Orbital Stability of Hierarchical 3 and 4-Body Systems with Inclination: Results for Kepler-1625, 1708, and HD 23079

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Introduction

As the number of potential exomoon candidates grows, there is a heightened motivation of pursing orbital stability analyses. In this work, we provide an in-depth investigation into 4-body systems, consisting of a star, planet, moon, and submoon by using the N-body simulator `rebound`. Particularly, we focus on the system of Kepler-1625, where evidence of a possible exomoon has been obtained. We investigate the 3-body star–planet–moon system for the proposed exomoon parameters allowing us to identify stable regions associated with most of the space parameters. Thereafter, we consider a 4-body system including a potential submoon. We find that there are both stable and unstable regions, as expected, as well as resonance patterns that are further explored using numerical and analytical methods including secular perturbation theory. We are able to identify these resonances as secular in nature. In addition, we investigate 3-body versions of two other systems, Kepler-1708 and HD 23079, while also studying a 4-body version of HD 23079. Our work may serve as a generalized framework for exploring other planet–moon cases in the future while noting that the current 4-body study may be an incentive for studying further exomoon and submoon systems. The extended version of this work is published in Monthly Notices (Patel, Quarles, and Cuntz, 2025).

Methodology

- We use the N-body software, `rebound`, to run orbital stability simulations in 3-body (star-planet-moon) systems for 3 key systems (Kepler-1625, Kepler-1708, and HD 23079)
- Simulations of 100k years with a stability limit put in place if the exomoon leaves the Hill radius
- 2D parameter space over exomoon's initial semi-major axis and inclination
  - For Kepler-1625 (Teachey & Kipping, 2018) and 1708 (Kipping et al., 2022), moon's initial conditions are based on the literature (no available parameters for HD 23079)
- We utilize the maximum eccentricity of the exomoon as a proxy for stability (plotted in parameter space plots)
- From the 3-body parameter space, we choose one stable case from Kepler-1625 and HD 23079 for the exomoon to use in the 4-body case
- 4-body simulations (including a submoon) are then run utilizing a similar parameter space (see Table 1 for details)
- We then utilize methods to test for resonances in our 4-body parameter space plots (i.e., analyzing apsidal precession rates, using the chaos indicator MEGNO in `rebound`, and using secular perturbation theory to test for secular resonances)

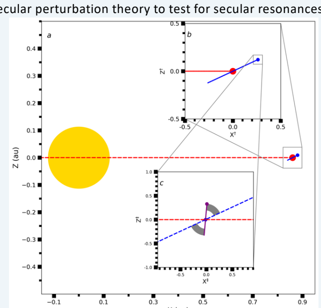

Fig. 1 – 4-body system setup for Kepler-1625 (gray areas: parameter space)

Results

3-Body Parameter Space

- Fig. 2 shows the results from our 3-body tests exploring exomoon stability, using maximum eccentricity as a proxy
- The black lines in Fig. 2a and 2c represent the initial conditions from the literature while the white dots represent the chosen parameters used in the 4-body case
- Fig. 2a and 2b show an instability gradient starting at an inclination around 45° corresponding with the von Zeipel-Lidov-Kozai (ZLK) effect
- There is also a gradient for Fig. 2a and 2b as the initial semi-major axis increases which can be attributed to the exomoon being further away from the planet leading to increased perturbations from the host star
- Fig. 2c shows more stratification with respect to inclination due to the exomoon being much closer to the planet
- We can conclude that most of the observed parameter ranges fall in stable areas except for those in the high inclination areas starting at around 40° or 45°

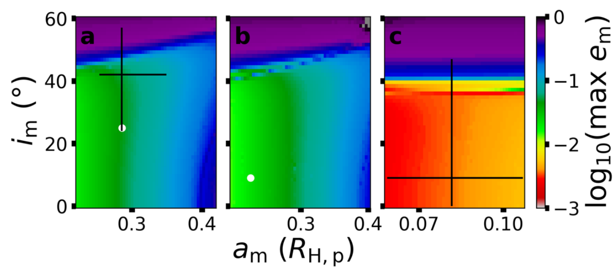

Fig. 2 – 3-body stability plots for (a) Kepler-1625, (b) HD 23079, and (c) Kepler-1708

	M_* ($M_\odot$)	M_p (M_J)	a_p (au)	e_p	M_m ($M_\oplus$)	a_m (au)	i_m (°)	M_{sm} ($M_\oplus$)	a_{sm} (R_H)	i_{sm} (°)
HD 23079	1.01	2.41	1.586	0.087	1.000	0.0260–0.0530	0–60	...	...	...
						0.0298	9	1.67×10^{-4}	0.20–0.33	0–60
Kepler-1625	1.04	2.28	0.863	0	10.218	0.0170–0.0320	0–60	...	...	...
						0.0219	25	1.67×10^{-4}	0.20–0.33	0–60
Kepler-1708	1.088	1.00	1.640	0.40	17.148	0.0038–0.0070	0–60	...	...	...

Table 1 – Initial conditions for the 3-body (first row of each respective system) and 4-body (second row) simulations

4-Body Parameter Space

- We extend our tests to consider a potential submoon using our results for stable exomoons in the Kepler-1625 and HD 23079 systems
- We find similar unstable regions starting at higher inclinations (~40°) due to the ZLK effect
  - Due to a smaller hill radius and more submoon orbits in the same timescale, the submoon is stripped away (gray) at high inclinations in contrast to the exomoon in the 3-body plots (blue/purple)
- Fig. 3a shows 3 distinct blue/cyan curves which may correspond to secular resonances (Fig. 3b also shows similar curves)

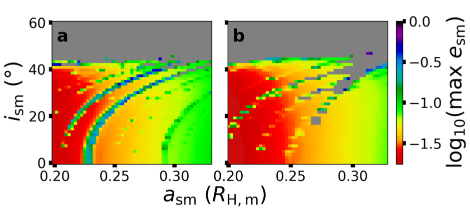

Fig. 3 – 4-body stability plots for (a) Kepler-1625 and (b) HD 23079

Secular Resonance Tests

- We look at apsidal precession rates for the moon (1.22° yr^{-1}) and submoon (8.026° yr^{-1}) and find a potential 13:2 secular resonance
- Using 100 year MEGNO simulations, we find the 2 significant curves in Fig. 4 that match the ones found in Fig. 3a which strongly suggests a secular resonance (the third curve may show up at longer timescales)
- In our final test, we use secular perturbation theory to compare with our `rebound` simulations
  - Utilizing equations found in Murray & Dermott (2000), we compare the time evolution of inclination to the `rebound` case
  - Shown in Fig. 5, we find that the secular inclination matches our `rebound` case up to a frequency shift (from non-secular effects) leading to our final confirmation of a secular resonance

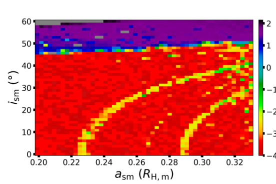

Fig. 4 – MEGNO parameter space for a submoon in the Kepler-1625 system

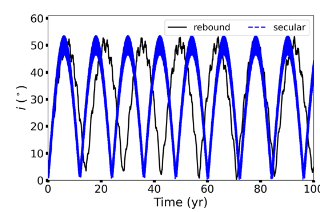

Fig. 5 – Comparison of submoon inclination from secular and N-body simulations

Conclusions

- We find that the exomoon parameter estimates given for Kepler-1625 (Teachey & Kipping, 2018) and Kepler-1708 (Kipping et al., 2022) are mostly situated in stable regions
  - Major exceptions for higher inclination parameters which are mostly unstable as expected due to the ZLK effect
- From our 4-body tests, we find stable regions for putative submoons in the Kepler-1625 and HD 23079 systems
  - There are also unstable resonance curves throughout the stable regions that are identified as secular in nature
- Our work may be useful for future studies, both regarding the exploration of exomoon candidates, as well as investigations of the possible existence of submoons

References

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