Topic 1

Real Business Cycle Theory: Part 1

The Stochastic Neoclassical Growth Model

with Variable Labor Supply

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Outline

Part 1.

Stochastic growth model with variable labor supply: preferences and technology

Planning solution

Decentralized solution

Steady state

Part 2.

Loglinear approximation

Business cycle dynamics: key properties

Calibration and quantitative performance

Shortcomings

Business cycle accounting: sources of deviations from the data

Background

- RBC developed as a response to the failure of large macroeconometric models during the late 1960s and early 1970s.

- Objective: derive a model of fluctuations purely from first principles, where only exogenous restrictions involve preferences and technology.

- Candidate model: stochastic neoclassical growth model: Among the virtues: a unified theory of the cycle and the trend.

- Side-product: "calibration" introduced as a way to assign values to model parameters. Involves bringing in independent information.

- In the end: RBC is a failure as a model of business fluctuations.

- But RBC is an important methodological advance. Modern macro models build on RBC by adding "frictions" need to confront data.

- Calibration also controversial, but has had influence.

Model Setup

Background Environment:

A stochastic intertemporal general equilibrium with capital and variable labor supply.

Representative household (equivalently, continuum of measure unity identical households).

Markets are competitive, complete and frictionless. (1st and 2nd welfare theorems apply).

Baseline: no growth (output constant in steady state). Then consider growth.

Model Setup (con't)

Preferences.

$$E_t \left[\sum_{i=0}^{\infty} \beta^{t+i} [u(C_{t+i}) - v(L_{t+i})] \right]$$

with

$$u(C) = \frac{1}{1-\gamma} C^{1-\gamma}$$
$$= \log C \text{ iff } \gamma = 1$$

$$v(L) = \frac{1}{1+\varphi} L^{1+\varphi}$$

with $0 < \beta < 1$; $\gamma > 0$; $\varphi > 0$

$\gamma \equiv$ coefficient of relative risk aversion; $\varphi \equiv$ Frisch elasticity of labor supply

where $C_t \equiv$ consumption, $L_t \equiv$ labor supply.

Model Setup (con't)

Technology:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$$

where $Y_t \equiv$ output, $A_t^{1-\alpha} \equiv$ total factor productivity, $K_t \equiv$ capital, $L_t \equiv$ labor input.

Resource Constraint ($\rightarrow$ Law of Motion for Capital):

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t$$

where $0 < \delta < 1$ is the depreciation rate and where TFP obeys

$$A_t/\overline{A}_t = (A_{t-1}/\overline{A}_{t-1})^{\rho} e^{\epsilon_t}$$

$$\overline{A}_t/\overline{A}_{t-1} = G = 1 + g \geq 1$$

where $\overline{A}_t =$trend TFP, $0 \leq \rho < 1$ and ϵ_t is i.i.d. with mean zero.

Baseline: $G = 1$.

Planning problem

With frictionless markets and no externalities, the planning problem and the decentralized problem yield the same (Pareto efficient) allocation in equilibrium.

Combine production function and resource constraints to eliminate $Y_t \rightarrow$

Social planner's sequence problem given initial state (K_t, A_t) :

$$V(K_t, A_t) = \max_{\{C_{t+i}, L_{t+i}, K_{t+1+i}\}_{i \geq 0}} E_t \left[\sum_{i=0}^{\infty} \beta^{t+i} \left(\frac{1}{1-\gamma} C_{t+i}^{1-\gamma} - \frac{1}{1+\varphi} L_{t+i}^{1+\varphi} \right) \right]$$

subject to

$$C_t + K_{t+1} = K_t^{\alpha} (A_t L_t)^{1-\alpha} + (1-\delta) K_t$$

$$A_t / \overline{A} = (A_{t-1} / \overline{A})^{\rho} e^{\epsilon_t}$$

$$K_0 = K$$

$$A_0 = A$$

Planning problem: Bellman Equation

$$V(K_t, A_t) = \max_{C_t, L_t, K_{t+1}} \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{1}{1+\varphi} L_t^{1+\varphi} + \beta E_t\{V(K_{t+1}, A_{t+1})\}$$

subject to

$$C_t + K_{t+1} = K_t^{\alpha}(A_t L_t)^{1-\alpha} + (1-\delta)\, K_t$$

The solution yields the policy functions $C(K_t, A_t), L(K_t, A_t), K_{t+1}(K_t, A_t)$

Note: any two policy functions combined with the resource constraint implies the third.

To solve: (i) use the resource constrant to eliminate C_t in the objective; (ii) optimize w.r.t. (K_{t+1}, L_t);(iii) use the envelope theorem to find $V_1(K_t, A_t)$.

Solution

To solve: (i) use the resource constrant to eliminate C_t in the objective; (ii) optimize w.r.t. (K_{t+1}, L_t);(iii) use the envelope theorem to find $V_1(K_t, A_t)$. $\rightarrow$

$$C_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} + (1-\delta)\, K_t - K_{t+1}$$

FONC w.r. K_{t+1}

$$C_t^{-\gamma} = \beta E_t\{V_1(K_{t+1}, A_{t+1}\}$$

Envelope theorem

$$V_1(K_t, A_t) = C_t^{-\gamma}[\alpha(\frac{K_t}{A_t L_t})^{\alpha-1} + 1 - \delta]$$

$\rightarrow$

$$V_1(K_{t+1}, A_{t+1}) = C_{t+1}^{-\gamma}[\alpha(\frac{K_{t+1}}{A_{t+1} L_{t+1}})^{\alpha-1} + 1 - \delta]$$

Necessary and sufficient conditions for optimality

First order condition for consumption saving:

$$C_t^{-\gamma} = E_t\{\beta C_{t+1}^{-\gamma} R_{t+1}\}$$

where $R_{t+1} \equiv$ gross return on capital:

$$R_{t+1} = \alpha(\frac{K_{t+1}}{A_{t+1}L_{t+1}})^{\alpha-1} + (1-\delta)$$

First order condition for labor supply

$$(1-\alpha)A_t(\frac{K_t}{A_t L_t})^{\alpha} C_t^{-\gamma} = L_t^{\varphi}$$

Transversality condition

$$\lim_{t\to\infty} \beta^t C_t^{-\gamma} K_{t+1} = 0$$

Complete Model

Endogenous variables: $(Y_t, L_t, C_t, R_{t+1}, K_{t+1})$

Predetermined states: (K_t, A_t)

output:: $$Y_t = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$$

labor: $$(1-\alpha) A_t (\frac{K_t}{A_t L_t})^{\alpha} = \frac{L_t^{\varphi}}{C_t^{-\gamma}}$$

consumption/saving: $$C_t^{-\gamma} = E_t\{\beta C_{t+1}^{-\gamma} R_{t+1}\}$$

gross return on capital: $$R_{t+1} = \alpha (\frac{K_{t+1}}{A_{t+1} L_{t+1}})^{\alpha-1} + 1 - \delta$$

resource constraint : $$K_{t+1} = Y_t + (1-\delta) K_t - C_t$$

evolution of technology: $$A_t/\overline{A} = (A_{t-1}/\overline{A})^{\rho} e^{\epsilon_t}$$

Cyclical driving force: fluctuations in A_t.

Decentralized Solution

Continuum of measure unity identical households

Household h consumes $C(h)$, supplies labor $L(h)$, saves capital $K(h)$ which it rents to firms.

Acts competively - takes real wage W and rental rate on capital Z as given

Continuum of measure unity firms with identical technologies

Firm f produces output using labor $L(f)$ and capital $K(f)$

Acts competively - takes real wage W and rental rate on capital Z as given

Market clearing determines W, Z and equilibrium quantities

Household Decision Problem

$\Gamma_t \equiv$ macro state (K_t, A_t);

$$V(K_t(h), \Gamma_t) = \max_{\{C(h), L(h)_t, K(h)_{t+1}\}} E_t \left[\sum_{i=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma} C_t(h)^{1-\gamma} - \frac{1}{1+\varphi} L_t(h)^{1+\varphi} \right) \right]$$

subject to the period budget constraint

$$C_t(h) + K_{t+1}(h) = W_t L_t(h) + (Z_t + 1 - \delta) K_t(h)$$

and a terminal condition on wealth that rules out "Ponzi" schemes

$$\lim_{\tau \to \infty} \beta^{\tau} \left(\frac{C_{\tau}(h)}{C_t(h)} \right)^{-\gamma} (Z_{\tau} + 1 - \delta) K_{\tau}(h) \geq 0$$

Necessary conditions for household optimality

first order condition for consumption/saving

$$C_t(h)^{-\gamma} = E_t\{\beta C_{t+1}(h)^{-\gamma} R_{t+1}\}$$

with

$$R_{t+1} = Z_{t+1} + 1 - \delta$$

first order condition for labor supply

$$W_t = L_t(h)^{\varphi} / C_t(h)^{-\gamma}$$

W_t and Z_t (and hence R_t) determined in general equilibrium

Note: Identical households of measure unity $\rightarrow C_t(h) = C_t$, $L_t(h) = L_t, K_t(h) = K_t$

Firms

Continuum of measure unity firms

Firms hire labor and rent capital on a period by period basis

No factor adjustment costs $\longrightarrow$ factor demand is a static decision

Constant returns and competition $\longrightarrow$ zero profits

Firm decision problem

$$\max_{K_t(f),L_t(f)} \; Y_t(f) - Z_t K_t(f) - W_t L_t(f)$$

subject to

$$Y_t(f) = A_t^{1-\alpha} K_t(f)^{\alpha} L_t(f)^{1-\alpha}$$

Necessary conditions for firm optimality

First order condition for capital and labor:

$$\alpha(\frac{K(f)_t}{A_t L_t(f)})^{\alpha-1} = Z_t$$

$$(1-\alpha)A_t(\frac{K_t(f)}{A_t L_t(f)})^{\alpha} = W_t$$

Some implications

Identical K_t/L_t ratios across firms: from FONCs

$$\frac{K_t(f)}{L_t(f)} = \frac{K_t}{L_t} = \frac{W_t}{Z_t}\frac{\alpha}{1-\alpha}$$

Zero profits: FONCs $\rightarrow Z_t K_t(f) = \alpha Y_t(f)$ and $W_t L_t(f) = (1-\alpha)Y_t(f) \rightarrow$

$$Y_t(f) - Z_t K_t(f) - W_t L_t(f) = 0$$

Individual firm size indeterminate (though size of firm sector pinned down)

Equilibrium

Equilibrium: an allocation (C_t, L_t, Y_t, K_{t+1}) and prices (W_t, Z_t) are a competitive equilibrium iff households and firms are maximizing and all markets clear. Conditions:

output:: $$Y_t = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$$

labor market clearing: $$(1-\alpha) A_t (\frac{K_t}{A_t L_t})^{\alpha} = W_t = \frac{L_t^{\varphi}}{C_t^{-\gamma}}$$

capital rental: $$Z_t = \alpha (\frac{K_{t+1}}{A_{t+1} L_{t+1}})^{\alpha - 1}$$

consumption/saving $$C_t^{-\gamma} = E_t \{ \beta C_{t+1}^{-\gamma} (Z_{t+1} + 1 - \delta) \}$$

resource constraint $$K_{t+1} = Y_t + (1-\delta) K_t - C_t$$

technology $$A_t / \overline{A} = (A_{t-1} / \overline{A})^{\rho} e^{\epsilon_t}$$

Competitive equilibrium equivalent to planning solution (given frictionless markets and no externalities).

Deterministic Steady State: No Growth Case (g=0)

four variables: Y, K, C, L:

output $$Y = K^{\alpha}(\overline{A}L)^{1-\alpha}$$

labor market $$(1-\alpha)\frac{Y}{L} = L^{\varphi}/C^{-\gamma}$$

consumption/saving $$1 = \beta(\frac{C'}{C})^{-\gamma}[\alpha(\frac{K}{\overline{A}L})^{\alpha-1} + 1 - \delta] \rightarrow$$

$$\alpha(\frac{K}{\overline{A}L})^{\alpha-1} + 1 - \delta = \beta^{-1}(= R)$$

resource constraint $$K = Y + (1-\delta)K + C \rightarrow$$

$$Y = \delta K + C$$

transition dynamics:

$(K_t/\overline{A}L_t) < K/\overline{A}L \rightarrow \alpha(\frac{K_t}{\overline{A}L_t})^{\alpha-1} + 1 - \delta > \beta^{-1} \rightarrow$ increased saving $(\frac{C'}{C} \uparrow) \rightarrow$ $(K_t/\overline{A}L_t)$ converges to $K/\overline{A}L$.

$C \downarrow$ due to increase saving $\rightarrow L \uparrow \rightarrow Y \uparrow$ which speeds convergence.

Deterministic Steady State (con't)

express as $\frac{Y}{\overline{A}L}, \frac{K}{\overline{A}L}, \frac{C}{\overline{A}L}, L$ (convenient when we add growth)

$(\frac{Y}{\overline{A}L}, \frac{K}{\overline{A}L}, \frac{C}{\overline{A}L})$ determined by producton function, consumption/saving relation and resource constraint:

$$\frac{Y}{\overline{A}L} = (\frac{K}{\overline{A}L})^{\alpha}$$

$$\alpha(\frac{K}{\overline{A}L})^{\alpha-1} + 1 - \delta = \beta^{-1}$$

$$\frac{Y}{\overline{A}L} = \delta\frac{K}{\overline{A}L} + \frac{C}{\overline{A}L}$$

Labor market then determines L

$$(1-\alpha)\frac{Y}{L} = L^{\varphi}/C^{-\gamma} \rightarrow$$

$$(1-\alpha)\frac{Y}{\overline{A}L}(\frac{C}{\overline{A}L})^{-\gamma}\overline{A}^{1-\gamma} = L^{\gamma+\varphi}$$

Balanced Growth

Now suppose there is positive trend growth in TFP

$$\overline{A}_t/\overline{A}_{t-1} = G > 1$$

The steady state now corresponds to a balanced growth path where the quantities Y, C, K grow at the gross growth rate G, while L is constant (given that population is assumed to be constant).

In general allowing for trend TFP growth (and also population growth) leads to only minor changes in both the steady state and cyclical dynamics.

Allowing for growth does place restrictions on preferences, however.

Balanced Growth (con't)

Labor market equilibrium:

$$(1-\alpha)\frac{Y}{L}\cdot C^{-\gamma} = L^{\varphi}$$

$$MPL \cdot MUC = MDUL$$

To have a balanced growth path with constant labor, need $MPL \cdot MUC$ constant

Rewrite labor market equilibrium:

$$(1-\alpha)\frac{Y}{C}\cdot C^{1-\gamma} = L^{1+\varphi}$$

Given $\frac{Y}{C}$ constant in a balanced growth path, L constant requires $\gamma = 1$: $\rightarrow$

$$(1-\alpha)\frac{Y}{C} = L^{1+\varphi}$$

Balanced Growth (con't)

$\gamma = 1 \rightarrow$

$$u(C) - \nu(L) = \log C - \frac{1}{1+\varphi} L^{1+\varphi}$$

With logarithmic preferences, along a balanced growth path MUC $(= \frac{1}{C})$ declines at a rate that exactly offsets the increase in MPL $(= (1-\alpha)\frac{Y}{L})$ to keep the product constant.

Intuitively. with log preferences, in steady state wealth effect on supply (from $\frac{1}{C}$) exactly offsets substutution effect (from increasing $W = (1-\alpha)\frac{Y}{L}$).

With $\gamma > 1$, L will decline as output grows due to the wealth effect on labor supply (capture by $C^{-\gamma}$)

$$(1-\alpha)\frac{Y}{C} \cdot C^{1-\gamma} = (1-\alpha) Y \cdot C^{-\gamma} = L^{1+\varphi}$$

Preferences with $\gamma \neq 1$ that permit balanced growth

$$U(C,L) = \frac{1}{1-\gamma}[C^{\eta}(1-L)^{1-\eta}]^{1-\gamma}$$
$$= \log[C^{\eta}(1-L)^{1-\eta}] \text{ if } \gamma = 1$$

with $0 < \eta < 1$ and $\gamma > 0$

$$U_1(C,L) = \eta C^{\eta-1}(1-L)^{1-\eta}\frac{1}{1-\gamma}[C^{\eta}(1-L)^{1-\eta}]^{-\gamma}$$
$$U_2(C,L) = (1-\eta)C^{\eta}(1-L)^{-\eta}\frac{1}{1-\gamma}[C^{\eta}(1-L)^{1-\eta}]^{-\gamma}$$

labor market equilibrium:

$$(1-\alpha)\frac{Y}{L}U_1(C,L) = U_2(C,L) \rightarrow$$
$$(1-\alpha)\frac{Y}{C} = \frac{1-\eta}{\eta}\frac{L}{1-L}$$

$\rightarrow L$ constant along a balanced growth path

Steady state with balanced growth and log preferences

Determine $\frac{Y}{AL}, \frac{K}{AL}, \frac{C}{AL}, L$

$$\frac{Y}{AL} = (\frac{K}{AL})^{\alpha}$$
$$R = \alpha(\frac{K}{AL})^{\alpha-1} + 1 - \delta = \beta^{-1} \cdot (1+g)$$

given $\frac{C'}{C} = (1+g)$ and $\gamma = 1$.

$$\frac{Y}{AL} = (\delta + g)\frac{K}{AL} + \frac{C}{AL}$$

Labor market then determines L

$$(1-\alpha)\frac{Y}{L} \cdot \frac{1}{C} = L^{\varphi} \rightarrow$$
$$(1-\alpha)\frac{Y}{C} = L^{1+\varphi}$$

Road Ahead

- Loglinear approximation of model around deterministic steady state.
- "Calibrate" model parameters
- Evaluate business cycle dynamics versus data.