✓	✓	✗	✓	✓	✓	✗	✗	✗
RWR	✓	✗	✗	✗	✓	✓	✗	✗	✗
Simrank	✓	✗	✗	✗	✗	✗	✗	✗	✗
Proportional	✓	✗	✗	✓	✗	✓	✗	✗	✗

Data Sets

Graphs	# of People	# of Events
Southern Women	18	14
The Tempest	19	34
A Comedy of Errors	19	40
Macbeth	38	67
Reality Mining Bluetooth	104	326,248
Enron Emails	32,471	371,321

Completeness of Axioms 1-8

(Number of Ties **Not** Resolved by the Partial Order)

Dataset	Tie Pairs	Incomparable Pairs (%)
Southern Women	11,628	683 (5.87)
The Tempest	14,535	275 (1.89)
A Comedy of Errors	14,535	726 (4.99)
Macbeth	246,753	584 (0.23)
Reality Mining	13,794,378	1,764,546 (12.79)

- % of tie-pairs where different tie-strength functions can differ
  - Smaller is better
  - Generally, percentages are small
  - Large real-world networks have more unresolved ties

# of tie pairs = $\binom{\binom{n}{2}}{2}$

Take-away Point #1

- % of tie pairs on which different tie strength functions can differ is small.*

* Disclaimer: For ranking applications <u>and</u> tie-strength functions that satisfy the axioms

Two Tie-Strength Functions that Do **Not** Satisfy the Axioms

- Jaccard Index
  - Normalizes for how “social” *u* and *v* are

$$TS(u,v) = \frac{|\Gamma(u) \cap \Gamma(v)|}{|\Gamma(u) \cup \Gamma(v)|}$$

- Temporal Proportional
  - Increases with number of events
  - People spend time proportional to their tie-strength in a party
  - Events are ordered by time

$$TS(u,v,t) = \begin{cases} TS(u,v,t-1) & \text{if } u \text{ and } v \text{ do not attend } P_t \\ \epsilon \frac{1}{|P_t|} + (1-\epsilon) \frac{TS(u,v,t-1)}{\sum_{w \in P_t} TS(u,w,t-1)} & \text{otherwise} \end{cases}$$

Soundness of Axioms 1-8

(Number of Conflicts Between the Partial Order and Tie-Strength Functions **Not** Satisfying the Axioms)

Dataset	Tie Pairs	Jaccard (%)	Temporal (%)
Southern Women	11,628	1,441 (12.39)	665 (5.72)
The Tempest	14,535	488 (3.35)	261 (1.79)
A Comedy of Errors	14,535	1,114 (7.76)	381 (2.62)
Macbeth	246,753	2,638 (1.06)	978 (0.39)
Reality Mining	13,794,378	290,934 (0.02)	112,546 (0.01)

- % of tie-pairs in conflict with the partial order
  - Smaller is better
  - Generally, percentages are small
  - They decrease as the dataset increases

More on Soundness

- **Question 1:**

  Are the number of conflicts, between the partial order and tie-strength functions not satisfying the axioms, small because most of the tie-strengths are zeros (sparsity of real graph)?

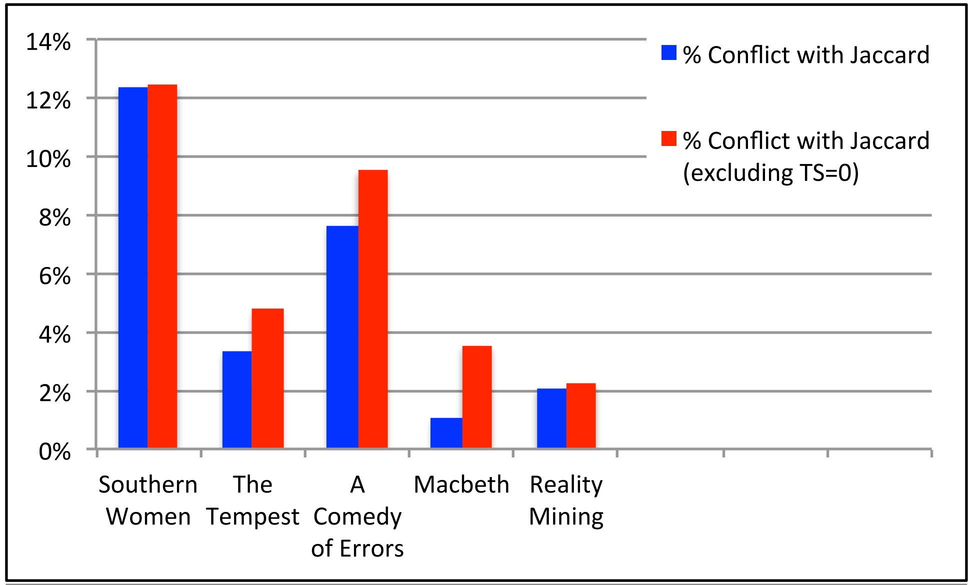
- **Answer:**
  - This is partially true.
  - For some pairs, the tie-strength being set to zero is caused by the axioms.
  - It may or may not be true that all these pairs have tie-strength zero in the actual function used.
    - For example, this won't be true for some self-referential functions like Simrank, Random Walk with Restart, etc.

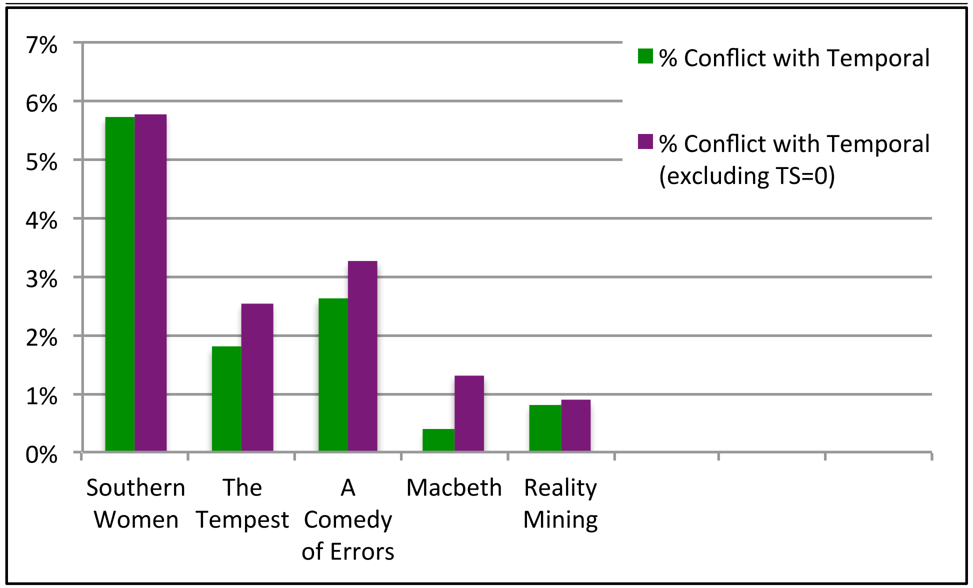
Even More on Soundness

- **Question 2:** How do the conflict numbers change if we only looked at tie pairs that have nonzero tie-strengths?
- **Answer:** The percentages go up but not by much.

Dataset	Tie Pairs	Tie Pairs (excluding TS=0)	Jaccard	Temporal
Southern Women	11,628	11,537	1,441	665
The Tempest	14,535	10,257	488	261
A Comedy of Errors	14,535	11,685	1,114	381
Macbeth	246,753	74,175	2,638	978
Reality Mining	13,794,378	12,819,272	290,934	112,546

Even More on Soundness





Take-away Point #2

- % of conflicts between our axioms and tie-strength functions that do not satisfying our axioms is small.*

* Disclaimer: For ranking applications

Put Take-away Points #1 & #2 Together

1. % of tie pairs on which different tie-strength functions can differ is small
2. % of conflicts between our axioms and tie-strength functions not satisfying our axioms is small

If your application is ranking, just pick the most computationally efficient tie-strength measure (e.g. common neighbor).

Disclaimer: For ranking applications

Related Work

- Strength of ties
  - Spread of information in social networks [Granovetter, 1973]
  - Use external information to learn strength of tie
    - [Gilbert & Karahalios, 2009], [Kahanda & Neville, 2009]
- Link prediction
  - No axiomatic work that I know of
  - See a theoretical justification of popular link prediction heuristics [Sarkar, Chakrabarti, Moore, 2010]

Summary

- Presented an axiomatic approach to the problem of inferring implicit social networks by measuring tie strength
- There are relatively few axiomatic work on graph measures
    1. Central positions in social networks [Brandes, 2020]
    2. Axioms for centrality [Boldi & Vigna, 2014]
    3. An axiomatic approach to trust-based recommendation systems [Andersen et al, 2008]
    4. PageRank axiomatization [Altman & Tennenholtz, 2005]
    5. Information theoretic measure of similarity [Lin, 1998]