Topic 5

Financial Market Frictions and Real Activity:

Part 2: Multi-period Contracts

and

General Equilibrium

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Multi-period Version of Costly Enforcement Problem

- Entrepreneur survives multiple periods in expectation
  - $\sigma \equiv$ probability of surviving from t to $t+1$
  - $\rightarrow$ expected horizon $= \frac{1}{1-\sigma}$
- Entrepreneur is risk neutral
  - Consumes all retained earnings upon exit
  - $\rightarrow$ Objective: maximize expected retained earnings upon exit
- Entrepreneur manages capital investments
  - Finances capital for $t+1$ with retained earnings and borrowing at t
  - Borrowing is in the form of short term non-contingent debt
- Moral hazard problem: entrepreneur may divert a fraction of assets for own use

Balance Sheet and Flow of Funds

$k \equiv$capital, $Q \equiv$ price of capital, $n \equiv$ net worth, $b \equiv$ borrowing

- Balance Sheet

$$Q_t k_{t+1} = n_t + b_t$$

- Flow of Funds (evolution of retained earnings)

$$\begin{aligned} n_{t+1} &= R_{kt+1} Q_t k_{t+1} - R_{t+1} b_t \\ &= (R_{kt+1} - R_{t+1}) Q_t k_{t+1} + R_{t+1} n_t \end{aligned}$$

with $R_k \equiv$ gross return on capital, $R \equiv$ gross borrowing rate

Entrepreneur's Objective

- Entrepreneur chooses (k_t, b_t, n_{t+1}) to maximize expected discounted terminal earnings, given by

$$V_t = \max E_t \{ \sum_{i=0} (1-\sigma)\sigma^i \beta^{1+i} n_{t+1+i} \}$$

- Expressing in recursive form:

$$V_t = \max E_t \{ \beta [(1-\sigma) n_{t+1} + \sigma V_{t+1}] \}$$

Agency Problem

- Agency Problem: After the entrepreneur borrows funds at the end of period t, it may divert the fraction θ of total assets for own use.

- If the entrepreneur does not honor its debt, lenders can liquidate the intermediate and obtain the fraction $1-\theta$ of initial assets

- Incentive Constraint:

$$V_t \geq \theta Q_t k_{t+1}$$

i.e. Under any financial arrangement, the value to the entrepreneur from operating honestly, V_t must be not less than the gain from diverting.$\theta Q_t k_t$

Entrepreneur's Optimization Problem:

- Simplify V_t :

$$V_t = \max E_t\{\beta[(1-\sigma)n_{t+1} + \sigma V_{t+1}]\}$$
$$= \max E_t \beta \Omega_{t+1} n_{t+1}$$

with

$$\Omega_{t+1} = 1 - \sigma + \sigma \frac{V_{t+1}}{n_{t+1}}$$

with $\frac{V_{t+1}}{n_{t+1}} \equiv$ shadow value of net worth (i.e. "Tobin's Q value")

- Simplify n_{t+1} and let $\phi_t = Q_t k_{t+1}/n_t \equiv$ leverage multiple

$$n_{t+1} = (R_{kt+1} - R_{t+1})Q_t k_{t+1} + R_{t+1} n_t$$
$$= [(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]n_t$$

Note volatility of n_t increase in ϕ_t

Optimization Problem (con't):

- Combining $\longrightarrow$:

$$V_t = \max_{\phi_t} E_t\{\beta\Omega_{t+1}[(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]n_t\}$$

subject to

- Incentive constraint:

$$E_t\{\beta\Omega_{t+1}[(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]n_t\} \geq \theta\phi_t n_t$$

Note that the problem is homogenous in n_t : $\longrightarrow$ Choice of ϕ_t independent of n_t

We verify later that $\Omega_{t+1} = 1 - \sigma + \sigma\frac{V_{t+1}}{n_{t+1}}$ is independent of firm-specific variables (so the entrepreneur takes it as given).

Optimization Problem (con't):

$\lambda_t \equiv$ multiplier on IC; $\mu_t \equiv E_t \beta \Omega_{t+1}(R_{kt+1} - R_{t+1}) =$ discounted excess return

$\nu_t \equiv E_t \beta \Omega_{t+1} R_{t+1} =$ discounted deposit cost

fonc ϕ_t

$$\mu_t = \frac{\lambda_t}{1+\lambda_t}\theta$$

fonc λ_t

$$V_t = (\mu_t \phi_t + \nu_t) n_t = \theta Q_t k_t$$

$\rightarrow$Since $Q_t k_t = \phi_t n_t$:

$$\mu_t \phi_t + \nu_t = \theta \phi_t$$

$\rightarrow$ solution for leverage multiple (when IC constraint binding)

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}$$

Solution: Case 1: IC never binding

- IC not binding $\rightarrow \lambda_t = 0 \rightarrow$

$$\mu_t = E_t\{\beta\Omega_{t+1}(R_{kt+1} - R_{t+1})\} = 0$$

- Given $\Omega_{t+1} = 1 - \sigma + \sigma\frac{V_{t+1}}{n_{t+1}}$ and given $\frac{V_{t+1}}{n_{t+1}} = 1$ when IC never binding:

$$\Omega_{t+1} = 1$$

- Combining:

$$E_t\{\beta(R_{kt+1} - R_{t+1})\} = 0$$

$\rightarrow$When incentive constraint not binding, excess returns driven to zero

$\rightarrow$No limits to arbitrage (i.e. capital market perfect).

Case 2: IC always binding

- Constraint binding (i.e. $\lambda_t > 0$) $\rightarrow \mu_t > 0$ and ϕ_t pinned down by IC

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}$$

with

$$\mu_t \equiv E_t \beta \Omega_{t+1} (R_{kt+1} - R_{t+1})$$
$$\nu_t \equiv E_t \beta \Omega_{t+1} R_{t+1}$$

Solution similar to 2 period case, except returns weighted by the multiplier Ω_{t+1}

- Given $\Omega_{t+1} = 1 - \sigma + \sigma \frac{V_{t+1}}{n_{t+1}}$ and $\frac{V_{t+1}}{n_{t+1}} = \theta \phi_{t+1}$ (given IC binds):

$$\Omega_{t+1} = 1 - \sigma + \sigma \theta \phi_{t+1}$$

Combining equations $\rightarrow$ nonlinear first order difference equation for ϕ_t

Case 2: IC always binding: solution

$$Qk_{t+1} = \phi_t n_t$$

$$\phi_t = \frac{E_t \beta \Omega_{t+1} R_{t+1}}{\theta - E_t \beta \Omega_{t+1}(R_{kt+1} - R_{t+1})}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\theta\phi_{t+1}$$

$$n_{t+1} = [(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]n_t$$

- Some observations
  - Limits to arbitrage: Qk_{t+1} constrained by n_t.
  - $\theta - E_t \beta \Omega_{t+1}(R_{kt+1} - R_{t+1}) > 0$ because, for constraint to bind, it must be that the marginal gain from diverting funds exceeds the excess return.
  - ϕ_t increasing in $E_t \beta \Omega_{t+1}(R_{kt+1} - R_{t+1})$, $E_t \beta \Omega_{t+1} R_{t+1}$; decreasing in θ
    * Intuition: gain from being honest increasing in μ_t and ν_t (since $V_t/n_t = (\mu_t\phi_t + \nu_t)$), while gain from diverting increasing in θ.

Case 3: IC not binding, but may bind in future

- Precautionary behavior possible:
  - Entrepreneur borrows less to reduce likelihood of low n_t when shadow value V_t/n_t is high .
  - Recall potential losses increasing in leverage multiple ϕ_t (since $n_{t+1} = [(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]n_t$)
- Example: constraint not binding at t but expected to bind at $t+1$:

$$E_t\{\beta\Omega_{t+1}(R_{kt+1} - R_{t+1})\} = 0$$
$$\Omega_{t+1} = 1 - \sigma + \sigma\frac{V_{t+1}}{n_{t+1}}$$

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  - Ω_{t+1} likely countercyclical (incentive constraints tighter in recessions $\rightarrow \frac{V_{t+1}}{n_{t+1}}$ higher in recessions)
  - $\rightarrow cov(\Omega_{t+1}, R_{kt+1} - R_{t+1}) < 0$ which reduces expected return $E_t\{\beta\Omega_{t+1}(R_{kt+}$ $R_{t+1})\}$

– $\longrightarrow$ Incentive to reduce ϕ_t to reduce negative covariance (precautionary reduction in leverage)

Aggregation

- Assume a measure unity of entrepreneurs.
  - As the fraction $1-\sigma$ exits each period, they are replaced by $1-\sigma$ entrants
  - Assume each entrant begins with $\frac{S}{1-\sigma}$ units of equity
- Since the leverage ratio ϕ_t does not depend on firm-specific factors, we can aggregate:

$$Q_t K_{t+1} = \phi_t N_t$$

- Evolution of Net Worth::

$$N_t = \sigma[(R_{kt} - R_t)\phi_t + R_t]N_{t-1} + S$$

with

$$R_{kt} = \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}$$

Investment Sector in Baseline NK Model

- Q investment theory

$$\frac{I_t}{K_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q_t})$$

- Perfect arbitrage between returns on bonds and capital

$$E_t\{\Lambda_{t,t+1}\,(r_t^n - E_t\pi_{t+1})\} = E_t\{\Lambda_{t,t+1}\,R_{kt+1}\}$$

with

$$R_{kt+1} = \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t}$$

- Investment varies positively with Q, which equals discounted cash flows (see Topic 4)
  - $\longrightarrow$ Financial structure irrelevant.
  - Asset price Q is summary statistic, does not directly affect real activity

Incorporating Financial Market Frictions

- Replace arbitrage condition with balance sheet constraint

$$Q_t K_{t+1} = \phi_t N_t$$

$$N_t = \sigma[(R_{kt} - R_t)\phi_t + R_t]N_{t-1} + S$$

$$R_{kt} = \frac{Z_t + (1-\delta)Q_t}{Q_t}$$

- Note positive feedback between financial sectors
  - $R_{kt} \downarrow \rightarrow N_t \downarrow \rightarrow Q_t K_{t+1} \downarrow$
  - Feedback: $Q_t K_{t+1} \downarrow \rightarrow Q_t \downarrow \rightarrow R_{kt} \downarrow$, and so on.
    - K_{t+1} also declines since I_t will fall, but percentage effect small
  - Transmission to real activity: $Q_t \downarrow \rightarrow I_t \downarrow$.
  - Overall strength of mechanism increasing in leverage multiple ϕ_t

Investment Sector with Financial Frictions

$$\frac{I_t}{K_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q_t})$$

$$Q_t K_{t+1} = \phi_t N_t$$

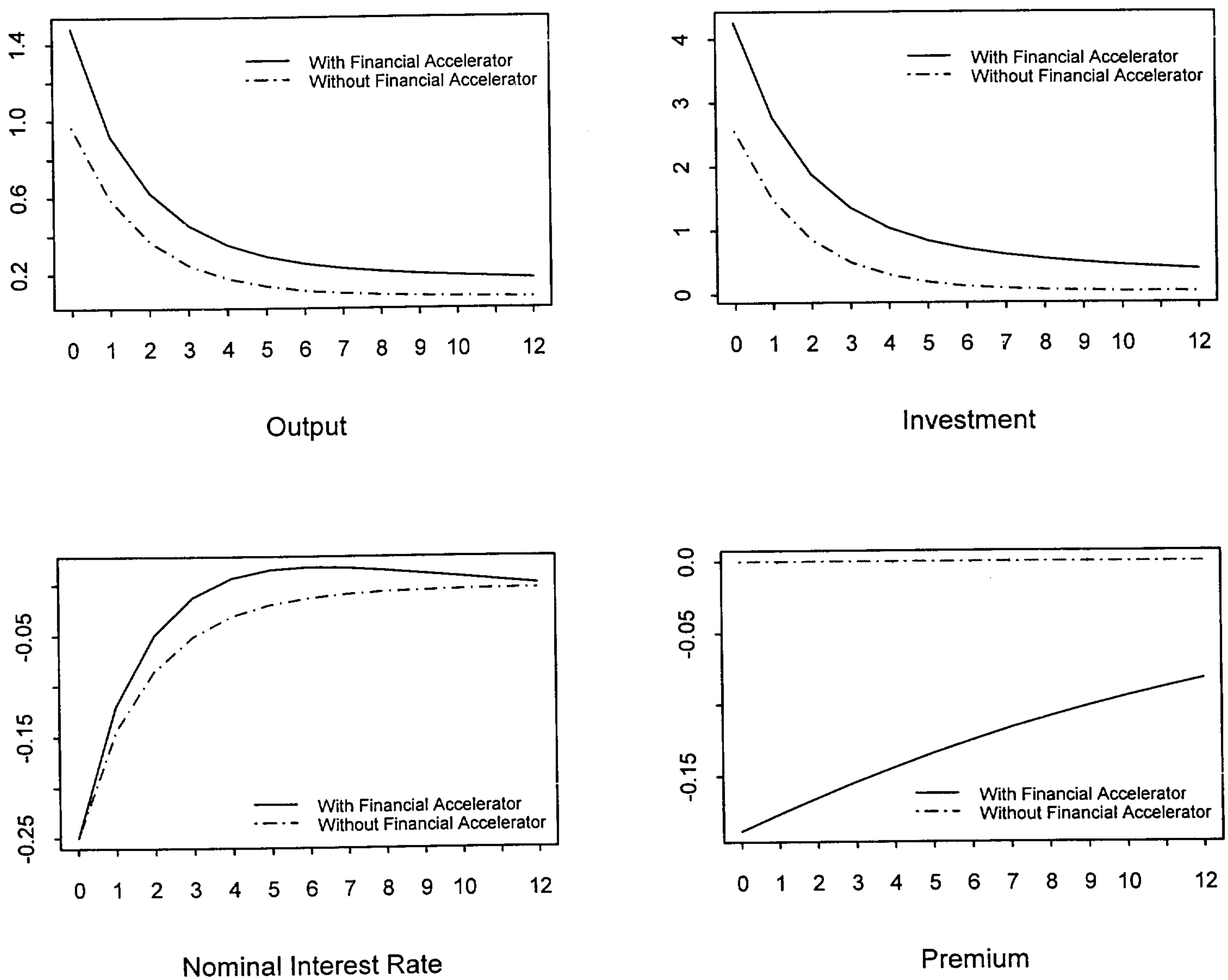
$$N_t = \sigma[\frac{Z_t + (1-\delta)Q_t}{Q_t} - (r_t^n - E_t\pi_{t+1})]\phi_t + (r_t^n - E_t\pi_{t+1})]N_{t-1} + S$$

- "Financial Accelerator": mutual feedback between financial and real sector
  - $I_t \downarrow \rightarrow Q_t \downarrow \rightarrow N_t \downarrow \longrightarrow Q_t K_{t+1} \downarrow \rightarrow Q_t \downarrow \rightarrow I_t \downarrow$, and so on.
- Shocks within financial sector also affect I_t.
  - e.g. $\phi_t \downarrow \longrightarrow Q_t K_{t+1} \downarrow \rightarrow Q_t \downarrow \rightarrow I_t \downarrow$, etc.
- Countercyclical excess returns

$$E_t\{R_{kt+1} - R_{t+1}\} = E_t\{\frac{Z_t + (1-\delta)Q_t}{Q_t} - (r_t^n - \pi_{t+1})\}$$

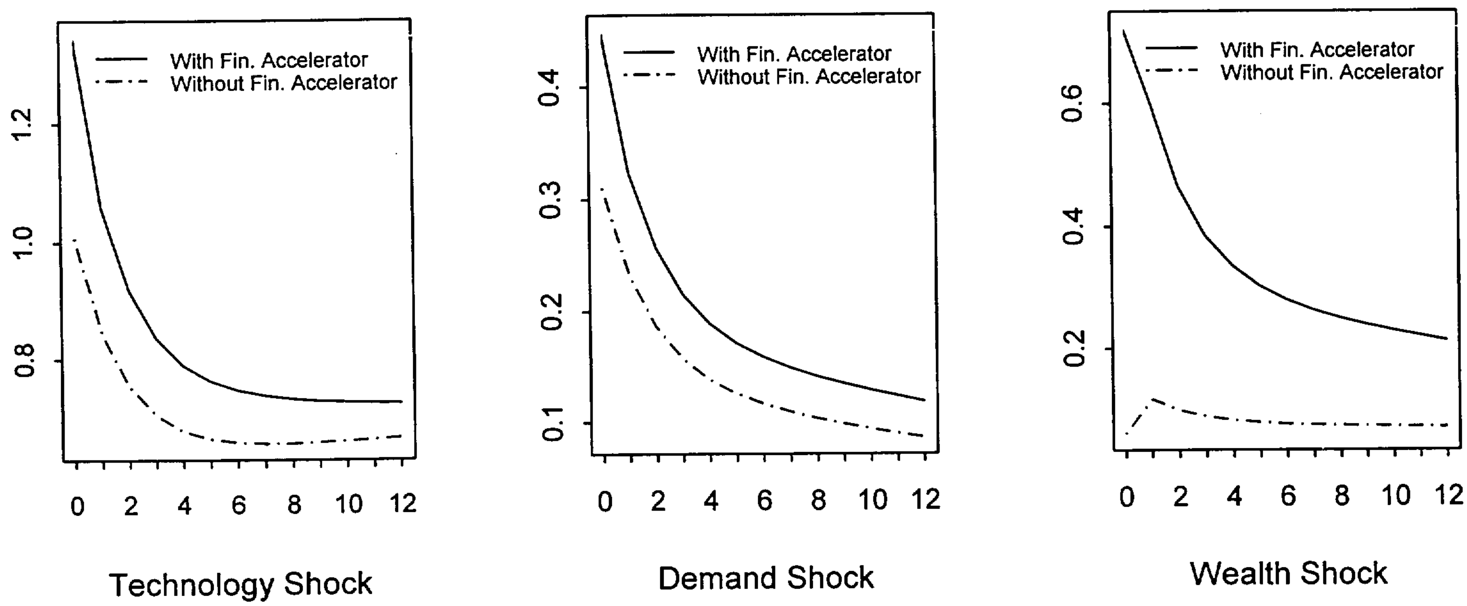
Q_t down in crisis raises excess returns.

Figure 3: Monetary Shock - No Investment Delay



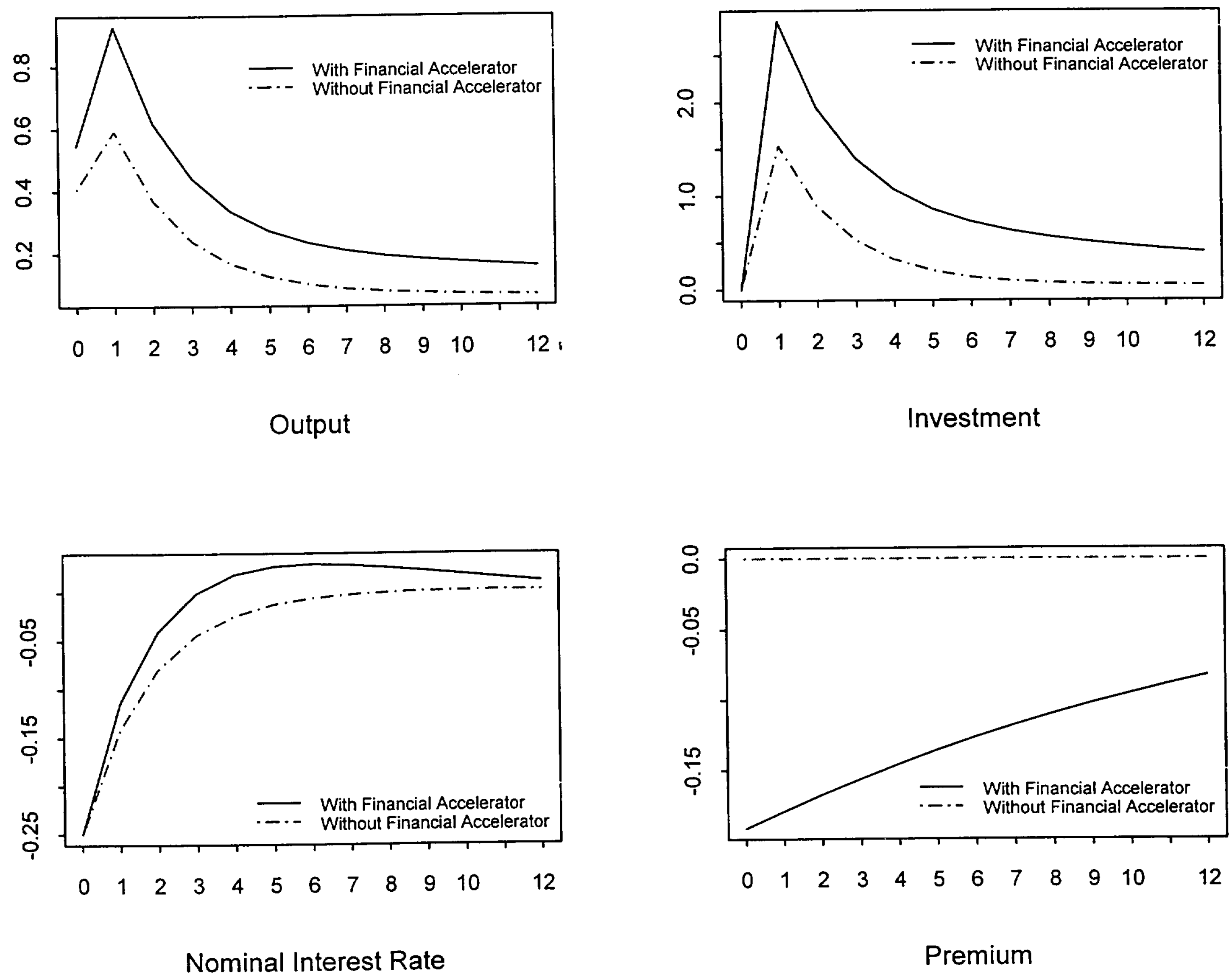
All Panels: Time Horizon in Quarters

Figure 4: Output Response - Alternative Shocks



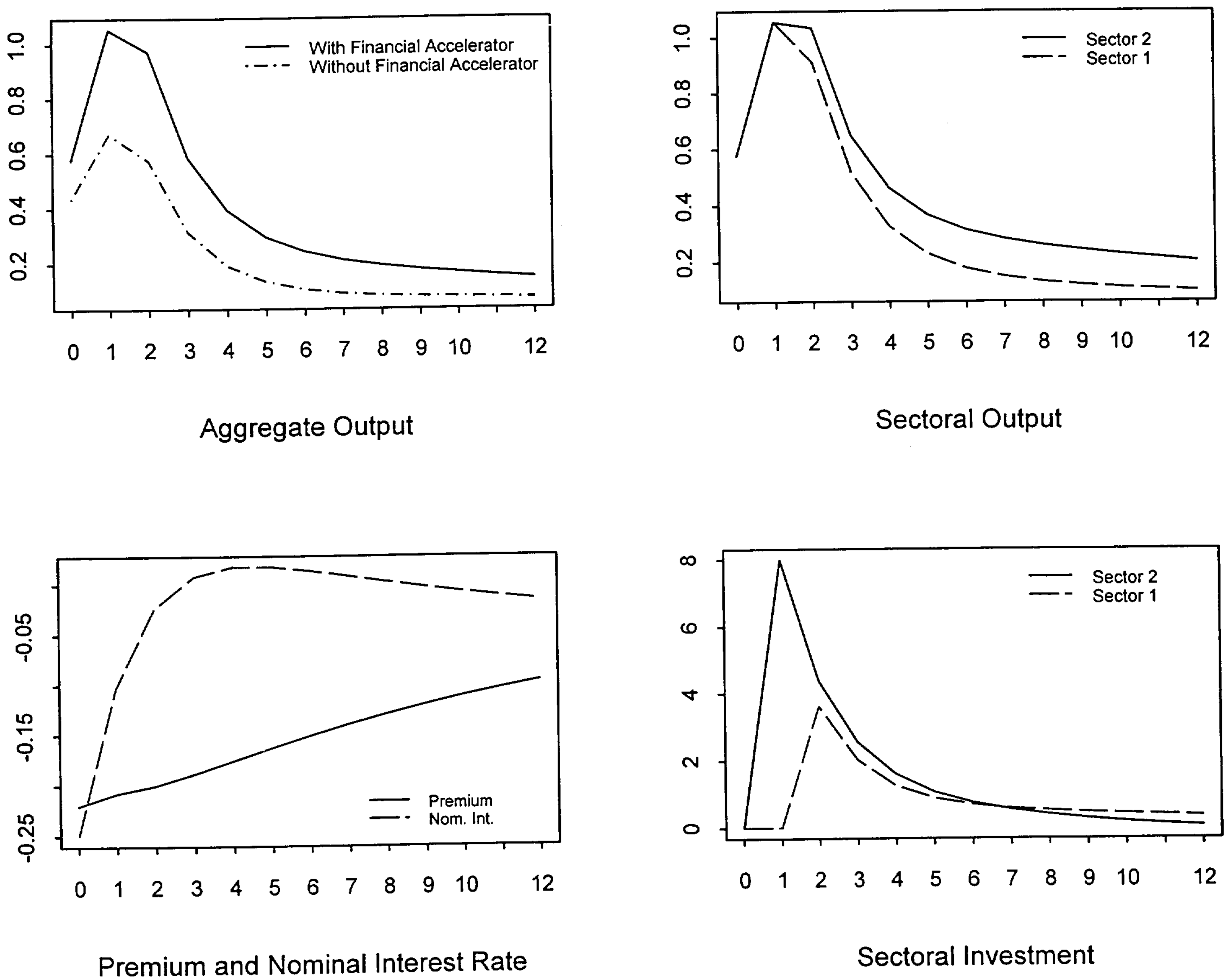
All Panels: Time Horizon in Quarters

Figure 5: Monetary Shock - One Period Investment Delay



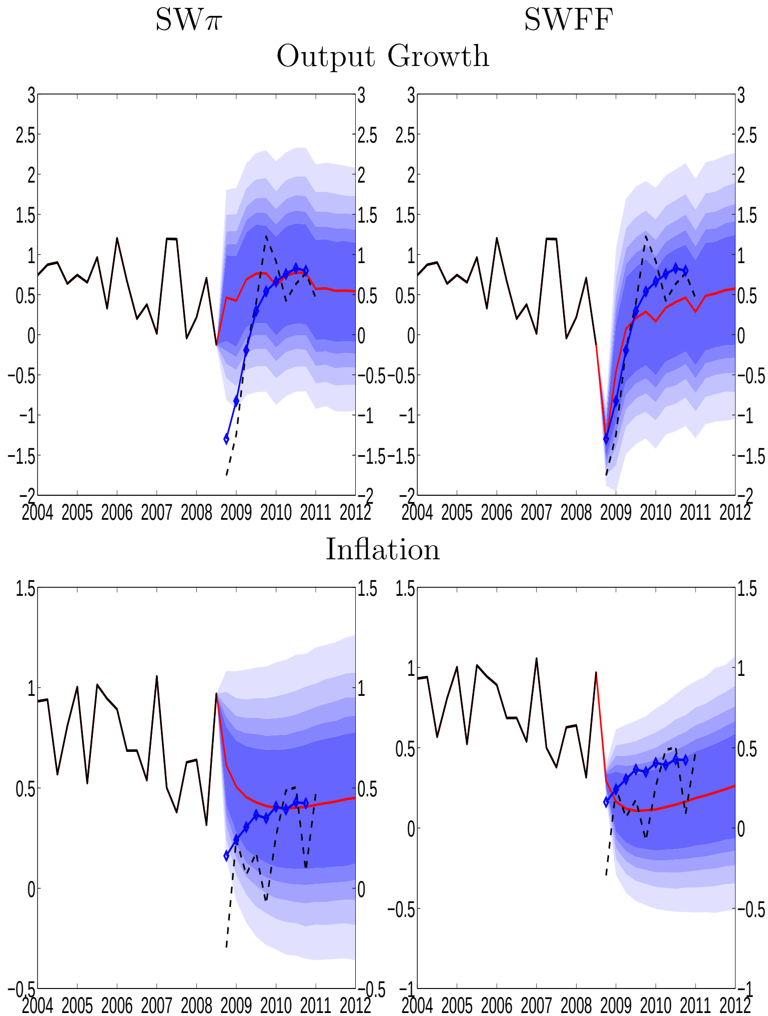
All Panels: Time Horizon in Quarters

Figure 6: Monetary Shock - Multisector Model with Investment Delays



All Panels: Time Horizon in Quarters; Panels 2-4: Model with Financial Accelerator.

Figure 1: DSGE forecasts of the Great Recession



Notes: The figure is taken from Del Negro and Schorfheide (2013). The panels show for each model/vintage the available real GDP growth (upper panel) and inflation (GDP deflator, lower panel) data (black line), the DSGE model's multi-step (fixed origin) mean forecasts (red line) and bands of its forecast distribution (shaded blue areas; these are the 50, 60, 70, 80, and 90 percent bands, in decreasing shade), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the May 2011 vintage (black dashed line) . All the data are in percent, Q-o-Q.shows the filtered mean of λ_t (solid black line) and the 50% , 68% and 90% bands in shades of blue.