Nonlinear Wave Dynamics in δ-Scuti Stars

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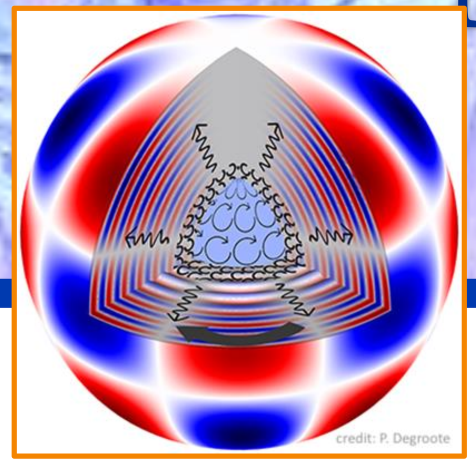
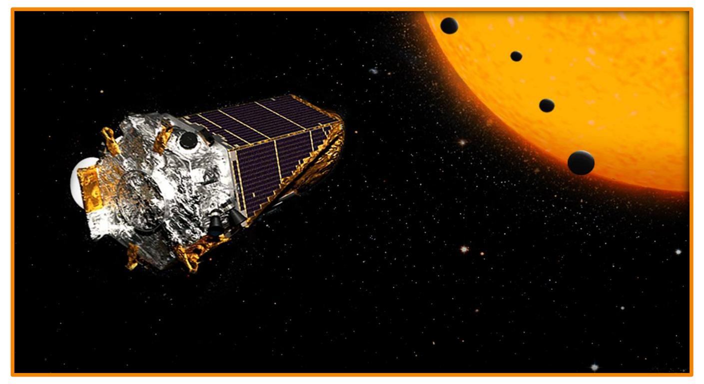


Abstract:

- ☆ Delta Scuti δ-Scuti are a type of variable star that show detectable brightness fluctuations obeying a period-luminosity relationship
- ☆ The brightness fluctuations are caused by unstable excited mode oscillations (fluid waves)
- ☆ The Kepler space telescope has observed around 1000 δ-Scuti stars, over half of which exhibit large amplitude variations and evidence for nonlinear mode coupling
- ☆ We demonstrated that this type of nonlinear coupling occurs often δ-Scuti models and can be utilized to explain these observations that contradict the linear theory.
- ☆ We utilized a second nonlinear mechanism to stabilize the excited waves and showed that we can study the evolution of these modes through time



Direct Coupling in δ-Scuti

- ☆ We construct 14 δ-Scuti models from four different mass stars using the stellar evolution program MESA
- ☆ With effective temperatures ranging from 7500 K to 8500 K and surface gravity 3.8-4.3 (cgs)

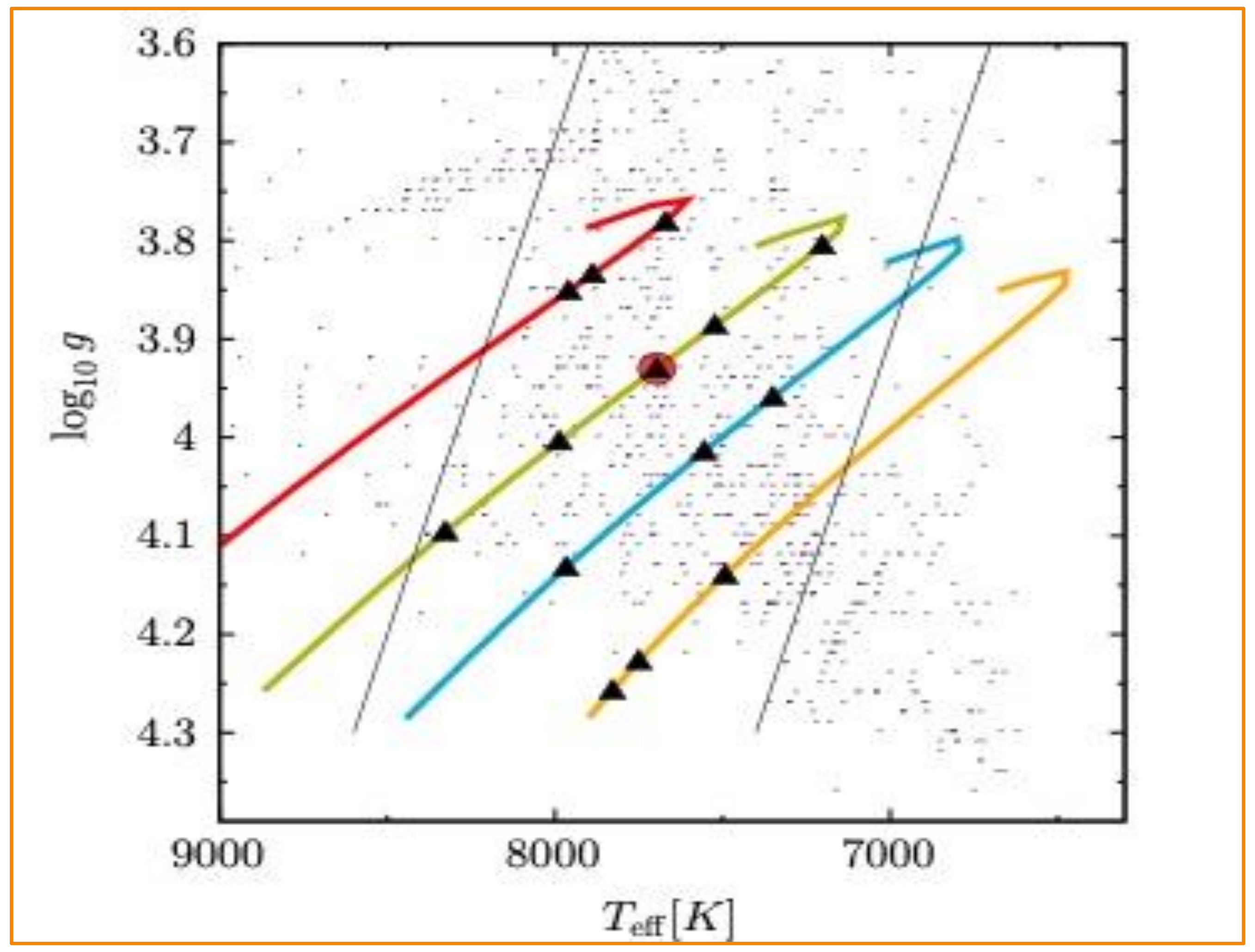


Fig 1 : Instability strip on gravity temperature planet, Our models in black triangles. The purple triangles in the background are observed δ Sct data (Bowman et al. 2016), the black lines are the theoretical bounds of the instability strip (Rodriguez & Breger 2001).

Mixed Coupling System

- ☆ Under the direct coupling nonlinear mechanism, the linearly unstable parents growth needs to be stopped in order to study the evolution of the system
- ☆ We propose the use of another nonlinear mechanisms, called the parametric instability, to stabilize the system
- ☆ This offers an elegant solution making use of nonlinear coupling only to study these δ-Scuti stars while conserving the observable properties of direct coupling

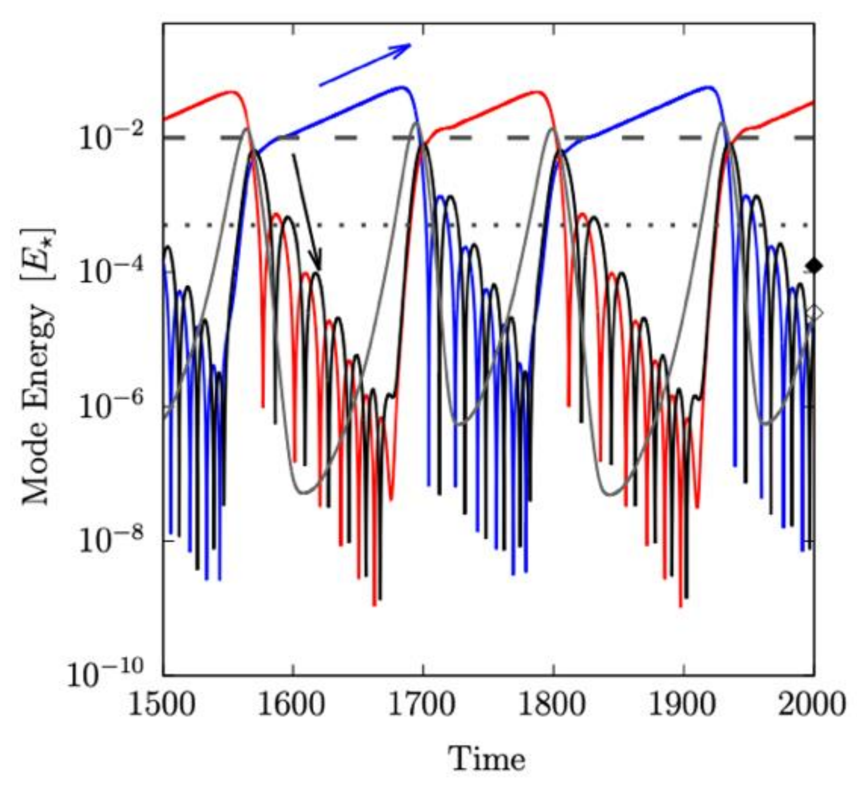
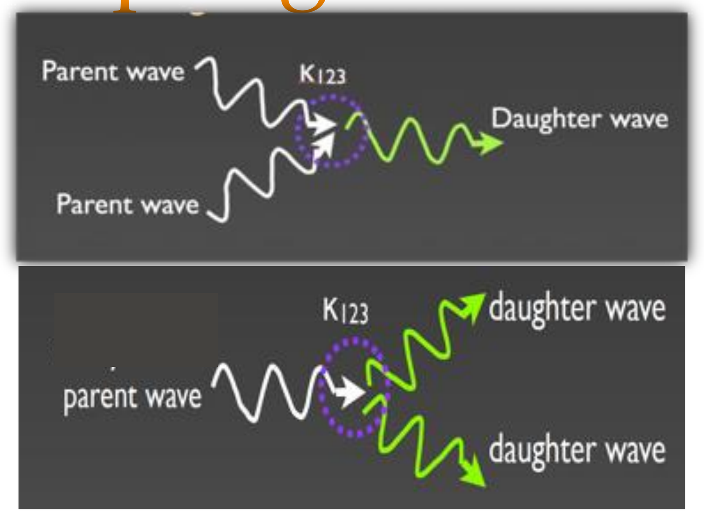


Fig. 2: From Mourabit & Weinberg 2025 (submitted ApJ), this figure demonstrates the stable limit cycle solution of a simple mixed coupling systems. With the mode amplitude adopting the behaviors we expect from observations

Nonlinear Mode Coupling:

- ☆ Direct coupling consists of two linearly driven parents (by the kappa-mechanism for δ-Scuti) nonlinearly exciting a third daughter wave



- ☆ For the parametric instability only one parent wave nonlinearly couples to two daughter waves
- ☆ In the mixed coupling model, the parents of the directly coupled triplets are stabilized by parametric daughters. This system requires no additional assumption than for the direct coupling.
- ☆ The mixed coupling system also retains the amplitude relationship between the three wave that is observable in these stars:

$$|A_a| = \mu |A_b||A_c|$$

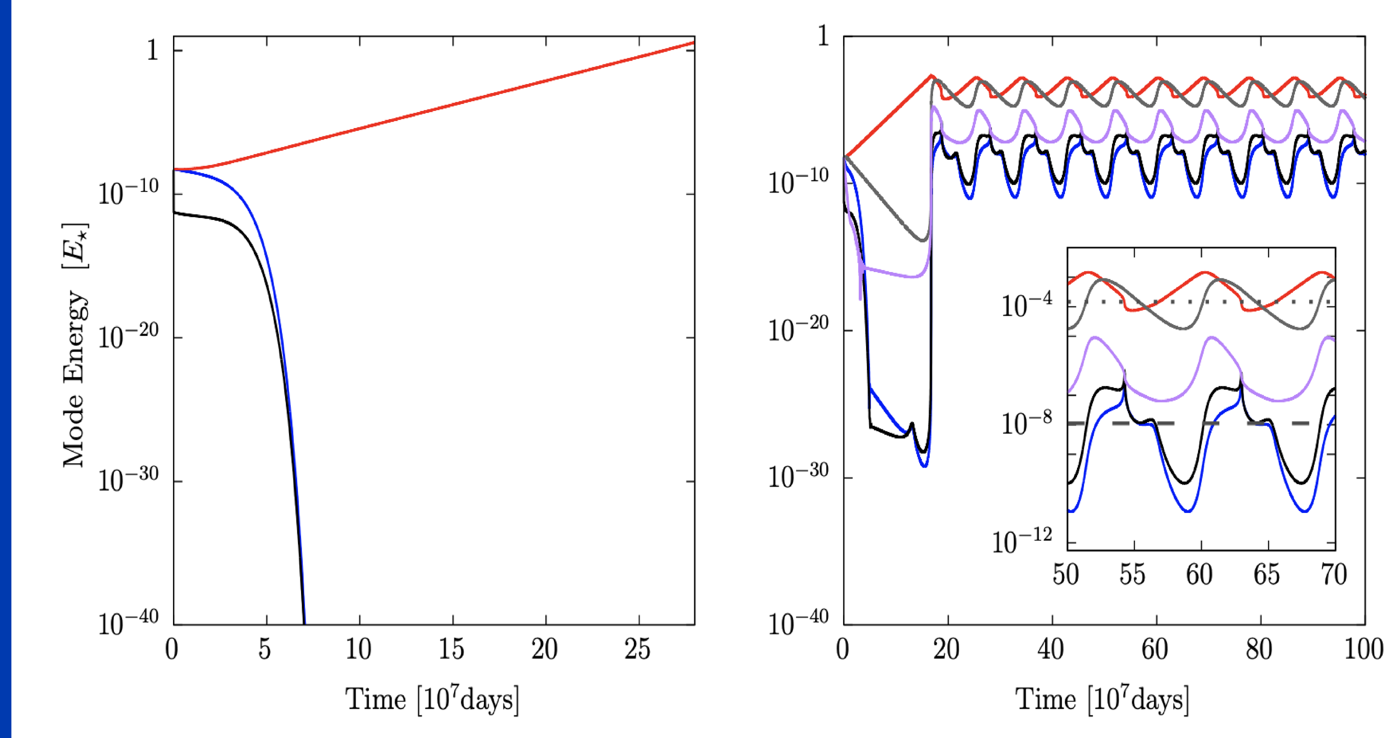


Fig. 3: Mode coupling with δ-Scuti model parameters. The left panel show the evolution of a simple directly coupled system. The right panel show the evolution of a mixed coupling system as a stable limit cycle

Results:

- ☆ We find that a large number of triplets have coupling strength $\mu > 10^3$ for all models
- ☆ Mixed coupling systems in stabilizing the direct coupling mechanism and conserving its observable quality

Conclusion:

- ☆ In all of our δ Sct models, we find many strongly coupled triplets (μ > **10^3**)
- ☆ These results are broadly consistent with the coupling strengths from Kepler observations (Breger et al. 2014, Bowman et al. 2016)
- ☆ The Mixed Coupling system allows us to study the evolution of nonlinearly coupled modes without any additional assumptions for δ-Scuti stars.
- ☆ These two works represent a proof of concept for nonlinear wave coupling in δ-Scuti stars and as a following project we will model mixed coupling as a large network of hundreds of modes and their evolution, to simulate the condition of δ-Scuti stars,
- ☆ We are in the process of analyzing Kepler and TESS data for nonlinear coupling

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