An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining*

Mark Gertler*, Luca Sala** and Antonella Trigari**
*New York University, **Bocconi University, IGIER

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Abstract

We develop and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, employment adjustment is on the extensive margin and the employment of existing workers is efficient. Wage rigidity, however, affects the hiring of new workers. The former is introduced via the staggered Nash bargaing setup of Gertler and Trigari (2006). A robust finding is that the model with wage rigidity provides a better description of the data than does a flexible wage version. Overall, the model fits the data roughly as well as existing quantitative macroeconomic models, such as Smets and Wouters (2007) or Christiano, Eichenbaum and Evans (2005). More work is necessary, however, to ensure a robust identification of the key labor market parameters.

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1 Introduction

This paper develops and estimates a quantitative macroeconomic framework that incorporates labor market frictions. Our starting point is the now conventional monetary DSGE model developed by Christiano, Eichenbaum and Evans (CEE, 2005), Smets and Wouters (SW, 2007) and others. We introduce labor market frictions with a variant of the Mortensen and Pissarides search and matching framework. This variant allows for staggered Nash wage bargaining, as in Gertler and Trigari (GT, 2006).

Our motivation is twofold. First, there are some compelling theoretical considerations. The recent vintage of monetary DSGE models typically has employment adjusting on the intensive margin along with staggered nominal wage contracting. The latter feature, further, is important for the quantitative performance of the model: The wage stickiness helps accounting for the volatility of hours. However, as a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firms and workers have an on-going relationship. If wages are not allocational then wage rigidity does not influence model dynamics. By contrast, in the model we present, firms adjust employment along the extensive margin[1]. In this instance, wage rigidity affects employment by influencing the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005a), since new workers have yet to form on-going relationships with firms, in this kind of setting the Barro's critique does not apply.

Second, within the search and matching literature, there is a debate over how well the baseline Mortensen and Pissarides framework can account for labor market volatility, or whether it may be necessary to introduce additional features such as wage rigidity, etc. (e.g. Shimer, 2005, Hall, 2005a, Hagedorn and Manovskii, 2006, Mortensen and Nagypal, 2007). A typical approach in the literature has been to develop a calibrated model, subject it to productivity shocks, and then examine model moments against moments of the data, with various features such as wage rigidity or on-the-job search shut on and then shut off. We instead estimate a complete macroeconomic framework using Bayesian methods. Doing so allows us to formally evaluate the significance of different mechanisms such as wage rigidity to overall model performance. In addition, our full information procedure permits us to account for the complete range of shocks that hit the economy.

In section 2 we develop the model. The basic framework follows CEE and SW closely. The only significant difference involves the treatment of the labor market. As in GT, we incorporate a variation of Mortensen/Pissarides that retains the empirically appealing feature of Nash bargaining, but replace the assumption of period-by-period wage negotiations to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and its existing workforce: Workers hired in-between

[1] In appendix F we describe how to extend the model to allow for the intensive margin (i.e., variable hours per worker.) Because the firm and its workers have an on-going relation, hours per worker are determined efficiently.

contract settlements receive the existing wage.[2] In the language of Hall (2005a), the existing contract wage provides a wage norm for the workers hired in-between contracting periods. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it surely would be preferable to fully endogenize the contract structure, the payoff from our shortcut is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. In this instance, a key primitive parameter of the model is the average frequency of wage adjustment. Whereas in GT this parameter is calibrated to existing evidence on wage contract length, here we are able to estimate it.

Another significant difference from GT, which is a purely real model, is that wage contracting is in nominal terms. However, as in CEE and SW, we allow for indexing of wages to past inflation and estimate the degree of indexing. This consideration is important for the following reasons. As indexing to past inflation becomes complete, nominal wage rigidity begins to approximate real wage rigidity. As Blanchard and Galí (2006) emphasize, real wage rigidity complicates the short run output/inflation trade-off that the central bank faces, beyond what would arise from simple nominal rigidities.

In section 4 we describe our estimation procedure and then present the model estimates. We use Bayesian methods, following closely SW and Primiceri, Schaumburg and Tambalotti (PST, 2006). We present a variety of diagnostics to evaluate the overall model performance and in particular the role of wage rigidity in our framework.

Before proceeding, we emphasize that there have been a number of papers related to ours. Trigari (2008) and Walsh (2005) were among the first to integrate a search and matching setup within a monetary DSGE model with nominal price rigidities. Blanchard and Galí (2006) develop a qualitative version of this model with a simple form of real wage rigidities. Christoffel et al. (2006) have also estimated a monetary DSGE model with labor market frictions and wage rigidity. They employ a setup with right-to-manage bargaining as in Trigari (2006), where ex post hours may be inefficient. Since part our interest is to address the Barro critique, we employ the setup of GT, which has efficient bargaining along with staggered wage setting. Thus, within our setup the employment of existing workers is fully efficient: wage rigidity affects hiring at the extensive margin. In addition, while Christoffel et al. (2006) model wage rigidity by introducing adjustment costs of wage changes for a representative firm, we do so by having staggered contracting.[3] We also differ in the details of the precise model we estimate, as well as the exact estimation procedure and data.

[2] A number of authors argue that the wages of new hires are more flexible that those of existing workers. See, for example, the survey by Pissarides (2007). Gertler, Huckfeldt and Trigari (2008), however, use data from the Survey of Income and Program Participation and show that after controlling for compositional effects, the evidence that new workers' wages are more flexible disappears.

[3] More recently, Christiano, Motto and Rostagno (2007) have also incorporated the GT variation of the MP model into the CEE/SW framework to study stock market boom bust cycles. The focus of our paper is instead in parameter estimation and model comparison.

2 The Model

As we discussed, the model is a variant of the conventional monetary DSGE framework. It has the key features that many have found useful for capturing the data. These include habit formation, costs of adjusting the flow of investment, variable capital utilization, nominal price and wage rigidities, and so on. The key changes involve the labor market. Rather than having hours vary on the intensive margin, we introduce variation on the extensive margin and unemployment. We do so by introducing search and matching in the spirit of Mortensen and Pissarides and others. Further, to introduce nominal wage rigidity, we use the staggered Nash bargaining approach of GT.

We note that in an earlier version of this paper we also allowed for variation in hours on the intensive margin. We drop this feature for two reasons. First, as Figure 1 shows, most of the cyclical variation in hours in the U.S. is on the extensive margin. Second, our earlier estimates confirmed that the intensive margin was unimportant to cyclical variation. We estimated a Frisch elasticity close to zero, which is certainly in line with the microeconomic evidence.[4]

Our model has three types of agents: households, wholesale firms, and retail firms. We use a representative family construct, similar to Merz (1995) in order to introduce complete consumption insurance. Production takes place at wholesale firms. These firms are competitive. They hire workers and negotiate wage contracts with them. Retail firms buy goods from wholesalers and then repackage them as final goods. Retailers are monopolistic competitors and set prices on a staggered basis. We separate retailers from wholesalers to keep the wage bargaining problem tractable.[5]

Finally, following SW, we introduce a number of exogenous shocks that correspond exactly to the number of data series we consider in our estimation.

2.1 Households

There is a representative household with a continuum of members of measure unity. The number of family members currently employed is n_t. Employment is determined through a search and matching process that we describe shortly. The family provides perfect consumption insurance for its members, implying that consumption is the same for each person, regardless of whether he or she is currently employed.

Except for the treatment of labor supply, the household's decision problem is identical to that in CEE and SW. In the latter, the household receives utility from leisure and it varies labor only on

[4] One problem is that there is considerable low frequency variation in hours per worker due to factors outside the model such as demographics. This could explain our difficulty in obtaining reasonable estimates of the Frisch elasticity. We are continuing to explore this issue.

[5] To keep the bargaining problem tractable, it is necessary to have constant returns at the firm level. This is to make the average and marginal worker the same, thus avoiding bargaining spillovers among workers. Introducing staggered price setting requires that firms face downward sloping demand curves, implying differences between average and marginal.

the intensive margin. Here there is no utility gain from leisure. Individuals not currently working are searching for jobs.

Accordingly, conditional on n_t, the household chooses consumption c_t, government bonds B_t, capital utilization ν_t, investment i_t, and physical capital k_t^p to maximize the utility function

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log\left(c_{t+s} - hc_{t+s-1}\right), \tag{1}$$

where h is the degree of habit persistence in consumption preferences and where ε_t^b is a preference shock with mean unity that obeys

$$\log \varepsilon_t^b = \rho^b \log \varepsilon_{t-1}^b + \varsigma_t^b, \tag{2}$$

and where all primitive innovations, including ς_t^b, are zero-mean i.i.d. random variables.

Let Π_t be lump sum profits, T_t lump sum transfers, p_t the nominal price level, and r_t the one period nominal interest rate (specifically, the central bank policy instrument). Then the household's budget constraint is

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t)\, b_t + r_t^k \nu_t k_{t-1}^p + \Pi_t + T_t - \mathcal{A}(\nu_t)\, k_{t-1}^p + \frac{B_{t-1}}{p_t}. \tag{3}$$

Households own capital and choose the capital utilization rate, ν_t, which transforms physical capital into effective capital according to

$$k_t = \nu_t k_{t-1}^p. \tag{4}$$

Effective capital is rented to the firms at the rate r_t^k. The cost of capital utilization per unit of physical capital is $\mathcal{A}(\nu_t)$. We assume that $\nu_t = 1$ in the steady state, $\mathcal{A}(1) = 0$ and $\mathcal{A}'(1)/\mathcal{A}''(1) = \eta_\nu$.

The physical capital accumulation equation is

$$k_t^p = (1 - \delta)\, k_{t-1}^p + \varepsilon_t^i \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right] i_t, \tag{5}$$

where we assume $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and $\mathcal{S}''(\gamma_z) = \eta_k > 0$ where γ_z is the economy's steady state growth rate.

ε_t^i is an investment specific technological shock with mean unity affecting the efficiency with which consumption goods are transformed into capital. We assume ε_t^i follows the exogenous stochastic process

$$\log \varepsilon_t^i = \rho^i \log \varepsilon_{t-1}^i + \varsigma_t^i. \tag{6}$$

The first order necessary conditions yield:

(c_t)

$$\lambda_t = \frac{\varepsilon_t^b}{c_t - hc_{t-1}} - \beta h E_t \frac{\varepsilon_{t+1}^b}{c_{t+1} - hc_t}, \tag{7}$$

(B_t)

$$\lambda_t = r_t \beta E_t \left(\frac{\lambda_{t+1} p_t}{p_{t+1}} \right), \tag{8}$$

(ν_t)

$$r_t^k = \mathcal{A}' \left(\nu_t \right), \tag{9}$$

(i_t)

$$q_t^k \varepsilon_t^i \left[1 - \mathcal{S} \left(\frac{i_t}{i_{t-1}} \right) \right] = q_t^k \varepsilon_t^i \mathcal{S}' \left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} - \beta E_t q_{t+1}^k \varepsilon_{t+1}^i \frac{\lambda_{t+1}}{\lambda_t} \mathcal{S}' \left(\frac{i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 + 1, \tag{10}$$

(k_t^p)

$$q_t^k = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\delta) q_{t+1}^k + r_{t+1}^k \nu_{t+1} - a \left(\nu_{t+1} \right) \right], \tag{11}$$

where $\lambda_t = u'(c_t)$ and q_t^k is the value of installed capital in consumption units.

Again, except for the treatment of the labor supply decision, the household sector is conventional.

2.2 Unemployment, Vacancies and Matching

At time t, each firm posts $v_t(i)$ vacancies in order to attract new workers and employs $n_t(i)$ workers. The total number of vacancies and employed workers are $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. All unemployed workers at t look for jobs. Our timing assumptions are such that unemployed workers who find a match go to work immediately within the period. Accordingly, the pool of unemployed workers searching for a job at t, u_t, is given by the difference between unity (the total population of workers) and the number of employed workers at the end of period $t-1$, n_{t-1}:

$$u_t = 1 - n_{t-1}. \tag{12}$$

The number of new hires or "matches", m_t, is a function of searching workers and vacancies, as follows:

$$m_t = \sigma_m u_t^\sigma v_t^{1-\sigma}. \tag{13}$$

The probability a firm fills a vacancy in period t, q_t, is given by

$$q_t = \frac{m_t}{v_t}. \tag{14}$$

Similarly, the probability a searching worker finds a job, s_t, is given by

$$s_t = \frac{m_t}{u_t}. \tag{15}$$

Both firms and workers take q_t and s_t as given.

Finally, each period, firms exogenously separate from a fraction $1-\rho$ of their existing workforce $n_{t-1}(i)$. Workers losing their job at time t are not allowed to search until next period. Accordingly, within our framework fluctuations in unemployment are due to cyclical variation in hiring as opposed to separations. Both Hall (2005b,c) and Shimer (2005, 2007) present evidence in support of this phenomenon.

2.3 Wholesale Firms

Each period, wholesale firms produce output y_{it} using capital, k_{it}, and labor, n_{it}:

$$y_{it} = (k_{it})^{\alpha} (z_t n_{it})^{1-\alpha}, \tag{16}$$

where z_t is a common labor-augmenting productivity factor. We assume $\varepsilon_t^z = z_t/z_{t-1}$ obeys the following exogenous stochastic process

$$\log \varepsilon_t^z = (1-\rho^z)\log \varepsilon^z + \rho^z \log \varepsilon_{t-1}^z + \varsigma_t^z. \tag{17}$$

Note that the steady state value ε^z corresponds to the economy's growth rate γ_z. Thus, following PST, we are allowing technology be non-stationary in levels, though stationary in growth rates.

For simplicity, we assume that capital is perfectly mobile across firms and that there is a competitive rental market in capital. These assumptions ensure constant returns to scale at the firm level, which greatly simplifies the wage bargaining problem (see the discussion in GT).

It is useful to define the hiring rate x_{it} as the ratio of new hires $q_t v_{it}$ to the existing workforce n_{it-1}:

$$x_{it} = \frac{q_t v_{it}}{n_{it-1}}. \tag{18}$$

Observe that due to the law of large numbers the firm knows x_{it} with certainty at time t since it knows the likelihood q_t that each vacancy it posts will be filled. The hiring rate is thus effectively the firm's control variable.

The total workforce, in turn, is the sum of the number of surviving workers ρn_{it-1} and new hires $x_{it} n_{it-1}$:

$$n_{it} = (\rho + x_{it})\, n_{it-1}. \tag{19}$$

Equation (19) reflects the timing assumption that new hires go to work immediately.[6]

Let p_t^w be the relative price of intermediate goods, w_{it}^n the nominal wage, r_t^k the rental rate of capital, and $\beta E_t \Lambda_{t,t+1}$ be the firm's discount rate, where the parameter β is the household's subjective discount factor and where $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$. Then, the value of the firm, $F_t(w_{it}^n, n_{it-1})$, may be expressed as:

$$F_t(w_{it}^n, n_{it-1}) = p_t^w y_{it} - \frac{w_{it}^n}{p_t} n_{it} - \frac{\kappa_t}{2} x_{it}^2 n_{it-1} - r_t^k k_{it} + \beta E_t \Lambda_{t,t+1} F_{t+1}\left(w_{it+1}^n, n_{it}\right), \tag{20}$$

[6] Blanchard and Gali (2006) use a similar timing.

with

$$\kappa_t = \kappa z_t. \tag{21}$$

As wage dispersion is present, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs, given here by $\frac{\kappa_t}{2} x_{it}^2 n_{it-1}$. As in GT, we have quadratic costs of hiring as opposed to fixed costs of post a vacancy for purely technical reasons. Because the contract structure leads to temporary wage dispersion and because (to simplify the bargaining problem) we have constant returns at the firm level, quadratic costs are required to keep capital and labor from shifting en mass to the low wage firms.[7] Finally, we allow adjustment costs to drift proportionately with productivity in order to maintain a balanced steady state growth path (otherwise adjustment costs become relatively less important as the economy grows.)

At any time, the firm maximizes its value by choosing the hiring rate (by posting vacancies) and its capital stock, given its existing employment stock, the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future.

We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain. The first order condition for capital is simply:

$$r_t^k = p_t^w \alpha \frac{y_{it}}{k_{it}} = p_t^w \alpha \frac{y_t}{k_t}. \tag{22}$$

Given Cobb-Douglas technology and perfect capital mobility, all firms choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios

Firms choose n_{it} by setting x_{it} or, equivalently, v_{it}. The firm's hiring decision yields:

$$\kappa_t x_{it} = p_t^w a_{it} - \frac{w_{it}^n}{p_t} + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}\left(w_{it+1}^n, n_{it}\right)/\partial n_{it}, \tag{23}$$

with

$$a_{it} = (1-\alpha)\frac{y_{it}}{n_{it}} = (1-\alpha)\frac{y_t}{n_t} = a_t. \tag{24}$$

By making use of the envelope theorem to obtain $F_t\left(w_{it}^n, n_{it-1}\right)/\partial n_{it-1}$ and combining equations, we obtain

$$\kappa_t x_{it} = p_t^w a_t - \frac{w_{it}^n}{p_t} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{it+1}^2 + \rho \beta E_t \Lambda_{t,t+1} \kappa_{t+1} x_{it+1}. \tag{25}$$

The hiring rate thus depends on the discounted stream of earnings and savings on adjustment costs.

[7] We could have instead introduced quadratic costs of posting vacancies as opposed to adding workers. We chose the latter because it leads to a simpler formulation.

Finally, for the purpose of the wage bargain it is useful to define $J_t(w_{it}^n)$, the value to the firm of having another worker at time t after new workers have joined the firm, i.e., after adjustment costs are sunk. Differentiating $F_t(w_{it}^n, n_{it-1})$ with respect to n_{it}, taking x_{it} as given yields:

$$J_t(w_{it}^n) = p_t^w a_t - \frac{w_{it}^n}{p_t} + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}(w_{it+1}^n, n_{it}) / \partial n_{it}. \tag{26}$$

By making use of the hiring rate condition and the relation for the evolution of the workforce, $J_t(w_{it}^n)$ may be expressed as expected average profits per worker net of the first period adjustment costs, with the discount factor accounting for future changes in workforce size:

$$J_t(w_{it}^n) = p_t^w a_t - \frac{w_{it}^n}{p_t} - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{it+1}^2 + E_t[\rho + x_{it+1}] \beta \Lambda_{t,t+1} J_{t+1}(w_{it+1}^n). \tag{27}$$

2.4 Workers

In this sub-section we develop an expression for a worker's surplus from employment, which is a critical determinant of the outcome of the wage bargain.

Let $V_{it}(w_{it}^n)$ be the value to a worker of employment at firm i and let U_t be the value of unemployment. These values are defined after hiring decisions at time t have been made and are in units of consumption goods. $V_t(w_{it}^n)$ is given by

$$V_t(w_{it}^n) = \frac{w_{it}^n}{p_t} + \beta E_t \Lambda_{t,t+1} \left[\rho V_{t+1}(w_{it+1}^n) + (1-\rho) U_{t+1}\right]. \tag{28}$$

To construct the value of unemployment, we first define $V_{x,t}$ as the average value of employment conditional on being a new worker at t:

$$V_{x,t} = \int_0^1 V_t(w_{it}^n) \frac{x_{it} n_{it-1}}{x_t n_{t-1}} di, \tag{29}$$

where $x_{it} n_{it-1}$ is total new workers at firm i and $x_t n_{t-1}$ is total new workers at t.[8] Next, let b_t be the flow value from unemployment, including unemployment benefits, as well as other factors that can be measured in units of consumption goods. As before, let s_t be the probability of finding a job for the subsequent period. Then U_t may be expressed as

$$U_t = b_t + \beta E_t \Lambda_{t,t+1} \left[s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1}\right], \tag{30}$$

with

$$b_t = b k_t^p,$$

[8] $V_{x,t}$ is thus distinct from the unconditional average value of employment $V_t = \int_0^1 V_t(w_{it}^n) \frac{n_{it}}{n_t} di$. One technical aspect is that there is no steady-state distribution of employment shares across firms. One solution to this issue would be to take averages integrating over the distribution of wages across workers, which is well-defined in the steady state. However, as this would not affect the loglinearized equilibrium, we choose to sidestep this issue here in order to keep the exposition simple. See Gertler and Trigari (2006) for details.

and where k_t^p is the economy-wide capital stock. We assume that b_t grows proportionately to k_t^p in order to maintain a balanced growth; otherwise b_t would become a smaller fraction of labor income as the economy grows. The value of unemployment thus depends on the current flow value b_t and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that is currently unemployed is $V_{x,t+1}$, the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.[9]

$H_t\left(w_{it}^n\right)$ and $H_{x,t}$ are given by:

$$H_t\left(w_{it}^n\right) = V_t\left(w_{it}^n\right) - U_t, \tag{31}$$

and

$$H_{x,t} = V_{x,t} - U_t. \tag{32}$$

It follows that:

$$H_t\left(w_{it}^n\right) = \frac{w_{it}^n}{p_t} - b_t + \beta E_t \Lambda_{t,t+1}\left[\rho H_{t+1}\left(w_{it+1}^n\right) - s_{t+1} H_{x,t+1}\right]. \tag{33}$$

2.5 Nash Bargaining and Wage Dynamics

As we noted earlier, we introduce staggered Nash wage bargaining as in GT, but with the following difference: Because we have a monetary model, we allow for nominal wage contracting, though following CEE and SW we also allow for the possibility of indexing to past inflation.

We introduce staggered multi-period wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability $1-\lambda$ that it may re-negotiate the wage.[10] This adjustment probability is independent of its history, making it unnecessary to keep track of individual firms' wage histories. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at $1/(1-\lambda)$. The coefficient λ is thus a measure of the degree of wage stickiness. In GT the parameter λ is calibrated. Here we are able to estimate it.

Since we allow for the possibility of indexing to past inflation, π_{t-1}, the fraction λ of firms that cannot renegotiate their contract set their nominal wages w_{it}^n following the indexation rule:

$$w_{it}^n = \overline{\gamma} w_{it-1}^n \pi_{t-1}^{\gamma}, \tag{34}$$

where $\pi_t = p_t/p_{t-1}$, $\overline{\gamma} = \gamma_z \pi^{1-\gamma}$ and where $\gamma \in [0,1]$ reflects the degree of indexing to past inflation. We also estimate the parameter γ. The term $\overline{\gamma}$ in the indexing rule provides an adjustment for trend productivity growth and trend inflation.

[9] There is accordingly no directed search. Note, however, that wage differentials across firms are only due to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

[10] This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

Firms that enter a new wage agreement at t negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage.[11] When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the existing contracting wage (i.e. last period nominal wage adjusted for possibly indexing.)[12] As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for worker who arrive in between contracting periods.[13] Of course, the newly hired workers recognize that they will be able to re-negotiate the wage at the next round of contracting.

Let w_t^{*n} denote the wage of a firm that renegotiates at t. Given constant returns, all sets of renegotiating firms and workers at time t face the same problem, and thus set the same wage. As we noted earlier, the firm negotiates with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the contract wage w_t^{*n} is chosen to solve

$$\max H_t\left(w_{it}^n\right)^{\eta_t} J_t\left(w_{it}^n\right)^{1-\eta_t}, \tag{35}$$

s.t.

$$w_{it+j}^n = \begin{cases} \overline{\gamma} w_{it+j-1}^n \pi_{t+j-1}^{\gamma} \text{ with probability } \lambda \\ w_{t+j}^{n*} \text{ with probability } 1-\lambda \end{cases}$$

$\forall\ j \geq 1$, where $\eta_t \in [0,1]$ is the worker's relative bargaining power, and where $J_t\left(w_{it}^n\right)$ and $H_t\left(w_{it}^n\right)$ are given by equations (27) and (33).

In the conventional search and matching framework the bargaining power parameter is constant. Here, in order to allow for an error term in the wage equation we allow this parameter to evolve exogenously according to

$$\eta_t = \eta \varepsilon_t^{\eta}, \tag{36}$$

[11] To be clear, with constant returns, one could either think of the firm bargaining with each marginal worker individually or bargaining with a union that wishes to maximixe average worker surplus.

[12] As we noted earlier, in Gertler, Huckfeldt and Trigari (2008) we present evidence in support of this assumption. Bewley (1999) also presents survey evidence that the wages of new hires are linked to those of existing workers. Other studies of the cyclical behavior of wages for new hires (e.g. Bils, 1985) do not examine the link with existing workers wages (due to data limitations) and thus do not speak to our hypothesis. We think that explaining the facts in these studies will require introducing heterogeneity into our framework.

[13] In addition to scale economies in bargaining, there are several complementary justifications for why hires in between contracts receive the existing contract wage. First, as we noted earlier, Bewley (1999) argues that internal equity constrains workers of similar productivity to receive similar wages. Second, Menzio and Moen (2006) show how the trade-off between efficient provision of insurance to senior workers and efficient recruitment of junior ones links the wages of new and existing workers in response to small and negative productivity shocks. Third, consistent with Hall (2005a), one might interpret the existing contract wage as the "wage norm" for workers hired in between contracts.

where ε_t^{η} is a mean-unity bargaining power shock that follows the stochastic process

$$\log \varepsilon_t^{\eta} = \rho^{\eta} \log \varepsilon_{t-1}^{\eta} + \varsigma_t^{\eta}. \tag{37}$$

The first order necessary condition for the Nash bargaining solution is given by

$$\eta \epsilon_t J_t \left(w_t^{*n}\right) = (1-\eta)\, \mu_t \left(w_t^{*n}\right) H_t \left(w_t^{*n}\right), \tag{38}$$

where $\epsilon_t = p_t \partial H_t \left(w_{it}^n\right) / \partial w_{it}^n$ is the effect of a rise in the real wage on the worker's surplus, $\mu_t \left(w_{it}^n\right) = -p_t \partial J_t \left(w_{it}^n\right) / \partial w_{it}^n$ is minus the effect of a rise in the real wage on the firm's surplus, and where

$$\epsilon_t = 1 + E_t \Lambda_{t,t+1} \left(\rho \lambda \beta\right) \frac{p_t}{p_{t+1}} \overline{\gamma} \pi_t^{\gamma} \epsilon_{t+1}, \tag{39}$$

and

$$\mu_t \left(w_{it}^n\right) = 1 + E_t \Lambda_{t,t+1} \left[\rho + x_{t+1}(\bar{\gamma} \pi_t^{\gamma} w_{it}^n)\right] (\lambda \beta) \frac{p_t}{p_{t+1}} \overline{\gamma} \pi_t^{\gamma} \mu_{t+1} \left(\bar{\gamma} \pi_t^{\gamma} w_{it}^n\right). \tag{40}$$

Observe that ϵ_t is effectively the cumulative discount factor the worker uses to value the contract wage stream, while $\mu_t \left(w_{it}^n\right)$ is that for the firm. Since the hiring rate must be non-negative at all times, $\mu_t \left(w_{it}^n\right) \geq \epsilon_t$, implying that the firm places a greater weight on the future than does the worker. Intuitively, the firm has a longer horizon than the worker because it cares about the effect of the current wage contract on payments not only to the existing workforce, but also to new workers who enter under the terms of the existing contract. A worker, on the other hand, only cares about wages during his or her tenure at the firm.

The first order condition for wages, then, can be rewritten as:

$$\chi_t \left(w_t^{*n}\right) J_t \left(w_t^{*n}\right) = \left[1 - \chi_t \left(w_t^{*n}\right)\right] H_t \left(w_t^{*n}\right), \tag{41}$$

with

$$\chi_t \left(w_{it}^n\right) = \frac{\eta_t}{\eta_t + (1-\eta_t)\, \mu_t \left(w_{it}^n\right) / \epsilon_t}. \tag{42}$$

Equation (41) is a variation of the conventional sharing rule where the relative share $\chi_t \left(w_t^{*n}\right)$ depends not only on the bargaining power but also on the different horizon over which the worker and the firm value the impact of the contract wage. In the limiting case of $\lambda = 0$, $\mu_t \left(w_t^{*n}\right) / \epsilon_t = 1$ and $\chi_t \left(w_t^{*n}\right) = \eta_t$ as in the conventional period-by -period case. With $\lambda > 0$, however, $\mu_t \left(w_t^{*n}\right) / \epsilon_t > 1$ and $\chi_t \left(w_t^{*n}\right) < \eta_t$. Intuitively, because it makes firms effectively more patient than workers, the "horizon effect" works to raise the effective bargaining power of firms from $1 - \eta_t$ to $1 - \chi_t \left(w_t^{*n}\right)$. This link between horizon and bargaining power is similar to what occurs in Binmore, Rubinstein and Wolinsky (1986).

Finally, the average nominal wage across workers is given by

$$w_t^n = \int_0^1 w_{it}^n \frac{n_{it}}{n_t} di. \tag{43}$$

Given that the probability of wage adjustment is i.i.d., the law of large numbers implies that the evolution of the average nominal wage is a linear contract of the target nominal wage and last period's nominal wages of non-adjusters, after factoring in indexing arrangements:

$$w_{t+1}^{n} = (1-\lambda)w_{t+1}^{*n} + \lambda \int_{0}^{1} (\bar{\gamma}\pi_t^{\gamma} w_{it}^{n}) \frac{\rho + x_{t+1}(\bar{\gamma}\pi_t^{\gamma} w_{it}^{n})}{\rho + x_{t+1}} \frac{n_{it}}{n_t} di. \tag{44}$$

2.6 Retailers

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. Retailers buy intermediate goods from the wholesale firms described in the previous section. They in turn differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, then re-sell them to the households. In addition, they set prices on a staggered basis.

Following SW, we permit each firm's elasticity to depend inversely on its relative market share, using a formulation due to Kimball (1995). The endogenous elasticity introduces a strategic complementarity in price setting (or real rigidity) that makes it easier for the model to match the micro evidence on price adjustment (see, e.g., the discussion in Woodford, 2003.)

The Kimball formulation is based on a generalization of the Dixit/Stiglitz aggregator of individual goods into a composite that allows for a varying elasticity of substitution between goods. Let y_{jt} be the quantity of output sold by retailer j and let p_{jt} be the nominal sale price. Under the Kimball formulation, accordingly, final goods, denoted with y_t, are a composite of individual retail goods, defined as follows:

$$\int_{0}^{1} \mathcal{G}\left(\frac{y_{jt}}{y_t}, \varepsilon_t^{p}\right) dj = 1, \tag{45}$$

where the function $\mathcal{G}(\cdot)$ is increasing and strictly concave, with $\mathcal{G}(1) = 1$. ε_t^{p} in an exogenous non-negative random variable that influences the elasticity of demand and thus the frictionless price markup. We assume that ε_t^{p} evolves as follows:

$$\log \varepsilon_t^{p} = (1-\rho^{p}) \log \varepsilon^{p} + \rho^{p} \log \varepsilon_{t-1}^{p} + \varsigma_t^{p}. \tag{46}$$

Cost minimization then leads to the follow demand curve facing each retailer j:

$$y_{jt} = \mathcal{G}'^{-1}\left(\frac{p_{jt}}{p_t}\tau_t\right) y_t, \tag{47}$$

where p_t is the aggregate price index and

$$\tau_t = \int_{0}^{1} \mathcal{G}'\left(\frac{y_{jt}}{y_t}\right) \frac{y_{jt}}{y_t} dj. \tag{48}$$

We assume that prices are staggered as in Calvo and we also allow for indexing. Unlike with wages, however, we only allow for indexing to trend inflation and not lagged inflation. The case for indexing to lagged inflation is less compelling for prices than for wages. Let $1 - \lambda^p$ be the probability a firm adjusts its price, where λ^p is a parameters that we estimate. Firms not adjusting their target price obey the following indexing rule:

$$p_{jt} = \overline{\gamma}^p p_{jt-1}, \tag{49}$$

where $\overline{\gamma}^p = \pi$ is an adjustment for trend inflation.

Firms that are able to adjust their price at t, set the price to maximize expected discounted profits subject to the constraints on the frequency of price adjustment. Since all firms that are resetting their price are identical ex ante, they all choose the same price, p_t^*. It follows that by combining equations (45) and (47) and applying the law of large numbers, we can express the price index as

$$p_t = (1 - \lambda^p) p_t^* \mathcal{G}'^{-1} \left(\frac{p_t^*}{p_t} \tau_t \right) + \lambda^p \overline{\gamma}^p p_{t-1} \mathcal{G}'^{-1} \left(\frac{\overline{\gamma}^p p_{t-1}}{p_t} \tau_t \right). \tag{50}$$

Re-optimizing retailers choose a target price, p_t^*, to maximize the following discounted stream of future profits:

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[\frac{p_t^*}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma}^p \right) - p_{t+s}^w \right] y_{jt+s}, \tag{51}$$

subject to the demand curve given by equation (47). Note that the sequence of discount factors depend on λ^p, the probability that the price remains fixed in the subsequent period.

The first order condition for the target price is given by

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[(1 + \Theta_t) \frac{p_t^*}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma}^p \right) - \Theta_t p_{t+s}^w \right] y_{jt+s} = 0, \tag{52}$$

with

$$\Theta_t = \left[\mathcal{G}'^{-1} \left(\tau_t p_t^* / p_{t+s} \right) \right]^{-1} \frac{\mathcal{G}' \left[\mathcal{G}'^{-1} \left(\tau_t p_t^* / p_{t+s} \right) \right]}{\mathcal{G}'' \left[\mathcal{G}'^{-1} \left(\tau_t p_t^* / p_{t+s} \right) \right]}.$$

By loglinearizing this condition, one can show that p_t^* depends on an expected discounted stream of the retailers nominal marginal cost, given by the nominal wholesale price $p_t p_t^w$.

By inverting the hiring condition derived earlier, one can obtain an expression for the retailers real marginal cost, p_t^w:

$$p_t^w = \frac{1}{a_t} \left[\frac{w_{it}}{p_t} + \kappa_t x_{it} - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{it+1}^2 - \rho \beta E_t \Lambda_{t,t+1} \kappa_{t+1} x_{it+1} \right]. \tag{53}$$

Real marginal costs thus depend on unit labor costs, plus terms that correct for the adjustment costs of hiring workers.

Observe that since we have normalized the relative price of final output at unity, the retailer's markup is given by $\mu_t^p = 1/p_t^w$. Since final goods prices are sticky and wholesale prices are flexible, this markup will in general exhibit cyclical behavior, with the direction depending on the nature of the disturbances hitting the economy, as well as other features of the model.

Finally, as we show in the appendix, by loglinearizing expressions for the price index and for the optimal reset price, equations (50) and (52), one can obtain a relation for consumer price inflation that is a variation of the conventional hybrid New Keynesian Phillips curve relating inflation to movements in real marginal cost, expected future inflation and lagged inflation. In this instance real marginal cost is simply the relative price of wholesale goods, p_t^w. The slope coefficient on real marginal cost, further, depends inversely on the degree of price stickiness, measured by λ^p. It also depends inversely on ξ, the percent change in the firm's price elasticity with respect to a one percent change in its relative price.[14] Everything else equal, a positive value of ξ reduces the firm's desired adjustment of its relative price. As discussed in Woodford (2003), this introduces a pricing complementarity that induces price adjusters to keep their relative prices closer to those of non-adjusters. This in turn reduces the sensitivity of inflation to movements in real marginal cost.

2.7 Government

Monetary policy obeys the following simple Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho^s} \left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{y_t}{y_{nt}}\right)^{r_y}\right]^{(1-\rho^s)} \varepsilon_t^r, \tag{54}$$

where

$$\log \varepsilon_t^r = \rho^r \log \varepsilon_{t-1}^r + \varsigma_t^r. \tag{55}$$

Government spending obeys:

$$g_t = \left(1 - \frac{1}{\varepsilon_t^g}\right) y_t, \tag{56}$$

where

$$\log \varepsilon_t^g = (1 - \rho^g) \ln \varepsilon^g + \rho^g \log \varepsilon_{t-1}^g + \varsigma_t^g. \tag{57}$$

2.8 Resource Constraint

The resource constraint divides output between consumption, investment and adjustment and utilization costs:

$$y_t = c_t + i_t + g_t + (\kappa_t/2) \int_0^1 x_{it}^2 n_{it-1} di + \mathcal{A}(\nu_t)\, k_{t-1}^p. \tag{58}$$

This completes the description of the model.

[14] Let η be the firm's elasticity of demand with respect to shifts in its market share. Then $\xi = \frac{\partial \eta}{\partial (p_{jt}/p_t)} \frac{p_{jt}/p_t}{\eta}$.

3 Wage and Hiring Dynamics

The key features of the model that differentiates it from the conventional monetary DSGE model involve wage and hiring dynamics. In this section, accordingly, we derive loglinear relationships for these variables. The analysis closely follows GT. An explicit derivation is in the appendix.

Let $w_t^o(w_t^{*n})$ be the target nominal wage at t, i.e., the wage the firm and its workers would agree to under period-by-period bargaining, given that firms and workers elsewhere remain on staggered multi-period wage contracts. Loglinearizing the first order condition for Nash bargaining, equation (41), we obtain the following loglinear difference equation for real contract wage, $\widehat{w}_t^*$:

$$\widehat{w}_t^* = \left[(1-\tau)\,\widehat{w}_t^o\left(w_t^{*n}\right) + \tau E_t\left(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^z\right)\right] + \tau E_t\widehat{w}_{t+1}^*. \tag{59}$$

where $\widehat{y}_t$ denotes the percent deviation of variable y_t from its steady state value and with $\psi = \chi\beta\lambda\mu + (1-\chi)\,\rho\beta\lambda\epsilon$ and $\tau = \psi/(1+\psi)$. The contract wage thus depends on the current and expected future path of the target wage $\widehat{w}_t^o(w_t^{*n})$ and terms that reflect adjustments for indexing. As the appendix shows,

$$\begin{aligned}\widehat{w}_t^o\left(w_t^{*n}\right) &= \varphi_a\left(\widehat{p}_t^w + \widehat{a}_t\right) + \varphi_x E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (1/2)\,\widehat{\Lambda}_{t,t+1}\right] \\ &\quad + \varphi_s E_t\left[\widehat{s}_{t+1} + \widehat{\widehat{H}}_{xt+1} + \widehat{\Lambda}_{t,t+1}\right] + \varphi_b\widehat{b}_t + \varphi_\chi\left[\widehat{\chi}_t\left(w_t^{*n}\right) - (\rho\beta)\,E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)\right],\end{aligned} \tag{60}$$

where the coefficients φ_a, φ_x, φ_s, φ_b, and φ_χ depend on the primitive model parameters and steady state values, as shown in the appendix.

Let $\widehat{w}_t^o$ be the wage that would arise if all firms and workers were negotiating wages period-by-period, i.e., the economy-wide target wage. The link between $\widehat{w}_t^o(w_t^{*n})$ and $\widehat{w}_t^o$ is given by

$$\widehat{w}_t^o\left(w_t^{*n}\right) = \widehat{w}_t^o + \frac{\tau_1}{1-\tau}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right) + \frac{\tau_2}{1-\tau}\left(\widehat{w}_t - \widehat{w}_t^*\right), \tag{61}$$

where, as the appendix shows, τ_1 and τ_2 are functions of the primitive parameters and steady state values of the model and measure the spillover effects of wages elsewhere in the economy on the wage bargain. The economy target wage $\widehat{w}_t^o$ (which is the spillover-free component of the target wage $\widehat{w}_t^o(w_t^{*n})$) is given by:

$$\begin{aligned}\widehat{w}_t^o &= \varphi_a\left(\widehat{p}_t^w + \widehat{a}_t\right) + \left(\varphi_s + \varphi_x\right)E_t\widehat{x}_{t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_b\widehat{b}_t \\ &\quad + \left(\varphi_s + \varphi_x/2\right)E_t\widehat{\Lambda}_{t,t+1} + \varphi_\chi\left[\widehat{\chi}_t - (\rho - s)\,\beta E_t\widehat{\chi}_{t+1}\right] + \widehat{\varepsilon}_t^w,\end{aligned} \tag{62}$$

with

$$\widehat{\varepsilon}_t^w = \varphi_\eta\left[1 - (\rho - s)\,\beta\rho^\eta\right]\widehat{\varepsilon}_t^\eta,$$

where, similarly, the coefficient φ_η depends on the primitive model parameters and steady state values, as shown in the appendix.

As in GT, there is both a direct and an indirect spillover effect on the target wage. The direct effect, captured by the second term in equation (61), reflects the impact of market wages on the worker's outside option. If, everything else equal, $E_t\widehat{w}_{t+1}$ exceeds $E_t\widehat{w}^*_{t+1}$, opportunities are unusually good for workers expecting to move into employment next period, and vice-versa if $E_t\widehat{w}_{t+1}$ is below $E_t\widehat{w}^*_{t+1}$. By influencing the worker's outside option in this way, the expected average market wage at $t+1$ induces a direct spillover effect on the wage bargain. The indirect effect, captured by the third term, depends on several factors. Overall, it is much smaller in absolute magnitude than the direct effect.

The loglinearized real wage index is in turn given by

$$\widehat{w}_t = (1-\lambda)\,\widehat{w}^*_t + \lambda\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}^z_t\right). \tag{63}$$

Combining these equations then yields the following second order difference equation which governs the evolution of the average wage:

$$\widehat{w}_t = \gamma_b\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}^z_t\right) + \gamma_o\widehat{w}^o_t + \gamma_f E_t\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}^z_{t+1}\right), \tag{64}$$

where

$$\begin{aligned}
\gamma_b &= (1+\tau_2)\,\phi^{-1}, \\
\gamma_o &= \varsigma\phi^{-1}, \\
\gamma_f &= \left(\tau\lambda^{-1} - \tau_1\right)\phi^{-1}, \\
\phi &= 1 + \tau_2 + \varsigma + \tau\lambda^{-1} - \tau_1, \\
\varsigma &= (1-\lambda)(1-\tau)\,\lambda^{-1}.
\end{aligned}$$

with $\gamma_b + \gamma_o + \gamma_f = 1$. Note the forcing variable in the difference equation is the economy-wide target wage $\widehat{w}^o_t$.

Due to staggered contracting, $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$ as well as the expected future wage $E_t\widehat{w}_{t+1}$. Solving out for the reduced form will yield an expression that relates the wage to the lagged wage and a discounted stream of expected future values of $\widehat{w}^o_t$. Note that the spillover effects, measured by τ_1 and τ_2 work to reduce the relative importance of the expected future wage relative to the lagged wage (by reducing γ_f relative to γ_b). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way as to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, for example, Woodford, 2003).

Note also that as we converge to $\lambda = 0$ (the case of period by period wage bargaining), both γ_b and γ_f go to zero, implying that $\widehat{w}_t$ simply tracks $\widehat{w}^o_t$ in this instance. Further, as we noted earlier,

$\widehat{w}_t^o$, becomes identical to the wage in the flexible case. The model thus nests the conventional period-by-period wage bargaining setup.

Next, loglinearizing the equation for the hiring rate and aggregating yields:

$$\widehat{x}_t = \varkappa_a \left(\widehat{p}_t^w + \widehat{a}_t\right) - \varkappa_w \widehat{w}_t + \varkappa_\lambda E_t \widehat{\Lambda}_{t,t+1} + \beta E_t \widehat{x}_{t+1} \qquad (65)$$

where the expressions for the coefficients are reported in the appendix. The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

4 Model Estimation

4.1 Estimation Procedure

We consider seven variables in our estimation. To facilitate comparison with the literature, we employ the seven quarterly series used in CEE, SW and PST.[15] Thus, the variables we use include: (1) per capita real GDP; (2) per capita real personal consumption expenditures of nondurables; (3) per capita real investment equal to the sum of per capita real private investment plus per capita real personal consumption of durables; (4) hours of all persons in the non-farm business sector divided by the population times the ratio of total employment to employment in non-farm business sector; (5) the real wage (compensation per hour in the nonfarm business sector); (6) inflation as measured by the quarter to quarter growth rate of the GDP deflator; and (7) the Federal Funds rate expressed in quarterly terms. To convert any nominal variable to real terms, we always use the GDP deflator. The sample goes from 1960Q1 to 2005Q1.

We first log-linearize the model around a deterministic balanced growth steady state. The appendix contains the complete log-linear model, as well as the steady state. The coefficients of the log-linear model depend on the primitive parameters of the model, as well as steady state values of variables. We use the steady state conditions of the model to solve out for a number of parameters. The model also contains seven exogenous shocks, one corresponding to each variable. In our estimation, further, we allow for the fact that the quantity variables (output, investment, consumption, etc.) are non-stationary due to the presence of a unit root in the technology shock.

There are twenty-one parameters, not including the parameters that characterize the exogenous shocks. Of the twenty-one, there are five new parameters that arise from our modification of the labor market. These include: the worker's relative bargaining power, η, the elasticity of new

[15] There are some slight differences in the series used in SW versus CEE and PST. As in CEE and PST, we include consumer durables in investment. As in SW, we use an economy-wide measure of hours based on an adjustment of non-farm business hours.

matches with respect to labor market tightness, σ, the job survival rate, ρ, the steady state job finding rate, s, and the steady state flow value of unemployment as fraction of the contribution of the worker to the job, $\tilde{b}$, defined as

$$\tilde{b} = \frac{\bar{b}}{p^w \bar{a} + (\beta/\gamma_z)(\kappa/2) x^2},$$

where $\bar{y}$ denotes y_t/z_t evaluated at steady state for a generic variable y_t.

Because we are not adding any new variables but are adding new parameters, in this first pass at the data we calibrate three of the five labor market parameters, for which there exists independent evidence. In particular, we choose the average monthly separation rate $1-\rho$ based on the observation that jobs last about two years and a half. Therefore, we set $\rho = 1 - 0.105$. We choose the elasticity of matches to unemployment, σ, to be equal to 0.5, the midpoint of the evidence typically cited in the literature.[16] In addition, this choice is within the range of plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then set $s = 0.95$ to match recent estimates of the U.S. average monthly job finding rate (Shimer, 2005a).

The two labor market parameters we estimate are $\tilde{b}$ and η. Both these parameters are critical determinants of the effective elasticity of labor supply along the extensive margin in the flexible wage case. As emphasized by Hagedorn and Manovskii (2006) and others, the closer $\tilde{b}$ is to unity, the better able is the period-by-period Nash bargaining framework to capture unemployment and wage dynamics. For example, when $\tilde{b}$ is very close to unity, there is little difference between the value of employment versus unemployment to the worker. Effectively, labor supply along the extensive margin is very elastic in this instance. The response of wages to a shift in the value of a worker to the firm is dampened because in this case a small change in the wage has a large percentage effect on the relative gains to the worker from employment versus unemployment. Indeed, Hagedorn and Manovskii (2006) show that a model calibrated with $\tilde{b}$ close to unity can capture the relative volatilities of labor market variables. This calibration however is quite controversial: Shimer (2005a) argues in favor of 0.4 based on the interpretation of $\tilde{b}$ as unemployment insurance, while Hall (2008) suggests 0.7, based on a broader interpretation that permits utility from leisure. Given the critical role of this parameter, it seems to be a prime candidate to estimate. The worker's bargaining parameter η is similarly important. The smaller is η, the less sensitive are wages to movements in the shadow value of labor, and thus the more sensitive is employment. Indeed, the Hagedorn and Manovskii calculation requires not only a high value of $\tilde{b}$ but also a low value of η.

There are four "conventional" parameters that we calibrate: the discount factor, β, the depreciation rate, δ, the "share" parameter on capital in the Cobb-Douglas production function, α, the steady state ratio of government consumption to output, $\bar{g}/\bar{y}$, and the sensitivity of firm demand

[16] The values for σ used in the literature are: 0.4 in Blanchard and Diamond (1989), Andolfatto (1994), Merz (1995) and Farmer (2006), 0.45 in Mortensen and Nagypal (2007), 0.5 in Hagedorn and Manovskii (2006), 0.72 in Shimer (2005a).

elasticity to market share (the Kimball aggregator), ξ. We use conventional values for all these parameters: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.33$, $\bar{g}/\bar{y} = 0.2$ and $\xi = 10$.[17]

Insert Table 1 here

The conventional parameters we estimate include: the elasticity of the utilization rate to the rental rate of capital, η_ν[18]; the elasticity of the capital adjustment cost function, η_k; the habit persistence parameter, h; the steady state price markup ε^P; the wage and price rigidity parameters, λ and λ^p; the wage indexing parameter, γ; and the Taylor rule parameters, r_π, r_y and ρ_s. In addition, we estimate the first order autocorrelations of all the exogenous disturbances, as well as their respective standard deviations.

We estimate the model with Bayesian methods (see An and Schorfheide, 2007, for a comprehensive survey). We combine the likelihood function of the model, $L(\theta, Y)$, with priors for the parameters to be estimated, $p(\theta)$, to obtain the posterior distribution, $L(\theta, Y)p(\theta)$. Draws from the posterior distribution are generated with the Random-Walk Metropolis Hastings (RWMH) algorithm.

4.2 Estimation Results

Table 2 reports the prior and the posterior distributions for each parameter. The table also reports the parameter configuration which maximizes the posterior (called Max), along with the mean and the values at the 5 and 95 percent tails. Similarly, Table 3 presents the estimates of the prior and posterior distribution of the shock processes.

For the conventional parameters, for the most part we use the same priors as in PST, which in turn follow closely those employed by SW. We proceed this way in order to facilitate comparison with the literature. Note that we propose uniform prior for the wage indexing parameter γ, based on the view that existing theory and evidence offer no guidance for the appropriate value of this parameters. Further, a priori, values of zero and unity for this parameter seem equally plausible. Finally, we note that in all instances the priors that we choose are reasonably loose.

As noted, we estimate two new labor market parameters, η and $\tilde{b}$. There is little direct evidence on the worker's bargaining power parameter η. In their survey paper, Mortensen and Nagypal (2007) propose a value of 0.5, which appears to reflect conventional thinking in the literature. Accordingly we set the mean of the prior for this variable at 0.5, with a standard deviation of unity.

[17] Note in contrast to the frictionless labor market model, the term $1-\alpha$ does not correspond to the labor share and will depend on the outcome of the bargaining process. However, because a wide rage of values of the bargaining power imply a labor share just below $1-\alpha$, here we simply follow convention by setting $1-\alpha = 2/3$. In our calculations, $1-\alpha$ equals 0.667 and the labor share 0.666.

[18] We follow SW, define ψ_ν such that $\eta_\nu = \frac{1-\psi_\nu}{\psi_\nu}$ and estimate ψ_ν. When $\psi_\nu = 1$, it is very costly to change the capital utilization rate and the utilization rate does not vary. When $\psi_\nu = 0$, the marginal cost of changing the capital utilization rate is constant and, as a result, the rental rate of capital does not vary.

We choose a similar prior for $\tilde{b}$. As we noted earlier, Shimer proposes 0.4 as a "generous" value for $\tilde{b}$, while Hall suggests 0.7 if one permits a broader interpretation of this variable. Both these estimates fall within the 5th and the 95th percentile of our prior.

Insert Tables 2 and 3 here

Since the estimates of the "conventional" parameters are consistent with other studies, we focus on the new parameters we consider here.[19] In particular, we estimate a very reasonable degree of wage rigidity. The estimate of λ is 0.72, which suggests a mean of just over three quarters between wage contracting periods.[20] The evidence from micro-data (Gottshalk, 2006), suggests a modal adjustment time of one year, though is silent (to our knowledge) about medians and means. In addition, the estimates suggest a high degree of indexing of wages to past inflation: The estimate of the indexing parameter is almost 1, which suggests a high degree of effective real wage rigidity. We also tried estimating the model with the restriction of no indexing. While most of the parameter estimates did not change, the model did not fit the data as well in certain dimensions, particularly those involving inflation dynamics. Apparently, wage indexing is critical in accounting for the apparent inertia in inflation.

The estimate of the key parameter $\tilde{b}$, the flow value of unemployment, is 0.73. What it suggests is that in units of consumption goods, the flow value of unemployment is seventy percent of the worker's marginal flow value to the firm. This percentage is close to the value proposed by Hall (2008) who, as we suggested earlier, motivates $\tilde{b}$ as reflecting not only unemployment insurance benefits but also utility gains from leisure. We also note that this estimate is well below the near unity value required for the conventional flexible model to account for the data. Thus, the data seem to prefer a combination of highly sticky wages and (effectively) inelastic labor supply along the extensive margin.

Finally, the estimate of the worker's bargaining power parameter is 0.9. This value lies above the range considered in the literature, typically 0.5 - 0.7. As we noted earlier, however, there is virtually no direct evidence on what an appropriate value of this parameter should be. One possibility, is that within our framework it is very difficult to separately identify $\tilde{b}$ and η. Both parameters enter the loglinear system via their respective impact on the steady state wage (see the

[19] In Appendix D we present estimates of the SW model. The precise version of the model we estimate is due to PST. Their formulation differs from SW only in some minor details.

[20] It is true that our estimate of the degree of price rigidity ($\lambda^p = 0.85$) is somehwat higher than that for wage rigidity. Several points: First, our estimate is similar to what one obtains with the SW model (see appendix D). Second, our measure of inflation is based on the GDP deflator, which consists of producer prices, which are stickier than consumer prices. In this regard, our estimate of λ^p suggests a median duration of price changes of roughly four quarters, which is not too far above Nakamura and Steinsson (2007) estimates of three quarters for producer prices. Note that the Poisson process for price adjustment leads to the mean exceeding the median, since the constant hazard process suggests that some prices may not be adjusted for a long time.

appendix). It may be that to achieve a good identification of these parameters, we may need to introduce additional labor market information.

On the other hand, the estimates are consistent with the following plausible scenario: At the low and medium frequencies wages co-move with productivity as a consequence of worker's strong bargaining power. At the high frequency, however, wages adjust with a lag to the movement in productivity due to the staggered contracting structure.

It is also worth noting that within our framework, the lion's share of the serial correlation in the real wages is accounted for by the wage contracting structure. The exogenous shock to the wage equation (modeled as a shock to bargaining power) has a first order serial coefficient of only 0.26.

We next consider the model without wage rigidity. Table 4 presents the parameter estimates and Table 5 presents the estimates of the shock processes in this case.

Insert Tables 4 and 5 here

The estimates of the conventional parameters do not change much. There is however now a large change is the estimates of the two key labor market parameters: $\tilde{b}$ increases to 0.98 and η falls to 0.6. The former is close to the value we described earlier that Hagedorn and Manovskii used to argue that a flexible wage model could account for labor market volatility.[21] Our estimates confirm that absent wage rigidity, it is necessary to have highly elastic labor supply along the extensive margin to account for the facts.[22]

One virtue of the Bayesian approach is that it is straightforward to compare the fit of the baseline model versus the model without wage rigidity Table 6 reports the log marginal likelihoods for the two models.

Insert Table 6 here

The baseline model clearly is preferred to the flex wage model. The difference in marginal likelihood is forty loglikelihood points, which is a significant difference.

Another way to assess how the model captures the data is to portray the autocovariance function of the model variables against the data. Figure 2 reports this information. The solid line in each panel reports the autocovariance function of the data. The dashed lines are ninety percent posterior intervals of the model autocovariances.[23] Overall, the baseline model does well. For the most part, the empirical autocovariance functions lie within the model standard error bands. In this regard,

[21] Hagedorn and Manvoski employ a slightly smaller value of $\tilde{b}$, 0.95, as opposed to 0.98, and much smaller value of η, 0.05, as opposed to 0.62. Note that our prior on η is sufficiently loose so as not to exclude values well below 0.5.

[22] The large value of $\tilde{b}$ suggests a huge response of employment to changes in unemployment insurance benefits, as Hornstein, Krusell and Gianluca Violante (2005) and others have noted.

[23] The posterior intervals are computed as follows. We sample 500 points from the posterior and for each of them we generate 160 observations, which is the length of the data sample, 100 times. Then, for each draw we compute autocovariances and we report the fifth and the ninety-fifth percentile.

the model does particularly well in capturing the reduced form dynamics of output, hours, wages and inflation.

As a further check on the model, we explore how well it is able to account for the dynamics of unemployment. As the bottom row shows, the empirical autocovariance function for unemployment lies within the posterior intervals of the model generated sample moments. We interpret this as a strong test of the model given that we did not use information on unemployment to fit model parameters.

Figure 3 presents the autocovariance function for the model without wage rigidity. Overall this model doesn't do as well. In contrast to the baseline model, the empirical autocovariances in places lie well outside the model-generated bands. Note that the flex wage model generates too much volatility in output, hours and unemployment. The reason, however, is that this model relies on more persistent exogenous forcing processes to capturing the data. As we show below, after controlling for the exogenous shocks, the wage rigidity works to amplify the effects of shocks on output, hours and unemployment relative to the response of inflation.

We next illustrate the properties of the model economy by simulating the response to several key shocks. We analyze the role of wage rigidity, in particular, by examining both our benchmark model and the same model with staggered contracting replaced by period by period wage negotiations. As Table 7 shows, the estimates suggest that the main driving force is the investment shock which, strictly speaking is interpretable as a shock to investment-specific technological change. It accounts for more than half the variation in output growth on impact and more than forty percent at all horizons. This finding is consistent with both SW and PST. Next in importance is the disembodied productivity shock which accounts for roughly seventeen percent of the variation on impact and more than thirty percent at horizons of a year or greater.

The shocks that are important in driving output are also the main factors determining the variation in the labor market variables. Tables 8 and 9 report the variance decompositions for hours and vacancies respectively. As with output, investment shocks are the dominant factor and shocks to total factor productivity are next in importance. The same is true for unemployment, though we do not report the results here.

Insert Tables 7, 8 and 9 here

The recent literature on unemployment fluctuations that we alluded to in the introduction almost uniformly treats productivity shocks as the main driving force. Thus for purposes of comparison, we begin with this disturbance. In particular, Figure 4 illustrates the response of the model economy to a productivity shock. The thick line is the model with wage rigidity. The dotted line has wage rigidity turned off.[24] Notice that the response of output and employment is significantly greater with wage rigidity than without. Conversely, due to the staggered contracting the response

[24] To shut off wage rigidity we simply set the probability that wages do not adjust, λ, equal to zero.

of wages is much smoother. The smooth response of wages, of course, implies a larger response of profits to the technology shock than otherwise. This leads to a stronger response of output and employment relative to the flexible wage case. Note also that there is an immediate drop in inflation following the productivity shock, which is in line with the evidence in Altig, Christiano, Eichenbaum and Evans (2004). This does not come as puzzle in this framework: with wage rigidity the rise in productivity reduces marginal costs and hence inflation.

We next turn to the investment shock, as portrayed in Figure 5. In contrast to the case of the productivity shock, in this instance the response of output and employment is very similar across the two models. Note, however, that the responses of wages, w, and inflation, π, are quite different. In the case with wage rigidity shut off, the responses of these variables appear counterfactually large. In absolute value, the response of wages is three times as large as the response of output and the response of inflation (annualized) is nearly half as large. To our knowledge there do not exist series for wages or inflation that display this kind of volatility relative to output.[25] Intuitively, the investment shock shifts output demand without directly affecting factor productivities. With nominal price rigidities, markups decline and employment adjusts to meet demand. In the staggered contracting model, because labor costs are sticky, the employment response is associated with only a modest change in real wages and inflation. With wage rigidity shut off, however, there is a sharp increase in wages which boosts up real marginal costs and hence inflation. In sum, in the case of investment demand shocks, the wage rigidity smooths out the response of wages and inflation to the disturbances.

In Figure 6 we report the results for a contractionary monetary policy shock. The baseline delivers a humped shaped contraction in output with minimal response of inflation, as is consistent with conventional wisdom. Without wage rigidity, inflation declines nearly as much as output. Thus, as with the investment shock, wage rigidity helps capture simultaneously the large output and hours response and the small inflation response to a monetary policy shock.

Next, we briefly consider how well our model fits the data as compared to SW. In appendix D we report the parameter estimates for the SW model. By and large, the estimates for the "non-labor market" parameters are similar across the two models. To explore relative fit, one possibility would be to compare the marginal likelihoods, as we did we the sticky and flex wage versions of our model. In what we did earlier, however, one model (the sticky wage) nested the other (the flex wage) and the priors used in the estimation are identical. Neither of these conditions applies in this instance. While in principle this should not be a problem, in practice it may be, given the sensitivity of the marginal likelihood to the priors. Accordingly, to get a sense of relative fit we simply compare the autocovariance functions of each model. As Figure A1 in the appendix shows, the model standard error bands of our model overlap closely with those of the SW model. We conclude that from a

[25] It is true that the experiment here is conditional on the investment shock and the evidence to which we are alluding consists of unconditional moments. However, the investment shock accounts for nearly fifty percent of the variation in output growth within the model. If the flex wage model were true then we should observe relatively volatile behavior of real wages and price markups.

practical standpoint, the ability of our model to characterize the data is very similar to that of SW.

The advantage of our model, however, is that we can capture the behavior of unemployment, something not possible in the SW model. In addition, given that we have a structural model we can characterize the gap between the unemployment and its natural level, a variable clearly of interest to policy makers. In particular, we can evaluate the model under flexible prices and wages to obtain a time series of the natural rate of unemployment. We then take the difference between unemployment and the natural rate series to obtain a measure of the unemployment gap. Figure 7 reports the results. Interestingly, in all the recessions before 1984, the unemployment gap becomes large. This is consistent with the notion that tightening of monetary policy in response to inflationary pressures played a significant role in these downturns. Post 1984 the unemployment gap still increases in downturns, though the peak is much smaller than in the earlier recessions. While tight money may have been less important in these downturns, it is possible that the model may be understating the decline in the natural rate over this period, due to the absence of demographic factors in the framework.[26]

5 Concluding Remarks

We have developed and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, the employment of existing workers is efficient. Thus, the model is immune to the Barro's (1977) critique that models relying on wage rigidity to have allocative effects in situations where firms and workers have on-going relationships ignore mutual gains from trade. In our model, in contrast, wage rigidity affects the hiring of new workers. The former is introduced via the staggered Nash bargaining setup of Gertler and Trigari (2006). A robust finding is that the model with rigidity provides a better description of the data than does a flexible wage version. Further, our model appears to capture the moments of the data as well as Smets and Wouters (2007). In addition, while the conventional model is silent about the behavior of unemployment, our model generates dynamics for this variable that are in line with the data.

More work is necessary, however, to ensure a robust identification of the key labor market parameters. Our preliminary estimates of the degree of wage rigidity and the flow value of unemployment appear to be quite reasonable. The estimate of worker's bargaining power lies above conventional wisdom, though there is little direct evidence on what this parameter should be. One possibility is that it may be difficult to separately identify some of the key labor market parameters that influence employment volatility. Accordingly, it may be necessary to introduce additional labor market information.

[26] See Sala, Soderström and Trigari (2008) for further analysis.

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Appendix A

Steady state calculation

Let $\bar{y}$ denote y_t/z_t evaluated at steady state for any variable y_t.

Consumption and savings

$$1 = (\beta/\gamma_z)\left(1-\delta+r^k\right)$$

Capital/employment ratio

$$r^k = \alpha\left(\bar{k}/n\right)^{-(1-\alpha)}$$

Marginal product of labor

$$\bar{a} = (1-\alpha)\left(\bar{k}/n\right)^{\alpha}$$

Investment

$$q^k = 1$$

Rates

$$x = 1-\rho$$

Flows

$$xn = su$$

Unemployment

$$u = 1-n$$

Matching

$$su = \sigma_m u^{\sigma} v^{1-\sigma}$$

Hiring

$$\kappa x = p^w \bar{a} - \bar{w} + \beta\frac{\kappa}{2}x^2 + \beta\rho\kappa x$$

Wages

$$\bar{w} = \chi\left(\bar{a} + \beta\frac{\kappa}{2}x^2 + \beta\kappa s x\right) + (1-\chi)\,\bar{b}$$

where

$$\chi = \frac{\eta}{\eta + (1-\eta)\,\mu/\epsilon} \qquad \mu = \frac{1}{1-\lambda\beta} \qquad \epsilon = \frac{1}{1-\rho\lambda\beta}$$

Resource constraint

$$1 = \frac{\bar{c}}{\bar{y}} + \frac{\bar{g}}{\bar{y}} + \frac{\bar{\imath}}{\bar{y}} + \frac{\kappa}{2}x^2\frac{n}{\bar{y}}$$

where

$$n/\bar{y} = \left(\bar{k}/n\right)^{-\alpha} \qquad \bar{\imath}/\bar{y} = \left(1-\frac{1-\delta}{\gamma_z}\right)\gamma_z\left(\bar{k}/n\right)^{(1-\alpha)}$$

APPENDIX B

Staggered Nash bargaining

- Consider a renegotiating firm and its workers. Given that next period's nominal wage w_{t+1}^{n} equals this period nominal wage adjusted for indexing $\bar{\gamma}\pi_t^{\gamma} w_t^{*n}$ with probability λ and next period's nominal target wage w_{t+1}^{*n} with probability $1-\lambda$, we can write the worker surplus $H_t\left(w_t^{*n}\right)$ and the firm surplus $J_t\left(w_t^{*n}\right)$ as

$$H_t\left(w_t^{*n}\right) = \frac{w_t^{*n}}{p_t} - b + \beta E_t \Lambda_{t,t+1}\left[\rho\lambda H_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) + \rho\left(1-\lambda\right) H_{t+1}\left(w_{t+1}^{*n}\right) - s_{t+1} H_{x,t+1}\right] \tag{B1}$$

$$\begin{aligned} J_t\left(w_t^{*n}\right) &= p_t^w a_t - \frac{w_t^{*n}}{p_t} - \beta E_t \Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}\left[\lambda x_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)^2 + \left(1-\lambda\right) x_{t+1}\left(w_{t+1}^{*n}\right)^2\right] \\ &\quad + \beta E_t \Lambda_{t,t+1}\lambda\left[\rho + x_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)\right] J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) \\ &\quad + \beta E_t \Lambda_{t,t+1}\left(1-\lambda\right)\left[\rho + x_{t+1}\left(w_{t+1}^{*n}\right)\right] J_{t+1}\left(w_{t+1}^{*n}\right) \end{aligned} \tag{B2}$$

where

$$\kappa_t x_t\left(w_t^{*n}\right) = J_t\left(w_t^{*n}\right) \tag{B3}$$

- Let

$$\begin{aligned} \epsilon_t &= p_t\left[\partial H_t\left(w_t^{*n}\right)/\partial w_t^{*n}\right] \\ \mu_t\left(w_t^{*n}\right) &= -p_t\left[\partial J_t\left(w_t^{*n}\right)/\partial w_t^{*n}\right] \\ E_t\epsilon_{t+1} &= E_t p_{t+1}\left[\partial H_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)/\partial w_t^{*n}\right] \\ E_t\mu_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) &= -E_t p_{t+1}\left[\partial J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)/\partial w_t^{*n}\right] \\ E_t\mu_{t+1}\left(w_{t+1}^{*n}\right) &= -E_t p_{t+1}\left[\partial J_{t+1}\left(w_{t+1}^{*n}\right)/\partial w_{t+1}^{*n}\right] \end{aligned}$$

- The first order condition for Nash bargaining is

$$\chi_t\left(w_t^{*n}\right) J_t\left(w_t^{*n}\right) = \left[1 - \chi_t\left(w_t^{*n}\right)\right] H_t\left(w_t^{*n}\right) \tag{B4}$$

with

$$\chi_t\left(w_t^{*n}\right) = \frac{\eta_t}{\eta_t + \left(1-\eta_t\right)\mu_t\left(w_t^{*n}\right)/\epsilon_t} \tag{B5}$$

where

$$\epsilon_t = 1 + E_t\Lambda_{t,t+1}\rho\beta\lambda\frac{p_t}{p_{t+1}}\bar{\gamma}\pi_t^{\gamma}\epsilon_{t+1} \tag{B6}$$

$$\mu_t\left(w_t^{*n}\right) = 1 + E_t\Lambda_{t,t+1}\left[\rho + x_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)\right]\beta\lambda\frac{p_t}{p_{t+1}}\bar{\gamma}\pi_t^{\gamma}\mu_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) \tag{B7}$$

Contract wage

Worker surplus

- Write the worker surplus as

$$\begin{aligned} H_t\left(w_t^{*n}\right) &= \frac{w_t^{*n}}{p_t} - \left[b_t + \beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1}\right] + \rho\beta E_t \Lambda_{t,t+1} H_{t+1}\left(w_{t+1}^{*n}\right) \\ &+ \beta\rho\lambda E_t \Lambda_{t,t+1}\left[H_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - H_{t+1}\left(w_{t+1}^{*n}\right)\right] \end{aligned}$$

- Write the term $E_t\left[H_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - H_{t+1}\left(w_{t+1}^{*n}\right)\right]$ in the second row of the equation above as

$$\begin{aligned} E_t\left[H_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - H_{t+1}\left(w_{t+1}^{*n}\right)\right] &= E_t\left(\frac{\bar{\gamma}\pi_t^{\gamma} w_t^{*n}}{p_{t+1}} - \frac{w_{t+1}^{*n}}{p_{t+1}}\right) \\ &+ \beta\rho\lambda E_t \Lambda_{t+1,t+2}\left[H_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma}\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - H_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma} w_{t+1}^{*n}\right)\right] \end{aligned}$$

- Loglinearizing, iterating forward and collecting terms

$$E_t\left[\widehat{H}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - \widehat{H}_{t+1}\left(w_{t+1}^{*n}\right)\right] = \left(\bar{w}/\bar{H}\right)\epsilon E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)$$

  where $\bar{y}$ denote y_t/z_t evaluated at steady state for any variable y_t.

- Loglinearizing the worker surplus and substituting the expression just found gives

$$\begin{aligned} \widehat{H}_t\left(w_t^{*n}\right) &= \left(\bar{w}/\bar{H}\right)\left[\widehat{w}_t^* + \left(\beta\rho\lambda\right)\epsilon E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)\right] \\ &- \beta s E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1} + \widehat{\Lambda}_{t,t+1}\right] - \left(\bar{b}/\bar{H}\right)\widehat{b}_t + \rho\beta E_t\left[\widehat{H}_{t+1}\left(w_{t+1}^{*n}\right) + \widehat{\Lambda}_{t,t+1}\right] \end{aligned}$$

Firm surplus

- Combining (B2) and (B3) we obtain

$$\begin{aligned} J_t\left(w_t^{*n}\right) &= p_t^w a_t - \frac{w_t^{*n}}{p_t} + \beta E_t \Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}\left[\lambda x_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)^2 + (1-\lambda)\, x_{t+1}\left(w_{t+1}^{*n}\right)^2\right] \\ &+ \rho\beta E_t \Lambda_{t,t+1}\left[\lambda J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) + (1-\lambda)\, J_{t+1}\left(w_{t+1}^{*n}\right)\right] \end{aligned}$$

- Write the firm surplus as

$$
\begin{aligned}
J_t\left(w_t^{*n}\right) &= p_t^w a_t - \frac{w_t^{*n}}{p_t} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1} \left(w_{t+1}^{*n}\right)^2 \\
&\quad + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} \lambda \left[x_{t+1} \left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)^2 - x_{t+1} \left(w_{t+1}^{*n}\right)^2 \right] \\
&\quad + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}\left(w_{t+1}^{*n}\right) \\
&\quad + \rho\beta\lambda E_t \Lambda_{t,t+1} \left[J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - J_{t+1}\left(w_{t+1}^{*n}\right) \right]
\end{aligned}
$$

with

$$\kappa_t x_t\left(w_t^{*n}\right) = J_t\left(w_t^{*n}\right)$$

- Write the term $E_t\left[J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - J_{t+1}\left(w_{t+1}^{*n}\right)\right]$ as

$$
\begin{aligned}
E_t\left[J_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - J_{t+1}\left(w_{t+1}^{*n}\right)\right] &= -E_t\left(\frac{\bar{\gamma}\pi_t^{\gamma} w_t^{*n}}{p_{t+1}} - \frac{w_{t+1}^{*n}}{p_{t+1}}\right) \\
&\quad + \beta\lambda E_t \Lambda_{t+1,t+2} \frac{\kappa_{t+2}}{2} \left[x_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma}\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right)^2 - x_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma} w_{t+1}^{*n}\right)^2 \right] \\
&\quad + \rho\beta\lambda E_t \Lambda_{t+1,t+2} \left[J_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma}\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - J_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma} w_{t+1}^{*n}\right) \right]
\end{aligned}
$$

- Loglinearizing, iterating forward and collecting terms

$$
\begin{aligned}
E_t\left[\widehat{J}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - \widehat{J}_{t+1}\left(w_{t+1}^{*n}\right)\right] &= -\left(\bar{w}/\bar{J}\right)\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right) \\
E_t\left[\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma} w_t^{*n}\right) - \widehat{x}_{t+1}\left(w_{t+1}^{*n}\right)\right] &= -\left(\bar{w}/\bar{J}\right)\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)
\end{aligned}
$$

- Loglinearizing the firm surplus and substituting

$$
\begin{aligned}
\widehat{J}_t\left(w_t^{*n}\right) &= \left(p^w\bar{a}/\bar{J}\right)\left(\widehat{p}_t^w + \widehat{a}_t\right) - \left(\bar{w}/\bar{J}\right)\left[\widehat{w}_t^* + \beta\lambda\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)\right] \\
&\quad + x\beta E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (1/2)\,\widehat{\Lambda}_{t,t+1}\right] + \rho\beta E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^{*n}\right) + \widehat{\Lambda}_{t,t+1}\right]
\end{aligned}
$$

Contract wage

- The loglinear version of the Nash first order condition, equation (B4), is

$$\widehat{J}_t\left(w_t^{*n}\right) + (1-\chi)^{-1}\,\widehat{\chi}_t\left(w_t^{*n}\right) = \widehat{H}_t\left(w_t^{*n}\right)$$

- Substituting the loglinear expressions for $\widehat{J}_t\left(w_t^{*n}\right)$ and $\widehat{H}_t\left(w_t^{*n}\right)$ and using the Nash foc for next period to simplify yields

$$
\begin{aligned}
& \left(p^w \bar{a}/\bar{J}\right)\left(\widehat{p}_t^w + \widehat{a}_t\right) - \left(\bar{w}/\bar{J}\right)\left[\widehat{w}_t^* + \beta\lambda\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)\right] \\
& + x\beta E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (1/2)\widehat{\Lambda}_{t,t+1}\right] + (1-\chi)^{-1}\widehat{\chi}_t\left(w_t^{*n}\right) \\
= \; & \left(\bar{w}/\bar{H}\right)\left[\widehat{w}_t^* + \rho\beta\lambda\epsilon E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right)\right] \\
& - \left(\bar{b}/\bar{H}\right)\widehat{b}_t - \beta s E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1} + \widehat{\Lambda}_{t,t+1}\right] \\
& + \rho\beta(1-\chi)^{-1} E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)
\end{aligned}
$$

Rearranging and collecting terms

$$
\begin{aligned}
& \widehat{w}_t^* + \psi E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right) \\
= \; & \chi p^w\left(\bar{a}/\bar{w}\right)\left(\widehat{p}_t^w + \widehat{a}_t\right) \\
& + \chi x\beta\left(\bar{J}/\bar{w}\right) E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (1/2)\widehat{\Lambda}_{t,t+1}\right] \\
& + (1-\chi)\left(\bar{b}/\bar{w}\right)\widehat{b}_t + (1-\chi)\left(\bar{H}/\bar{w}\right)\beta s E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1} + \widehat{\Lambda}_{t,t+1}\right] \\
& + \chi\left(\bar{J}/\bar{w}\right)(1-\chi)^{-1}\left[\widehat{\chi}_t\left(w_t^{*n}\right) - \rho\beta E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)\right]
\end{aligned}
$$

where

$$\psi = \chi\beta\lambda\mu + (1-\chi)\rho\beta\lambda\epsilon$$

- Finally, we can write

$$\widehat{w}_t^* = \left[(1-\tau)\widehat{w}_t^o\left(w_t^{*n}\right) + \tau E_t\left(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^z\right)\right] + \tau E_t\widehat{w}_{t+1}^*$$

where τ is given by

$$\tau = \frac{\psi}{1+\psi}$$

and where $\widehat{w}_t^o\left(w_t^{*n}\right)$ is the target wage given by

$$
\begin{aligned}
\widehat{w}_t^o\left(w_t^{*n}\right) = \; & \varphi_a\left(\widehat{p}_t^w + \widehat{a}_t\right) + \varphi_x E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (1/2)\widehat{\Lambda}_{t,t+1}\right] \\
& + \varphi_s E_t\left[\widehat{s}_{t+1} + \widehat{H}_{x,t+1} + \widehat{\Lambda}_{t,t+1}\right] + \varphi_b\widehat{b}_t + \varphi_\chi\left[\widehat{\chi}_t\left(w_t^{*n}\right) - \rho\beta E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)\right]
\end{aligned}
$$

with

$$\varphi_a = \chi p^w\left(\bar{a}/\bar{w}\right) \qquad \varphi_x = \chi x\beta\left(\bar{J}/\bar{w}\right) \qquad \varphi_b = (1-\chi)\,\bar{b}/\bar{w}$$

$$\varphi_s = (1-\chi)\, s\beta\left(\bar{H}/\bar{w}\right) \qquad \varphi_\chi = \chi\,(1-\chi)^{-1}\left(\bar{J}/\bar{w}\right)$$

Firm and worker discount factors

- The loglinear worker discount factor is

$$\widehat{\epsilon}_t = \rho\beta\lambda E_t\left(\widehat{\epsilon}_{t+1} + \widehat{\Lambda}_{t,t+1} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}^z_{t+1}\right)$$

- We now proceed to find a loglinear recursive expression for the firm discount factor.
- Loglinearizing equation (B7)

$$\widehat{\mu}_t\left(w_t^{*n}\right) = x\beta\lambda E_t\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) + \beta\lambda E_t\left[\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) + \widehat{\Lambda}_{t,t+1} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}^z_{t+1}\right]$$

$$\begin{aligned}
\widehat{\mu}_t\left(w_t^{*n}\right) \;=\;& (\beta\lambda)\, xE_t\left[\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) + (\beta\lambda)\,\widehat{x}_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma}\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) + \ldots\right] \\
& + (\beta\lambda)\, E_t\left(\widehat{\Lambda}_{t,t+1} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}^z_{t+1}\right) \\
& + (\beta\lambda)^2\, E_t\left(\widehat{\Lambda}_{t+1,t+2} + \gamma\widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} - \widehat{\varepsilon}^z_{t+2}\right) \\
& + (\beta\lambda)^3\, E_t\left(\widehat{\Lambda}_{t+2,t+3} + \gamma\widehat{\pi}_{t+2} - \widehat{\pi}_{t+3} - \widehat{\varepsilon}^z_{t+3}\right) \\
& + \ldots
\end{aligned}$$

- Recall from previous section

$$\begin{aligned}
& E_t\left[\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) - \widehat{x}_{t+1}\left(w_{t+1}^{*n}\right)\right] \\
=\;& -\varkappa_w\mu E_t\left(\widehat{w}_t^{*} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}^z_{t+1} - \widehat{w}_{t+1}^{*}\right)
\end{aligned}$$

$$\begin{aligned}
& E_t\left[\widehat{x}_{t+2}\left(\bar{\gamma}\pi_{t+1}^{\gamma}\bar{\gamma}\pi_t^{\gamma}w_t^{*n}\right) - \widehat{x}_{t+2}\left(w_{t+2}^{*n}\right)\right] \\
=\;& -\varkappa_w\mu E_t\left(\widehat{w}_t^{*} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}^z_{t+1} + \gamma\widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} - \widehat{\varepsilon}^z_{t+2} - \widehat{w}_{t+2}^{*}\right)
\end{aligned}$$

and so on, where $\varkappa_w = \bar{w}/\bar{J}$.

- Substituting and rearranging

$$\begin{aligned}
\widehat{\mu}_t\left(w_t^{*n}\right) &= (x\beta\lambda)\, E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) + (\beta\lambda)\,\widehat{x}_{t+2}\left(w_{t+2}^{*n}\right) + (\beta\lambda)^2\,\widehat{x}_{t+3}\left(w_{t+3}^{*n}\right) + ...\right] \\
&- (x\beta\lambda)\,(\varkappa_w\mu)\,\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z\right) \\
&+ (x\beta\lambda)\,(\varkappa_w\mu)\, E_t\widehat{w}_{t+1}^* \\
&+ (x\beta\lambda)\,(\varkappa_w\mu)\,(\beta\lambda)\, E_t\left(\widehat{w}_{t+2}^* - \gamma\widehat{\pi}_{t+1} + \widehat{\pi}_{t+2} + \widehat{\varepsilon}_{t+2}^z\right) \\
&+ (x\beta\lambda)\,(\varkappa_w\mu)\,(\beta\lambda)^2\, E_t\left(\widehat{w}_{t+3}^* - \gamma\widehat{\pi}_{t+1} + \widehat{\pi}_{t+2} + \widehat{\varepsilon}_{t+2}^z - \gamma\widehat{\pi}_{t+2} + \widehat{\pi}_{t+3} + \widehat{\varepsilon}_{t+3}^z\right) \\
&+... \\
&+ (\beta\lambda)\, E_t\left(\widehat{\Lambda}_{t,t+1} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z\right) \\
&+ (\beta\lambda)^2\, E_t\left(\widehat{\Lambda}_{t+1,t+2} + \gamma\widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} - \widehat{\varepsilon}_{t+2}^z\right) \\
&+ (\beta\lambda)^3\, E_t\left(\widehat{\Lambda}_{t+2,t+3} + \gamma\widehat{\pi}_{t+2} - \widehat{\pi}_{t+3} - \widehat{\varepsilon}_{t+3}^z\right) \\
&+...
\end{aligned}$$

- Write recursively as

$$\begin{aligned}
&\widehat{\mu}_t\left(w_t^{*n}\right) + (x\beta\lambda)\,(\varkappa_w\mu)\,\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z\right) \\
=\;& (x\beta\lambda)\,\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) \\
&+\beta\lambda E_t\left(\widehat{\Lambda}_{t,t+1} + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z\right) \\
&+ (x\beta\lambda)\,(\varkappa_w\mu)\, E_t\widehat{w}_{t+1}^* \\
&- (x\beta\lambda)\,(\beta\lambda)\,(\varkappa_w\mu)\,\mu E_t\left(\gamma\widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} - \widehat{\varepsilon}_{t+2}^z\right) \\
&+ (\beta\lambda)\, E_t\left[\widehat{\mu}_{t+1}\left(w_{t+1}^{*n}\right) + (x\beta\lambda)\,(\varkappa_w\mu)\,\mu\left(\widehat{w}_{t+1}^* + \gamma\widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} - \widehat{\varepsilon}_{t+2}^z\right)\right]
\end{aligned}$$

- Finally, rearrange as

$$\begin{aligned}
\widehat{\mu}_t\left(w_t^{*n}\right) &= (x\beta\lambda)\, E_t\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right) - (x\beta\lambda)\,(\varkappa_w\mu)\,\mu E_t\left(\widehat{w}_t^* + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z - \widehat{w}_{t+1}^*\right) \\
&+ (\beta\lambda)\, E_t\left[\widehat{\mu}_{t+1}\left(w_{t+1}^{*n}\right) + \gamma\widehat{\pi}_t - \widehat{\pi}_{t+1} - \widehat{\varepsilon}_{t+1}^z + \widehat{\Lambda}_{t,t+1}\right]
\end{aligned}$$

The spillover effects

- The target wage is

$$\begin{aligned}\widehat{w}_t^o\left(w_t^{*n}\right) &= \varphi_a\left(\widehat{p}_t^w+\widehat{a}_t\right)+\varphi_x E_t\left[\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right)+(1/2)\,\widehat{\Lambda}_{t,t+1}\right] \\ &\quad +\varphi_s E_t\left[\widehat{s}_{t+1}+\widehat{H}_{x,t+1}+\widehat{\Lambda}_{t,t+1}\right]+\varphi_b\widehat{b}_t+\varphi_\chi\left[\widehat{\chi}_t\left(w_t^{*n}\right)-\rho\beta E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)\right]\end{aligned}$$

- Let's find expressions for $\widehat{x}_{t+1}\left(w_{t+1}^{*n}\right)$, $\widehat{\chi}_t\left(w_t^{*n}\right)$, $\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right)$ and $\widehat{H}_{x,t+1}$ in terms of gaps between contract and average wages.

- Applying the same procedure as above

$$E_t\left[\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)-\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)\right]=-\varkappa_w\mu E_t\left(\widehat{w}_t^*-\widehat{w}_t\right)$$

- Consider the non recursive loglinear expressions for $\widehat{\mu}_t\left(w_t^{*n}\right)$ and $\widehat{\mu}_t\left(w_t^n\right)$

$$\widehat{\mu}_t\left(w_t^{*n}\right)=\left(x\beta\lambda\right)E_t\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)+\beta\lambda E_t\left[\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)+\widehat{\Lambda}_{t,t+1}+\gamma\widehat{\pi}_t-\widehat{\pi}_{t+1}-\widehat{\varepsilon}_{t+1}^z\right]$$

$$\widehat{\mu}_t\left(w_t^n\right)=\left(x\beta\lambda\right)E_t\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)+\beta\lambda E_t\left[\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)+\widehat{\Lambda}_{t,t+1}+\gamma\widehat{\pi}_t-\widehat{\pi}_{t+1}-\widehat{\varepsilon}_{t+1}^z\right]$$

- Taking differences, substituting and iterating forward

$$\begin{aligned}\widehat{\mu}_t\left(w_t^{*n}\right)-\widehat{\mu}_t\left(w_t^n\right) &= \left(x\beta\lambda\right)E_t\left[\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)-\widehat{x}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)\right] \\ &\quad +\beta\lambda E_t\left[\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)-\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)\right] \\ &= -\left(x\beta\lambda\right)\varkappa_w\mu\left(\widehat{w}_t^*-\widehat{w}_t\right)+\beta\lambda E_t\left[\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^{*n}\right)-\widehat{\mu}_{t+1}\left(\bar{\gamma}\pi_t^\gamma w_t^n\right)\right] \\ &= -\left(x\beta\lambda\right)\left(\varkappa_w\mu\right)\mu\left(\widehat{w}_t^*-\widehat{w}_t\right)\end{aligned}$$

- Now we have

$$\widehat{\chi}_t\left(w_t^{*n}\right)=\left(1-\chi\right)\left[\widehat{\epsilon}_t-\widehat{\mu}_t\left(w_t^{*n}\right)\right]+\left(1-\chi\right)\left(1-\eta\right)^{-1}\widehat{\varepsilon}_t^\eta$$

- Taking differences with the average

$$\begin{aligned}\widehat{\chi}_t\left(w_t^{*n}\right)-\widehat{\chi}_t\left(w_t^n\right) &= -\left(1-\chi\right)\left[\widehat{\mu}_t\left(w_t^{*n}\right)-\widehat{\mu}_t\left(w_t^n\right)\right]+\left(1-\chi\right)\left(1-\eta\right)^{-1}\widehat{\varepsilon}_t^\eta \\ &= \left(1-\chi\right)\left(x\beta\lambda\right)\left(\varkappa_w\mu\right)\mu\left(\widehat{w}_t^*-\widehat{w}_t\right)+\left(1-\chi\right)\left(1-\eta\right)^{-1}\widehat{\varepsilon}_t^\eta\end{aligned}$$

with

$$\widehat{\chi}_t\left(w_t^n\right) = (1-\chi)\left[\widehat{\epsilon}_t - \widehat{\mu}_t\left(w_t^n\right)\right]$$

- Similarly

$$\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right) - \widehat{\chi}_{t+1}\left(w_{t+1}^{n}\right) = (1-\chi)\left(x\beta\lambda\right)\left(\varkappa_w\mu\right)\mu E_t\left(\widehat{w}_{t+1}^{*} - \widehat{w}_{t+1}\right) + (1-\chi)\left(1-\eta\right)^{-1}\widehat{\varepsilon}_{t+1}^{\eta}$$

- Using the results in previous section

$$E_t\left[\widehat{H}_{t+1}\left(w_{t+1}^{*n}\right) - \widehat{H}_{t+1}\left(w_{t+1}^{n}\right)\right] = (1-\chi)\,\chi^{-1}\varkappa_w\epsilon E_t\left(\widehat{w}_{t+1}^{*} - \widehat{w}_{t+1}\right)$$
$$E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^{*n}\right) - \widehat{J}_{t+1}\left(w_{t+1}^{n}\right)\right] = -\varkappa_w\mu E_t\left(\widehat{w}_{t+1}^{*} - \widehat{w}_{t+1}\right)$$

Start from the Nash foc next period

$$E_t\widehat{J}_{t+1}\left(w_{t+1}^{*n}\right) + (1-\chi)^{-1}E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{*n}\right) = E_t\widehat{H}_{t+1}\left(w_{t+1}^{*n}\right)$$

Substitute and rearrange to obtain

$$\begin{aligned} & E_t\widehat{J}_{t+1}\left(w_{t+1}^{n}\right) + (1-\chi)^{-1}E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{n}\right) + (1-\eta)^{-1}\widehat{\varepsilon}_{t+1}^{\eta} \\ = \; & E_t\widehat{H}_{t+1}\left(w_{t+1}^{n}\right) + \Gamma E_t\left(\widehat{w}_{t+1}^{*} - \widehat{w}_{t+1}\right) \end{aligned}$$

with

$$\Gamma = \left[1-\eta\left(x\beta\lambda\right)\mu\right]\eta^{-1}\mu\varkappa_w$$

- Using finally

$$\widehat{x}_t\left(w_t^n\right) = \widehat{J}_t\left(w_t^n\right)$$

we have

$$E_t\widehat{H}_{t+1}\left(w_{t+1}^{n}\right) = E_t\widehat{x}_{t+1}\left(w_{t+1}^{n}\right) - \Gamma E_t\left(\widehat{w}_{t+1}^{*} - \widehat{w}_{t+1}\right) + (1-\chi)^{-1}E_t\widehat{\chi}_{t+1}\left(w_{t+1}^{n}\right) + (1-\eta)^{-1}\widehat{\varepsilon}_{t+1}^{\eta}$$

where $E_t\widehat{H}_{t+1}\left(w_{t+1}^{n}\right) = E_t\widehat{H}_{x,t+1}$.

- Substituting in the target wage and rearranging we obtain

$$\widehat{w}_t^o\left(w_t^{*n}\right)=\widehat{w}_t^o+\frac{\tau_1}{1-\tau}E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)+\frac{\tau_2}{1-\tau}\left(\widehat{w}_t-\widehat{w}_t^*\right)$$

where

$$\begin{aligned}\tau_1 &= \left[\varkappa_w\mu\varphi_x+\varphi_\chi\left(1-\chi\right)\left(x\beta\lambda\right)\left(\varkappa_w\mu\right)\mu\left(\rho\beta\right)+\varphi_s\Gamma\right]\left(1-\tau\right)\\ \tau_2 &= -\left(\varkappa_w\mu\right)\varphi_\chi\left(1-\chi\right)\left(x\beta\lambda\right)\mu\left(1-\tau\right)\end{aligned}$$

and

$$\begin{aligned}\widehat{w}_t^o &= \varphi_a\left(\widehat{p}_t^w+\widehat{a}_t\right)+\left(\varphi_s+\varphi_x\right)E_t\widehat{x}_{t+1}\left(w_{t+1}^n\right)+\varphi_sE_t\widehat{s}_{t+1}+\varphi_b\widehat{b}_t\\ &\quad+\left(\varphi_s+\varphi_x/2\right)E_t\widehat{\Lambda}_{t,t+1}+\varphi_\chi\left[\widehat{\chi}_t\left(w_t^n\right)-\left(\rho-s\right)\beta E_t\widehat{\chi}_{t+1}\left(w_{t+1}^n\right)\right]+\widehat{\varepsilon}_t^w\end{aligned}$$

and

$$\widehat{\varepsilon}_t^w=\varphi_\eta\left[1-\left(\rho-s\right)\beta\rho^\eta\right]\widehat{\varepsilon}_t^\eta$$
$$\varphi_\eta=\varphi_\chi\left(1-\chi\right)\left(1-\eta\right)^{-1}$$

- Next we present the complete loglinear model. We will write $\widehat{x}_t\left(w_t^n\right)$, $\widehat{\mu}_t\left(w_t^n\right)$, and $\widehat{\chi}_t\left(w_t^n\right)$ simply as $\widehat{x}_t$, $\widehat{\mu}_t$, and $\widehat{\chi}_t$.

APPENDIX C

The complete loglinear model

- Technology

$$\widehat{y}_t = \alpha \widehat{k}_t + (1-\alpha)\,\widehat{n}_t \tag{C1}$$

- Resource constraint

$$\widehat{y}_t = y_c \widehat{c}_t + y_i \widehat{i}_t + y_g \widehat{g}_t + y_\nu \widehat{\nu}_t + y_x\,(2\widehat{x}_t + \widehat{n}_{t-1}) \tag{C2}$$

where $y_c = \bar{c}/\bar{y}$, $y_i = \bar{\imath}/\bar{y}$, $y_g = \overline{g}/\bar{y}$, $y_\nu = r^k \bar{k}/\bar{y}$ and $y_x = (\kappa/2)\left(x^2 n/\bar{y}\right)$.

- Matching

$$\widehat{m}_t = \sigma \widehat{u}_t + (1-\sigma)\,\widehat{v}_t \tag{C3}$$

- Employment dynamics

$$\widehat{n}_t = \widehat{n}_{t-1} + (1-\rho)\,\widehat{x}_t \tag{C4}$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \tag{C5}$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \tag{C6}$$

- Unemployment

$$\widehat{u}_t = -\,(n/u)\,\widehat{n}_{t-1} \tag{C7}$$

- Effective capital

$$\widehat{k}_t + \widehat{\varepsilon}^z_t = \widehat{\nu}_t + \widehat{k}^p_{t-1} \tag{C8}$$

- Physical capital dynamics

$$\widehat{k}^p_t = \xi\left(\widehat{k}^p_{t-1} - \widehat{\varepsilon}^z_t\right) + (1-\xi)\left(\widehat{i}_t + \widehat{\varepsilon}^i_t\right) \tag{C9}$$

where $\xi = \frac{1-\delta}{\gamma_z}$

- Aggregate vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_{t-1} \tag{C10}$$

- Consumption-saving

$$0 = E_t \widehat{\Lambda}_{t,t+1} + (\widehat{r}_t - E_t \widehat{\pi}_{t+1}) - E_t \widehat{\varepsilon}^z_{t+1} \tag{C11}$$

- Marginal utility

$$\left(1-\tilde{h}\right)\left(1-\beta\tilde{h}\right)\widehat{\lambda}_t \;=\; \tilde{h}\left(\widehat{c}_{t-1}-\widehat{\varepsilon}^z_t\right)-\left(1+\beta\tilde{h}^2\right)\widehat{c}_t+ \beta\tilde{h}E_t\left(\widehat{c}_{t+1}+\widehat{\varepsilon}^z_{t+1}\right)+\left(1-\tilde{h}\right)\left(\widehat{\varepsilon}^b_t-\beta\tilde{h}E_t\widehat{\varepsilon}^b_{t+1}\right) \quad \text{(C12)}$$

where h measures the degree of habit persistence in consumption and where $\tilde{h}=h/\gamma_z$

- Capital utilization

$$\widehat{\nu}_t=\eta_\nu\widehat{r}^k_t \quad \text{(C13)}$$

where $\eta_\nu=\mathcal{A}'(1)/\mathcal{A}''(1)=\frac{1-\psi_\nu}{\psi_\nu}$

- Investment

$$\widehat{i}_t=\frac{1}{1+\beta}\left(\widehat{i}_{t-1}-\widehat{\varepsilon}^z_t\right)+\frac{1/\left(\eta_k\gamma_z^2\right)}{1+\beta}\left(\widehat{q}^k_t+\widehat{\varepsilon}^i_t\right)+\frac{\beta}{1+\beta}E_t\left(\widehat{i}_{t+1}+\widehat{\varepsilon}^z_{t+1}\right) \quad \text{(C14)}$$

where $\eta_k=\mathcal{S}''(\gamma_z)$

- Capital renting

$$\widehat{p}^w_t+\widehat{y}_t-\widehat{k}_t=\widehat{r}^k_t \quad \text{(C15)}$$

- Tobin's q

$$\widehat{q}^k_t=\tilde{\beta}(1-\delta)E_t\widehat{q}^k_{t+1}+\left[1-\tilde{\beta}(1-\delta)\right]E_t\widehat{r}^k_{t+1}-\left(\widehat{r}_t-E_t\widehat{\pi}_{t+1}\right) \quad \text{(C16)}$$

where $\tilde{\beta}=\beta/\gamma_z$

- Aggregate hiring rate

$$\widehat{x}_t=\varkappa_a\left(\widehat{p}^w_t+\widehat{a}_t\right)-\varkappa_w\widehat{w}_t+\varkappa_\lambda E_t\widehat{\Lambda}_{t,t+1}+\beta E_t\widehat{x}_{t+1} \quad \text{(C17)}$$

where $\varkappa=(\kappa x)^{-1}$, $\varkappa_a=\varkappa p^w\bar{a}$, $\varkappa_w=\varkappa\bar{w}$ and $\varkappa_\lambda=\beta(1+\rho)/2$

- Marginal product of labor

$$\widehat{a}_t=\widehat{y}_t-\widehat{n}_t \quad \text{(C18)}$$

- Weight in Nash bargaining

$$\widehat{\chi}_t=-(1-\chi)\left(\widehat{\mu}_t-\widehat{\epsilon}_t\right) \quad \text{(C19)}$$

with

$$\widehat{\epsilon}_t=(\rho\lambda\beta)E_t\left(\widehat{\Lambda}_{t,t+1}-\widehat{\pi}_{t+1}+\gamma\widehat{\pi}_t+\widehat{\epsilon}_{t+1}-\widehat{\varepsilon}^z_{t+1}\right) \quad \text{(C20)}$$

$$\widehat{\mu}_t \;=\; (x\lambda\beta)E_t\widehat{x}_{t+1}-(x\lambda\beta)(\varkappa_w\mu)\mu E_t\left(\widehat{w}_t+\gamma\widehat{\pi}_t-\widehat{\pi}_{t+1}-\widehat{\varepsilon}^z_{t+1}-\widehat{w}_{t+1}\right) +(\lambda\beta)E_t\left(\widehat{\mu}_{t+1}+\widehat{\Lambda}_{t,t+1}+\gamma\widehat{\pi}_t-\widehat{\pi}_{t+1}-\widehat{\varepsilon}^z_{t+1}\right) \quad \text{(C21)}$$

- Spillover-free target wage

$$
\begin{aligned}
\widehat{w}_t^o &= \varphi_a\left(\widehat{p}_t^w + \widehat{a}_t\right) + \left(\varphi_s + \varphi_x\right) E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \varphi_b \widehat{b}_t \\
&\quad + \left(\varphi_s + \varphi_x/2\right) E_t \widehat{\Lambda}_{t,t+1} + \varphi_\chi\left(\widehat{\chi}_t - (\rho - s)\beta\widehat{\chi}_{t+1}\right) + \widehat{\varepsilon}_t^w
\end{aligned} \tag{C22}
$$

where

$$
\varphi_a = \chi p^w \bar{a}\bar{w}^{-1} \qquad \varphi_x = \chi\beta\kappa x^2 \bar{w}^{-1} \qquad \varphi_b = (1-\chi)\,\bar{b}\bar{w}^{-1}
$$

$$
\varphi_s = (1-\chi)\,s\beta\bar{H}\bar{w}^{-1} \qquad \varphi_\chi = \chi\,(1-\chi)^{-1}\,\kappa x \bar{w}^{-1}
$$

$$
\begin{aligned}
\widehat{\varepsilon}_t^w &= \varphi_\eta\left[1 - (\rho - s)\,\beta\rho^\eta\right]\widehat{\varepsilon}_t^\eta \\
\varphi_\eta &= \varphi_\chi\,(1-\chi)\,(1-\eta)^{-1}
\end{aligned}
$$

- Aggregate wage

$$
\widehat{w}_t = \gamma_b\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^z\right) + \gamma_o\widehat{w}_t^o + \gamma_f E_t\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^z\right) \tag{C23}
$$

where

$$
\gamma_b = (1+\tau_2)\,\phi^{-1} \qquad \gamma_o = \varsigma\phi^{-1} \qquad \gamma_f = \left(\tau\lambda^{-1} - \tau_1\right)\phi^{-1}
$$

$$
\phi = (1+\tau_2) + \varsigma + \left(\tau\lambda^{-1} - \tau_1\right) \qquad \varsigma = (1-\lambda)\,(1-\tau)\,\lambda^{-1}
$$

$$
\begin{aligned}
\tau_1 &= \left[\varkappa_w\mu\varphi_x + \varphi_\chi\,(1-\chi)\,(x\beta\lambda)\,(\varkappa_w\mu)\,\mu\,(\rho\beta) + \varphi_s\Gamma\right](1-\tau) \\
\tau_2 &= -\left(\varkappa_w\mu\right)\varphi_\chi\,(1-\chi)\,(x\beta\lambda)\,\mu\,(1-\tau)
\end{aligned}
$$

$$
\Gamma = (1 - \eta x\beta\lambda\mu)\,\eta^{-1}\mu\varkappa_w
$$

- Phillips curve

$$
\widehat{\pi}_t = \iota_b\widehat{\pi}_{t-1} + \iota_o\left(\widehat{p}_t^w + \widehat{\varepsilon}_t^p\right) + \iota_f E\widehat{\pi}_{t+1} \tag{C24}
$$

where

$$
\iota_b = \gamma^p\,(\phi^p)^{-1} \qquad \iota_o = (\varsigma^p/\tau^p)\,(\phi^p)^{-1} \qquad \iota_f = \beta\,(\phi^p)^{-1}
$$

$$
\phi^p = 1 + \beta\gamma^p \qquad \varsigma^p = (1-\lambda^p)\,(1-\lambda^p\beta)\,(\lambda^p)^{-1} \qquad \tau^p = 1 + (\varepsilon^p - 1)\,\xi
$$

- Tayor rule

$$
\widehat{r}_t = \rho_s\widehat{r}_{t-1} + (1-\rho_s)\left[r_\pi\widehat{\pi}_t + r_y\left(\widehat{y}_t - \widehat{y}_{nt}\right)\right] + \widehat{\varepsilon}_t^r \tag{C25}
$$

- Government spending

$$
\widehat{g}_t = \widehat{y}_t + \frac{1 - y_g}{y_g}\widehat{\varepsilon}_t^g \tag{C26}
$$

- Market tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \tag{C27}$$

- Benefits

$$\widehat{b}_t = \widehat{k}^p_t \tag{C28}$$

APPENDIX D

Estimates of the SW model

Note that the parameters ω and ε^w are, respectively, the Frisch elasticity of labor supply and the gross steady state wage markup. The parameters ρ_w and σ_w refer to a wage markup shock.

Insert Tables A1 and A2 here

APPENDIX E

The measurement equation

The model is completed by the following measurement equation that relates a set of observables (on the left-hand side) to the corresponding model variables (on the right-hand side), as follows:

$$\begin{bmatrix} 100\Delta\log y_t \\ 100\Delta\log c_t \\ 100\Delta\log i_t \\ 100\Delta\log w_t^n/p_t \\ \log n_t - \overline{\log n} \\ \pi_t - \overline{\pi} \\ r_t - \overline{r} \end{bmatrix} = \begin{bmatrix} 100(\gamma_z - 1) \\ 100(\gamma_z - 1) \\ 100(\gamma_z - 1) \\ 100(\gamma_z - 1) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\ \hat{c}_t - \hat{c}_{t-1} + \hat{z}_t \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t \\ \hat{w}_t - \hat{w}_{t-1} + \hat{z}_t \\ \hat{n}_t \\ \hat{\pi}_t \\ \hat{r}_t \end{bmatrix}$$

where the bar over a variable indicate the sample mean. We have exploited the relation between sample means and steady states to estimate γ_z, while we have demeaned $\log n_t, \pi_t$ and r_t.

APPENDIX F

The model with variable hours per worker

Here we briefly outline how it is possible to amend the model to allow hours per worker to vary.

As before, there is a representative household with a continuum of members of measure unity. The number of family members currently employed is now $\bar{n}_t$,which is no longer the same as hours n_t. Now each employed member works ψ_t hours, which is determined via decentralized bargaining between firms and workers.

Accordingly, conditional on $\bar{n}_t$ and ψ_t, the household chooses consumption c_t, government bonds B_t, capital utilization ν_t, investment i_t, and physical capital k_t^p to maximize the utility function

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \left[\log \left(c_{t+s} - h c_{t+s-1} \right) - \frac{\varepsilon_{t+s}^{\psi}}{1+\omega} \psi_{t+s}^{1+\omega} \bar{n}_{t+s} \right], \tag{F1}$$

where ε_t^{ψ} is a shock to the supply of hours, with

$$\log \varepsilon_t^{\psi} = (1-\rho^{\psi}) \log \varepsilon^{\psi} + \rho^{\psi} \log \varepsilon_{t-1}^{\psi} + \varsigma_t^{\psi}. \tag{F2}$$

Total hours is given by

$$n_t = \psi_t \bar{n}_t. \tag{F3}$$

Relative to our baseline model, the one additional variable to be determined id hours per worker, ψ_t. While the hiring margin is affected by rigidity, the hours margin is not, due to the on-going relation between the firm and its existing workforce. Accordingly at each point in time the two parties agree to an efficient allocation of hours. In particular, hours adjust to the point where the marginal value product $p_t^w a_t$ equals a worker's marginal rate of substitution between consumption and leisure $\varepsilon_t^{\psi} \psi_t^{\omega} / \bar{\lambda}_t$:

$$p_t^w a_t = \frac{\varepsilon_t^{\psi} \psi_t^{\omega}}{\bar{\lambda}_t}, \tag{F4}$$

with

$$\bar{\lambda}_t = \lambda_t / \varepsilon_t^b. \tag{F5}$$

Equation (F4) determines hours per worker. After allowing for variable hours per worker, the rest of the model is the same as in the text.

Table 1: Calibrated parameters

β	δ	α	$\bar{g}/\bar{y}$	ξ	σ	ρ	s
0.99	0.025	0.33	0.2	10	0.5	0.895	0.95

Table 2: Prior and posterior distribution of structural parameters

		Prior distribution	Posterior distribution Max	Mean	5%	95%
Utilization rate elasticity	ψ_ν	Beta (0.5,0.1)	0.695	0.700	0.603	0.761
Capital adjustment cost elasticity	η_k	Normal (4,1.5)	2.425	2.375	1.639	3.457
Habit parameter	h	Beta (0.5,0.1)	0.727	0.708	0.672	0.773
Bargaining power parameter	η	Beta (0.5,0.1)	0.907	0.907	0.868	0.946
Relative flow value of unemployment	$\tilde{b}$	Beta (0.5,0.1)	0.726	0.723	0.656	0.790
Calvo wage parameter	λ	Beta (0.75,0.1)	0.717	0.717	0.656	0.782
Calvo price parameter	λ_p	Beta (0.66,0.1)	0.848	0.846	0.804	0.887
Wage indexing parameter	γ	Uniform (0,1)	0.816	0.815	0.689	0.915
Steady-state price markup	ε^p	Normal (1.15,0.05)	1.405	1.408	1.360	1.455
Taylor rule response to inflation	r_π	Normal (1.7,0.3)	2.015	2.006	1.916	2.157
Taylor rule response to output gap	r_y	Gamma (0.125,0.1)	0.333	0.332	0.272	0.421
Taylor rule inertia	ρ_s	Beta (0.75,0.1)	0.773	0.772	0.728	0.810
Steady-state growth rate	γ_z	Uniform (1,1.5)	1.004	1.004	1.003	1.005

This table reports the prior and posterior distribution of the estimated structural parameters. For the uniform distribution, the two numbers in parentheses are the lower and upper bounds. Otherwise, the two numbers are the mean and the standard deviation of the distribution.

Table 3: Prior and posterior distribution of shock parameters

		Prior distribution	Posterior distribution Max	Mean	5%	95%
(a) Autoregressive parameters						
Technology	ρ_z	Beta (0.5,2)	0.140	0.096	0.071	0.198
Preferences	ρ_b	Beta (0.5,2)	0.713	0.724	0.639	0.764
Investment	ρ_i	Beta (0.5,2)	0.605	0.599	0.517	0.674
Price markup	ρ_p	Beta (0.5,2)	0.808	0.814	0.744	0.857
Bargaining power	ρ_w	Beta (0.5,2)	0.264	0.261	0.200	0.349
Government	ρ_g	Beta (0.5,2)	0.991	0.993	0.984	0.995
Monetary	ρ_r	Beta (0.5,2)	0.207	0.179	0.133	0.299
(b) Standard deviations						
Technology	σ_z	IGamma (0.15,0.15)	1.039	1.025	0.966	1.090
Preferences	σ_b	IGamma (0.15,0.15)	0.362	0.334	0.278	0.502
Investment	σ_i	IGamma (0.15,0.15)	0.166	0.165	0.121	0.229
Price markup	σ_p	IGamma (0.15,0.15)	0.062	0.06	0.048	0.080
Bargaining power	σ_w	IGamma (0.15,0.15)	0.578	0.586	0.516	0.651
Government	σ_g	IGamma (0.15,0.15)	0.357	0.358	0.331	0.396
Monetary	σ_r	IGamma (0.15,0.15)	0.224	0.226	0.208	0.251

This table reports the prior and posterior distribution of the estimated parameters of the exogenous shock processes. The two numbers in parentheses are the mean and the standard deviation of the distribution.

Table 4: Prior and posterior distribution of structural parameters - $\lambda = 0$

		Prior distribution	Posterior distribution Max	Mean	5%	95%
Utilization rate elasticity	ψ_ν	Beta (0.5,0.1)	0.861	0.852	0.783	0.911
Capital adjustment cost elasticity	η_k	Normal (4,1.5)	1.023	1.179	0.803	1.635
Habit parameter	h	Beta (0.5,0.1)	0.801	0.803	0.760	0.840
Bargaining power parameter	η	Beta (0.5,0.1)	0.616	0.589	0.451	0.726
Relative flow value of unemployment	$\tilde{b}$	Beta (0.5,0.1)	0.983	0.982	0.975	0.987
Calvo wage parameter	λ	Beta (0.75,0.1)	–	–	–	–
Calvo price parameter	λ_p	Beta (0.66,0.1)	0.574	0.575	0.512	0.630
Wage indexing parameter	γ	Uniform (0,1)	–	–	–	–
Steady-state price markup	ε^p	Normal (1.15,0.05)	1.347	1.351	1.298	1.407
Taylor rule response to inflation	r_π	Normal (1.7,0.3)	1.927	1.999	1.748	2.297
Taylor rule response to output gap	r_y	Gamma (0.125,0.1)	0.013	0.019	0.003	0.043
Taylor rule inertia	ρ_s	Beta (0.75,0.1)	0.685	0.700	0.648	0.746
Steady-state growth rate	γ_z	Uniform (1,1.5)	1.003	1.003	1.001	1.004

This table reports the prior and posterior distribution of the estimated structural parameters when $\lambda = 0$. For the uniform distribution, the two numbers in parentheses are the lower and upper bounds. Otherwise, the two numbers are the mean and the standard deviation of the distribution.

Table 5: Prior and posterior distribution of shock parameters - $\lambda = 0$

		Prior distribution	Posterior distribution Max	Mean	5%	95%
(a) *Autoregressive parameters*						
Technology	ρ_z	Beta (0.5,2)	0.287	0.282	0.193	0.378
Preferences	ρ_b	Beta (0.5,2)	0.351	0.363	0.231	0.507
Investment	ρ_i	Beta (0.5,2)	0.865	0.852	0.812	0.891
Price markup	ρ_p	Beta (0.5,2)	0.916	0.909	0.866	0.946
Bargaining power	ρ_w	Beta (0.5,2)	0.984	0.984	0.977	0.990
Government	ρ_g	Beta (0.5,2)	0.987	0.987	0.981	0.992
Monetary	ρ_r	Beta (0.5,2)	0.249	0.250	0.162	0.334
(b) *Standard deviations*						
Technology	σ_z	IGamma (0.15,0.15)	1.071	1.083	0.992	1.188
Preferences	σ_b	IGamma (0.15,0.15)	0.686	0.738	0.436	1.146
Investment	σ_i	IGamma (0.15,0.15)	0.066	0.074	0.059	0.094
Price markup	σ_p	IGamma (0.15,0.15)	0.093	0.093	0.076	0.115
Bargaining power	σ_w	IGamma (0.15,0.15)	0.250	0.274	0.183	0.369
Government	σ_g	IGamma (0.15,0.15)	0.352	0.355	0.324	0.389
Monetary	σ_r	IGamma (0.15,0.15)	0.237	0.241	0.218	0.267

This table reports the prior and posterior distribution of the estimated parameters of the exogenous shock processes when $\lambda = 0$. The two numbers in parentheses are the mean and the standard deviation of the distribution.

Table 6: Log marginal likelihood

Baseline	Flex wage
-1191	-1234

Table 7: Variance decomposition for the growth rate of output at different horizons

Shocks	Horizons			
	on impact	1 year	4 years	long run
Technology (ς_z)	16.7	32.5	31.0	31.0
Monetary (ς_r)	6.1	5.0	5.4	5.4
Preferences (ς_b)	11.1	9.2	9.5	9.5
Investment (ς_i)	54.8	41.9	42.4	42.4
Government (ς_g)	9.4	8.7	8.2	8.2
Price Markup (ς_p)	1.9	2.5	3.2	3.2
Bargaining power (ς_w)	0.0	0.2	0.3	0.3

This table reports the forecast error variance decomposition for the growth rate of output, computed at the mean of the posterior distribution.

Table 8: Variance decomposition for vacancies at different horizons

Shocks	Horizons			
	on impact	1 year	4 years	long run
Technology (ς_z)	0.14	0.18	0.37	0.42
Monetary (ς_r)	0.07	0.09	0.06	0.05
Preferences (ς_b)	0.12	0.11	0.07	0.06
Investment (ς_i)	0.56	0.48	0.34	0.31
Government (ς_g)	0.01	0.07	0.07	0.07
Price Markup (ς_p)	0.00	0.03	0.06	0.07
Bargaining power (ς_w)	0.09	0.04	0.03	0.03

This table reports the forecast error variance decomposition for vacancies, computed at the mean of the posterior distribution.

Table 9: Variance decomposition for total hours (employment) at different horizons

Shocks	Horizons			
	on impact	1 year	4 years	long run
Technology (ς_z)	0.14	0.13	0.40	0.45
Monetary (ς_r)	0.07	0.10	0.06	0.05
Preferences (ς_b)	0.12	0.12	0.06	0.05
Investment (ς_i)	0.56	0.51	0.31	0.28
Government (ς_g)	0.01	0.08	0.08	0.08
Price Markup (ς_p)	0.00	0.03	0.07	0.08
Bargaining power (ς_w)	0.09	0.03	0.02	0.01

This table reports the forecast error variance decomposition for total hours, computed at the mean of the posterior distribution.

Table A1: Prior and posterior distribution of structural parameters - SW model

		Prior distribution	Posterior distribution			
			Max	Mean	5%	95%
Utilization rate elasticity	ψ_ν	Beta (0.5,0.1)	0.667	0.657	0.546	0.765
Capital adjustment cost elasticity	η_k	Normal (4,1.5)	2.922	3.593	2.286	5.297
Habit parameter	h	Beta (0.5,0.1)	0.746	0.772	0.703	0.839
Inverse of Frish elasticity	ω	Gamma (2,0.75)	3.910	4.041	2.829	5.452
Calvo wage parameter	λ	Beta (0.75,0.1)	0.881	0.865	0.783	0.925
Calvo price parameter	λ_p	Beta (0.66,0.1)	0.856	0.854	0.816	0.890
Wage indexing parameter	γ	Uniform (0,1)	0.796	0.763	0.574	0.938
Steady-state price markup	ε^p	Normal (1.15,0.05)	1.392	1.391	1.336	1.448
Steady-state wage markup	ε^w	Normal (1.15,0.05)	1.138	1.127	1.042	1.210
Taylor rule response to inflation	r_π	Normal (1.7,0.3)	2.057	2.053	1.725	2.413
Taylor rule response to output gap	r_y	Gamma (0.125,0.1)	0.307	0.320	0.224	0.437
Taylor rule inertia	ρ_s	Beta (0.75,0.1)	0.807	0.813	0.765	0.855
Steady-state growth rate	γ_z	Uniform (1,1.5)	1.004	1.004	1.003	1.005

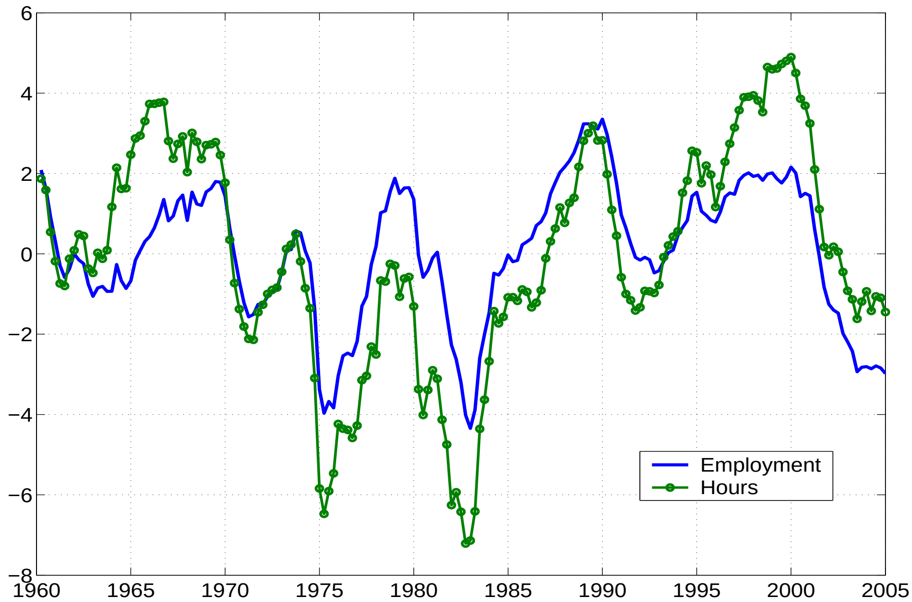
This table reports the prior and posterior distribution of the estimated structural parameters in the SW model. For the uniform distribution, the two numbers in parentheses are the lower and upper bounds. Otherwise, the two numbers are the mean and the standard deviation of the distribution.

Table A2: Prior and posterior distribution of shock parameters - SW model

		Prior distribution	Posterior distribution			
			Max	Mean	5%	95%
(a) *Autoregressive parameters*						
Technology	ρ_z	Beta (0.5,0.15)	0.129	0.143	0.067	0.226
Preferences	ρ_b	Beta (0.5,0.15)	0.698	0.662	0.527	0.777
Investment	ρ_i	Beta (0.5,0.15)	0.531	0.508	0.385	0.631
Price markup	ρ_p	Beta (0.5,0.15)	0.810	0.799	0.746	0.851
Wage markup	ρ_w	Beta (0.5,0.15)	0.296	0.309	0.195	0.420
Government	ρ_g	Beta (0.5,0.15)	0.991	0.989	0.982	0.995
Monetary	ρ_r	Beta (0.5,0.15)	0.241	0.254	0.155	0.359
(b) *Standard deviations*						
Technology	σ_z	IGamma (0.15,0.15)	1.022	1.041	0.952	1.141
Preferences	σ_b	IGamma (0.15,0.15)	0.420	0.583	0.337	0.999
Investment	σ_i	IGamma (0.15,0.15)	0.195	0.241	0.154	0.354
Price markup	σ_p	IGamma (0.15,0.15)	0.060	0.063	0.048	0.080
Wage markup	σ_w	IGamma (0.15,0.15)	0.197	0.197	0.166	0.231
Government	σ_g	IGamma (0.15,0.15)	0.359	0.362	0.331	0.396
Monetary	σ_r	IGamma (0.15,0.15)	0.228	0.231	0.211	0.253

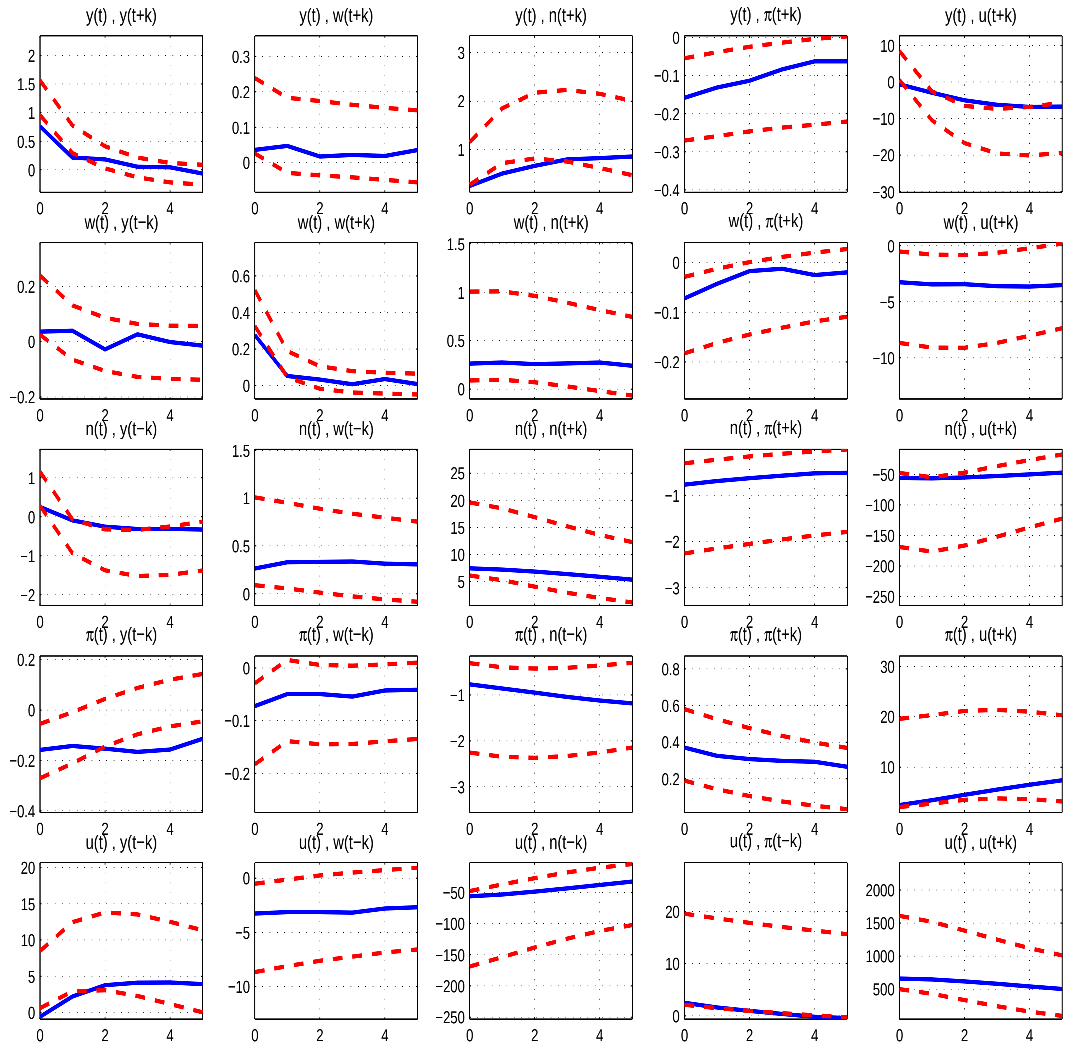
This table reports the prior and posterior distribution of the estimated parameters of the exogenous shock processes in the SW model. The two numbers in parentheses are the mean and the standard deviation of the distribution.

Figure 1: Employment and Hours



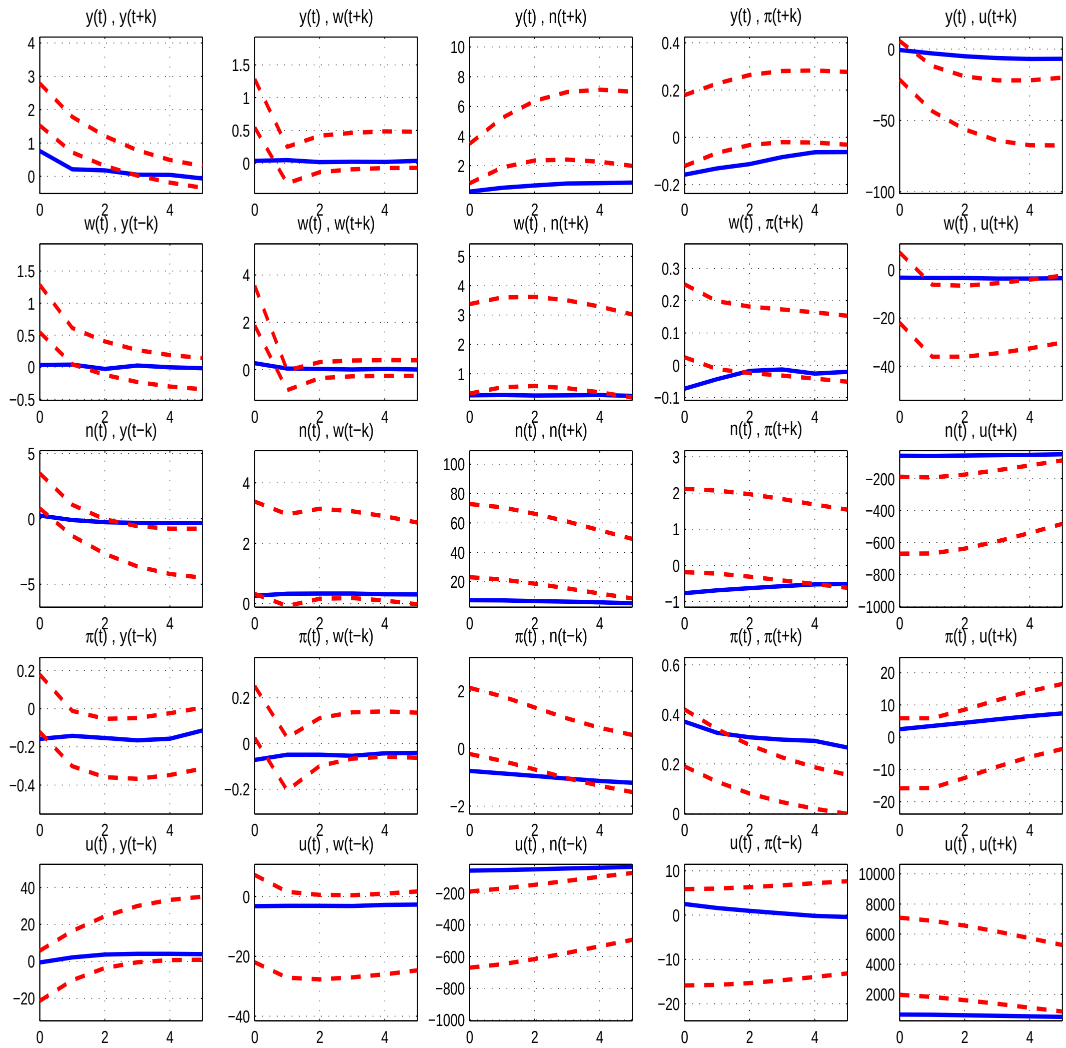
This figure shows total hours per capita and employment from 1960 to 2005. Total hours per capita is the log of hours of all persons in the non-farm business sector divided by population times the ratio of total employment to employment in non-farm business sector. Employment is the log of employment over 16 divided by population, detrended with a linear trend.

Figure 2: Autocovariances of selected variables - U.S. data and estimated model



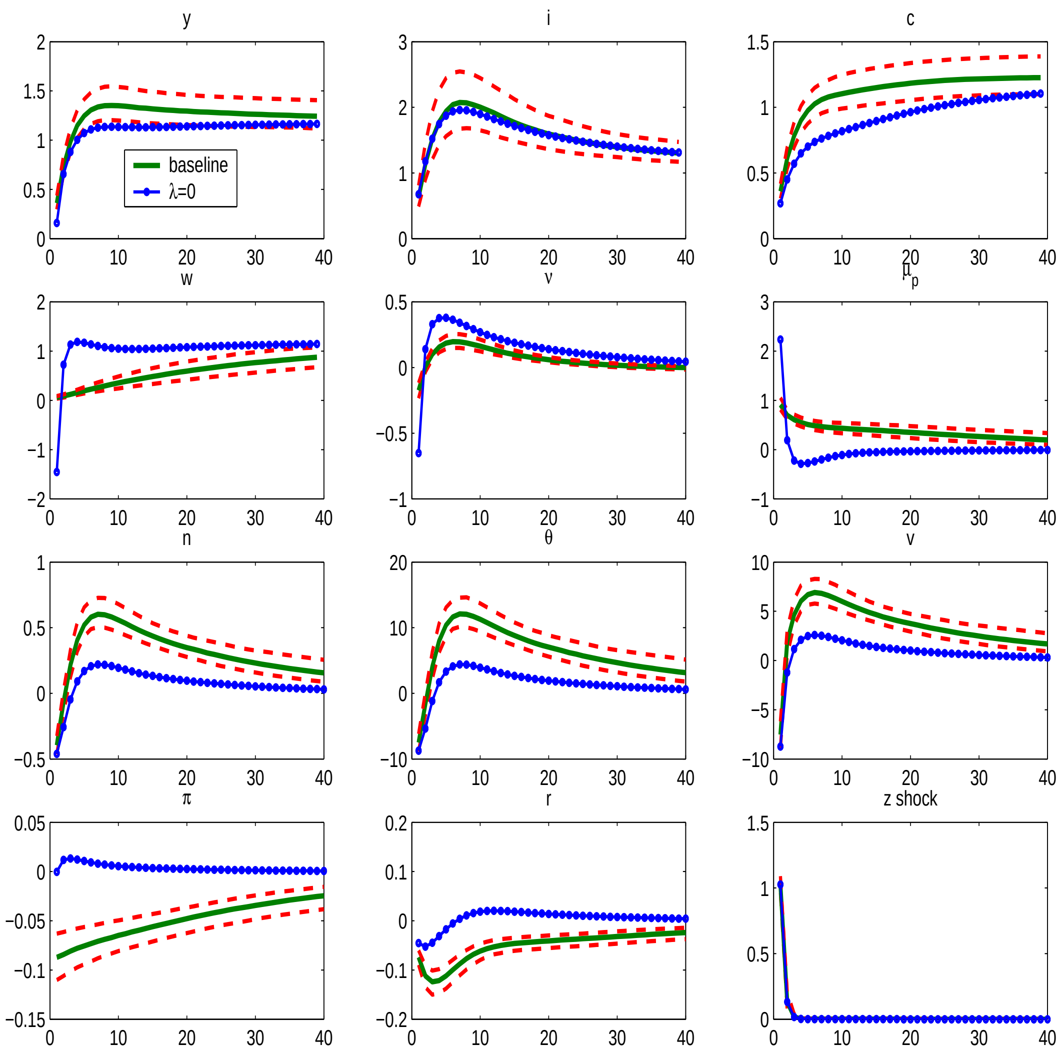
This figure shows the autocovariance function of the growth rates of output and the real wage, and the level of employment (total hours), inflation and unemployment in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5-th and 95-th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 180 observations for each draw).

Figure 3: Autocovariances of selected variables - U.S. data and estimated model ($\lambda = 0$)



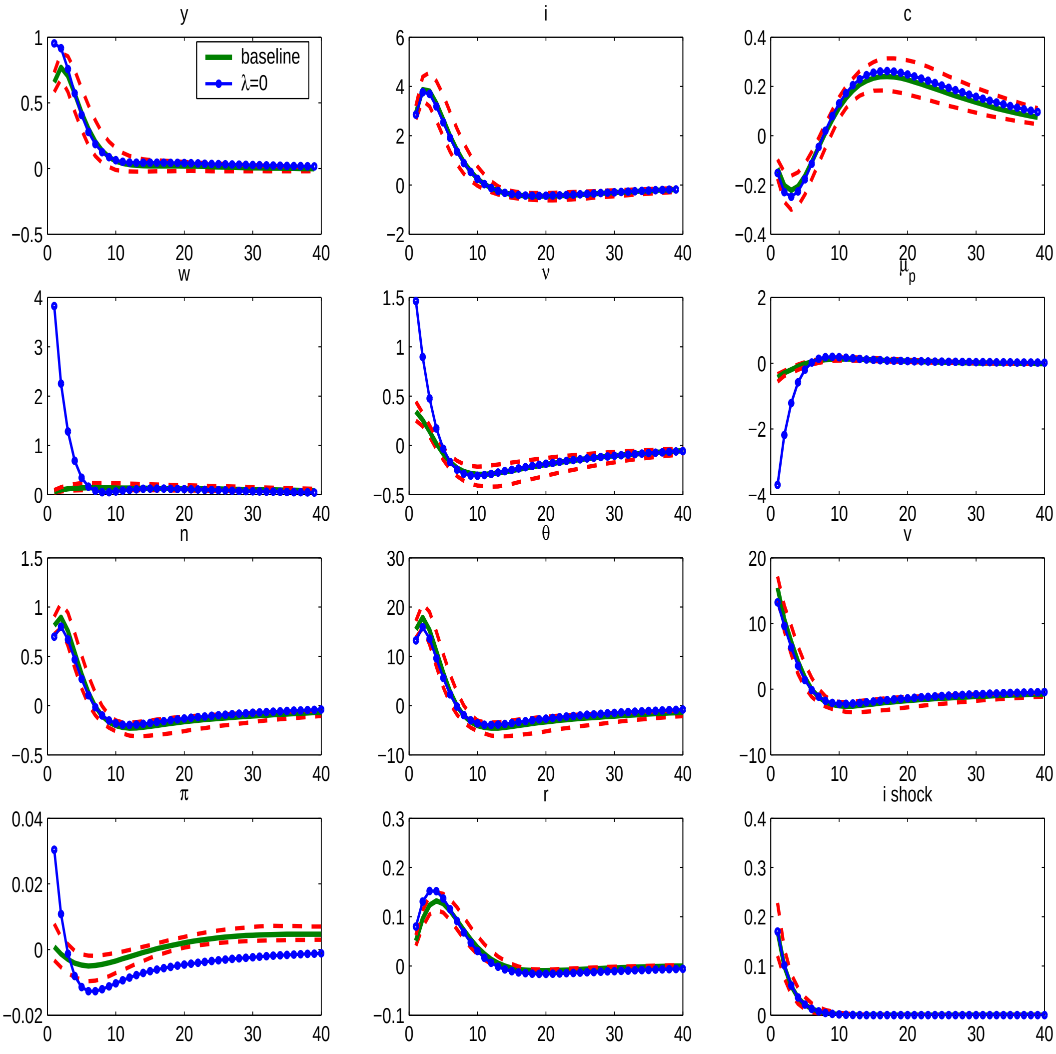
This figure shows the autocovariance function of the growth rates of output and the real wage, and the level of employment (total hours), inflation and unemployment in U.S. data (solid lines) and in the estimated model with $\lambda = 0$ (dashed lines, representing the 5-th and 95-th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 180 observations for each draw).

Figure 4: Impulse responses to a technology shock ς_z



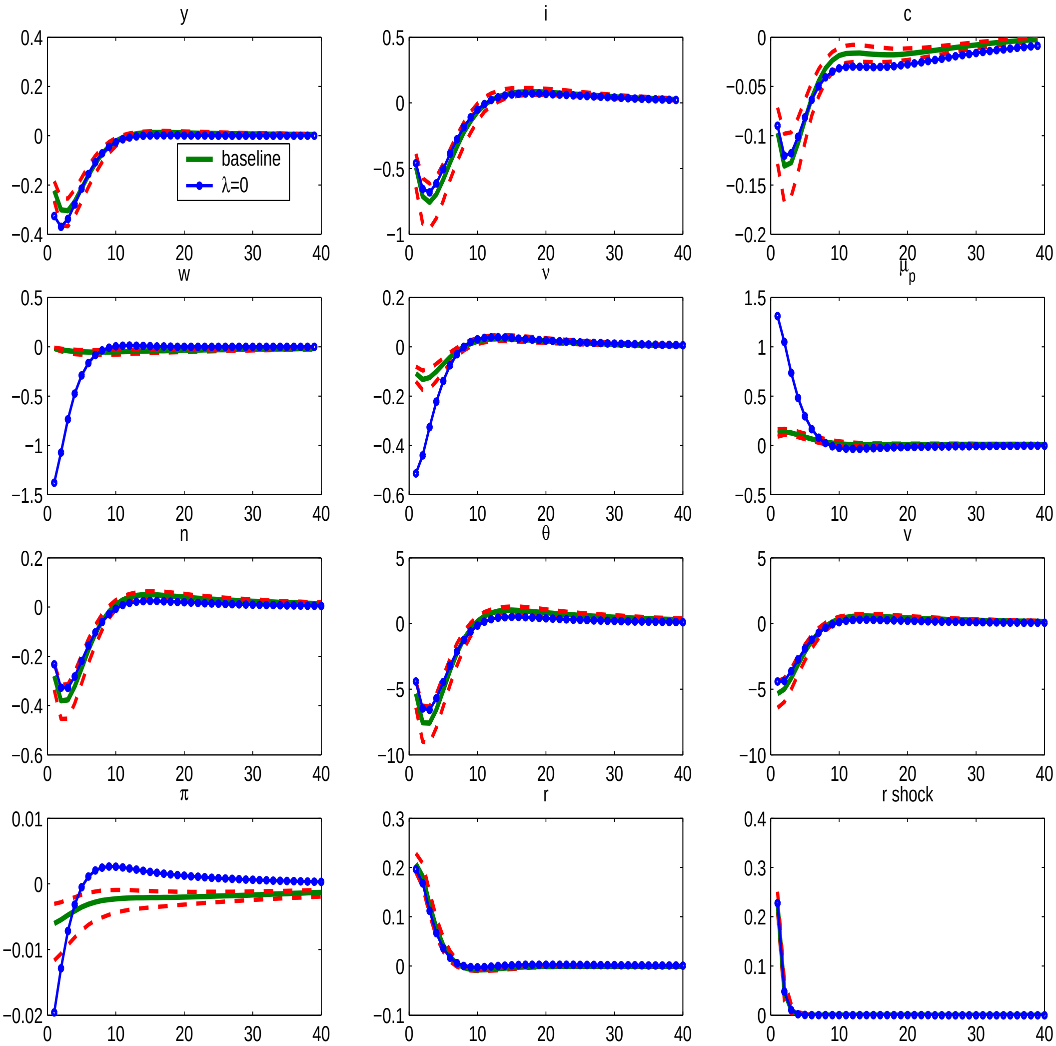
This figure shows the impulse responses to a technology shock. The solid line is the median impulse response. The dashed lines are the 5-th and 95-th percentile of the posterior distribution. The dotted line is the median impulse response obtained by setting $\lambda = 0$.

Figure 5: Impulse responses to an investment specific shock ς_i



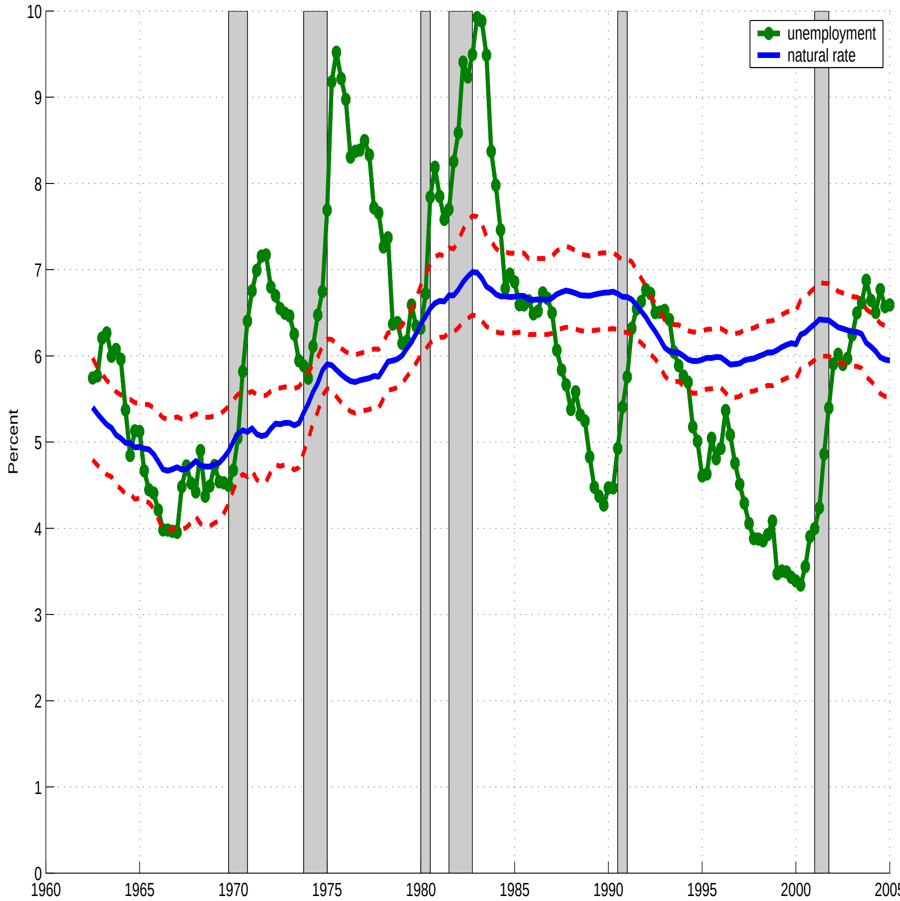
This figure shows the impulse responses to an investment specific shock. The solid line is the median impulse response. The dashed lines are the 5-th and 95-th percentile of the posterior distribution. The dotted line is the median impulse response obtained by setting $\lambda = 0$.

Figure 6: Impulse response to a monetary policy shock ς_r



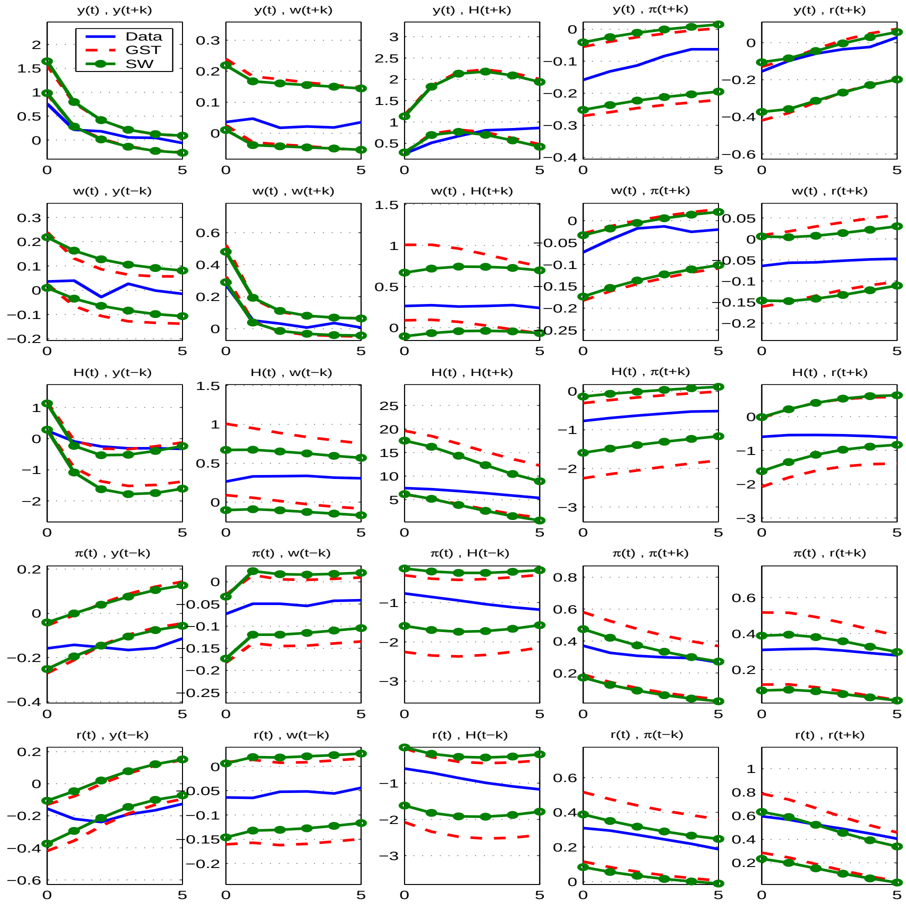
This figure shows the impulse responses to a monetary policy shock. The solid line is the median impulse response. The dashed lines are the 5-th and 95-th percentile of the posterior distribution. The dotted line is the median impulse response obtained by setting $\lambda = 0$.

Figure 7: Estimated actual and natural rate of unemployment



This figure shows the estimated path for the actual and natural rates of unemployment. The unemployment rates have been calculated assuming a steady-state rate of unemployment of 6%. For the natural rate, the thick line is the median and the dashed lines are the 5-th and 95-th percentiles of the empirical distribution, taking into account parameter uncertainty and Kalman filter uncertainty. Shaded areas correspond to recessions dated by the NBER.

Figure A1: Autocovariances of selected variables - U.S. data and estimated model (GST vs. SW)



This figure shows the autocovariance function of the growth rates of output and the real wage, and the level of total hours, inflation and nominal interest rate in U.S. data (solid lines), in the estimated GST model (dashed lines) and in the estimated SW model (solid-dotted lines). The dashed and the solid-dotted lines represent the 5-th and 95-th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 180 observations for each draw, respectively in the GST and SW model).