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Acyclic Mechanism Design for Freight Consolidation

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Freight consolidation is a logistics practice that improves the cost-effectiveness and efficiency of transportation operations, and also reduces energy consumption and carbon footprint. A "fair" shipping cost sharing scheme is indispensable to help establish and sustain the cooperation of a group of suppliers in freight consolidation. In this paper, we design a truthful acyclic mechanism to solve the cost-sharing problem in a freight consolidation system with one consolidation center and one common destination. Applying the acyclic mechanism, the consolidation center decides which suppliers' demands ship via the consolidation center and their corresponding cost shares based on their willingness to pay for the service. The proposed acyclic mechanism is designed based on bin packing solutions that are also strong Nash equilibria for a related non-cooperative game. We study the budget-balance of the mechanism both theoretically and numerically. We prove a 2-budget-balance guarantee for the mechanism in general and better budget-balance guarantees under specific problem settings. Empirical tests on budget-balance show that our mechanism performs much better than the guaranteed budget-balance ratio. We also study the economic efficiency of our mechanism numerically to investigate its impact on social welfare under different conditions.

Key words: freight consolidation, cost allocation, acyclic mechanism, bin packing, game theory
History:

1. Introduction

The recent rapid growth of e-commerce and the accompanying increased demand for moving commodities in a reliable, efficient and secure logistics network have stimulated the transportation sector in the United States. The transportation sector contributed 8.9% to the Gross Domestic Product (GDP) of the U.S. economy in 2016 (Bureau of Transportation Statistics 2018). The logistics sector, a part of this transportation sector, however, operates neither efficiently nor sustainably (Montreuil 2011). For instance, statistics show that trailers on the road are only approximately

60% full on average and about 20% of the trailers are traveling completely empty (Montreuil 2011). Moreover, in 2015, approximately 27% of the total United States CO_2 emissions, the biggest source of greenhouse gases (GHG) emissions, came from transportation, primarily due to burning fossil fuel for cars, trucks, ships, trains and planes (U.S. Environmental Protection Agency 2017). As a result, cost-effective, efficient and environmentally friendly logistics practices are urgently needed. Freight consolidation, the process of assembling and transporting small shipments together to take advantage of lower freight rates, is an effective strategy to increase capacity utilization, improve cost-effectiveness of operations, and reduce energy consumption and carbon footprint. To this end, more and more companies are consolidating and shipping their demands using shared transportation capacity.

Both academic research and industry practices have demonstrated the effectiveness of freight consolidation in reducing carbon footprint. Using a discrete-time-based shipment consolidation strategy, Ülkü (2012) showed that freight consolidation directly helps reduce the emissions of CO_2. At the strategic level, Pan, Ballot, and Fontane (2009) concluded that freight consolidation can reduce CO_2 emissions by 14% based on real data from two French retailers, and this reduction would be 52% if rail transport is also considered. Moreover, a successful implementation of freight consolidation between two pharmaceutical companies, UCB and Baxter, achieved approximately a 50% reduction in CO_2 emissions (Mènèdéme 2011, Van Breedam, Vannieuwenhuyse, and Verstrepen 2011).

The economic incentive to consolidate freight is significant. Transportation costs play an important role in the success of various industries because they often account for a substantial portion of the product costs. For instance, large portions of revenues have long been paid to transport products in the agriculture industry (Nguyen et al. 2013). However, the constrained transportation capacity and the ever-increasing demand for logistics services with high service levels have contributed to the rise in transportation costs (Ward et al. 2019). Therefore, it is a priority for companies to reduce their transportation costs in order to remain competitive in the market. Significant cost savings have been reported through freight consolidation in various industries, (e.g. Cruijssen et al. 2010, Vanovermeire et al. 2014). Nguyen et al. (2013) also concluded that a freight consolidation practice could save $20 million on transportation cost per year for California cut flower growers.

Although the environmental and economic initiatives to consolidate freight are widely acknowledged, various concerns for establishing successful collaborations have slowed the implementation of freight consolidation among companies. Cruijssen, Cools, and Dullaert (2007) conducted a survey with approximately 1500 representative logistics service providers in Belgium and concluded that a lack of a fair cost/profit sharing scheme is one of the main hurdles that make individual logistics

service providers decide not to form collaborations. Freight consolidation often takes place among companies that produce similar products or provide similar services in the same geographical region and therefore they are also competitors in the market. Establishing cooperation among them is not possible unless there exists a perceived "fair" way to share the benefit of collaboration to ensure that each company maintains its competitive advantage over time in the collaboration. Considering this, solving the consolidation shipping cost allocation problem for potential collaborators is a fundamental step to encourage freight consolidation with cooperation.

One approach to solve the cost allocation problem is cost-sharing mechanism design. In a cost-sharing game, there is a set of players who are interested in using a common service from a provider. Each player has a private valuation of the service and submits its willingness to pay for the service as its bid to the provider. Players do not know the other players' valuations or bids. Using the service costs and the bids solicited from the players, a cost-sharing mechanism helps the provider decide which players to serve and how much to charge each player for service. In a binary demand cost-sharing game, each player's service request is either served or rejected, while in a general demand game, each player can receive different levels of service.

Cost-sharing mechanisms are usually designed to possess certain desired properties: *truthfulness*, *budget-balance* and *economic efficiency*. A truthful cost-sharing mechanism guarantees that the solicited bids from the players are their true willingness to pay. In other words, no player can be better off by submitting false bids. This is important because the cost is shared based on the information given by the bids. This characteristic of the mechanism is particularly important for establishing and sustaining cooperation. A cost-sharing mechanism is budget-balanced if the total cost charged to players recovers the cost of providing the service. An economically efficient cost-sharing mechanism maximizes the social welfare of all players. Unfortunately, it has been proven by Green, Kohlberg, and Laffont (1976) and Roberts (1979) that it is not possible to design a cost-sharing mechanism that is simultaneously truthful, budget-balanced and economically efficient. When budget-balance is impossible to achieve, we can design approximately budget-balanced mechanisms to recover as much of the incurred cost as possible. When the mechanism cannot be economically efficient, we can design the mechanism to maximize the social welfare as much as possible. However, many mechanisms yield zero or negative social welfare and thus it is difficult to make meaningful relative comparisons between mechanisms in terms of economic efficiency. An alternative measure of economic efficiency, social cost, which is always nonnegative, was proposed by Roughgarden and Sundararajan (2009) to quantify the inefficiency of a mechanism.

One of the frameworks that help design truthful and approximately budget-balanced cost-sharing mechanisms is the Moulin mechanism, which was introduced by Moulin (1999) and Moulin and Shenker (2001). The Moulin mechanism determines the players to serve and their cost shares using

a cost-sharing method in an iterative process. A cost-sharing method is a function that assigns a nonnegative cost share to each player in a set of players to be served. The iterative process starts with all players in the set to be served. A player is removed from the service set if its cost share is greater than its bid. The process halts when the cost shares of all players in the service set are smaller than or equal to their bids. The truthfulness of the Moulin mechanism is enforced by requiring a cost-sharing method to be cross-monotonic – in other words, a player's cost shares do not decrease when another player is removed from the service set. As a result, players are offered a sequence of nondecreasing cost shares through the iterations of a Moulin mechanism. Approximately budget-balanced Moulin mechanisms have been applied in a wide variety of applications, such as scheduling (e.g. Brenner and Schäfer 2007, Bleischwitz and Monien 2009), network design (e.g. Gupta et al. 2007, Roughgarden and Sundararajan 2007, Roughgarden and Schrijvers 2016), facility location (e.g. Könemann, Leonardi, and Schäfer 2005, Immorlica, Mahdian, and Mirrokni 2008)), logistics (e.g. Xu and Yang 2009) and online selection problems (e.g. Elmachtoub and Levi 2014).

Applying the Moulin mechanism framework to a cost-sharing problem in freight consolidation was recently studied by Zhang et al. (2018). In the consolidation system they considered, a set of suppliers in the same geographical region can use a non-profit consolidation center to have their demand shipped to a common faraway destination using trucks. Suppliers' demands can be split and packed together to fill a truck as much as possible at the consolidation center. The cost of using a truck depends on the demand volume packed in the truck, the less-than-truckload (LTL) rate, and the full-truckload (FTL) rate. They found that it is not possible to obtain a simultaneously truthful and budget-balanced Moulin mechanism. By using a two-slope piecewise linear function to model the cost function at the consolidation center, they designed a truthful and approximately budget-balanced Moulin mechanism. Using the social cost as the measure of economic efficiency, they analyzed the economic efficiency of the mechanism computationally.

In this paper, we study a similar cost-sharing problem in freight consolidation, but with the restriction that the entire less-than-truckload demand of each supplier must be shipped in a single truck at the consolidation center. That is, the demand cannot be split among multiple trucks if it can fit into a single truck. There are several reasons to make this assumption. On one hand, suppliers typically want their demand to be delivered in a single shipment and prefer less handling of their products to avoid unnecessary damage. For instance, agricultural products – e.g. flowers, eggs – are prone to damage during handling. Furthermore, a consolidation center can save on the extra handling costs resulting from separating and combining the demands of different suppliers. This small change in the problem setting greatly complicates the decisions that need to be made by the consolidation center. In the problem that Zhang et al. (2018) studied, demands at the

consolidation center are packed aggregately, so the way the demand of the suppliers is packed into trucks does not affect the total outbound shipping cost and cost shares. However, in the problem we study, the total outbound shipping cost, which is the sum of the shipping costs of each used truck, heavily depends on how the suppliers' demands are packed into trucks. Different combinations of demands in trucks (i.e. packing solutions) may result in different total outbound shipping costs and cost shares. However, the number of possible packing plans grows exponentially as the number of suppliers grows. Therefore, designing a cost-sharing mechanism under this setting is a challenging task.

Instead of a Moulin mechanism, we design an *acyclic mechanism* for the cost-sharing problem described above. An acyclic mechanism (Mehta, Roughgarden, and Sundararajan 2009) is another scheme that leads to a truthful and approximately budget-balanced cost-sharing mechanism. It is a strict generalization of the Moulin mechanism. Unlike the Moulin mechanism, an acyclic mechanism offers cost shares to the players in each iteration according to a pre-defined order instead of simultaneously. This additional ordering protocol allows the construction of truthful mechanisms to be no longer dependent on cross-monotonic cost-sharing methods, which are necessary to induce truthful Moulin mechanisms. Mehta, Roughgarden, and Sundararajan (2009) point out that a large number of primal-dual algorithms naturally induce acyclic mechanisms with non-ascending prices. Meanwhile, acyclic mechanisms have better budget-balance and economic efficiency than the Moulin mechanisms for several classes of basic cost-sharing problems, e.g. vertex cover, set cover, no-metric/metric uncapacitated facility location. Finally, the acyclic mechanism framework can be extended to solve cost-sharing problems with general demand settings (e.g. Balireddi and Uhan 2012), in which every player bids for each of the service levels it may receive. Although acyclic mechanisms have the aforementioned advantages compared to the Moulin mechanism, they achieve a weaker notion of truthfulness than the Moulin mechanism. The Moulin mechanism achieves a strong notion of truthfulness – *group strategyproofness* (GSP) – that ensures that not only can an individual player not be better off by false bidding, but also a subset of players can never strictly increase the utility of one of its members without decreasing the utility of some other member by coordinating false bids. The acyclic mechanism achieves *weak group strategyproofness* (WGSP), which ensures that there must exist a member whose utility remains the same, i.e. an indifferent member, if a coordinated false bid can strictly increase the utility of one of its members.

The contribution of this paper is two-fold. We advance the research on cost allocations in freight consolidation by designing an acyclic mechanism, which has been rarely applied to transportation settings. Our acyclic mechanism possesses several desirable properties. The packing solutions of our acyclic mechanism are strong Nash equilibria in a related non-cooperative game. This result provides an incentive to the suppliers to participate in freight consolidation. Our acyclic mechanism

is 2-budget-balanced in general, and better budget-balance guarantees can be achieved under specific problem settings. Furthermore, our empirical tests show that our acyclic mechanism usually performs at much better budget-balance ratios than 2. Finally, using social cost as the measure of economic efficiency, we show that the outcomes of our acyclic mechanism have social cost gaps of less than 3.8% in our numerical experiments under various problem settings.

The rest of the paper is organized as follows. In Section 2, we formally define our problem. In Section 3, we briefly review the acyclic mechanism and introduce the design of an acyclic mechanism based on bin packing solutions. We study the budget-balance of our proposed cost-sharing mechanism both theoretically and computationally in Section 4. In Section 5, we investigate the economic efficiency of the mechanism. We conclude our work in Section 6.

2. Problem Definition

The freight consolidation system we consider consists of a set of suppliers, a consolidation center and a common destination. Let N denote the set of suppliers who are interested in reducing their transportation costs by shipping their demands via a consolidation center to a common destination. These suppliers produce similar products and are all located in the same region. Each supplier i has a positive demand d_i (measured in units such as ft^3 or pounds) and a private valuation v_i for the service provided by the consolidation center for shipping demand d_i. Suppliers are self-interested. Each supplier has two shipping options. One is to ship the demand directly to the destination. We call this option *direct shipping* and the corresponding shipping cost *stand-alone cost*. The other is to ship the demand via the consolidation center. Supplier i expresses its willingness to pay to the consolidation center by submitting bid q_i at the planning phase of the consolidation. The consolidation center then charges p_i to the selected supplier i. If supplier i is selected to be served by the consolidation center, then supplier i pays for the *inbound shipping* (from supplier to the consolidation center) and is charged p_i as its share of the cost of *outbound shipping* (from the consolidation center to the destination) with other selected suppliers. We assume that the suppliers' utility functions are quasi-linear: that is, supplier i's utility is $u_i = v_i x_i - p_i$, where $x_i = 1$ if it receives service, and $x_i = 0$ otherwise. When selected to participate in the consolidation, suppliers require their less-than-truckload demand to be shipped in one truck for the outbound shipping.

The consolidation center, as the central planner of the consolidation service, is not profit-driven in our setting. Its goal is three-fold. First, the consolidation center enables consolidation by implementing a cost-sharing mechanism and incentivizing suppliers to participate. Second, the consolidation center aims to minimize the incurred outbound shipping cost recovered through the prices charged as much as possible to maintain operational efficiency, though it can be subsidized by the government or associated organizations to provide the service. Third, the consolidation center

intends to achieve good social welfare via consolidation. The consolidation center provides binary service to the suppliers, i.e. either a supplier's entire demand is served or none of its demand is served. Although suppliers need to bid for the consolidation service frequently over time, (e.g. daily, weekly) the truthfulness of the cost-sharing mechanism allows us to rely on the mechanism to solicit suppliers' truthful bids instead of learning their preferences over time. Each supplier i submits a bid for shipping its demand volume d_i and receives service for shipping exactly d_i if selected. So, no supplier has incentive to misreport their shipping demands. We also assume that the bidding demand d_i of supplier i is decided beforehand and remains fixed within the context of the mechanism. As a result, we can assume that the shipping volume of each supplier is known to the consolidation center. Based on the above assumptions, we model our problem as a one-time cost-sharing game with binary demand.

Both the suppliers and the consolidation center use trucks to ship demands to the destination. However, we assume in our setting that neither the suppliers nor the consolidation center owns a fleet of trucks. Instead, they ship using third-party logistics, and so the suppliers' and consolidation center's shipping costs primarily depend on two parameters: the less-than-truckload (LTL) rate c_L and the full-truckload (FTL) rate c_F. The LTL rate is the shipping cost per unit and the FTL rate is a fixed cost for using an entire truck. The shipping cost of a truck first increases linearly at the LTL rate as the shipping demand volume increases from 0. When the shipping demand volume in one truck exceeds a threshold value b, the shipping cost of a truck is always the FTL rate regardless of the actual shipping volume. In other words, shipping demand b or more in one truck costs the same as if the full truckload is used. We call the threshold value b the FTL equivalent volume and it satisfies $c_F = bc_L$. Mathematically, the shipping cost of one truck is

$$c(d) = \begin{cases} dc_L & \text{if } 0 < d < b, \\ c_F & \text{if } b \le d \le k_F, \end{cases}$$

where k_F denotes the capacity of a truck. We assume that the values of the LTL rate and the FTL rate are only mileage dependent. Larger distances between the origin and the destination of a shipment induce greater LTL and FTL rates. Given a destination, let c_{L1} and c_{F1} denote the LTL rate and the FTL rate for the outbound shipping, respectively. The FTL equivalent volume at the consolidation center is $b_C = \frac{c_{F1}}{c_{L1}}$.

Suppliers and the consolidation center are assumed to have the same shipping cost structure, but they do not necessarily share or know each other's cost parameters or the FTL equivalent volume. We assume the suppliers have the same cost parameters for inbound shipping and direct shipping because of their proximity in location. Let g_{L0} and g_{F0} denote the LTL rate and the FTL rate for inbound shipping, respectively, and let g_{L1} and g_{F1} denote the LTL rate and FTL rate for

direct shipping, respectively. Because the destination is always farther from the suppliers than the consolidation center, we have $g_{L1} > g_{L0}$ and $g_{F1} > g_{F0}$. We further assume that the FTL equivalent volume for inbound shipping and direct shipping is the same for the suppliers. Consequently, every supplier has FTL equivalent volume $b_G = \frac{g_{F1}}{g_{L1}} = \frac{g_{F0}}{g_{L0}}$.

Based on the above cost structure, suppliers with multiple truckloads of demand should not submit bids for full truckloads of demand because they can ship them cheaper on their own at the direct FTL rate. As a result, these suppliers only submit bids for their remaining less-than-truckload demand to see if they can save on shipping cost through consolidation. From the consolidation center's perspective, full truckloads do not contribute to consolidation but require extra handling to ship from the consolidation center. Moreover, it is trivial to decide whether full truckloads should be shipped via the consolidation center because such operations are beneficial for suppliers only when the savings from the outbound shipping can cover the inbound shipping cost of full truckloads. Based on the above discussion, the kind of demand profile that is worth studying is the one in which each supplier has less-than-truckload demand to send through the consolidation center and consolidating can make a significant difference for them. In addition, we assume suppliers would bid for all the remaining less-than-truckload demand to achieve operational efficiency.

In this paper, we solve the cost-sharing problem for freight consolidation for a set of suppliers N, whose demands satisfy $d_i < k_F$, $\forall i \in N$. Because each supplier's demand cannot be split and must be shipped in a single truck at the consolidation center, determining whose demands to pack into one truck is a critical decision that affects the outbound shipping cost and thus influences the cost share of each selected supplier. Let $T_1, T_2 \dots T_l$ denote a packing solution for the selected set of suppliers $S \subseteq N$ using l trucks. In particular, each set T_k, $k \in \{1, 2 \dots l\}$ contains the indices of suppliers whose demands are assigned to truck k. Let $D(T_k)$ denote the total demand volume packed in truck k. Then we define the outbound shipping cost of truck k as:

$$Z(T_k) = \begin{cases} D(T_k)c_{L1}, & \text{if } D(T_k) < b_C, \\ c_{F1}, & \text{if } D(T_k) \geq b_C. \end{cases}$$

As a result, the outbound shipping cost incurred at the consolidation center for shipping the demand of suppliers in S using the packing solution $T_1, T_2 \dots T_l$ is $\sum_{k \in \{1,2 \dots l\}} Z(T_k)$.

3. Acyclic Mechanism Based on Bin Packing

3.1. Acyclic Mechanisms

Acyclic mechanisms were first introduced by Mehta, Roughgarden, and Sundararajan (2009) as an alternative to the Moulin mechanism for designing truthful and approximately budget-balanced cost-sharing mechanisms. An acyclic mechanism is induced by a cost-sharing method χ and a corresponding valid offer function τ. An offer function $\tau(i, S)$ is a mapping from any given subset

$S \subseteq N$ and player $i \in S$ to a nonnegative offer time. The offer times reveal the sequence that cost shares will be offered to the players in each iteration. Players with lower offer times are offered cost shares earlier than the players with higher offer times. Players with equal offer times are offered cost shares simultaneously. Although the cost-sharing method χ in an acyclic mechanism is defined in the same way as in the Moulin mechanism, it is not required to be cross-monotonic to induce a truthful mechanism. This flexibility comes from the offer function. In particular, the order of offers can be designed to suppress the non-cross-monotonicity of the cost-sharing method so that a player is still offered a sequence of nondecreasing cost shares as the iterations progress. As a result, designing such an offer function τ for a specific cost-sharing method χ is critical in acyclic mechanism design. For a subset $S \subseteq N$ and a player $i \in S$, let $L(i,S)$, $E(i,S)$, and $G(i,S)$ denote the players of S whose offer times are strictly less than, equal to, and strictly greater than $\tau(i,S)$, respectively. A valid offer function τ for a cost-sharing method χ is defined in Mehta, Roughgarden, and Sundararajan (2009) as follows:

Definition 1. An offer function τ is *valid* for the cost-sharing method χ if for every subset $S \subseteq N$ and player $i \in S$,

(a) $\chi(i, S \setminus W) = \chi(i,S)$ for every subset $W \subseteq G(i,S)$

(b) $\chi(i, S \setminus W) \geq \chi(i,S)$ for every subset $W \subseteq G(i,S) \cup (E(i,S) \setminus \{i\})$

From the above definition, we can see that the cost shares for a player i cannot decrease if the players in $G(i,S)$ and $E(i,S)$ are removed from the service set. These two conditions ensure that cost shares for player i are cross-monotonic when players in $G(i,S)$ and $E(i,S)$ are removed. However, the definition does not restrict how the cost shares change when players in $L(i,S)$ are removed.

With a cost-sharing method χ and a corresponding valid offer function τ, an acyclic mechanism can be defined as follows.

Definition 2. An *acyclic mechanism* is a mechanism $M(\chi,\tau)$ induced by a cost-sharing method χ and an offer function τ that is valid for χ. $M(\tau,\chi)$ operates as follows (Mehta, Roughgarden, and Sundararajan 2009):

1. Collect a bid q_i from each player $i \in N$.

2. Initialize $S := N$.

3. If $q_i \geq \chi(i,S)$ for every $i \in S$, then stop. Return the set S. Each player $i \in S$ is charged the price $p_i = \chi(i,S)$.

4. If there exist some players $j \in J$, $J \subseteq S$ such that $q_j < \chi(j,S)$, choose $j^* \in J$ such that $\tau(j^*,S) \leq \tau(j,S)\, \forall j \in J$, set $S := S \setminus \{j^*\}$ and return to Step 3.

In Step 4, when there exist players whose cost shares are strictly greater than their bids, the acyclic mechanism removes the player with the earliest offer time and if there is a tie, then breaks

the tie arbitrarily. If the cost shares for suppliers are always raised to q_i for all $i \in S$, then based on Step 3 all suppliers in set S will be served by the consolidation center with $\chi(i, S) = q_i$.

3.2. Cost-Sharing Mechanism Based on Bin Packing (BBP)

As we have shown in the above section, designing an acyclic mechanism requires a cost-sharing method and a corresponding valid offer function. Since each supplier's demand cannot be split among different trucks in shipping, the outbound shipping cost, which will be shared among the consolidation participating suppliers, is determined by how suppliers' demands are eventually packed in trucks among the numerous possibilities. Therefore, the packing solution of suppliers' demands is crucial to the design of cost-sharing mechanism because it affects the outbound shipping cost and thus influences each supplier's cost share and the selection of the suppliers to be served. To obtain this packing solution, we model our packing problem as a bin packing problem. In what follows, we will derive the cost-sharing method and the corresponding valid offer function for our mechanism from the subset sum algorithm – the algorithm we use to solve the bin packing problem.

In a standard bin packing problem, given a list of items L, each with a nonnegative size, and the capacity of each bin H, we want to find a way to pack all the items using as few bins as possible. Similarly, the consolidation center wants to pack suppliers' demands using as few trucks as possible to induce a smaller outbound shipping cost. A smaller outbound shipping cost leads to smaller cost shares for suppliers and thus encourages more suppliers to consolidate. The bin packing problem is known to be NP-hard. We solve it using a heuristic approach called the subset sum algorithm (ss).

The subset sum algorithm is an intuitive way to solve the bin packing problem. It iteratively fills one bin to its fullest using the unpacked items. Mathematically, in each iteration we solve the optimization problem:

$$\begin{aligned} \max \quad & \sum_{i \in L} x_i h_i \\ \text{s.t.} \quad & \sum_{i \in L} x_i h_i \le H, \\ & x_i \in \{0,1\}, \ i \in L, \end{aligned}$$

where $x_i = 1$ means to pack item i in the current bin and 0 otherwise, h_i denotes the size of item i and L is the list of currently unpacked items. This is a special case of the 0-1 knapsack problem in which the value of each item equals its size, also known as the subset sum problem. Although the subset sum algorithm is not a polynomial-time algorithm, it is shown in Pisinger and Toth (1998) that the subset sum problem can be solved to optimality efficiently even for lists with a very large number of items.

Given a set of suppliers N, each with a positive demand, and the capacity of a truck k_F, we apply the subset sum algorithm to solve our packing problem as follows. Let $ss(N)$ denote the number of trucks required for outbound shipping for the suppliers in N using the subset sum algorithm. The output of the subset sum algorithm is a packing solution presented in a sequence of ordered sets $T_1, T_2, \ldots, T_{ss(N)}$; each set contains the indices of suppliers whose demands are assigned to the same truck.

Algorithm 1 Subset sum algorithm

$U \leftarrow N$
$k \leftarrow 1$
while $U \neq \emptyset$ **do**
  $T_k \leftarrow \arg\max_{P \subseteq U} \{\sum_{i \in P} d_i : \sum_{i \in P} d_j \leq k_F\}$
  $U \leftarrow U \setminus T_k$
  $k \leftarrow k + 1$
end while
return $T_1, T_2, \ldots, T_{k-1}$

The subset sum algorithm packs one truck per iteration. T_k is packed in the kth iteration. We call the returned packing solution $T_1, T_2, \ldots, T_{ss(N)}$ for supplier set N the subset sum packing solution. Note that the subset sum packing solution $T_1, T_2 \ldots T_{ss(N)}$ for any set of suppliers N are ordered such that $D(T_1) \geq D(T_2) \ldots \geq D(T_{ss(N)})$.

Having obtained the packing solution, we can now define the cost-sharing method. There are many ways to define cost-sharing methods to address different requirements or preferences. In our study, we define a truck-based cost-sharing method that shares the outbound shipping cost of each truck among its corresponding suppliers proportional to their demand. Although this way of cost-sharing may not be universally preferred in practice, this method applies directly in cases where each truck is required to be paid separately, as would be the case, for example, if the trucks are operated by independent contractors.

With the subset sum packing solution T_k, $k \in \{1, \ldots ss(S)\}$ for any set of suppliers $S \subseteq N$, and each set T_k of suppliers' demands is shipped using one truck, we formally define our cost-sharing method χ, which assigns a nonnegative cost share to each supplier $i \in S$ for every $S \subseteq N$, as follows:

$$\chi(i,S) = \begin{cases} c_{L1} d_i, & \text{if} \quad i \in T_k, \quad D(T_k) < b_C, \\ \frac{d_i}{D(T_k)} c_{F1}, & \text{if} \quad i \in T_k, \quad D(T_k) \geq b_C. \end{cases}$$

In addition, we leverage the packing order that is naturally derived from the subset sum packing solution to define our offer function $\tau(i,S)$, which determines the sequence in which the cost shares are revealed for the set of suppliers S:

$$\tau(i,S)=k \quad \text{s.t.} \quad i\in T_k,\ \forall i\in S.$$

Because k indicates the iteration in which the supplier's demand is packed, this offer function implies that suppliers whose demands are assigned in the earlier iterations in the subset sum algorithm are offered cost shares earlier than those assigned in the later iterations. The suppliers whose demands are assigned to the same truck are offered the cost shares at the same time. Note that this sequence is only used internally by the mechanism to decide which suppliers' demands to serve and how much cost they share. The final service decision is shared with suppliers simultaneously. In other words, the suppliers will not observe this sequence of decision making.

Next, we will show the defined offer function τ is valid for our cost-sharing method χ following Definition 1. We first analyze how the removal of some suppliers influences our subset sum packing solutions in Lemma 1 and Lemma 2. (For all proofs, please see the appendix.)

LEMMA 1. *Suppose $W\subseteq G(i,S)$ is removed from S, for some subset $S\subseteq N$ and supplier $i\in S$. Let $T_k, k\in\{1,\dots ss(S)\}$ and $T'_l, l\in\{1,\dots ss(S\setminus W)\}$ be subset sum packing solutions for S and $S\setminus W$, respectively. Then, for every supplier $j\in L(i,S)\cup E(i,S)$, if $j\in T_k$ and $j\in T'_l$, then $T_k=T'_l$.*

LEMMA 2. *Suppose $W\subseteq (E(i,S)\setminus\{i\})$ is removed from S, for some subset $S\subseteq N$ and supplier $i\in S$. In addition, suppose supplier i is packed in truck T in a subset sum packing solution for S, and in truck T' in a subset sum packing solution for $S\setminus W$. Then $D(T')\le D(T)$.*

Now that we know how the removal of suppliers affects the subset sum packing solution, we show that the offer function τ is valid for the cost-sharing method χ.

PROPOSITION 1. *The offer function τ is valid for the cost-sharing method χ.*

With a valid offer function τ for the cost-sharing method χ, we now formally define the cost-sharing mechanism Based on Bin Packing (BBP) as the acyclic mechanism $M(\chi,\tau)$ induced by χ and τ. Mehta, Roughgarden, and Sundararajan (2009) showed that every acyclic mechanism is weakly group strategyproof (WGSP). Therefore, our cost-sharing mechanism BBP also achieves truthfulness.

PROPOSITION 2. *Cost-sharing mechanism Based on Bin Packing (BBP) is weakly group strategyproof (WGSP).*

Since suppliers bid for outbound shipping cost, we assume that supplier i's valuation of the consolidation service v_i is its stand-alone cost minus its inbound shipping cost and thus is its bid q_i submitted under cost-sharing mechanism BBP.

3.3. Cost-Sharing Mechanism BBP from the Selfish Bin Packing Perspective

The subset sum packing solutions produced by the cost-sharing mechanism BBP not only help to induce an truthful acyclic mechanism, they also provide an insight from a non-cooperative game theoretic perspective: the subset sum packing solution for the selected suppliers is a strong Nash equilibrium. Consider a bin packing game in which each demand is controlled by a self-interested supplier. Each supplier has complete information about the other suppliers and a set of strategies corresponding to which truck to pack its demand for each possible packing of all the other suppliers' demands. The cost of using each truck is shared proportional to demand among the suppliers whose demands are packed in the same truck.

This game was first introduced and studied by Bilò (2006) and is referred to as the selfish bin packing problem. Bilò (2006) proved that there always exists a pure Nash equilibrium to the bin packing game defined above. A strategy profile is a Nash equilibrium if no supplier can strictly reduce its shared shipping cost by moving its demand to another truck while the packing of other demands remains the same. A stronger notion is strong Nash equilibrium (Aumann 2016), in which any subset of suppliers cannot strictly reduce the shared shipping costs of every member by moving their demands while the other demands are packed in the same way. Epstein and Kleiman (2011) proved that the packing solutions yielded by the subset sum algorithm for this bin packing game are always strong Nash equilibria.

Relating the above result to our cost-sharing mechanism BBP, we can conclude that the packing solutions from our cost-sharing mechanism BBP are strong Nash equilibria in the setting even when suppliers are allowed to pick or change the truck in which they pack their demands. No subset of suppliers can move their demands to benefit every member of the coalition. In other words, every supplier should be satisfied with the subset sum packing solutions provided by the cost-sharing mechanism BBP. This outcome further motivates the use of the subset sum algorithm to produce packing solutions.

4. Budget-Balance of Cost-Sharing Mechanism BBP

In this section, we study the budget-balance guarantee of the cost-sharing mechanism BBP under different conditions and problem settings. In the literature, there exist different perspectives for defining (approximate) budget-balance for cost-sharing mechanisms. One stream of literature (e.g. Moulin (1999), Brenner and Schäfer (2007), Zhang et al. (2018)) quantifies approximate budget-balancedness by comparing the total charged price to the minimum possible cost. Here, a cost-sharing method χ is β-budget-balanced if $C(S)/\beta \leq \sum_{i\in S} \chi_i \leq C(S)$, $\beta \geq 1$, for any outcome set S, where $C(S)$ is the minimum cost of serving set S. Another stream of literature (e.g. Roughgarden and Sundararajan (2009), Mehta, Roughgarden, and Sundararajan (2009), Brenner and Schäfer

(2008), Könemann, Leonardi, and Schäfer (2005), Balireddi and Uhan (2012)) quantifies approximated budget-balancedness by comparing the total charged price to the actual cost incurred and the optimal cost to measure the cost recovery and competitiveness, respectively. We follow this stream of literature and define a cost-sharing method χ as β-budget-balanced if $C_M(S) \leq \sum_{i \in S} \chi_i \leq \beta C(S)$, $\beta \geq 1$, for any outcome set S, where $C_M(S)$ is the cost of a feasible solution for serving set S output by the mechanism. Note that in this definition, the lower bound of the total charged price is $C_M(S)$. This means that we focus on studying the no-deficit cost-sharing methods and quantifying approximate budget-balancedness from a competitiveness perspective.

4.1. Theoretical Results on Budget-Balance Ratio

Intuitively, packing using a smaller number of trucks leads to a smaller cost. So the minimum outbound shipping cost should be induced by the optimal bin packing solution. However, this is not necessarily true with our trucking cost structure. For example, let $c_{L1} = \$1$, $b_C = 13$, $k_F = 14$, so $c_{F1} = \$13$. Assume we have 13 suppliers with 5 units of demand and 6 suppliers with 3 units of demand. One optimal packing solution, which uses 7 trucks, is to pack 5, 5 and 3 units of demand in each of the first 6 trucks and 5 units of demand in the last truck. The outbound shipping cost of this optimal packing solution is \$83. The subset sum packing solution of our cost-sharing mechanism uses 8 trucks. It packs 5, 3, 3 and 3 units of demand in each of the first two trucks, 5 and 5 units of demand in the next 5 trucks and 5 units of demand in the last truck. The outbound shipping cost of this subset sum packing solution is \$81. From the above example, we can see that although the optimal packing solution uses one fewer truck, it costs more to ship the total demand.

This phenomenon is due to our trucking cost structure, in which shipping 13 units or more in one truck costs the same. The subset sum packing solution ships two more units of demand in the first two trucks without paying more. Because of this phenomenon, it is not easy to determine the minimum outbound shipping cost and use it to study the budget-balance ratio of the cost-sharing mechanism BBP. However, the outbound shipping cost for a set of suppliers when their demands can be split and consolidated to fill a truck as much as possible, is a lower bound for the minimum outbound shipping cost when suppliers' demands cannot be split. If we compare the outbound shipping cost incurred by the subset sum packing solution to the lower bound of minimum outbound shipping cost, we can obtain an upper bound on the budget-balance ratio for our cost-sharing mechanism BBP.

For the convenience of analysis, we define *the budget-balance ratio of* S as $\beta(S) = \frac{C_M(S)}{C(S)}$ and therefore, $\beta = \max_S \left\{ \frac{C_M(S)}{C(S)} \right\}$. Let $C^*(S)$ denote the lower bound of the minimum outbound shipping cost for the supplier set S. Its value only depends on the total demand volume for a given set of cost parameters:

$$C^*(S) = \begin{cases} \left\lfloor \frac{\sum_{i\in S} d_i}{k_F} \right\rfloor c_{F1} + \left(\sum_{i\in S} d_i - k_F \left\lfloor \frac{\sum_{i\in S} d_i}{k_F} \right\rfloor \right) c_{L1} & \text{if } \sum_{i\in S} d_i - k_F \left\lfloor \frac{\sum_{i\in S} d_i}{k_F} \right\rfloor < b_C, \\ \left(\left\lfloor \frac{\sum_{i\in S} d_i}{k_F} \right\rfloor + 1 \right) c_{F1} & \text{if } \sum_{i\in S} d_i - k_F \left\lfloor \frac{\sum_{i\in S} d_i}{k_F} \right\rfloor \geq b_C. \end{cases}$$

Next we find an upper bound for β using $C^*(S)$. By definition, $C^*(S) \leq C(S)$ for any supplier set S.

LEMMA 3. *For any set of suppliers S, let m be such that $(m-1)k_F < \sum_{i\in S} d_i \leq mk_F$. Then $ss(S) \leq 2m-1$, and this inequality is tight.*

PROPOSITION 3. *The cost-sharing mechanism BBP is 2-budget-balanced.*

Although we have obtained an upper bound on β for our cost-sharing mechanism, we use the lower bound of the minimum outbound shipping cost $C^*(S)$ to obtain this lower bound and sometimes this cost can be much lower than the minimum outbound shipping cost for our problem. We could possibly obtain better bounds on the value of β by using the minimum outbound shipping cost that is induced by a packing solution in which the suppliers' demands are not split. We cannot easily find this minimum outbound shipping cost in general, but we can restrict our attention to special cases for which we can determine this minimum outbound shipping cost. For example, we can look at certain input demand profiles that produce structured subset sum packing solutions such that the outcome of the cost-sharing mechanism BBP is budget-balanced. We look into two scenarios below.

PROPOSITION 4. *For a given supplier set S, if the subset sum packing solution uses no more than two trucks, then $\beta(S) = 1$.*

LEMMA 4. *Caprara and Pferschy (2004) Given a set of suppliers S, if no three suppliers' demands fits in one truck, then $ss(S) = opt(S)$, where $opt(S)$ denotes the minimum number of trucks that have to be used in order to ship the demands of suppliers in S.*

PROPOSITION 5. *Given a set of suppliers S, if no three supplier's demands fit in one truck, then the subset sum packing solution for supplier set S induces the minimal outbound shipping cost for supplier set S.*

With the result in Proposition 5, we can easily draw the conclusion in Corollary 1.

COROLLARY 1. *If no three suppliers' demands fit in one truck in supplier set S, then $\beta(S) = 1$.*

As a summary of the above results, we present Proposition 6.

PROPOSITION 6. *Cost-sharing mechanism BBP is budget-balanced for the demand profiles in which (1) no three suppliers' demands fit in one truck or (2) the corresponding subset sum packing solutions use no more than two trucks.*

We can also determine the minimum outbound shipping cost when we have specific values of b_C. Recall the example that shows the minimum outbound shipping cost is not necessarily induced by the optimal bin packing solution for a given set of suppliers. If we change b_C to 7 and c_{F1} to \$7 in that example, we can see now the outbound shipping cost of the optimal bin packing solution is \$47, which is smaller than that of the subset sum packing solution \$54. This example seems to indicate that with a smaller b_C the optimal bin packing solutions yield the minimum outbound shipping cost. Next, we show a sufficient condition for the optimal bin packing solutions to yield the minimum outbound shipping cost.

Let $B_1, \dots B_m$ denote a bin packing solution using m bins, $D(B_k)$, $k = 1, \dots, m$ denote the total item size packed in B_k and H denote the capacity of bins. For the sake of analysis, we define *nontrivial bin packing solutions*.

DEFINITION 3. A bin packing solution $B_1, \dots B_m$ is nontrivial if there are at least $m-1$ bins half filled.

The outcomes of bin packing algorithms are often nontrivial bin packing solutions. The optimal bin packing solutions must be nontrivial. If they are not, we can easily reduce the solution by one bin by simply combining the items in two bins that are both less than half filled, contradicting the fact that the packing solution is optimal. The subset sum packing solutions are also nontrivial. If there are two bins less than half filled, the subset sum algorithm should pack all the items in these two bins in one bin to maximize the total size instead of keeping them in separate bins.

PROPOSITION 7. *When $b_C \leq \frac{1}{2}k_F$, the minimum outbound shipping cost $C(S)$ is induced by an optimal bin packing solution for supplier set S.*

Note that the optimal bin packing solution for a supplier set may not be unique. If there are multiple optimal bin packing solutions, the outbound shipping cost we refer to as $C(S)$ is always induced by the one whose least filled truck has the smallest total demand among all optimal bin packing solutions.

As we see above, when $b_C \leq \frac{1}{2}k_F$, the number of trucks used plays an important role in determining the outbound shipping costs. In order to study β to induce α, we want to know how many more trucks the subset sum packing solution uses compared to the optimal bin packing solution for any set of suppliers. For the convenience of the analysis we define *the worst-case ratio* $R_{ss}(S)$ for a given supplier set S as the ratio between the number of trucks used by the subset sum packing solution $ss(S)$ and the number of trucks used by an optimal bin packing solution $opt(S)$, i.e.

$R_{ss}(S) = \frac{ss(S)}{opt(S)}$. The *absolute worst-case ratio* is $R_{ss} = \max_S\{R_{ss}(S)\}$, which has been proven to be 1.6067 by Epstein, Kleiman, and Mestre (2009).

With $R_{ss} \approx 1.6067$, we can easily calculate the maximum possible number of trucks used by the subset sum algorithm when given the optimal number of trucks. For a given set of suppliers S, the possible number of trucks used by the subset sum algorithm $ss(S) \in \{k|opt(S) \le k \le \lfloor 1.6067 opt(S) \rfloor, k \in \mathbb{Z}^+\}$. Next we analyze the instance budget-balance ratio for supplier set S for which $opt(S) \in \{3, \ldots, 8\}$.

PROPOSITION 8. *For a given supplier set S, if $b_C \le \frac{1}{2}k_F$ and $3 \le opt(S) \le 8$, then $\beta(S) < \frac{11}{6}$.*

PROPOSITION 9. *For a given supplier set S, if $b_C \le \frac{1}{2}k_F$, $\beta < 1.8075$ when $opt(S) \ge 9$.*

Combining Proposition 8 and Proposition 9, we obtain the following Corollary 2.

COROLLARY 2. *For any given supplier set S, if $b_C \le \frac{1}{2}k_F$, then $\beta < \frac{11}{6}$.*

As a result of Corollary 2, we obtain Theorem 1.

THEOREM 1. *When $b_C \le \frac{1}{2}k_F$, the cost-sharing mechanism BBP is $\frac{11}{6}$-budget-balanced.*

4.2. Numerical Results on Budget-Balance Ratio

Although we have obtained an upper bound on the budget-balancedness of the cost-sharing mechanism BBP, this upper bound on β may not truly reflect the performance of our cost-sharing mechanism in practice. In order to reveal a more accurate picture of the budget-balance ratio of our cost-sharing mechanism, we study the ratio numerically. For each given demand profile, we obtain a budget-balance ratio by calculating the minimum outbound shipping cost and the outbound shipping cost incurred by our cost-sharing mechanism. As we have mentioned before, the minimum outbound shipping cost is not always induced by the optimal bin packing solution, but this does not mean that we cannot obtain the minimum outbound shipping cost numerically. The following proposition helps us find the minimum outbound shipping cost for any demand profile using the first-fit algorithm.

PROPOSITION 10. *For any given supplier set S, the packing solution that induces the minimum outbound shipping cost for set S can be obtained by applying the first-fit algorithm to a specific order of the demand profiles in S.*

Because of Proposition 10, we can always obtain the minimum outbound shipping cost for a supplier set by packing the demands using the first-fit algorithm on every possible ordering of the demand profiles. The number of possible ordering of demand profiles for supplier set S is $|S|!$ and the first-fit algorithm can be implemented in $O(|S|\log|S|)$ elementary operations. Therefore, we can exactly compute the minimum outbound shipping cost for moderate $|S|$.

We compare the minimum outbound shipping costs and the outbound shipping costs incurred by our cost-sharing mechanism for the demand profiles with 3, 6 and 10 suppliers. For each number of suppliers, we generate 100 demand profiles. Each supplier has less than truckload demand randomly generated from the uniform distribution on $(0, k_F)$. The parameters we used to generate the demand profiles and calculate the shipping cost is shown in Table 1.

Table 1 Experiment parameters

k_F (ft^3)	g_{L0} (\$)	g_{L1} (\$)
4000	1	15

Because the value of b_C also influences the minimum shipping cost, we run the experiments with three b_C values while setting $b_C = b_G$. Given the number of suppliers and the value of b_C, we summarize the results over 100 demand profiles by reporting the statistics of the budget-balance ratios, which we define as $\frac{C(S)}{C_M(S)}$ for supplier set S.

Table 2 Budget-balance ratio for demand profiles with 3 suppliers

	Min ratio	Max ratio	Avg. ratio	# of same cost
$b_C = \frac{1}{4}k_F$	1	1	1	100
$b_C = \frac{1}{2}k_F$	1	1	1	100
$b_C = \frac{3}{4}k_F$	1	1	1	100

Table 3 Budget-balance ratio for demand profiles with 6 suppliers

	Max ratio	Min ratio	Avg. ratio	# of same cost
$b_C = \frac{1}{4}k_F$	1.0507	1	1.0005	99
$b_C = \frac{1}{2}k_F$	1.2284	1	1.0042	98
$b_C = \frac{3}{4}k_F$	1.1001	1	1.0021	94

Table 4 Budget-balance ratio for demand profiles with 10 suppliers

	Max ratio	Min ratio	Avg. ratio	# of same cost
$b_C = \frac{1}{4}k_F$	1.2000	1	1.0132	93
$b_C = \frac{1}{2}k_F$	1.1937	1	1.0160	78
$b_C = \frac{3}{4}k_F$	1.0913	1	1.0109	62

In Tables 2, 3 and 4, we present the maximum, minimum and average budget-balance ratios among the 100 demand profiles results for each combination of numbers of suppliers and b_C values.

In addition, we also show the number of demand profiles whose minimum outbound shipping cost equals the outbound shipping cost of the subset sum packing solution.

From the results in Table 2, we empirically see that when there are three suppliers in the demand profiles, the subset sum packing solution always yields the minimum outbound shipping cost. This result aligns with Proposition 4. In Tables 3 and 4, the maximum ratios are much smaller than the upper bounds we found for β and the average ratios are all bounded above by 1.016. This empirically demonstrates that our cost-sharing mechanism BBP has good budget-balance performance by only charging slightly more than the minimum shipping cost on average. Moreover, we see that the subset sum packing solutions often induce the minimum outbound shipping costs. However, the number of instances with the same cost decreases as the value of b_C or number of suppliers increases.

5. Economic Efficiency of Cost-Sharing Mechanism BBP

We have shown that our cost-sharing mechanism BBP is truthful and thoroughly studied its budget-balance guarantees in various problem settings. The desired property left to explore is economic efficiency. The economic efficiency of a cost-sharing mechanism is usually measured by social welfare. The outcome of an economically efficient cost-sharing mechanism maximizes the *social welfare*, defined as $W(S) = V(S) - C(S)$, where $V(S)$ denotes the total valuation of the suppliers in S and $C(S)$ denotes the total cost of serving suppliers in S. Intuitively, social welfare quantifies the savings from providing the common service to a selected set of suppliers. However, it has been shown that truthful and approximately budget-balanced cost-sharing mechanisms often yield outcomes with zero or negative social welfare when there exist outcomes with strictly positive social welfare (Feigenbaum et al. 2003). Therefore, it is difficult to relatively compare the economic efficiency of cost-sharing mechanisms using social welfare.

Social cost $\pi(S)$ was proposed as an alternative way to measure the economic efficiency of a mechanism (Roughgarden and Sundararajan 2009). Instead of quantifying the savings, social cost is equal to the summation of the cost of serving the selected suppliers S and the total valuation of players in $N \setminus S$. Mathematically, $\pi(S) = C(S) + V(N \setminus S)$, where $V(N \setminus S)$ denotes the total valuation of the suppliers not in S. By definition, social cost is always a positive value and thus makes it easier to relatively compare the economic efficiency of mechanisms with the same budget-balance guarantee. In addition, social cost can be obtained from social welfare using an affine transformation: $\pi(S) = -W(S) + V(N)$. This means that minimizing social cost is equivalent to maximizing social welfare. The outcome of economically efficient cost-sharing mechanisms should minimize the social cost.

In this section, we study the economic efficiency of the cost-sharing mechanism BBP by comparing the social cost of the cost-sharing mechanism BBP to the minimum social cost of our problem.

The social cost in our problem is the total shipping cost of all suppliers. We minimize the social cost of our problem using a mixed integer optimization model, in which each supplier ships all its demand either to the consolidation center or to the destination directly. Each supplier's demand is delivered in its entirety, without splitting. The parameters, decision variables, and the model are presented below.

Parameters:

N: set of suppliers.

T: set of trucks available at the consolidation center.

G_i^0: Inbound shipping cost for the entire demand of supplier i $\forall i \in N$.

G_i^1: Stand-alone shipping cost for the entire demand of supplier i $\forall i \in N$.

Decision variables:

x_{CF}^j: Binary variable. If outbound shipping of truck j uses the FTL rate, then $x_{CF}^j = 1$; otherwise, 0 $\forall j \in T$.

y_{CF}^j: Amount of demand of truck j shipped by the FTL rate from the consolidation center to the destination $\forall j \in T$.

y_{CL}^j: Amount of demand of truck j shipped by the LTL rate from the consolidation center to the destination $\forall j \in T$.

z_{ij}: Binary variable. If the entire demand of supplier i is shipped using truck j, then $z_{ij} = 1$; otherwise $z_{ij} = 0$.

Model:

$$\min \sum_{i\in N}\sum_{j\in T} z_{ij}G_i^0 + \sum_{i\in N}(1-\sum_{j\in T} z_{ij})G_i^1 + \sum_{j\in T}(c_{F1}x_{CF}^j + c_{L1}y_{CL}^j) \tag{1}$$

$$\text{s.t.} \quad y_{CF}^j \le k_F x_{CF}^j, \ \forall j \in T \tag{2}$$

$$y_{CL}^j \le b_C, \ \forall j \in T \tag{3}$$

$$y_{CF}^j + y_{CL}^j \le k_F, \ \forall j \in T \tag{4}$$

$$\sum_{j\in T} z_{ij} \le 1, \ \forall i \in N \tag{5}$$

$$\sum_{i\in N} z_{ij} d_i = y_{CF}^j + y_{CL}^j, \ \forall j \in T \tag{6}$$

$$x_{CF}^j, \ z_{ij} \in \{0,1\}, \ \forall i \in N, \forall j \in T \tag{7}$$

$$y_{CF}^j \ge 0, \ y_{CL}^j \ge 0, \forall j \in T \tag{8}$$

Constraints (2) and (3) ensure that trucks correctly incur the FTL rate or the LTL rate under our trucking cost structure, respectively. Constraint (4) ensures that the packed demand in each truck does not exceed k_F. Constraint (5) allows each supplier's demand to be packed in at most

one truck at the consolidation center. Constraint (6) makes sure that the consolidation center ships all demands packed in each truck. Constraints (7) and (8) enforce the corresponding decision variables to be binary and nonnegative reals. For supplier i, if $\sum_{j\in T} z_{ij} = 0$, then supplier i ships all its demand directly. If $\sum_{j\in T} z_{ij} = 1$, then supplier i's demand is shipped using truck j at the consolidation center. Therefore, in the objective function (1), $\sum_{i\in N}\sum_{j\in T} z_{ij}G_i^0$ represents the total inbound shipping cost for the suppliers who ship their demands via the consolidation center, $\sum_{i\in N}(1-\sum_{j\in T} z_{ij})G_i^1$ represents the total stand-alone cost for the suppliers who ship their demands directly to the destination, and $\sum_{j\in T}(c_{F1}x_{CF}^j + c_{L1}y_{CL}^j)$ is the total outbound shipping cost. Like our cost-sharing mechanism BBP, the optimization model (1) - (8) also decides which suppliers participate in consolidation and how their demands are packed in trucks, but with the objective of minimizing the system's total cost.

We compare the social cost of our cost-sharing mechanism BBP to that of the optimization model (1) - (8) for demand profiles with 3, 6, 10, and 15 suppliers, respectively. Again, for each number of suppliers, we generate 100 demand profiles. Each supplier has less than truckload demand randomly generated from the uniform distribution on $(0, k_F]$. We fix the values of k_F, b_C, b_G, c_{L1} and g_{L1} (in Table 5), and study how different numbers of suppliers and different $\frac{g_{L1}}{g_{L0}}$ ratios influence the social cost of cost-sharing mechanism BBP compared to the minimum social cost. Because the shipping rates are distance dependent, the larger $\frac{g_{L1}}{g_{L0}}$ is, the farther the destination is compared to the location of the consolidation center. Our choices of $\frac{g_{L1}}{g_{L0}}$ are 1.5, 2.4, 3.2, 4.8, 9 and 15. Other cost parameters are calculated accordingly for each selection of $\frac{g_{L1}}{g_{L0}}$.

Table 5 Fixed values for the parameters

k_F (ft^3)	b_C (ft^3)	b_G (ft^3)	c_{L1} (\$)	g_{L1} (\$)
4000	2000	2000	1	1

We present the number of instances for which the optimization model and the cost-sharing mechanism BBP result in the same solution and the average social cost gap in Table 6. For each $\frac{g_{L1}}{g_{L0}}$ ratio, the first column shows the number of same solutions and second column shows the average social cost gap. The social cost gap is defined as

$$\frac{\text{mechanism social cost} - \text{optimal social cost}}{\text{optimal social cost}}.$$

In our experiments, when $\frac{g_{L1}}{g_{L0}} = 1.5$, solutions from the optimization model and the cost-sharing mechanism BBP always shows zero participation. This demonstrates that consolidation is more likely to be beneficial for suppliers when the destination is far (long-haul transportation) compared with the location of the consolidation center. Hence, it is not surprising that the cost-sharing

Table 6 Comparison of social cost gaps for cost-sharing mechanism BBP

$\frac{g_{L1}}{g_{L0}}$	1.5		2.4		3.2		4.8		9		15	
3 suppliers	100	0%	97	0.72%	95	2.33%	95	1.97%	98	1.66%	97	0.33%
6 suppliers	100	0%	88	0.70%	86	1.51%	81	3.22%	80	3.79%	79	3.29%
10 suppliers	100	0%	72	0.54%	65	1.10%	64	2.19%	58	2.97%	55	3.17%
15 suppliers	100	0%	50	0.54%	47	1.27%	43	2.08%	45	3.44%	38	3.62%

mechanism BBP is economically efficient when $\frac{g_{L1}}{g_{L0}} = 1.5$. In terms of the number of same solutions, roughly speaking, it is less likely for the cost-sharing mechanism BBP to yield the social cost minimizing solution as the number of suppliers and the $\frac{g_{L1}}{g_{L0}}$ ratio increase. In particular, the number of same solutions decreases more quickly as the number of suppliers increases. In terms of social cost gap, all gaps presented in Table 6 are smaller than 3.8%. One interesting phenomenon of the average social cost gap is that the values are not monotonic in the number of suppliers or the $\frac{g_{L1}}{g_{L0}}$ ratios. For some demand profiles, we find that the packing solutions for two "consecutive" $\frac{g_{L1}}{g_{L0}}$ ratios (e.g. 1.5 and 2.4, 2.4 and 3.2) come from two completely different sets of suppliers. That is, it is not always true for a demand profile that as $\frac{g_{L1}}{g_{L0}}$ increases, zero participation becomes partial participation or partial participation becomes total participation. Moreover, the social cost changes are not monotonic in the number of suppliers either. A possible contributing factor of this phenomenon is that packing the demands in trucks is essentially a combinatorial problem, whose solution largely depends on the specific composition of the demand profiles rather than the number of suppliers. As a result, the changes in social cost gap may not be aligned with changes of the $\frac{g_{L1}}{g_{L0}}$ ratios and the number of suppliers. However, we can still expect that the social cost gap generally increases as the number of suppliers or the $\frac{g_{L1}}{g_{L0}}$ increases.

6. Conclusions

In this paper, we study the cost-sharing problem in a freight consolidation system with one consolidation center. Self-interested suppliers in the same region have the option to use this nearby consolidation center to ship their demands together to a common destination with trucks for lower transportation rates. The entire less-than-truckload demand of each supplier must be shipped in a single truck. We design an acyclic mechanism to solve this shipping cost allocation problem for freight consolidation. At the planning phase of each consolidation, the central planner of the consolidation collects bids from all the suppliers. Then applying a cost-sharing mechanism, the central planner decides the set of suppliers who participate in the consolidation and their corresponding cost shares based on the bids.

In our problem setting, a critical problem we need to solve first is how to pack a set of suppliers' less-than-truckload demands since the packing solution directly influences the selected set of suppliers and their outbound shipping cost shares. We formulate this problem as a bin packing problem

and obtain the packing solution using the subset sum algorithm. Based on the obtained packing solution, we derive the cost-sharing method and the offer function that induces our truthful acyclic mechanism – cost-sharing mechanism Based on Bin Packing (BBP). Our cost-sharing mechanism BBP is weakly group strategyproof. Additionally, we find that the packing solutions yielded by the subset sum algorithm are strong Nash equilibria from a non-cooperative game theory perspective. This outcome supports our use of the subset sum algorithm.

We first study the budget-balance guarantee of the cost-sharing mechanism BBP theoretically, and we prove that our cost-sharing mechanism BBP is 2-budget-balanced in general. However, the mechanism BBP is budget-balanced when the demand profiles satisfy either of the two following conditions: 1) the corresponding subset sum algorithm uses no more than two trucks, or 2) no three suppliers' demands can fit in one truck. Additionally, when $b_C \leq \frac{1}{2}k_F$, the cost-sharing mechanism BBP is $\frac{11}{6}$-budget-balanced. To comprehensively analyze the budget-balance ratio, we then investigate the ratio empirically. On average, the budget-balance ratio is only slightly above 1. Moreover, our cost-sharing mechanism BBP always recovers all the incurred costs.

Finally, we study the economic efficiency of the mechanism BBP numerically. We use social cost to quantify and compare the economic efficiency. We obtain the minimum social cost shipping solution of the consolidation system with a mixed integer optimization model. Compared with the minimum social cost, the outcomes of our cost-sharing mechanism BBP have an average social cost gap less than 3.8%. In addition, although the changes in social cost gap are not perfectly aligned with changes of the $\frac{g_{L1}}{g_{L0}}$ ratios and the number of suppliers, the social cost gap can be expected to increase in general as the $\frac{g_{L1}}{g_{L0}}$ ratio and the number of suppliers increase.

One interesting direction for future research is to model the possibility that suppliers bid for having only a fraction of their demand shipped. We have simple examples showing that it may be optimal for a supplier to bid less than their total demand; assuming the remainder is shipped directly, the supplier's total cost may be lower than either shipping everything directly or only through the consolidation center. This motivates additional interesting directions of research, as a supplier does not know other supplier's valuations or demand quantities in advance, and of course each supplier's cost depends on the other's quantities.

One possibility is to design a cost-sharing mechanism under a general demand setting (e.g. Mehta, Roughgarden, and Sundararajan (2009)). In a general demand setting, each supplier can provide different bids for different demand volumes or service levels. The mechanism chooses which level of service will each supplier receive and the corresponding cost shares. It would be interesting to understand when it is optimal for the suppliers to bid for their entire demand or only a fraction; one might try to design a mechanism that guarantees the former.

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Appendix

LEMMA 1. *Suppose $W \subseteq G(i,S)$ is removed from S, for some subset $S \subseteq N$ and supplier $i \in S$. Let $T_k, k \in \{1,\dots ss(S)\}$ and $T'_l, l \in \{1,\dots ss(S \setminus W)\}$ be subset sum packing solutions for S and $S \setminus W$, respectively. Then, for every supplier $j \in L(i,S) \cup E(i,S)$, if $j \in T_k$ and $j \in T'_l$, then $T_k = T'_l$.*

Proof. The above claim indicates that for subset sum packing solutions, the removal of suppliers who are packed in iterations later than supplier i does not affect the subset sum packing solutions of the suppliers who are packed before any of the removed suppliers. Because the demand of any supplier in $W \subseteq G(i,S)$ does not contribute to maximizing the total demand in the iterations in which the demand of supplier $j \in L(i,S) \cup E(i,S)$ is packed, the existence of $W \subseteq G(i,S)$ does not affect the subset sum packing solutions for the supplier $j \in L(i,S) \cup E(i,S)$. As a result, every supplier $j \in L(i,S) \cup E(i,S)$, including supplier i, ends up being packed with the same suppliers when we remove $W \subseteq G(i,S)$ from S. □

LEMMA 2. *Suppose $W \subseteq (E(i,S) \setminus \{i\})$ is removed from S, for some subset $S \subseteq N$ and supplier $i \in S$. In addition, suppose supplier i is packed in truck T in a subset sum packing solution for S, and in truck T' in a subset sum packing solution for $S \setminus W$. Then $D(T') \leq D(T)$.*

Proof. We prove by contradiction. If $D(T') > D(T)$, then according to the subset sum algorithm, supplier i should end up being in T' in a subset sum packing solution for S instead of T. This contradicts the fact that supplier i is packed in T in a subset sum packing solution for S. □

PROPOSITION 1. *The offer function τ is valid for the cost-sharing method χ.*

Proof. We first prove part (a) of Definition 1. Part (a) indicates that supplier i's cost share remains the same if we remove some suppliers who are offered cost shares after supplier i. According to Lemma 1, all suppliers $j \in L(i,S) \cup E(i,S)$, including supplier i, are packed with the same suppliers when $W \subseteq G(i,S)$ is removed from S. As a result, supplier i 's cost share remains the same when we remove $W \subseteq G(i,S)$ from S, i.e. $\chi(i, S \setminus W) = \chi(i,S)$.

For part (b), when $W \subseteq G(i,S) \cup (E(i,S) \setminus \{i\})$, we assume there exist two supplier sets P and Q such that $P \subseteq G(i,S)$, $Q \subseteq (E(i,S) \setminus \{i\})$, and $P \cup Q = W$. Based on the above argument, we can conclude that the removal of P results in $\chi(i, S \setminus P) = \chi(i,S)$. Since the cost shares remain the same for supplier i when P is removed, we can restrict our attention to the setting in which Q is removed from $S \setminus P$. Let T be the truck in which supplier i is packed in a subset sum packing solution for $S \setminus P$. According to Lemma 2, when Q is removed from $S \setminus P$, supplier i is packed in T' such that $D(T') \leq D(T)$. Since χ shares the outbound shipping cost proportional to demand, the cost share of supplier i dose not decrease when removing Q from $S \setminus P$. Therefore, $\chi(i, S \setminus W) \geq \chi(i, S \setminus P) = \chi(i,S)$. □

LEMMA 3. *For any set of suppliers S, let m be such that $(m-1)k_F < \sum_{i\in S} d_i \le mk_F$. Then $ss(S) \le 2m-1$, and this inequality is tight.*

Proof. When $m=1$, the subset sum algorithm packs all demands in one truck. So the claim holds. Next, when $m \ge 2$, we prove this claim in two steps. First, we prove the subset sum algorithm uses no more than $2m-1$ trucks. In any subset sum packing solution, the sum of the demands in any two trucks exceeds k_F. Assume the subset sum algorithm uses t trucks. Then there are $\frac{t(t-1)}{2}$ different pairs of trucks in the solution. Summing over all these pairs, we have more than $\frac{t(t-1)}{2}k_F$ units of demand packed in these $\frac{t(t-1)}{2}$ pairs of trucks. Each truck participates in exactly $t-1$ pairs. As a result, the total demand packed in these t trucks is strictly greater than $\frac{t}{2}k_F$. Since $\frac{t}{2}k_F < \sum_{i\in S} d_i \le mk_F$, we have $t < 2m$. Second, we prove that $2m-1$ is a tight upper bound. Without loss of generality, let $\sum_{i\in S} d_i = (m-\frac{1}{2})k_F + \delta$, $\delta \le \frac{1}{2}k_F$. We construct a demand profile with $|S| = 2m-1$, and $d_i = \frac{1}{2}k_F + \frac{\delta}{2m-1}$ for all $i \in S$. Since each supplier's demand is strictly greater than $\frac{1}{2}k_F$, the subset sum algorithm will pack each supplier's demand in a separate truck and thus use $2m-1$ trucks. □

PROPOSITION 3. *The cost-sharing mechanism BBP is 2-budget-balanced.*

Proof. Without loss of generality, assume we have a set of suppliers S such that $(m-1)k_F < \sum_{i\in S} d_i \le mk_F$. When the suppliers' demands can be split and consolidated into full truckloads, we have at least $m-1$ full trucks in this consolidated packing solution. Therefore, $C^*(S) > (m-1)c_{F1}$. If the subset sum packing solution uses at most $2m-2$ trucks for the same demand profile, then $\beta(S) = \frac{C_M(S)}{C(S)} \le \frac{(2m-2)c_{F1}}{C^*(S)} < \frac{(2m-2)c_{F1}}{(m-1)c_{F1}} = 2$. If the subset sum packing solution uses $2m-1$ trucks, we can conclude that $\sum_{i\in S} d_i > (m-\frac{1}{2})k_F$. This is because the first $2m-3$ trucks must at least be half filled and the total demand of the last two trucks must exceed k_F and so the total demand must be strictly greater than $(2m-3)\cdot\frac{1}{2}k_F + k_F = (m-\frac{1}{2})k_F$. As a result, the last truck in the consolidated packing solution when demands can be split must have more than $\frac{1}{2}k_F$. If $b_C \le \frac{1}{2}k_F$, then $C^*(S) = mc_{F1}$. If $\frac{1}{2}k_F < b_C \le k_F$, then $c_{F1} = b_C c_{L1} \le k_F c_{L1}$ and thus, $C^*(S) > (m-1)c_{F1} + \frac{1}{2}k_F c_{L1} \ge (m-\frac{1}{2})c_{F1}$. Consequently, when the subset sum packing solution uses $2m-1$ trucks, $\beta(S) = \frac{C_M(S)}{C(S)} \le \frac{(2m-1)c_{F1}}{C^*(S)} < \frac{(2m-1)c_{F1}}{(m-\frac{1}{2})c_{F1}} = 2$. To summarize, $\beta = \max_S\{\beta(S)\} < 2$. As a result, the cost-sharing mechanism BBP is 2-budget-balanced. □

PROPOSITION 4. *For a given supplier set S, if the subset sum packing solution uses no more than two trucks, then $\beta(S) = 1$.*

Proof. If the subset sum packing solution uses one truck to pack all demands in S, then obviously $\beta(S) = 1$. If the subset sum packing solution uses two trucks to pack all demands in S, then any other packing solution will use at least two trucks to pack all the demands in S. The outbound shipping cost for an arbitrary packing solution is $(\sum_{i\in S} d_i - \hat{D})c_{L1}$, where $\hat{D}$ is the total demand volume that exceeds b_C in each of the packed trucks. As $\hat{D}$ increases, the outbound shipping cost decreases. Let T_1^s and T_2^s denote the subset sum packing solution, and let $T_1^a, \ldots, T_p^a$ $(D(T_1^a) \ge \ldots \ge D(T_p^a), p \ge 2)$ denote any other packing solution for supplier set S. Let $\hat{D}_1$ and $\hat{D}_2$ denote the demand volume that exceeds b_C in T_1^s and T_2^s, respectively and let $\hat{D}'_1, \ldots, \hat{D}'_p$ denote the demand volume that exceeds b_C in $T_1^a, \ldots, T_p^a$, respectively. Then we have $\hat{D}_1 \ge \hat{D}'_1$

according to the subset sum algorithm. Let $0 \le q \le p$ denote the number of trucks that have demand volumes exceed b_C in $T_1^a, \ldots, T_p^a$, i.e. $\hat{D}'_1 \ge \ldots \ge \hat{D}'_q > 0$, and $\hat{D'_{q+1}} = \ldots = \hat{D}'_p = 0$.

When $\hat{D}_1 = 0$, then $\hat{D}'_1 = 0$. All demands are shipped using the LTL rate in both packing solutions. So the outbound shipping cost of T_1^s, T_2^s equals to the outbound shipping cost of $T_1^a, \ldots, T_p^a$.

When $\hat{D}_1 > 0$ and $\hat{D}_2 = 0$, if $q \le 1$, then the outbound shipping cost of $T_1^a, \ldots, T_p^a$ is greater than that of the subset sum packing solution because $\hat{D}_1 + \hat{D}_2 > \hat{D}'_1 + \ldots + \hat{D}'_q$. If $q \ge 2$, then we have,

$$\begin{aligned}
&\hat{D}_1 + b_C + D(T_2^s) = (\hat{D}'_1 + \ldots + \hat{D}'_q) + qb_C + \sum_{k=q+1}^{p} D(T_k^a) \\
&\implies \hat{D}_1 = (\hat{D}'_1 + \ldots + \hat{D}'_q) + (q-1)b_C - D(T_2^s) + \sum_{k=q+1}^{p} D(T_k^a) \\
&\implies \hat{D}_1 > \hat{D}'_1 + \ldots + \hat{D}'_q.
\end{aligned}$$

Therefore, the outbound shipping cost of $T_1^a, \ldots, T_p^a$ is greater than that of the subset sum packing solution.

When $\hat{D}_1 > \hat{D}_2 > 0$, if $q \le 1$, then the outbound shipping cost of $T_1^a, \ldots, T_p^a$ is greater than that of the subset sum packing solution following the same argument above. If $q \ge 2$, then we have,

$$\begin{aligned}
&\hat{D}_1 + \hat{D}_2 + 2b_C = (\hat{D}'_1 + \ldots + \hat{D}'_q) + qb_C + \sum_{k=q+1}^{p} D(T_k^a) \\
&\implies \hat{D}_1 + \hat{D}_2 = (\hat{D}'_1 + \ldots + \hat{D}'_q) + (q-2)b_C + \sum_{k=q+1}^{p} D(T_k^a) \\
&\implies \hat{D}_1 + \hat{D}_2 > \hat{D}'_1 + \ldots + \hat{D}'_q.
\end{aligned}$$

Therefore, the outbound shipping cost of $T_1^a, \ldots, T_p^a$ is greater than that of the subset sum packing solution.

Based on the above analysis, the subset sum packing solutions always cost no more than any other packing solutions. Consequently, when the subset sum packing solution uses no more than two trucks, it always yields the minimum outbound shipping cost, i.e. $\beta(S) = 1$. □

PROPOSITION 5. *Given a set of suppliers S, if no three supplier's demands fit in one truck, then the subset sum packing solution for supplier set S induces the minimal outbound shipping cost for supplier set S.*

Proof. We prove the above claim by proving that the subset sum packing solution does not cost more than any other packing solution for the same set of suppliers. Without loss of generality, let $T_1^{ss}, \ldots, T_M^{ss}$ denote the subset sum packing solution and $T_1, \ldots, T_K$ denote any packing solution that is different from $T_1^{ss}, \ldots, T_M^{ss}$. For the sake of analysis, we order the packing solution $T_1, \ldots, T_K$ so that $D(T_1) \ge D(T_2) \ldots \ge D(T_K)$. Because of Lemma 4, $M \le K$. Recall that each T_k, $k \in \{1, 2, \ldots, K\}$, in the solution set contains the suppliers' indices whose demands are packed in the kth truck. In order, we compare T_m^{ss} with T_m for $m \in \{1, 2, \ldots M\}$. Let $m^* \le M$ be the smallest index such that $T_m^{ss} = T_m$, $m \in \{1, 2 \ldots m^*\}$. Therefore, up to the m^*th truck, $T_1^{ss}, \ldots, T_M^{ss}$ and $T_1, \ldots, T_K$ have the same exact packing solution, and thus incur the same outbound shipping cost. Since $T_m^{ss} = T_m$, $m \in \{1, 2 \ldots m^*\}$, the packing solutions $T_{m^*+1}^{ss}, \ldots, T_M^{ss}$ and $T_{m^*+1}, \ldots, T_K$ contain the demands of the same set of suppliers, whose demands are not packed in the first m^* trucks. Consequently,

$D(T^{ss}_{m^*+1}) \geq D(T_{m^*+1})$. We consider the following four cases based on the relationships between $D(T^{ss}_{m^*+1})$, $D(T_{m^*+1})$ and b_C.

Case 1: when $D(T_{m^*+1}) \leq D(T^{ss}_{m^*+1}) \leq b_C$, packing solutions $T^{ss}_{m^*+1}, \ldots, T^{ss}_M$ and $T_{m^*+1}, \ldots, T_K$ incur the same outbound shipping cost because the demands are all shipped at the LTL rate.

Case 2: when $D(T^{ss}_{m^*+1}) > b_C \geq D(T_{m^*+1})$, the outbound shipping cost incurred by $T^{ss}_{m^*+1}, \ldots, T^{ss}_M$ is strictly lower because it ships at least $D(T^{ss}_{m^*+1})$ at the flat FTL rate while $T_{m^*+1}, \ldots, T_K$ ships all demands at the LTL rate.

Case 3: when $D(T^{ss}_{m^*+1}) = D(T_{m^*+1}) > b_C$, we prove that we can reconfigure the packing solutions so that $T^{ss}_{m^*+1} = T_{m^*+1}$ while retaining the same outbound shipping cost of both packing solutions.

When $T^{ss}_{m^*+1} = \{i\}$ and $T_{m^*+1} = \{j\}$, but $i \neq j$, there must exist $T_{m^*+\Delta}$, $\Delta \in \{2, \ldots K - m^*\}$ so that $T_{m^*+\Delta} = \{i\}$. Otherwise, if supplier i's demand is packed with another supplier's demand, this contradicts the fact that $D(T^{ss}_{m^*+1}) \geq D(T_{m^*+1})$ and $D(T_{m^*+1}) \geq \ldots \geq D(T_K)$. If we switch T_{m^*+1} and $T_{m^*+\Delta}$, we have $T^{ss}_{m^*+1} = T_{m^*+1}$.

When $T^{ss}_{m^*+1} = \{i\}$ and $T_{m^*+1} = \{j, u\}$, but $i \neq j \neq u$, there must exist $T_{m^*+\Delta}$, $\Delta \in \{2, \ldots K - m^*\}$ so that $T_{m^*+\Delta} = \{i\}$ for the same reason in the above argument. If we switch $T_{m^*+\Delta}$ with T_{m^*+1}, we have $T^{ss}_{m^*+1} = T_{m^*+1}$. Similarly, when $T^{ss}_{m^*+1} = \{j, u\}$ and $T_{m^*+1} = \{i\}$, we can have $T^{ss}_{m^*+1} = T_{m^*+1}$ as well.

When $T^{ss}_{m^*+1} = \{i, j\}$ and $T_{m^*+1} = \{u, v\}$ but $\{i, j\} \neq \{u, v\}$, consider the following.

If $T^{ss}_{m^*+1}$ and T_{m^*+1} do not share suppliers, i.e. $\{i, j\} \cap \{u, v\} = \emptyset$, we can always have a truck that contains the demands of supplier u and v in the subset sum packing solution without changing the outbound shipping cost. If there exists $T^{ss}_{m^*+\Delta}$, $\Delta \in \{2, \ldots M - m^*\}$ so that $T^{ss}_{m^*+\Delta} = \{u, v\}$, then the above claim holds. If the demands of supplier u and v are not packed in the same truck, then either d_u or d_v should be packed in a truck whose total demand equals $d_u + d_v$; otherwise the demands of supplier u and v should be packed together by the subset sum algorithm to yield a truck with greater total demand. Assume d_l is packed with d_u in $T^{ss}_{m^*+\Delta}$, $\Delta \in \{2, \ldots M - m^*\}$ such that $d_l + d_u = d_u + d_v$. Therefore, $d_v = d_l$. If we switch d_v and d_l, we obtain a $T^{ss}_{m^*+\Delta}$ that contains the demands of supplier u and v and retain the outbound shipping cost of the packing solution. By switching $T^{ss}_{m^*+1}$ and $T^{ss}_{m^*+\Delta}$, we have $T^{ss}_{m^*+1} = T_{m^*+1}$.

If $T^{ss}_{m^*+1}$ and T_{m^*+1} share one common supplier – WLOG, we assume $i = u$ – then $d_j = d_v$. d_j must be packed in one of the trucks in $T_{m^*+2}, \ldots, T_K$. If we switch d_v and d_j in $T_{m^*+1}, \ldots, T_K$, we have $T^{ss}_{m^*+1} = T_{m^*+1}$.

Note that all the swaps in Case 3 only change the ordering of trucks with equal demand volume or the packings of equal demands. The resulting packing solutions are essentially equivalent to $T^{ss}_{m^*+1}, \ldots, T^{ss}_M$ and $T_{m^*+1}, \ldots, T_K$. Therefore, their outbound shipping costs do not change.

Case 4: when $D(T^{ss}_{m^*+1}) > D(T_{m^*+1}) > b_C$, we prove that we can always change $T_{m^*+1}, \ldots, T_K$ to have $T^{ss}_{m^*+1} = T_{m^*+1}$ without increasing the outbound shipping cost of $T_{m^*+1}, \ldots, T_K$. First, we prove when $D(T^{ss}_{m^*+1}) > D(T_{m^*+1}) > b_C$, there must be two suppliers' demands in $T^{ss}_{m^*+1}$. If $T^{ss}_{m^*+1}$ contains only one supplier's demand d_i, then d_i must be packed in one of $T_{m^*+1}, \ldots, T_K$. This contradicts $D(T^{ss}_{m^*+1}) > D(T_{m^*+1})$ and $D(T_{m^*+1}) \geq \ldots \geq D(T_K)$. Therefore, $T^{ss}_{m^*+1}$ must contain two suppliers' demands d_i and d_j. Since $D(T^{ss}_{m^*+1}) > D(T_{m^*+1})$, d_i and d_j are not packed in the same truck in $T_{m^*+1}, \ldots, T_K$. Let's assume d_i is packed with d_p and d_j is packed with d_q somewhere in the packing solution $T_{m^*+1}, \ldots, T_K$. Because d_i and

d_j are packed in $T^{ss}_{m^*+1}$, $d_i + d_j > d_i + d_p$ and $d_i + d_j > d_j + d_q$. Thus, $d_j > d_p$ and $d_i > d_q$. Now suppose we modify $T_{m^*+1}, \ldots, T_K$ to pack d_i and d_j together in one truck and pack d_p and d_q together in another truck. Because $d_i + d_j$ is the largest demand volume that can be packed in one truck, T_{m^*+1} now becomes $\{i, j\}$. If $d_i + d_p \geq b_C$, $d_j + d_q \geq b_C$, and $d_p + d_q \geq b_C$, then packing d_i and d_j together in $T_{m^*+1}, \ldots, T_K$ does not change the outbound shipping cost of $T_{m^*+1}, \ldots, T_K$. If $d_i + d_p \geq b_C$, $d_j + d_q \geq b_C$, and $d_p + d_q < b_C$, then packing d_i and d_j together in $T_{m^*+1}, \ldots, T_K$ reduces the outbound shipping cost of $T_{m^*+1}, \ldots, T_K$ from $2c_{F1}$ to $c_{F1} + (d_p + d_q)c_{L1}$. If $d_i + d_p \geq b_C$ and $d_j + d_q < b_C$, then $d_p + d_q < b_C$ because $d_j > d_p$. Therefore, the cost for shipping d_i, d_j, d_p, d_q decreases from $c_{F1} + (d_j + d_q)c_{L1}$ to $c_{F1} + (d_p + d_q)c_{L1}$ after packing d_i and d_j together. Similarly, if $d_i + d_p < b_C$ and $d_j + d_q \geq b_C$, the cost of shipping $T_{m^*+1}, \ldots, T_K$ decreases as well after packing d_i and d_j together. If $d_i + d_p < b_C$, $d_j + d_q < b_C$ and $d_p + d_q < b_C$, the cost of shipping d_i, d_j, d_p, d_q decreases from $(d_i + d_p + d_j + d_q)c_{L1}$ to $c_{F1} + (d_p + d_q)c_{L1}$ after packing d_i and d_j together. Thus, having $T^{ss}_{m^*+1} = T_{m^*+1}$ by switching demands in $T_{m^*+1}, \ldots, T_K$ does not increase the outbound shipping cost of the packing solution. Finally, if d_i or d_j is packed in a truck alone in $T_{m^*+1}, \ldots, T_K$, the above conclusion also holds because it can be seen as a special case of the above situation where $d_p = 0$ or $d_q = 0$.

Comparing $T^{ss}_{m^*+\Delta}$ and $T_{m^*+\Delta}$, $\Delta \in \{2, \ldots M - m^*\}$, if at any time, $D(T^{ss}_{m^*+\Delta})$ and $D(T_{m^*+\Delta})$ satisfy the conditions in cases 1 or 2, we can conclude that the outbound shipping cost of $T^{ss}_1, \ldots, T^{ss}_M$ is no more than $T_1, \ldots, T_K$. If $D(T^{ss}_{m^*+\Delta})$ and $D(T_{m^*+\Delta})$ satisfy the conditions in cases 3 or 4, we set $m^* = m^* + 1$ and repeat the above procedures again. If $D(T^{ss}_{m^*+\Delta})$ and $D(T_{m^*+\Delta})$ always fall into cases 3 or 4, we will end up with $T^{ss}_m = T_m$, $m \in \{1, \ldots, M\}$. Because we do not increase the outbound shipping cost every time we perform the above procedures to change $T_1, \ldots, T_K$ toward $T^{ss}_1, \ldots, T^{ss}_M$, we can conclude that $T^{ss}_1, \ldots, T^{ss}_M$ costs no more than $T_1, \ldots, T_K$. □

PROPOSITION 7. *When $b_C \leq \frac{1}{2}k_F$, the minimum outbound shipping cost $C(S)$ is induced by an optimal bin packing solution for supplier set S.*

Proof. Let $opt(S)$ denote the number of trucks that the optimal bin packing solution uses for supplier set S. Because optimal bin packing solutions are nontrivial bin packing solutions, based on Definition 3, there are at least $opt(S) - 1$ trucks in the optimal bin packing solutions half filled. Since $b_C \leq \frac{1}{2}k_F$, we have $(opt(S) - 1)c_{F1} < C(S) \leq opt(S)c_{F1}$. Other nontrivial bin packing solutions that are not optimal use at least $opt(S) + 1$ bins and therefore, cost strictly more than $opt(S)c_{F1}$ to ship all demands. As a result, the outbound shipping cost induced by an optimal bin packing solution is the smallest among all packing solutions for supplier set S. □

PROPOSITION 8. *For a given supplier set S, if $b_C \leq \frac{1}{2}k_F$ and $3 \leq opt(S) \leq 8$, then $\beta(S) < \frac{11}{6}$.*

Proof. Assume $opt(S) = 3$, then $ss(S) = \{3, 4\}$. We prove $\beta(S) < \frac{3}{2}$. If $opt(S) = ss(S) = 3$ and $b_C \leq \frac{1}{2}k_F$, then $\beta(S) = \frac{2c_{F1} + Z(T^{ss}_3)}{2c_{F1} + Z(T^{opt}_3)} < \frac{3c_{F1}}{2c_{F_1}} = \frac{3}{2}$, where $Z(T_k)$ denotes the outbound shipping cost of truck T_k. If $opt(S) = 3$ and $ss(S) = 4$. Because of how the subset sum algorithm works, $D(T^{opt}_1) \leq D(T^{ss}_1)$. Then it must be true that $D(T^{opt}_2) + D(T^{opt}_3) \geq \sum_{i=2}^{4} D(T^{ss}_i)$. Additionally, the total demand volume of any two trucks in a subset sum packing solution exceeds k_F, e.g. $D(T^{ss}_2) + D(T^{ss}_3) > k_F$. Then we have $D(T^{ss}_2) + D(T^{ss}_3) > k_F$, $D(T^{ss}_3) + D(T^{ss}_4) > k_F$ and $D(T^{ss}_2) + D(T^{ss}_4) > k_F$. Summing up these three inequalities, we obtain

$D(T_2^{ss})+D(T_3^{ss})+D(T_4^{ss})>\frac{3}{2}k_F$. Therefore, $D(T_2^{opt})+D(T_3^{opt})>\frac{3}{2}k_F$. This implies that $D(T_3^{opt})>\frac{1}{2}k_F$. As a result, if $opt(S)=3$, $ss(S)=4$ and $b_C\leq\frac{1}{2}k_F$, then $\beta(S)=\frac{3c_{F1}+Z(T_4^{ss})}{3c_{F1}}<\frac{4c_{F1}}{3c_{F_1}}=\frac{4}{3}$. Finally, we can conclude that when $opt(S)=3$, $\beta(S)<\frac{3}{2}$.

Applying the exact same technique when $4\leq opt(S)\leq 8$, we are able to obtain the corresponding instance budget-balance ratios that are summarized in Table 7:

Table 7 Instance budget-balance ratio summary

$opt(S)$	$ss(S)$	$\beta(S)$
4	$4,5,6$	$\frac{5}{3}$
5	$5,6,7,8$	$\frac{7}{4}$
6	$6,7,8,9$	$\frac{9}{5}$
7	$7,8,9,10,11$	$\frac{11}{6}$
8	$8,9,10,11,12$	$\frac{12}{7}$

To summarize, when $3\leq opt(S)\leq 8$ and $b_C\leq\frac{1}{2}k_F$, $\beta(S)=\max\{\frac{3}{2},\frac{5}{3},\frac{7}{4},\frac{9}{5},\frac{11}{6},\frac{12}{7}\}=\frac{11}{6}$. □

Proposition 9. *For a given supplier set S, if $b_C\leq\frac{1}{2}k_F$, $\beta<1.8075$ when $opt(S)\geq 9$.*

Proof. Following the same argument in Proposition 8,

$$\begin{aligned}\beta(S)&=\frac{(ss(S)-1)c_{F1}+Z(T^{ss}_{ss(S)})}{(opt(S)-1)c_{F1}+Z(T^{opt}_{opt(S)})}\\&<\frac{ss(S)c_{F1}}{(opt(S)-1)c_{F1}}\\&=\frac{ss(S)}{opt(S)-1}\\&\leq\frac{\lfloor 1.6067opt(S)\rfloor}{opt(S)-1}\\&\leq\frac{1.6067opt(S)}{opt(S)-1}.\end{aligned}$$

The value of $f(x)=\frac{x}{x-1}$ decreases as x increases. When $opt(S)\geq 9$, the maximum value of $\frac{1.6067opt(S)}{opt(S)-1}$ is obtained when $opt(S)=9$. As a result, $\beta(S)<1.8075$ when $opt(S)\geq 9$. □

Proposition 10. *For any given supplier set S, the packing solution that induces the minimum outbound shipping cost for set S can be obtained by applying the first-fit algorithm to a specific order of the demand profiles in S.*

Proof. Let $T_1,\ldots,T_k$ be the packing solution that induces the minimum outbound shipping cost $C(S)$. Assume T_m, $m\in\{1,\ldots,k\}$ are ordered such that $D(T_1)\geq\ldots\geq D(T_k)$. The packing solution that induces the minimum outbound shipping cost may not be unique. We prove that applying the first-fit algorithm on a specific order of demand profiles leads us to one such packing solution. Among all the packing solutions that induce the minimum outbound shipping cost, there must exist one $T_1^*,\ldots,T_k^*$ such that each demand in T_m^*, $m\in\{1,\ldots k\}$, cannot be moved to $T^*_{m-\Delta}$, $\Delta\in\{1,\ldots,m-1\}$ without moving other packed demands.

Otherwise, we can create a packing solution that induces less shipping cost. If we order the demand profiles of suppliers S in a sequence of how they are placed in $T_1^*, \ldots, T_k^*$, the packing solution of the first-fit algorithm for this sequence of demand profiles is $T_1^*, \ldots, T_k^*$ by the definition of the first-fit algorithm. □

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