Obtaining a Position Fix Using Doppler Shift Measurements

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Abstract

The original *Transit* Doppler navigation system used a complex communications system[5] to augment and significantly enhance the position fix algorithm. The system, which was the precursor to GPS, used two carrier frequencies to remove ionospheric effects, with both transmissions broadcasting orbital data to provide accurate fixes. This paper describes a simple algorithm to obtain a position fix given Doppler shift measurements from arbitrary satellite constellations. Simulations are described to demonstrate the performance of the algorithm.

Problem Statement

Obtaining a position fix is simple exercise in optimization. The algorithm determines the position on earth that minimizes the difference between the measured Doppler shift and the expected Doppler shift at that location. This is a simple concept, but in most cases an algebraic solution does not exist. Therefore numerical optimization must be used. Additionally, algebraic expressions for the expected Doppler shifts aren't obtainable, which results in an expensive-to-evaluate cost function, or function we would like to minimize to find our position.

Optimization Algorithms

Optimization algorithms are divided into two categories – stochastic methods and iterative methods. Stochastic methods are not guaranteed to produce an exact solution, but frequently have large search spaces.[6] Iterative methods are locally convergent, but frequently find only local extrema. Figure 1 shows the "view" from an iterative method - it is very clear which direction to go in, but it is not clear whether or not the minimum will be global. Figure 2 shows the result from a stochastic algorithm - it is clear which region contains the global minima, but the precise location is unclear.

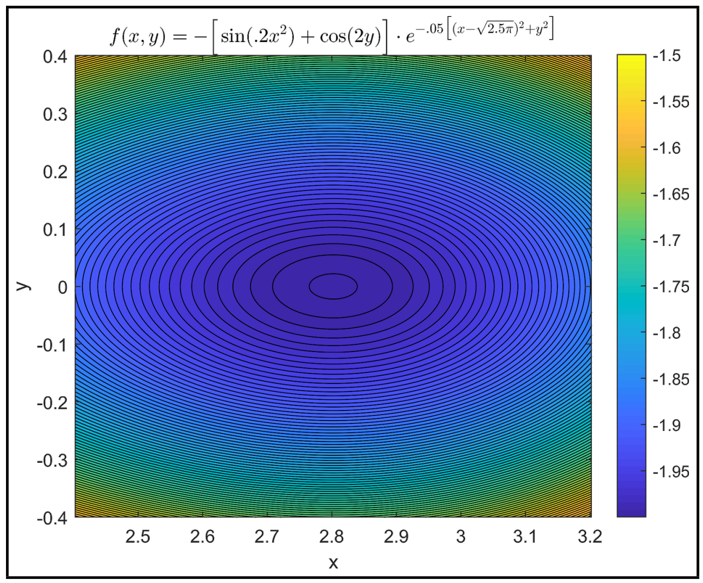

Figure 1: Rastrigin Function - Contour Plot

Newton's Method

Newton's method is a well-studied root finding algorithm with quadratic convergence.[2] It requires a continuous first derivative, and requires an initial guess that is in the neighborhood of a root. While the first requirement isn't an issue in most cases, the second requirement is a major shortcoming of Newton's method. Consider the function $f(x) = \arctan(x)$. Figure 3 shows two lines, $|x_k|$ and $|x_{k+1}|$, for $f(x) = \arctan(x)$, where x_k is the current

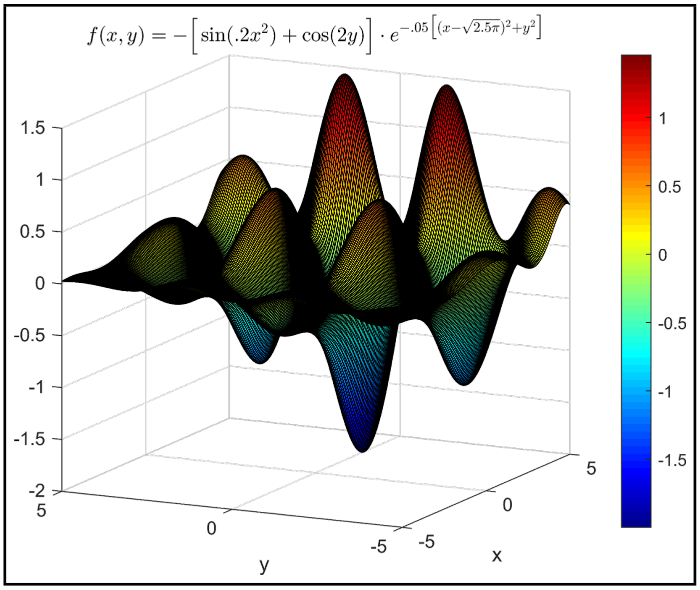

Figure 2: Rastrigin Function

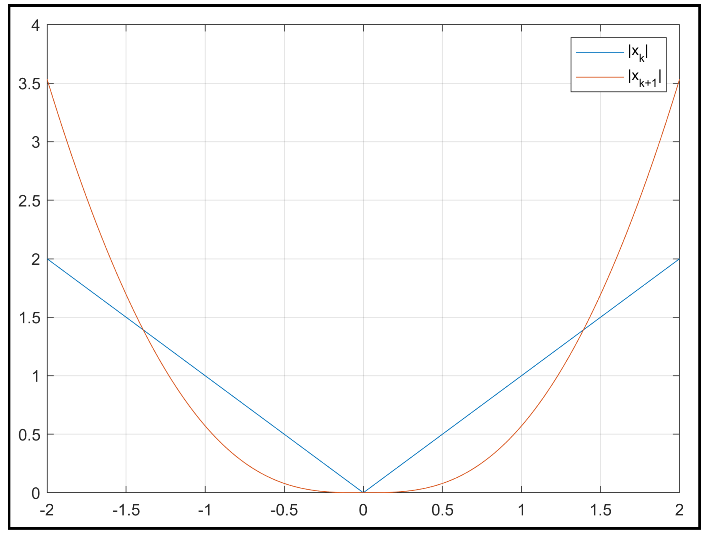

Figure 3: Region of Convergence - $f(x) = \arctan(x)$

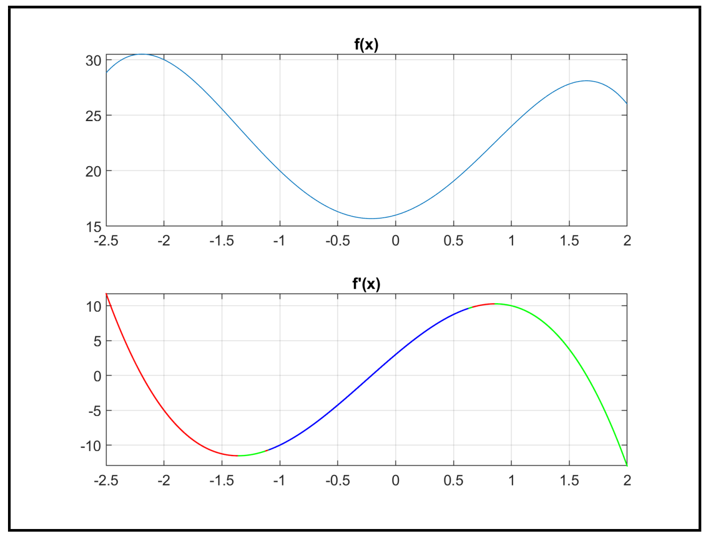

Figure 4: Region of Convergence - $f(x) = -x^4 - x^3 + 7x^2 + 3x - 16$

approximation of the root, and x_{k+1} is the new approximation after one step of Newton's method. If $|x_k| \geq 1.39$, then $|x_{k+1}| > |x_k|$ and the algorithm moves further away from the solution. This means that the initial guess must satisfy $|x_0| < 1.39$. If we were given a black-box with $f(x) = \arctan(x)$, we would need to start extremely close to the solution in order to find the solution. This is circular logic.

The need to provide an initial guess near the solution is far-reaching, and has serious implications for optimization. Figure 4 shows a simple fourth degree polynomial $f(x) = -x^4 - x^3 + 7x^2 + 3x - 16$ and its derivative. The colored lines denote the basins of attraction for each root of the derivative. To find the local minimum, the initial guess must be in the blue region, otherwise the algorithm will converge to a local maximum. In my initial Doppler analysis simulations, I experienced extremely poor convergence to the global minimum using Newton's method. As a result, I began a search for optimization techniques less prone to premature convergence to local extrema.

Differential Evolution

While researching algorithms to avoid local extrema, I stumbled upon the Differential Evolution (DE) algorithm. DE is a simple stochastic algorithm designed to provide "an efficient framework to solve complicated optimization tasks."[3] It works by iteratively improving a set of candidate solutions by comparing the candidates to random linear combinations of other candidates. That is, a new candidate solution is generated, and if the new candidate is an improvement, it replaces the original population member. This happens for every member of the population for each iteration. DE converges much slower than Newton's method, but is extremely simple to implement, doesn't require knowledge of the gradient of the cost function, and can search large solution spaces. In the example from Figure 4, my implementation of Differential Evolution found the global minimum in every trial.

The main disadvantage of DE algorithms is the need to evaluate the cost function many times, while the advantage is the ability to search large spaces for extrema.

Adaptations

DE is also prone to converging to local extrema. According to an experiment in [3], the default DE algorithm converged to the global minimum in just 2% of trials. As a result, the algorithm was adapted to maintain population diversity. Population diversity prevents premature convergence (contraction of the solution space) to a local extrema.[4] The proposed modification was to replace the worst candidates with new random candidates from the original solution space. In the same experiment, this modification improved convergence from 2% to 83%[3]. Replacing the worst candidates with random samples ensures that the solution space will never shrink prematurely, if at all, and significantly improves the ability to jump out of a basin of attraction. Additionally, I further modified the algorithm to replace any candidates outside a desired solution space.

My Algorithm

The task was to optimize an expensive-to-evaluate cost function, with no algebraic representation, for a reasonably large number of data points. Differential Evolution provides the ability to search a large solution space, many kilometers wide. To reduce the computational burden of DE, I use only the most significant data points in the initial search. In theory, one can pinpoint the location of a receiver in the orbital plane using just the point of inflection in the Doppler curve.[1] The angle to the orbital plane can be determined by the width of the Doppler curve. Only the first two measurements, the last two measurements, and two midpoints of the Doppler curve are used in the initial DE computations. This provides a crude estimate of the receiver position. This initial guess is used as the starting point for Newton's method with all of the data points, resulting in quick and precise convergence of Newton's method to the global extrema approximated by the DE algorithm.

Initial Results

To simulate the system, I generated Doppler data for a single satellite pass for a fixed receiver position, then added 1Hz of Additive White Gaussian Noise, or AWGN, to the measurements. I then ran the optimization algorithm on the simulated data. Results should be taken with a grain of salt, as the simulation assumed that the TLE data was error-free. TLE error directly translates to position fix error. To mitigate this issue, I plan to use multiple satellites to average out the TLE error and obtain a more accurate fix. With an initial guess provided within 2° of longitude and latitude, the algorithm converged 74/75 times using the simulated data.

Summary

The problem was to globally optimize an expensive-to-evaluate cost function. While countless optimization techniques exist, many are prone to premature convergence, or convergence to local extrema. Optimization algorithms can generally be divided into two categories – fast, local methods, and slow, global methods. I used the exploratory capabilities of the modified Differential Evolution[3] algorithm to avoid local extrema; and took advantage of the extremely fast, local convergence of Newton's method to pinpoint the global extrema from the "best guess" of the DE algorithm. This produced a reliable algorithm that can be used to generate an accurate position fix from Doppler shift measurements.

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